Noise sources of high-Mach-number jets at low frequencies studied with a phased-array approach based on LES database

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1. Motivation and objectives

Since the phased-microphone-array techniques have become popular in aeroacoustics, source identification of jet noise has been examined with a microphone array in several research projects (Dougherty 1999; Suzuki & Butler 2002; Venkatesh, Polak & Narayanan 2003; Dougherty, Panda & Lee 2005a; Lee & Bridges 2005; 2006; Suzuki 2006, Papamoschou & Dadvar 2006). When the Mach number is relatively low, the conventional beam-forming technique, in which a free-space monopole is assumed for the reference solution, provides noise source distributions consistent with results obtained with former experimental approaches. However, as the Mach number is increased or the jet is heated, the directivity of sound radiation greatly increases and the solution to the Helmholtz equation (i.e., in a quiescent field) is probably no longer valid. It may be difficult to apply beam-forming algorithms if the sound intensity is oriented to a narrow range of solid/zenithal angles.

To construct valid reference solutions for higher-Mach-number jets and heated jets, we must understand sound radiation patterns, particularly phase variation, for these flow conditions. We can consider two possible mechanisms that can create highly directive radiation patterns. One is refraction due to large velocity gradient in a mixing layer, and the other is the Mach-wave radiation owing to supersonic noise-source convection. The refraction effect tends to focus acoustic rays toward the critical angle, which separates the zone of silence from the “geometrical acoustic” region (Howe 1970). In the geometrical acoustic region, spherical waves propagate; therefore, as long as the microphones are located in this region, conventional phase tracking is applicable. On the other hand, Mach waves create general plane-wave patterns in the downstream direction (Tam & Burton 1984a; 1984b), and the sound source is less likely compact. These features restrict the capabilities of the conventional beam-forming. To be precise, ray trajectories also become parallel in the zone of silence, even for subsonic jets (Howe (1970) or Suzuki & Lele (2002), referred to as “refracted arrival” waves). Although the dominant sound-generation mechanism of supersonic jets may be clearer than that of subsonic jets, the applicability of the phased-array technique appear to be limited for higher-Mach-number jets or heated ones. The key to success to the source identification for these conditions is to capture the radiation patterns and to relate them with the near-field fluctuations.

While phased-array pressure data from an experiment may be available, it is difficult to verify the source model unless the near-field information is well understood. On the other hand, a computational database allows us to access detailed flow information. In particular, phase information of coherent disturbances can be readily extracted from

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computational results. At realistically high Reynolds numbers, Large-Eddy Simulation (LES) is considered to be a prospective tool to produce a jet-flow database. Bodony & Lele (2005a) have computed a turbulent round jet with LES and performed detailed comparisons with experimental data for various flow conditions. This is our motivation to apply a phased-array technique to a database acquired by the LES. This study is a step toward the practical use of phased-array approaches.

Thus, the objective of this study is to provide an insight into the noise source of supersonic jets computed with LES from the view of phased-microphone-array techniques. Namely, from pressure time histories at discrete points in the far field we attempt to identify the noise sources and to relate them with the near-field flow properties. The modes obtained with the Proper Orthogonal Decomposition (POD) in the frequency domain visualize the coherent-wave-propagation patterns, and we compare these POD pressure fields with rays sent back from the far-field observer points as well as the noise-source maps produced by the beam-forming technique. We then discuss the possibility and limitation of phased-microphone-array techniques applied to high-Mach-number jets.

2. LES jet flow data

We use the LES database pre-computed by Bodony & Lele (2005a) in this study. The set of simulations ranges from subsonic to supersonic turbulent round jets at the Reynolds numbers on the order of $Re = O(10^5) \sim O(10^6)$. The domain of $x \in [0, 31D] \times r \in [0, 12.5D] \times \phi \in [0, 2\pi)$ (where $D$ denotes the jet diameter, $x$ is the downstream direction and $\phi$ is the azimuthal angle) including the buffer zone is discretized with $256 \times 128 \times 32$ grid points (refer to Fig. 1 for the computational domain). We specifically choose two $M_\infty = 1.47$ cases, one is unheated and the other is heated, which exhibit satisfactory results in comparison with experiments (Tanna 1970; Troutt & McLaughlin 1982; Viswanathan 2004). The detailed flow conditions are shown in Table 1, where the set points follow the notation by Tanna (1977). For details of the computational methods and additional results, refer to Bodony & Lele (2005a).

