A dynamic global-coefficient subgrid-scale model for compressible turbulence in complex geometries

By D. You AND P. Moin

1. Motivation and objectives

The "local-equilibrium" hypothesis that assumes a local balance between the viscous dissipation and the subgrid-scale (SGS) dissipation at the same physical location in turbulent flow has been commonly used for turbulence modeling. For example, based on the local equilibrium hypothesis, Germano *et al.* (1991) and Moin *et al.* (1991) developed dynamic procedures for determining the model coefficients of the Smagorinsky-model-based subgrid-scale eddy viscosity as a function of space and time. The local equilibrium assumption results in both favorable and unfavorable consequences to large-eddy simulation. The dynamic modeling procedure allows vanishing eddy viscosity by the model coefficient vanishing in regions where the flow is laminar or the eddy viscosity should be zero. However, the dynamic model coefficient can cause numerical instability since its value often becomes negative and/or highly fluctuates in space and time. Therefore, the numerical instability has been remedied by additional numerical procedures such as an averaging of the model coefficient over statistically homogeneous directions or an *ad hoc* clipping procedure (*e.g.,* Meneveau *et al.* 1996). However, the numerical stabilization procedure becomes complicated when the dynamic model is applied to a complex flow configuration in which there are no homogeneous directions.

The shortcoming of the dynamic Smagorinsky models based on the local-equilibrium hypothesis was overcome by You & Moin (2007*a*) and Park *et al.* (2006). They proposed a dynamic procedure for determining the model coefficient of an eddy-viscosity model developed by Vreman (2004) utilizing a "global equilibrium" hypothesis that assumes a global balance between the subgrid-scale dissipation and the viscous dissipation. In the global-equilibrium approaches (You & Moin 2007*a*; Park *et al.* 2006), the model coefficient is determined to be globally uniform in space but to vary in time, and does not require any *ad hoc* numerical stabilization or clipping operations. Even with a non-zero constant model coefficient, the global-coefficient models still guarantee zero eddy viscosity in the laminar and fully resolved flow regions by the inherent advantage of Vreman's eddy-viscosity model (2004) in which vanishing subgrid-scale dissipation for various laminar shear flows is theoretically guaranteed.

The global-equilibrium-based modeling approach of You & Moin (2007*a*) was generalized for large-eddy simulation of turbulent flow with scalar transport, especially in complex configurations (You & Moin 2007*b*). The dynamic procedure assumes "global equilibrium" between the subgrid-scale scalar diffusion and the molecular diffusion. Similarly to the dynamic procedure for determining eddy viscosity, the model necessitates only a single-level test filter and does not require any numerical stabilization procedures. Therefore the proposed model is suitable for large-eddy simulation of turbulent flow with scalar transport in complex configurations.

In this study, the dynamic global-coefficient model by You & Moin (2007*a*) is generalized to compressible turbulence. The governing equations and the derivation of the dynamic SGS model for compressible flows are given in section 2. A brief summary of the study and related future plans are discussed in section 3

2. Mathematical formulation

We consider the filtered continuity, momentum, and internal energy equations for the large-scale flow field:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho v_k}}{\partial x_k} = 0, \tag{2.1}$$

$$\frac{\partial \overline{\rho v_k}}{\partial t} + \frac{\partial \overline{\rho v_k v_l}}{\partial x_l} = -\frac{\partial \overline{p}}{\partial x_k} + \frac{\partial \overline{\sigma}_{kl}}{\partial x_l}, \tag{2.2}$$

$$\frac{\partial}{\partial t}(\overline{C_V \rho T}) + \frac{\partial}{\partial x_k}(\overline{C_V \rho T v_k}) = -\overline{p\frac{\partial v_k}{\partial x_k}} + \overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}} + \frac{\partial}{\partial x_k}(\overline{k\frac{\partial T}{\partial x_k}}), \tag{2.3}$$

where

$$\overline{\sigma}_{kl} = -\frac{2}{3}\overline{\mu\frac{\partial v_j}{\partial x_j}}\delta_{kl} + \overline{\mu\left(\frac{\partial v_k}{\partial x_l} + \frac{\partial v_l}{\partial x_k}\right)},$$

ρ is the density, v_k is the velocity component in the k-direction, p is the pressure, T is the temperature, C_V is the specific heat at constant volume, k is the thermal conductivity, and μ is the molecular viscosity.

