The radiated noise from isotropic turbulence and heated jets

By G. M. Lilley

1. Motivation and objectives

Our understanding of aerodynamic noise has its foundations in the work of Sir James Lighthill (1952), which was the first major advance in acoustics since the pioneering work of Lord Rayleigh in the last century. The combination of Lighthill's theory of aerodynamic noise as applied to turbulent flows and the experimental growing database from the early 1950's was quickly exploited by various jet propulsion engine designers in reducing the noise of jet engines at takeoff and landing to levels marginally acceptable to communities living in the neighborhoods of airports. The success in this noise containment led to the rapid growth of fast economical subsonic civil transport aircraft worldwide throughout the 1960's and has continued to the present day. One important factor in this success story has been the improvements in the engine cycle that have led to both reductions in specific fuel consumption and noise. The second is the introduction of Noise Certification, which specifies the maximum noise levels at takeoff and landing that all aircraft must meet before they can be entered on the Civil Aircraft Register. The growing interest in the development of a new supersonic civil transport to replace 'Concorde' in the early years of the next century has led to a resurgence of interest in the more challenging problem of predicting the noise of hot supersonic jets and developing means of aircraft noise reduction at takeoff and landing to meet the standards now accepted for subsonic Noise Certification.

The prediction of aircraft noise to the accuracy required to meet Noise Certification requirements has necessitated reliance upon experimental measurements and empirically derived laws based on the available experimental data bases. These laws have their foundation in the results from Lighthill's theory, but in the case of jet noise, where the noise is generated in the turbulent mixing region with the external ambient fluid, the complexity of the turbulent motion has prevented the full deployment of Lighthill's theory from being achieved. However, the growth of the supercomputer and its applications in the study of the structure of turbulent shear flows in both unbounded and wall bounded flows, which complements and in certain cases extends the work of the few dedicated experimental groups working in this field for the past forty years, provides an opportunity and challenge to accurately predict the noise from jets. Moreover a combination of numerical and laboratory experiments offers the hope that in the not too distant future the physics of noise generation and flow interaction will be better understood and it will then be possible to not only improve the accuracy of noise prediction but also to explore and optimize schemes for noise reduction. The present challenge is to provide time and space accurate numerical databases for heated subsonic and supersonic jets to provide information on the fourth-order space-time covariance of Lighthill's equivalent stress tensor, T_{ij}, which governs the characteristics of the farfield radiated noise and the total acoustic power. Validation with available experimental databases will establish how close Lighthill's theory is to the accurate prediction of the directivity and spectrum of jet noise and the total acoustic power, and the need, in the applications of the theory, to include the effects of flow-acoustic interaction.

2. Accomplishments

2.1 Lighthill's acoustic analogy

Our understanding of the theory of jet noise has its foundations in Lighthill's theory of aerodynamic noise (1952, 1954, 1962, 1963, 1978). Lighthill's theory is based on an acoustic analogy whereby the exact Navier-Stokes equations for fluid flow are rearranged, using an ingeneous technique, to form an inhomogeneous wave equation for the fluctuating fluid density. Since all disturbances created by a turbulent flow result in alternate compressions and expansions of a fluid element as it is convected by the flow, the time rate of change of this fluid element, δV, per unit volume of fluid, following the fluid is

$$\lim_{\delta V \to 0} \frac{1}{\delta V} \frac{D\delta V}{Dt} = -\frac{D \ln \rho}{Dt} = \bigtriangledown \cdot \boldsymbol{v} \tag{1}$$

and as a consequence noise is generated and radiated away from the fluid element with a propagation speed equal to the speed of sound. Although the dilatation, $\theta = \bigtriangledown \cdot \mathbf{v}$, in Eq. 1 is zero in an incompressible flow it is always finite in compressible flows, and similarly so is $\bigtriangledown \cdot \rho\mathbf{v}$. In order to ensure the finiteness of the latter throughout the flow in calculations concerning aerodynamic noise, Lighthill derived the inhomogeneous wave equation for the density fluctuations by eliminating $\bigtriangledown \cdot \rho\mathbf{v}$ between the equations of conservation of mass and momentum. The forcing function on its right-hand side represents a distribution of acoustic sources in the ambient flow at rest, replacing the complete unsteady flow. In Lighthill's theory $\partial^2 T_{ij}/\partial x_i \partial x_j$ is the strength of these acoustic sources per unit volume, where