3. Acoustic-field computation and Kirchhoff surface integration

To generate the acoustic field, we apply the Kirchhoff-Helmholtz integral theorem (refer to Pierce 1989). When we need pressure time histories at several discrete points, it is convenient to integrate the pressure data on the Kirchhoff surface in the time domain rather than the frequency domain. Assuming no flow and no temperature variation outside the Kirchhoff surface, the pressure time history at a point $x$ can be recovered as follows:

\begin{table}[h]
\centering
\begin{tabular}{cccccc}
\hline
Set Point & $M_\infty$ & $M_{jet}$ & $T_{jet}/T_\infty$ & $Re$ & $\Delta t$ \\
39 & 1.47 & 0.97 & 2.30 & $8.4 \times 10^4$ & 0.0025 \\
62 & 1.47 & 1.95 & 0.56 & $33.6 \times 10^4$ & 0.0050 \\
\hline
\end{tabular}
\caption{Flow conditions chosen for this study. $M_\infty \equiv U_{jet}/a_\infty$, $M_{jet} \equiv U_{jet}/a_{jet}$ and $Re \equiv U_{jet}D/\nu$, where $a$ denotes the speed of sound and the subscript $\infty$ denotes the ambient condition. $\Delta t$ denotes the non-dimensional time step based on the ambient speed of sound and the jet diameter for the original computation.}
\end{table}
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$p(t, x) = \int \int \left[ \frac{(x - y) \cdot n}{4\pi|x - y|^2 a_{\infty}} \frac{\partial p}{\partial t}(\tau, y) + \frac{(x - y) \cdot n}{4\pi|x - y|^3} p(\tau, y) - \frac{1}{4\pi|x - y|} \frac{\partial p}{\partial n}(\tau, y) \right] dy$,  

where the surface integral is performed with respect to $y$ and $n$ denotes the unit normal vector pointing outward from the Kirchhoff surface. The retarded time is defined as

$\tau \equiv t - \frac{|x - y|}{a_{\infty}}$.  

In this study, a cylinder with a radius of $r = 6D$ is taken as the Kirchhoff surface. The upstream and downstream ends of the computational domain are treated with a windowing function shown in Fig. 1, yet the cross-sectional end-planes are excluded in the domain of integration. Here, the origin is taken at the left computational boundary on the centerline (refer to Bodony & Lele 2005a for a discussion on the virtual origin, which is not considered here).

Pressure and its radial derivative at all the points on the surface are used to compute the integral at every tenth computational time step (i.e., $10\Delta t$). From each grid point, the contribution to the pressure time history at each microphone position is calculated based on the retarded time, and a value at a specific time in the pressure history is calculated by interpolation with four neighboring points in time. The time-derivative term is similarly treated. Pressure time histories in which some portions of the surface information are missing at the beginning or end are eliminated from use in the post-processing. A total of more than 8000 $10\Delta t$ steps are computed to generate a cross-spectral matrix.

To verify the implementation of the Kirchhoff surface integration scheme, we first set a monopole at $(x, y, z) = (8D, 0, 0)$ and recover pressure time histories at $r = 40D$ centered at the source position. The amplitude and the phase errors observed over a wide range of zenithal angles are plotted in Figs. 2 and 3, respectively. Here, the Strouhal number is defined based on the set point 39 in Table 1 (denoted by SP39 thereafter).
The amplitude is reasonably well recovered in the range of $30^\circ \leq \theta \leq 120^\circ$ ($\theta$ denotes the zenithal angle from the downstream axis). At Strouhal numbers higher than 0.5, the accuracy of the calculation significantly deteriorates. Since the source is positioned closer to the upstream end, the amplitude at higher angles rapidly decays. In the downstream direction, the amplitude can be recovered up to about the angle where the direct ray from the source to the observer intersects the Kirchhoff surface. The phase error, which is more important for the phased-array techniques, exhibits similar trends. Between $20^\circ \leq \theta \leq 140^\circ$, the phase error is estimated to be less than $|\Delta \varphi| < \pi/12$, beyond which phase tracking techniques are likely to fail.

Next, Fig. 4 displays the overall sound pressure level over a range of zenithal angles for SP39 at $r = 100D$. Compared with Fig. 21 of Bodony & Lele (2005a), which is computed with the same database using the Kirchhoff-Helmholtz method in the frequency domain, the overall magnitude agrees reasonably well. At higher angles, the sound pressure level of the time-domain approach becomes higher than the result of Bodony & Lele (2005a) up to about 10dB. This discrepancy may be caused by the difference of the spatial windowing functions or the effect of the time windowing in the frequency domain approach.