2.1. *Subgrid-scale model for the momentum equations*

In this study the formalisms of You & Moin (2007*a*) for the dynamic SGS model for incompressible flows are extended to compressible flows. We consider transport equations of the trace of the subgrid-scale stress tensor of compressible flows in the grid- and test-filter levels:

$$\frac{\partial \tau_{kk}}{\partial t} = \frac{\partial}{\partial x_l}\{-(\overline{\rho v_k v_k v_l} - \bar{\rho}\tilde{v}_k\tilde{v}_k\tilde{v}_l) - 2(\overline{v_l p} - \tilde{v}_l\overline{p}) + 2(\overline{v_k \sigma_{kl}} - \tilde{v}_k\tilde{\sigma}_{kl}) + 2\tilde{v}_k\tau_{kl}\}$$

$$+2\left(\overline{p\frac{\partial v_k}{\partial x_k}} - \bar{p}\frac{\partial \tilde{v}_k}{\partial x_k}\right) - 2\left(\overline{\sigma_{kl}\frac{\partial v_k}{\partial x_l}} - \tilde{\sigma}_{kl}\frac{\partial \tilde{v}_k}{\partial x_l}\right) - 2\tau_{kl}\frac{\partial \tilde{v}_k}{\partial x_l}, \tag{2.4}$$

$$\frac{\partial \mathcal{T}_{kk}}{\partial t} = \frac{\partial}{\partial x_l}\left\{-(\widehat{\overline{\rho v_k v_k v_l}} - \hat{\bar{\rho}}\check{v}_k\check{v}_k\check{v}_l) - 2(\widehat{\overline{v_l p}} - \check{v}_l\hat{\bar{p}}) + 2(\widehat{\overline{v_k \sigma_{kl}}} - \check{v}_k\check{\sigma}_{kl}) + 2\check{v}_k\mathcal{T}_{kl}\right\}$$

$$+2\left(\widehat{\overline{p\frac{\partial v_k}{\partial x_k}}} - \hat{\bar{p}}\frac{\partial \check{v}_k}{\partial x_k}\right) - 2\left(\widehat{\overline{\sigma_{kl}\frac{\partial v_k}{\partial x_l}}} - \check{\sigma}_{kl}\frac{\partial \check{v}_k}{\partial x_l}\right) - 2\mathcal{T}_{kl}\frac{\partial \check{v}_k}{\partial x_l}, \tag{2.5}$$

where $\tau_{kl} = \overline{\rho v_k v_l} - \bar{\rho}\tilde{v}_k\tilde{v}_l$ and $\mathcal{T}_{kl} = \widehat{\overline{\rho v_k v_l}} - \hat{\bar{\rho}}\check{v}_k\check{v}_l$ are SGS stress tensors in the grid- and test-filter levels, respectively. Favre-filtered variables are defined as $\tilde{f} = \overline{\rho f}/\bar{\rho}$ and $\check{f} = \widehat{\overline{\rho f}}/\hat{\bar{\rho}}$ in the grid- and test-filter levels, respectively.