$$T_{ij} = \rho v_i v_j - \tau_{ij} + (p - c_\infty^2 \rho)\delta_{ij} \tag{2}$$

is Lighthill's instantaneous applied acoustic stress tensor. p, ρ, c, and τ_{ij} are respectively the pressure, density, speed of sound, and the viscous stress tensor. In this acoustic analogy the equivalent acoustic sources may move but not the fluid.

Here we follow Lighthill's approach and derive the inhomogeneous wave equation for the fluctuating pressure in the form derived by Lilley (1973), where the only deviation from Lighthill's derivation is in the replacement of $\partial(p - \rho c_\infty^2)/\partial t$ by its equivalent terms from the total enthalpy, h_s, equation together with continuity, giving

$$\frac{\partial(p - c_\infty^2 \rho)}{c_\infty^2 \partial t} = -\frac{(\gamma - 1)}{2c_\infty^2} \frac{\partial \rho v^2}{\partial t} - \bigtriangledown \cdot \frac{\rho \boldsymbol{v}(h_s - h_\infty)}{h_\infty} + \frac{(\gamma - 1)}{c_\infty^2} \bigtriangledown \cdot (\boldsymbol{q} + \boldsymbol{v} \cdot \boldsymbol{\tau}). \tag{3}$$

where $\boldsymbol{q}$ is the heat flux vector and γ is the ratio of the specific heats. The suffix ∞ denotes ambient conditions. The resultant inhomogeneous wave equation is

$$\frac{\partial^2 p}{c_\infty^2 \partial t^2} - \nabla^2 p = \nabla \cdot (\nabla \cdot \rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{\tau}) - \frac{(\gamma-1)}{2c_\infty^2}\frac{\partial^2 \rho v^2}{\partial t^2} - \nabla \cdot \frac{\partial}{\partial t}\frac{\rho \boldsymbol{v}(h_s - h_\infty)}{h_\infty}$$

$$+\frac{(\gamma-1)}{c_\infty^2}\nabla \cdot \frac{\partial}{\partial t}(\boldsymbol{q} + \boldsymbol{v}\cdot\boldsymbol{\tau}) \equiv A(\boldsymbol{x}, t) \tag{4}$$

having the unbounded solution

$$(p - p_\infty)(\boldsymbol{x}, t) = \frac{1}{4\pi}\int_V [A(\boldsymbol{y}, t)]\frac{d^3\boldsymbol{y}}{|\boldsymbol{x} - \boldsymbol{y}|} \tag{5}$$

where the [..] denotes the function is evaluated at the retarded time, $\tau = t - |\boldsymbol{x} - \boldsymbol{y}|/c_\infty$. The far-field approximation, when $|\boldsymbol{x} - \boldsymbol{y}| \approx x$, is

$$(p - p_\infty)(\boldsymbol{x}, t) \approx \frac{1}{4\pi x c_\infty^2}\int_V d^3\boldsymbol{y}\frac{\partial^2}{\partial t^2}\Bigg(\rho u_x^2 - \tau_{xx}$$

$$-\frac{(\gamma-1)}{2}\rho v^2 + (\gamma-1)\frac{\rho u_x(h_s - h_\infty)}{c_\infty} - (\gamma-1)\frac{(q_x + u_k\tau_{kx})}{c_\infty}\Bigg) \tag{6}$$

where u_x is the component of the velocity in the direction joining the source at $\boldsymbol{y}$ to the far-field observer at $\boldsymbol{x}$. We find the integrand in Eq. 6 is identical with the component, (xx), of Lighthill's stress tensor, T_{ij}. Apart from the noise generated by the diffusive terms, $\boldsymbol{q}$ and $\boldsymbol{\tau}$, which at high Reynolds numbers is shown to be very small and can be neglected, the major sources of sound in a turbulent flow involve the fluctuations of the momentum flux, $\rho\boldsymbol{v}\boldsymbol{v}$, and the fluctuations of the total enthalpy flux, $\rho\boldsymbol{v}(h_s - h_\infty)$. The fluctuations of the kinetic energy, $\rho v^2/2$, make a small contribution to the radiated noise. (In an inviscid incompressible flow the time gradient of the integral of the kinetic energy would be zero.)