Figure 5 also shows the one-third octave sound pressure level at $\theta = 45^\circ$ for SP39 (compared to Fig. 22 in Bodony & Lele, 2005a). Similar to the previous result, the magnitude is close to the result of Bodony & Lele (2005a). The high-frequency decay is somewhat gentler in the current approach. This may be related to the discrepancy at higher zenithal angles, assuming that the high-frequency noise component is relevant to the upstream direction. Note that for both tests, the center is taken at the origin of the computational domain, which is $3.2D$ downstream of the center defined by Bodony & Lele (2005a), but this effect is considered negligible at $r = 100D$.

4. Ray trajectories and near-field pressure fluctuations

4.1. Microphone array and cross-spectral matrix

Referring to the analysis in the preceding section, we design the microphone distribution for the phased-array investigation as follows. At a higher Mach number ($M_\infty = 1.47$
in this study), the sound pressure level is peaked near $\theta = 30^\circ$. On the other hand, the accuracy of the Kirchhoff integral method is deteriorated at $\theta \leq 20^\circ$. Hence, we distribute 16 sets (i.e., rings) of microphones approximately between $20^\circ \leq \theta \leq 50^\circ$. They are equally spaced in the axial direction considering the possibility of capturing the Mach-wave radiation. Because the range of zenithal angles is narrower than typical caged-microphone arrays applied to source detection for jet noise, the resolution of the source maps in the axial direction would become worse. On the other hand, we can exclusively capture the dominant noise component of high-Mach-number or heated jets oriented toward the downstream direction.

In the azimuthal direction, six microphones are necessary to resolve up to $m = \pm 2$ (i.e., the second azimuthal mode). These six microphones are equally spaced, and their azimuthal angles of each ring are staggered from the adjacent rings. The distance from the origin is determined as $r = 35D$ so that spacings between microphones are narrow enough to resolve the Strouhal numbers up to $St = 0.5$. The resultant microphone distribution
is displayed in Fig. 6. It consists of 16 rings times 6 microphones per ring, for a total of 96 microphones.

To generate a cross-spectral matrix, pressure time histories are first Fourier transformed in time. Four time periods of the target Strouhal number are processed with the Hann window (corresponding to about one-sixth octave bandwidth). Subsequently, they are Fourier transformed in the azimuthal direction for each ring, generating a $16 \times 16$ matrix at each Strouhal number. It should be noted that the resolution in the azimuthal direction is somewhat limited due to a finite number of microphones; however, the contribution of the higher azimuthal modes to the far-field sound is expected to be small (which can be confirmed in §5). Therefore, we consider up to $m = 2$ modes in this study. Cross-spectral matrices are generated by averaging over at least 14 segments of the four-time-period interval with overlaps.

4.2. Ray tracing

From the primary eigenvector of the cross-spectral matrix for each $m$, we attempt to send acoustic rays back to the source location. From the phase information of the eigenvectors, its variation along the microphone positions on the $x - r$ plane is fitted with a fourth-order polynomial. Here, the ambiguity of $2\pi$ can be readily removed with a polynomial fit. The phase, $\varphi(x, r)$, is then differentiated along the microphone positions as follows:

$$\frac{d\varphi}{ds} = \frac{\partial\varphi}{\partial x} \frac{dx}{ds} + \frac{\partial\varphi}{\partial r} \frac{dr}{ds}. \quad (4.1)$$

Since there is no wave-number component in the azimuthal direction after Fourier transforms, the partial derivatives of the phase can be replaced by

$$\frac{\partial\varphi}{\partial x} = k_x \text{ and } \frac{\partial\varphi}{\partial r} = \sqrt{k^2 - k_x^2}, \quad (4.2)$$

where $k$ is the wavenumber and $k_x$ is its axial component. Thus, we can uniquely determine the wavefront normal at the microphone positions. It should be noted that the deviation in phase fitted with the polynomial is less $\Delta\varphi < 0.5^\circ$, but this error can be amplified in the calculation of the wavefront angle in (4.1) up to $\Delta\theta \approx 3^\circ$.

Based on these angles of the wavefront, straight acoustic rays are sent backward toward the jet axis. It is certainly possible to calculate ray trajectories more precisely based on the eikonal equation with the mean velocity and temperature fields (Keller & Lewis 1995). However, small errors in the initial wavefront angle cause large deviation of the trajectories near the source location, and such deviation is estimated to be greater than the error associated with refraction. Hence, only straight rays are drawn in the results. It should be noted, however, that the accuracy of the source localization is fatally deteriorated if the microphones were to be located outside the geometrical acoustic region.