From Eqs. (2.4) and (2.5), one obtains a transport equation for $L_{kk}(= \mathcal{T}_{kk} - \widehat{\tau}_{kk})$:

$$\frac{\partial L_{kk}}{\partial t} =$$

$$\frac{\partial}{\partial x_l}\left\{-(\widehat{\bar{\rho}\tilde{v}_k\tilde{v}_k\tilde{v}_l} - \hat{\bar{\rho}}\check{v}_k\check{v}_k\check{v}_l) - 2(\widehat{\tilde{v}_l\bar{p}} - \check{v}_l\hat{\bar{p}}) + 2(\widehat{\tilde{v}_k\tilde{\sigma}_{kl}} - \check{v}_k\check{\sigma}_{kl}) + 2(\check{v}_k\mathcal{T}_{kl} - \widehat{\tilde{v}_k\tau_{kl}})\right\}$$

$$+2\left(\widehat{\bar{p}\frac{\partial\tilde{v}_k}{\partial x_k}} - \hat{\bar{p}}\frac{\partial\check{v}_k}{\partial x_k}\right) - 2\left(\widehat{\tilde{\sigma}_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}} - \check{\sigma}_{kl}\frac{\partial\check{v}_k}{\partial x_l}\right) - 2\left(\mathcal{T}_{kl}\frac{\partial\check{v}_k}{\partial x_l} - \widehat{\tau_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}}\right). \tag{2.6}$$

Taking a volume integration of Eq. (2.6) assuming "a global equilibrium" finally results in

$$-\frac{1}{2}\left\langle\frac{\partial L_{kk}}{\partial t}\right\rangle_V + \left\langle\widehat{\bar{p}\frac{\partial\tilde{v}_k}{\partial x_k}} - \hat{\bar{p}}\frac{\partial\check{v}_k}{\partial x_k}\right\rangle_V - \left\langle\widehat{\tilde{\sigma}_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}} - \check{\sigma}_{kl}\frac{\partial\check{v}_k}{\partial x_l}\right\rangle_V = \left\langle\mathcal{T}_{kl}\frac{\partial\check{v}_k}{\partial x_l} - \widehat{\tau_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}}\right\rangle_V, \tag{2.7}$$

where redistribution terms are negligible.

We use the trace-free eddy-viscosity model developed by Vreman (2004) for the subgrid-scale stress tensors:

$$\tau_{kl} - \frac{1}{3}\lambda^2\delta_{kl} = -2C_g\bar{\rho}\Pi^{\tilde{g}}\left(\tilde{S}_{kl} - \frac{1}{3}\tilde{S}_{mm}\delta_{kl}\right),$$
$$\mathcal{T}_{kl} - \frac{1}{3}\Lambda^2\delta_{kl} = -2C_g\hat{\bar{\rho}}\Pi^{\check{t}}\left(\check{S}_{kl} - \frac{1}{3}\check{S}_{mm}\delta_{kl}\right), \tag{2.8}$$

where $\lambda^2 = \tau_{ii}$ and $\Lambda^2 = \mathcal{T}_{ii}$ are the isotropic parts of the SGS stress tensors, and $\tilde{S}_{kl}$ and $\check{S}_{kl}$ are the strain-rate tensors in the grid- and test-filter levels, respectively. $\Pi^{\tilde{g}}$ and $\Pi^{\check{g}}$ are defined in Eq. (2.12). λ^2 and Λ^2 are modeled using Vreman's kernel while they are absorbed into the pressure in the incompressible flow:

$$\lambda^2 = C_I\bar{\rho}\Pi^{\tilde{g}}|\tilde{S}|,$$
$$\Lambda^2 = C_I\hat{\bar{\rho}}\Pi^{\check{t}}|\check{S}|, \tag{2.9}$$

where the model coefficient C_I is computed using the Germano-identity and global-equilibrium concept:

$$\tau_{kk} = \overline{\rho v_k v_k} - \bar{\rho}\tilde{v}_k\tilde{v}_k = \lambda^2 = C_I\bar{\rho}\Pi^{\tilde{g}}|\tilde{S}|,$$

$$\mathcal{T}_{kk} = \widehat{\overline{\rho v_k v_k}} - \hat{\bar{\rho}}\check{v}_k\check{v}_k = \Lambda^2 = C_I\hat{\bar{\rho}}\Pi^{\check{t}}|\check{S}|,$$