2.1.1 The acoustic power output in isotropic turbulence

The intensity, $I(\boldsymbol{x})$, of the radiated sound in the far-field is proportional to the square of the fluctuating pressure and is defined by

$$I(\boldsymbol{x}, t) = \frac{<(p - p_\infty)^2>}{\rho_\infty c_\infty.} \tag{7}$$

Similarly the autocorrelation, $I(\boldsymbol{x}, t^*)$, for a stationary turbulent flow is

$$I(\boldsymbol{x}, t^*) = \frac{1}{16\pi^2 x^2 \rho_\infty c_\infty^5}\int_V d^3\boldsymbol{y}\int \frac{\partial^4}{\partial \tau^4}P_{xx,xx}(\boldsymbol{y}, \boldsymbol{r}, \tau))d^3\boldsymbol{r} \tag{8}$$

where t^* is the far-field time difference and the spectral density, $I(\boldsymbol{x}, \omega)$ is

$$I(\boldsymbol{x},\omega) = \frac{1}{2\pi}\int I(\boldsymbol{x},t^*)\exp(i\omega t^*)dt^*$$

$$= \frac{\pi\omega^4}{2x^2\rho_\infty c_\infty^5}\int_V P_{xx,xx}(\boldsymbol{y},\boldsymbol{k},\omega)d^3\boldsymbol{y} \tag{9}$$

where $\boldsymbol{r}$ is the spatial separation in fixed coordinates, τ is the retarded time difference defined by $\tau = t^* + \boldsymbol{x}\cdot\boldsymbol{r}/xc_\infty$, ω is the far-field circular frequency, and $P_{xx,xx}$ is the source, $(\boldsymbol{y})$,-observer, $(\boldsymbol{x})$, aligned space-retarded time covariance of T_{ij}, and

$$P_{xx,xx}(\boldsymbol{y},\boldsymbol{k},\omega) = \frac{1}{16\pi^4}\int \exp(i\boldsymbol{k}\cdot\boldsymbol{r})d^3\boldsymbol{r}\int\exp(i\omega\tau)P_{xx,xx}(\boldsymbol{y},\boldsymbol{r},\tau)d\tau \tag{10}$$

is the four-dimensional wavenumber-frequency spectrum function corresponding to the aligned space-retarded time covariance of T_{ij}. The frequency of the sound, ω, is the same as in the turbulence, and the wavenumber vector of the sound, $\boldsymbol{k} = -\omega\boldsymbol{x}/xc_\infty$, equals the wavenumber vector in the turbulence. In near incompressible flow, where the wavelength of sound is large, $|\boldsymbol{k}| \to 0$. In the turbulence small values of k_x receive contributions from all scales of turbulence.

The total acoustic power per unit volume of turbulence is found by integrating the intensity per unit volume at the given source position, $\boldsymbol{y}$, over a large spherical surface so that for isotropic turbulence

$$p_s = \frac{1}{4\pi\rho_\infty c_\infty^5}\int\frac{\partial^4}{\partial\tau^4}P_{xx,xx}(\boldsymbol{y},\boldsymbol{r},\tau)d^3\boldsymbol{r}. \tag{11}$$

When the acoustic sources are in uniform motion with the eddy convection speed, V_c, and the space-retarded time covariance of T_{ij} is measured in the moving frame, where the moving coordinates are defined by

$$\boldsymbol{\eta} = \boldsymbol{y} - c_\infty\boldsymbol{M}_c\tau \tag{12}$$

such that the source emits as it crosses the fixed point $\boldsymbol{y}$ at time $t = \tau$, the spectral density of the sound intensity per unit volume is given from the Lighthill-Ffowcs Williams eddy convection theory (1963) in the form

$$i(\boldsymbol{x},\omega) = \frac{\pi\omega^4}{2x^2\rho_\infty c_\infty^5}P_{xx,xx}(\boldsymbol{y},\boldsymbol{k},\omega_T). \tag{13}$$