4.3. Beam-forming algorithm

To evaluate the applicability of the beam-forming technique, we apply the conventional beam-forming algorithm (refer to, for example, Johnson & Dudgeon 1993) to the cross-spectral matrix mentioned above. To be precise, since the cross-spectral matrix has been already Fourier decomposed in the azimuthal direction, we must carefully determine the zenithal dependence so that the reference solution satisfies the governing wave equation (refer to Suzuki 2006). However, the range of the zenithal angle covered by the array is relatively limited; hence, we assume that the acoustic signals at the microphone positions are all in phase. We then set the reference solution for each $m$ as follows:
Taking Eq. 4.3 to define the steering vector and assuming that the source is localized on the jet axis, we perform the beam-forming technique to generate noise-source maps.

4.4. Near-field data processing

To capture coherent pressure signals in the near field, we apply the POD analysis in the frequency domain. Pressure data at all grid points are Fourier decomposed in the azimuthal direction. But, unlike the microphone data above, all 32 grid points in $\phi$ are used for the decomposition. These fields are then Fourier transformed in time with the same data processing as the far-field sound. Thus, the near-field pressure disturbances that are Fourier transformed in time are obtained for each azimuthal mode at a given Strouhal number.

Regarding such a Fourier-transformed pressure field as a snapshot (without overlapping), we apply the snapshot POD method to extract the most energetic mode. The first POD typically exhibits the representative coherent structure. Since this mode has been Fourier transformed in time, it contains both amplitude and phase information, which allows us to readily relate the near-field fluctuations to the acoustic-wave propagation.

In the conventional snapshot POD method, however, the information on periodicity is missing; hence, a large number of modes are required to represent wave-propagation characteristics (Freund & Colonius 2002). A typical phase field obtained with the POD in the frequency domain is displayed in Fig. 1.

5. Results and discussion

5.1. Heated jet (SP39)

First, we observe the SP39 case (heated jet). Figure 7 shows eigenvalue distributions of the cross-spectral matrices obtained from the microphone array at $St = 0.1$. They demonstrate that the first coherent mode dominates the low-frequency acoustic signal in the downstream direction. This justifies that we focus on the primary eigenmode of the cross-spectral matrix. The magnitude of the first mode decays with increasing azimuthal mode number (note that the only $m = 0$ case is scaled differently in Fig. 7).

Figure 8 displays the ray trajectories sent back from the microphone positions superposed with the phase contours of the first POD mode. We can confirm that the ray trajectories are approximately orthogonal to the wavefronts in the ambient region. We can clearly see organized hydrodynamics structures up to $x = 10D$ near the jet axis, and they seem to develop into turbulence downstream. Namely, the intensive spots in the amplitude contours of the POD modes in Fig. 9 indicate the end of the potential core. Since the POD method extracts coherent modes, these hydrodynamic structures are actually related to the dominant sound component. At low frequencies, we can observe some region in the streamwise extent where the ray trajectories become nearly parallel.

Noise-source maps produced by the beam-forming in Fig. 10 indicate that the source positions are approximately $x = 10D$ or slightly upstream for all azimuthal modes. Among three azimuthal modes, the axisymmetric mode is by far the greatest (the amplitude scale in Fig. 10(a) is different from the others although all three are equally normalized), which is consistent with the eigenvalue distribution in Fig. 7. Since the range of the zenithal angle of the array is relatively small, the lobe-width of the source...
Figure 7. Eigenvalue distribution of the azimuthally-decomposed cross-spectral matrices for SP39 at $St = 0.1$. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$. The eigenvalues are normalized by the greatest eigenvalue for the $m = 0$ mode.

Figure 8. Phase contours of the first POD mode taken from the LES flowfield with ray trajectories for SP39 at $St = 0.1$. Contour intervals are $\pi/4$. Microphone positions are denoted by $\circ$. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

Figure 9. Magnitude contours of the first POD mode taken from the LES flowfield for SP39 at $St = 0.1$. Contour intervals are 6dB. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

Figure 10. Noise-source maps generated by the beam-forming for SP39 at $St = 0.1$. Unit of the amplitude is arbitrary, but normalized consistently among the azimuthal modes. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

map tends to spread; thus, we should interpret the map as the approximate "center" of the source as opposed to the source "distribution."