$$L_{kk} = \mathcal{T}_{kk} - \widehat{\tau}_{kk} = \widehat{\bar{\rho}\tilde{v}_k\tilde{v}_k} - \hat{\bar{\rho}}\check{v}_k\check{v}_k = C_I\hat{\bar{\rho}}\Pi^{\check{t}}|\check{S}| - C_I\widehat{\bar{\rho}\Pi^{\tilde{g}}|\tilde{S}|},$$

$$C_I = \frac{\left\langle \widehat{\bar{\rho}\tilde{v}_k\tilde{v}_k} - \frac{1}{\hat{\bar{\rho}}}\widehat{\bar{\rho}\tilde{v}_k}\widehat{\bar{\rho}\tilde{v}_k} \right\rangle_V}{\left\langle \hat{\bar{\rho}}\Pi^{\check{t}}|\check{S}| - \widehat{\bar{\rho}\Pi^{\tilde{g}}|\tilde{S}|} \right\rangle_V}, \tag{2.10}$$

where $\langle\rangle_V$ indicates averaging over the entire computational domain. The model coefficient C_g in Eq. (2.8) can be determined using Eqs. (2.7), (2.8), and (2.10) as follows:

$$C_g =$$

$$\frac{\frac{1}{3}\left\langle \Lambda^2\delta_{kl}\frac{\partial\check{v}_k}{\partial x_l} - \lambda^2\widehat{\delta_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}} \right\rangle_V + \frac{1}{2}\left\langle \frac{\partial L_{kk}}{\partial t} \right\rangle_V - \left\langle \widehat{\bar{p}\frac{\partial\tilde{v}_k}{\partial x_k}} - \hat{\bar{p}}\frac{\partial\check{v}_k}{\partial x_k} \right\rangle_V + \left\langle \widehat{\tilde{\sigma}_{kl}\frac{\partial\tilde{v}_k}{\partial x_l}} - \check{\sigma}_{kl}\frac{\partial\check{v}_k}{\partial x_l} \right\rangle_V}{2\left\langle \hat{\bar{\rho}}\Pi^{\check{t}}\left(\check{S}_{kl} - \frac{1}{3}\check{S}_{mm}\delta_{kl}\right)\frac{\partial\check{v}_k}{\partial x_l} - \widehat{\bar{\rho}\Pi^{\tilde{g}}\left(\tilde{S}_{kl} - \frac{1}{3}\tilde{S}_{mm}\delta_{kl}\right)\frac{\partial\tilde{v}_k}{\partial x_l}} \right\rangle_V},$$

$$= \frac{C_g{}^{term1} + C_g{}^{term2} + C_g{}^{term3} + C_g{}^{term4}}{C_g{}^{term5}} \tag{2.11}$$

where

$$\Pi^{\tilde{g}} = \sqrt{\frac{B_\beta^{\tilde{g}}}{\tilde{\alpha}_{kl}\tilde{\alpha}_{kl}}},$$

$$B_\beta^{\tilde{g}} = \beta_{11}^{\tilde{g}}\beta_{22}^{\tilde{g}} - \beta_{12}^{\tilde{g}}\beta_{12}^{\tilde{g}} + \beta_{11}^{\tilde{g}}\beta_{33}^{\tilde{g}} - \beta_{13}^{\tilde{g}}\beta_{13}^{\tilde{g}} + \beta_{22}^{\tilde{g}}\beta_{33}^{\tilde{g}} - \beta_{23}^{\tilde{g}}\beta_{23}^{\tilde{g}},$$

$$\beta_{ij}^{\tilde{g}} = \sum_{m=1}^{3}\overline{\Delta}_m^2\tilde{\alpha}_{mi}\tilde{\alpha}_{mj},$$

$$\tilde{\alpha}_{ij} = \frac{\partial\tilde{u}_j}{\partial x_i}, \quad \tilde{u}_j = \frac{\overline{\rho u_j}}{\bar{\rho}}, \tag{2.12}$$

and

$$\Pi^{\check{t}} = \sqrt{\frac{B_\beta^{\check{t}}}{\check{\alpha}_{kl}\check{\alpha}_{kl}}},$$