$\boldsymbol{M}_c = \boldsymbol{V}_c/c_\infty$ is the vector convection Mach number. The radiated sound in the far-field at frequency, ω, arises from turbulence in the moving frame with frequency, ω_T, which is the *Döppler* shifted frequency, with $\omega_T = \omega(1-\boldsymbol{M}_c\cdot\boldsymbol{x}/x)$. The wavenumber in the turbulence, $\boldsymbol{k} = -\omega\boldsymbol{x}/xc_\infty$, and is unaffected by the eddy motion. When the direction to the far-field is near the Mach wave direction, where normal to the

Mach wave ($\boldsymbol{M}_c \cdot \boldsymbol{x}/x = 1$), detailed analysis shows that the relation between the frequencies in the turbulence and that of the radiated sound becomes

$$\omega_T = \omega \left(|1 - \boldsymbol{M}_c \cdot \boldsymbol{x}/x|^2 + S_T^2 M_T^2 \right)^{1/2} \tag{14}$$

where S_T and $M_T = v_T/c_\infty$ are respectively the characteristic Strouhal number and Mach number of the turbulence. The reference Strouhal number of the turbulence, which we assume to be a constant throughout a given turbulent flow and is of order unity, is defined by $S_T = \Omega L/v_T$, where L is the local integral scale of the turbulence and Ω is the reference frequency in the turbulence. The reference turbulent velocity is given as $v_T = \sqrt{2K/3}$, where K is the local kinetic energy of the turbulence. In isotropic turbulence v_T is equal to $\sqrt{< u^2 >}$, where u is the velocity component in any direction.

The corresponding result for the intensity per unit volume, found by integrating (13) over all frequencies, is

$$i(\boldsymbol{x}) = \frac{1}{16\pi^2 x^2 \rho_\infty c_\infty^5} \left(|1 - \boldsymbol{M}_c \cdot \boldsymbol{x}/x|^2 + S_T^2 v_T^2/c_\infty^2 \right)^{-5/2} \int \frac{\partial^4}{\partial \tau^4} P_{xx,xx}(\boldsymbol{y}, \boldsymbol{\delta}, \tau)) d^3\boldsymbol{\delta} \tag{15}$$

where $\boldsymbol{\delta}$ is the separation distance in the moving frame and τ is the corresponding retarded time difference, showing the preferential direction for sound radiation in the downstream direction of the convecting eddies with a sharp peak in the direction normal to the Mach angle when the eddy convection Mach number is supersonic.

2.1.2 The specific noise power in heated isotropic turbulence

We will assume the turbulence has a uniform density, ρ_0, and ratio of specific heats, γ_0, compared with the ambient medium values of ρ_∞, and γ_∞. The mean pressure in the turbulent flow is assumed equal to that of the external medium. We found above there were three dominant source terms in Lighthill's aligned stress tensor, T_{xx}, and if we further assume they are statistically independent, we find their separate contributions to the radiated sound power are in the case of stationary isotropic turbulence at rest

$$p_s^{(1)} = \frac{1}{4\pi} \frac{\rho_0^2 u^8 S_T^4}{\rho_\infty c_\infty^5 L} \int \frac{\partial^4}{\partial \tau^4} \frac{< (u_x)_A^2 (u_x)_B^2 - < u^2 >^2 >}{< u^2 >^2} d^3\boldsymbol{r} \tag{16}$$

$$p_s^{(2)} = \frac{1}{4\pi} \frac{\rho_0^2 u^8 S_T^4}{\rho_\infty c_\infty^5 L} \frac{(\gamma_0 - 1)^2}{4} \int \frac{\partial^4}{\partial \tau^4} \frac{< v_A^2 v_B^2 - < v^2 >^2 >}{< u^2 >^2} d^3\boldsymbol{r} \tag{17}$$