Even at a higher Strouhal number ($St = 0.3$), the trend in the eigenvalue distribution (see Fig. 11) is similar to the lower frequency case. The relation between the ray trajectories and the wavefronts is again consistent. However, the rays tend to focus nearly at $x = 5D$ for all azimuthal modes, and this position coincides with the intensive spots in the magnitude contours of the first POD modes in Fig. 13. These positions at $St = 0.3$ appear to be slightly upstream compared with those at $St = 0.1$. In fact, the noise-source maps in Fig. 14 also indicate the source positions to be close to $x = 5D$ for all $m$. Their
magnitudes decrease with increasing azimuthal mode number. These source positions are a few diameters upstream of those computed by Bodony & Lele (2005b) based on Lighthill’s source representation (refer to their Figs. 15 and 16).

At a still higher Strouhal number \((St = 0.5)\), Fig. 15 shows that the ratio of the first to the second eigenvalues becomes somewhat smaller, and their magnitudes among three azimuthal modes become the same order. Ray trajectories in Fig. 16 depict similar trends to the \(St = 0.3\) case, and the position of the focus is approximately the same \((x = 5D)\), which is consistent with the magnitude contours shown in Fig. 17 as well as the noise-source maps generated by the beam-forming in Fig. 18. This result also agrees with that of the Lighthill source in Bodony & Lele (2005b).

At higher Strouhal numbers, we can observe some phase discontinuities; e.g., in Fig. 16 (b), there is a line indicating a \(\pi\) shift nearly at \(\theta = 90^{\circ}\). In fact, there exists a valley
along this line in the magnitude contour in Fig. 17(b). As shown in Ghosh, Bridges & Hussain (1995) and Suzuki (2006), the sound directivity of low subsonic jets exhibits multipole-like radiation patterns at low Strouhal numbers. In particular, a free-space quadrupole for the $m = 1$ mode has a $\pi$ shift at $\theta = 90^\circ$. Thus, it is not surprising that phase discontinuities appear in coherent modes although they do not exactly follow the radiation patterns of the free-space spherical multipoles over all azimuthal modes. Unless a valid source model is available, it is difficult to apply advanced beam-forming algorithms for high-Mach-number jets.

5.2. Unheated jet (SP62)

Next, we consider the unheated case (SP62). Figures 19 and 20 show the ray trajectories with the phase contours of the first POD modes and their magnitude contours at $St =$
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Figure 19. Phase contours of the first POD mode taken from the LES flowfield with ray trajectories for SP62 at $St = 0.1$. Notation is the same as Fig. 8. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

Figure 20. Magnitude contours of the first POD mode taken from the LES flowfield for SP62 at $St = 0.1$. Notation is the same as Fig. 9. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

Figure 21. Noise-source maps generated by the beam-forming for SP62 at $St = 0.1$. Notation is the same as Fig. 10. (a) $m = 0$. (b) $m = 1$. (c) $m = 2$.

0.1, respectively. There is relatively large extent in which plane wavefronts propagate, especially for $m = 0$. Compared with the heated case in Fig. 8, the correlation between the hydrodynamic field and the acoustic one appears to be strong. Although the jet velocity relative to the ambient is the same between these two cases, $M_{jet}$ is greater for SP62 and the OASPL is also higher over the entire range of zenithal angles (Bodony & Lele 2005a). It is clear, at least at low frequencies, the coherent hydrodynamic disturbances generate intensive noise at shallow angles.

The intensive spots of the POD modes in Fig. 20 and the noise source positions in Fig. 21 both appear approximately $x = 15D$, which is more downstream compared with the heated case (SP39 in Figs. 9 and 10). This source shift is also captured by the study of Bodony & Lele (2005b). For an unheated jet, because the growth rate of the instability waves is low, the potential core extends downstream compared with a heated case.

Even at $St = 0.3$, these trends are similar, as shown in Figs. 22 and 23. These ray trajectories indicate that plane-wave patterns are generated upstream of the potential-core end and spherical waves are radiated near the end of the potential core (the $m = 2$ case is not be able to capture the wave-propagation patterns upstream well). The noise positions in Fig. 24 are slightly shifted upstream as the frequency is increased, but they are still downstream of the source positions for the heated case. The peak source position calculated by Bodony & Lele (2005b) for $St = 0.3$ is approximately $x = 12D$, which agrees with these results.

As the Strouhal number is increased (see Figs. 25, 26 and 27), the rays tend to be more closely focused, and the radiation pattern seems to be closer to the spherical waves.
The rays are focused in $5D < x < 10D$, while the intensive spots of the POD modes and the peaks of the noise-source maps are located about $x = 10D$ or slightly downstream. It should be noted, however, that the polynomial fit of the phase at the microphone position is most inaccurate in this case, and this may cause the discrepancy between the ray trajectories and the noise-source maps. The sound directivity is more inclined to downstream jet axis compared with the heated case (SP39).