$$B_\beta^{\check{t}} = \beta_{11}^{\check{t}}\beta_{22}^{\check{t}} - \beta_{12}^{\check{t}}\beta_{12}^{\check{t}} + \beta_{11}^{\check{t}}\beta_{33}^{\check{t}} - \beta_{13}^{\check{t}}\beta_{13}^{\check{t}} + \beta_{22}^{\check{t}}\beta_{33}^{\check{t}} - \beta_{23}^{\check{t}}\beta_{23}^{\check{t}},$$

$$\beta_{ij}^{\check{t}} = \sum_{m=1}^{3}\widehat{\overline{\Delta}}_m^2\check{\alpha}_{mi}\check{\alpha}_{mj},$$

$$\check{\alpha}_{ij} = \frac{\partial\check{u}_j}{\partial x_i}, \quad \check{u}_j = \frac{\widehat{\overline{\rho u_j}}}{\hat{\bar{\rho}}}. \tag{2.13}$$

2.2. *Subgrid-scale model for the energy equation*

The dynamic modeling procedure based on the global-equilibrium hypothesis is generalized to model the subgrid-scale heat flux. A transport equation for the temperature variance in the grid-filter level is derived as follows:

$$\frac{\partial}{\partial t}(C_V\rho TT) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V\rho TTv_k + 2kT\frac{\partial T}{\partial x_k}\right) - 2pT\frac{\partial v_k}{\partial x_k} + 2T\sigma_{ik}\frac{\partial v_k}{\partial x_i} - 2k\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}, \quad (2.14)$$

$$\frac{\partial}{\partial t}(C_V\bar{\rho}\tilde{T}\tilde{T}) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V\bar{\rho}\tilde{T}\tilde{T}\tilde{v}_k + 2\tilde{k}\tilde{T}\frac{\partial \tilde{T}}{\partial x_k} - 2C_V\tilde{T}q_k\right)$$

$$-2\tilde{T}\overline{p\frac{\partial v_k}{\partial x_k}} + 2\tilde{T}\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}} - 2\tilde{k}\frac{\partial \tilde{T}}{\partial x_k}\frac{\partial \tilde{T}}{\partial x_k} + 2C_Vq_k\frac{\partial \tilde{T}}{\partial x_k}, \quad (2.15)$$

$$\frac{\partial}{\partial t}(C_V\hat{\bar{\rho}}\check{T}\check{T}) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V\hat{\bar{\rho}}\check{T}\check{T}\check{v}_k + 2\check{k}\check{T}\frac{\partial \check{T}}{\partial x_k} - 2C_V\check{T}Q_k\right)$$

$$-2\check{T}\widehat{\overline{p\frac{\partial v_k}{\partial x_k}}} + 2\check{T}\widehat{\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}} - 2\check{k}\frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k} + 2C_VQ_k\frac{\partial \check{T}}{\partial x_k}. \quad (2.16)$$

Extracting Eq. (2.15) from the grid-filtered Eq. (2.14) leads to

$$\frac{\partial}{\partial t}(C_V(\overline{\rho TT} - \bar{\rho}\tilde{T}\tilde{T})) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V(\overline{\rho TTv_k} - \bar{\rho}\tilde{T}\tilde{T}\tilde{v}_k) + 2(\overline{kT\frac{\partial T}{\partial x_k}} - \tilde{k}\tilde{T}\frac{\partial \tilde{T}}{\partial x_k}) + 2C_V\tilde{T}q_k\right)$$

$$-2\left(\overline{Tp\frac{\partial v_k}{\partial x_k}} - \tilde{T}\overline{p\frac{\partial v_k}{\partial x_k}}\right) + 2\left(\overline{T\sigma_{ik}\frac{\partial v_k}{\partial x_i}} - \tilde{T}\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}\right)$$

$$-2\left(\overline{k\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}} - \tilde{k}\frac{\partial \tilde{T}}{\partial x_k}\frac{\partial \tilde{T}}{\partial x_k}\right) - 2C_Vq_k\frac{\partial \tilde{T}}{\partial x_k}. \quad (2.17)$$

Extracting Eq. (2.16) from the test-filtered Eq. (2.14) leads to

$$\frac{\partial}{\partial t}(C_V(\widehat{\overline{\rho T T}} - \hat{\bar{\rho}}\check{T}\check{T})) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V(\widehat{\overline{\rho T T v_k}} - \hat{\bar{\rho}}\check{T}\check{T}\check{v}_k) + 2(\widehat{\overline{kT\frac{\partial T}{\partial x_k}}} - \check{k}\check{T}\frac{\partial \check{T}}{\partial x_k}) + 2C_V\check{T}Q_k\right)$$

$$-2\left(\widehat{\overline{Tp\frac{\partial v_k}{\partial x_k}}} - \check{T}\widehat{\overline{p\frac{\partial v_k}{\partial x_k}}}\right) + 2\left(\widehat{\overline{T\sigma_{ik}\frac{\partial v_k}{\partial x_i}}} - \check{T}\widehat{\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}}\right)$$

$$-2\left(\widehat{\overline{k\frac{\partial T}{\partial x_k}\frac{\partial T}{\partial x_k}}} - \check{k}\frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k}\right) - 2C_V Q_k\frac{\partial \check{T}}{\partial x_k}. \tag{2.18}$$

Finally, extracting Eq. (2.18) from the test-filtered Eq. (2.17) yields

$$\frac{\partial}{\partial t}(C_V(\widehat{\bar{\rho}\tilde{T}\tilde{T}} - \hat{\bar{\rho}}\check{T}\check{T})) =$$

$$\frac{\partial}{\partial x_k}\left(-C_V(\widehat{\bar{\rho}\tilde{T}\tilde{T}\tilde{v}_k} - \hat{\bar{\rho}}\check{T}\check{T}\check{v}_k) + 2(\widehat{\tilde{k}\tilde{T}\frac{\partial \tilde{T}}{\partial x_k}} - \check{k}\check{T}\frac{\partial \check{T}}{\partial x_k}) + 2C_V(\check{T}Q_k - \widehat{\tilde{T}q_k})\right)$$

$$-2\left(\widehat{\overline{\tilde{T}p\frac{\partial \tilde{v}_k}{\partial x_k}}} - \check{T}\widehat{\overline{p\frac{\partial \check{v}_k}{\partial x_k}}}\right) + 2\left(\widehat{\overline{\tilde{T}\sigma_{ik}\frac{\partial v_k}{\partial x_i}}} - \check{T}\widehat{\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}}\right)$$

$$-2\left(\tilde{k}\widehat{\frac{\partial \tilde{T}}{\partial x_k}\frac{\partial \tilde{T}}{\partial x_k}} - \check{k}\frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k}\right) - 2C_V\left(Q_k\frac{\partial \check{T}}{\partial x_k} - q_k\widehat{\frac{\partial \tilde{T}}{\partial x_k}}\right). \tag{2.19}$$

Taking the volume average $\langle\rangle_V$ of the terms in Eq. (2.19) over the entire computational domain assuming that the volume average of the redistribution terms are negligible, yields

$$C_V\left\langle Q_k\frac{\partial \check{T}}{\partial x_k} - q_k\widehat{\frac{\partial \tilde{T}}{\partial x_k}}\right\rangle_V =$$

$$-\left\langle \widehat{\overline{\tilde{T}p\frac{\partial \tilde{v}_k}{\partial x_k}}} - \check{T}\widehat{\overline{p\frac{\partial \check{v}_k}{\partial x_k}}}\right\rangle_V + \left\langle \widehat{\overline{\tilde{T}\sigma_{ik}\frac{\partial v_k}{\partial x_i}}} - \check{T}\widehat{\overline{\sigma_{ik}\frac{\partial v_k}{\partial x_i}}}\right\rangle_V$$