It should be noted that for SP62, the orders of the intensity among three azimuthal modes are all comparable for these Strouhal numbers ($0 < St < 0.5$), while the axisymmetric mode is dominant for SP39. In the near field, instability waves greatly influence the length of the potential core. Assuming that the intensive spots of the POD modes in the near field indicate the end of the potential core, the shift of the source positions indicated in this study is likely related to the nature of instability waves, whose azimuthal mode balance can largely vary with the jet temperature (Suzuki & Colonius 2006). On the other hand, the difference in source positions over different azimuthal modes is negligible although their growth and decay rates in the streamwise direction substantially differ as a function of $m$.

6. Conclusions and future plans

We have analyzed the relation between acoustic fields and near-field pressure fluctuations based on the LES database for high-Mach-number turbulent round jets. Focusing on low Strouhal numbers ($0.1 \leq St \leq 0.5$), two cases for $M_\infty = 1.47$, one is heated (corresponding to SP39 for Tanna’s (1977) test matrix) and the other unheated (SP62),
have been considered, and the axisymmetric, first and second azimuthal modes (i.e., \( m = 0, 1, 2 \)) have been analyzed. Near-field pressure fluctuations have been Fourier decomposed in the azimuthal direction, and the first POD mode in the frequency domain has been displayed for each azimuthal mode at given frequencies. On the other hand, a microphone array in the computational domain has been designed so that it captures the dominant coherent-noise component. The array consists of 16 rings with 6 microphones each, a total of 96 microphones, resolving up to \( m = 2 \). Acoustic wavefronts at the microphone positions have been computed from the primary eigenvector of the cross-spectral matrix, and acoustic rays have been sent backward toward the jet axis. The conventional beam-forming approach has also been tested with the azimuthally decomposed cross-spectral matrix to generate a noise-source map for each \( m \). The ray trajectories have been compared with the phase fields of the first POD mode obtained from the near-field pressure fluctuations, and the intensive spots of these POD modes have been compared with the noise-source maps as well as Lighthill’s sources computed by Bodony & Lele (2005b).

The results show that the ray trajectories and the phase fields of the POD mode are consistent. The near-field POD modes demonstrate that the coherent structures in the hydrodynamic region are strongly related with the sound propagating to the angle of peak intensity. The intensive spots of the POD modes and the positions where the time-reversal rays are focusing agree reasonably well. These positions are also approximately the same as the peaks of the noise-source maps as well as the peaks of Lighthill’s sources.

The phase fields together with the ray trajectories may indicate that there exist two
types of radiation patterns. One is the spherical radiation centered near the end of the potential core, and the other is the general-plane-wave radiation, which may be associated with the Mach waves. The latter seems relevant for the unheated case (SP62) at lower frequencies. Although the accuracy of the wavefront angles at the microphone positions is not sufficient to distinguish these two wave patterns, the following conclusions can be deduced. At low Strouhal numbers, the hydrodynamic structures are correlated with the dominant sound propagating in the downstream direction. At the same time, relatively weak spherical waves are propagating upstream, and they are also correlated with the hydrodynamic disturbances.

In the future research, we should further investigate the coherent structures in the near field quantitatively. This study strongly suggests that the instability waves actually generate the low-frequency sound near the end of the potential core; however, to deduce definitive conclusion, instability-wave components must be extracted from near-field pressure disturbances. By quantifying the amplitude of the instability waves, we can compare their azimuthal-mode balance with that of the far-field sound.

Regarding the beam-forming applications, this study demonstrates that the dominant source position can be detected by distributing the microphones near the angle of peak intensity. For the two cases studied in the current research, we observe some indication of Mach waves, but they seem not to deteriorate the resolution or accuracy of the beam-forming technique significantly. In low subsonic cases, coherent sound propagates over a wide range of the zenithal angles, and they depict multipole-like radiation patterns (Ghosh, Bridges, & Hussain 1995; Suzuki 2006). This requires a reference solution that matches the lobe pattern, ideally with real sources. On the other hand, the POD phase contours show there is no phase discontinuity near the angle of peak intensity, which helps the conventional phase-matching algorithm to work in the current cases. However, further study is necessary to evaluate the detectability at supersonically convective Mach numbers, in which the Mach-wave radiation would become more relevant.

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