$$-\left\langle \tilde{k}\widehat{\frac{\partial \tilde{T}}{\partial x_k}\frac{\partial \tilde{T}}{\partial x_k}} - \check{k}\frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k}\right\rangle_V - \frac{1}{2}\left\langle\frac{\partial}{\partial t}(C_V(\widehat{\bar{\rho}\tilde{T}\tilde{T}} - \hat{\bar{\rho}}\check{T}\check{T}))\right\rangle_V, \tag{2.20}$$

where the subgrid-scale heat fluxes in the grid- and test-filter levels are modeled as

$$q_k = \overline{\rho v_k T} - \frac{1}{\bar{\rho}}\overline{\rho v_k}\,\overline{\rho T} = -\frac{\bar{\rho}\nu_T}{D_T}\frac{\partial \tilde{T}}{\partial x_k},$$

$$Q_k = \widehat{\overline{\rho v_k T}} - \frac{1}{\hat{\bar{\rho}}}\widehat{\overline{\rho v_k}}\,\widehat{\overline{\rho T}} = -\frac{\hat{\bar{\rho}}\check{\nu}_T}{D_T}\frac{\partial \check{T}}{\partial x_k}. \tag{2.21}$$

The model coefficient D_T is determined by inserting Eq. (2.21) into Eq. (2.20) as follows:

$$D_T = \frac{C_V \left\langle \hat{\bar{\rho}}\check{\nu}_T \frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k} - \widehat{\bar{\rho}\nu_T \frac{\partial \tilde{T}}{\partial x_k}}\frac{\partial \tilde{T}}{\partial x_k} \right\rangle_V}{\left\langle \widehat{\bar{p}\tilde{T}\frac{\partial \tilde{v}_k}{\partial x_k}} - \hat{\bar{p}}\check{T}\frac{\partial \check{v}_k}{\partial x_k} \right\rangle_V - \left\langle \widehat{\tilde{T}\tilde{\sigma}_{ik}\frac{\partial \tilde{v}_k}{\partial x_i}} - \check{T}\check{S}_{ik}\frac{\partial \check{v}_k}{\partial x_i} \right\rangle_V + \left\langle \widehat{\tilde{k}\frac{\partial \tilde{T}}{\partial x_k}\frac{\partial \tilde{T}}{\partial x_k}} - \check{k}\frac{\partial \check{T}}{\partial x_k}\frac{\partial \check{T}}{\partial x_k} \right\rangle_V + \frac{1}{2}\left\langle \frac{\partial}{\partial t}\left(C_V\left(\widehat{\bar{\rho}\tilde{T}\tilde{T}} - \hat{\bar{\rho}}\check{T}\check{T}\right)\right) \right\rangle_V}$$

$$= \frac{{D_T}^{term1}}{{D_T}^{term2} + {D_T}^{term3} + {D_T}^{term4} + {D_T}^{term5}}. \tag{2.22}$$

3. Summary and future work

The dynamic global-coefficient subgrid-scale eddy-viscosity model by You & Moin [Phys. Fluids **19**, 065110 (2007)] and You & Moin [CTR Ann. Res. Briefs, pp.169-182 (2007)] has been generalized for large-eddy simulation of compressible turbulent flow. The model coefficients for eddy viscosity and subgrid-scale heat flux which are globally constant in space but vary in time are dynamically determined based on the "global conservation" of transport equations for the trace of the Germano identity and the temperature variance, respectively. The present dynamic model is especially designed for large-eddy simulation of compressible turbulent flow in complex geometries, avoids any *ad hoc* spatial and temporal averaging or clipping of the model coefficient for numerical stabilization, and requires only a single-level test filter.

The predictive capability of the present model is being evaluated in a number of test cases. From the tests, the contribution of the dilatation (C_g^{term3} in Eq. (2.11) and D_T^{term2} in Eq. (2.22)) and time-derivative (C_g^{term2} in Eq. (2.11) and D_T^{term5} in Eq. (2.22)) terms to the model coefficients will be examined.

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