$$p_s^{(3)} = \frac{1}{4\pi} \frac{\rho_0^2 u^6 S_T^4}{\rho_\infty c_\infty^3 L} \left(\frac{\gamma_0 - 1}{\gamma_\infty - 1} \right)^2 \int \frac{\partial^4}{\partial \tau^4} \frac{< (u_x)_A (h')_A (u_x)_B (h')_B - < u_x h' >^2 >}{< u^2 > h_\infty^2} d^3\boldsymbol{r} \tag{18}$$

where v and h' are respectively the fluctuation of the velocity and enthalpy, and $< .. >$ denotes a mean value. Suffixes $\boldsymbol{A}$ and $\boldsymbol{B}$ denote the two source positions, distance $\boldsymbol{r}$ apart, forming the respective space-retarded time covariances.

Let us consider the evaluation of the aligned velocity squared space-retarded time covariance that appears in $p_s^{(1)}$ in (16).

$$P^{(1)}_{xx,xx}(\boldsymbol{r}) =< (u_A^2 u_B^2 - < u^2 >^2) > . \tag{19}$$

Now this fourth-order isotropic tensor can be shown to be a function of the longitudinal and lateral velocity squared covariances which are functions of r only. When the turbulence follows Gaussian statistics, as assumed by Proudman (1952), we find according to Millionshtchikov's hypothesis as given by Batchelor (1953) that the velocity squared covariances can be replaced by the sum of the squares of the corresponding second order covariances involving $f(r)$ and $g(r)$ where the second order longitudinal and lateral covariances are respectively

$$\overline{u_p(\boldsymbol{x})u_p(\boldsymbol{x}+\boldsymbol{r})} = u^2 f(r) \qquad 20)$$

and

$$\overline{u_n(\boldsymbol{x})u_n(\boldsymbol{x}+\boldsymbol{r})} = u^2 g(r). \tag{21}$$

Lighthill (1992) has shown more generally that the fourth- order longitudinal velocity covariance

$$(\overline{u_p(\boldsymbol{x})^2 u_p(\boldsymbol{x}+\boldsymbol{r})^2} - < u^2 >^2 = \left(\overline{u_p(\boldsymbol{x})u_p(\boldsymbol{x}+\boldsymbol{r})}\right)^2 \left(\frac{\overline{u^4}}{\overline{u^2}^2} - 1\right), \tag{22}$$

and a similar relation holds for the fourth-order lateral covariance by replacing the suffix, p, by the suffix, n. The relationship between the respective fourth and second-order covariances holds for the given retarded time difference, τ. The velocity flatness factor, $T_1 = \overline{u^4}/\overline{u^2}^2$ has the value 3 in Gaussian statistics, and is found by Townsend (1956) to be nearly 3 in decaying isotropic turbulence. A similar result was obtained in the (DNS) results of Sarkar and Hussaini (1993) and Dubois (1993).

In weakly compressible flows, the turbulent Mach number is very small, and in this case we may assume that the modulus of the wave-number $\boldsymbol{k}$ in the turbulence is small also. In terms of the longitudinal velocity correlation function, $f(r,\tau)$, the contribution to the acoustic power spectral density is

$$p_s^{(1)}(\omega) = \frac{\rho_\infty \omega^4 < u^2 >^2 (T_1 - 1)}{c_\infty^5} \frac{2}{15\pi} \int_0^\infty \cos\omega\tau d\tau \int_0^\infty r^4 \left(\frac{\partial f}{\partial r}\right)^2 dr. \tag{23}$$

as given by Lilley (1994). The integrals in (23) can only be evaluated when the distribution $f(r,\tau)$ is known.

Lilley (1994) used the (DNS) databases obtained by Sarkar and Hussaini (1993), Dubois (1993) and the (DNS) and (LES) databases obtained by Witkowska (1994) to obtain the spatial and temporal covariances. Thus using the data derived from these database the value of the Proudman constant, $\alpha_P^{(1)}$, in

$$p_s^{(1)} = \alpha_P^{(1)} \frac{\rho_0 u^8}{\rho_\infty c_\infty^5 L} \tag{24}$$

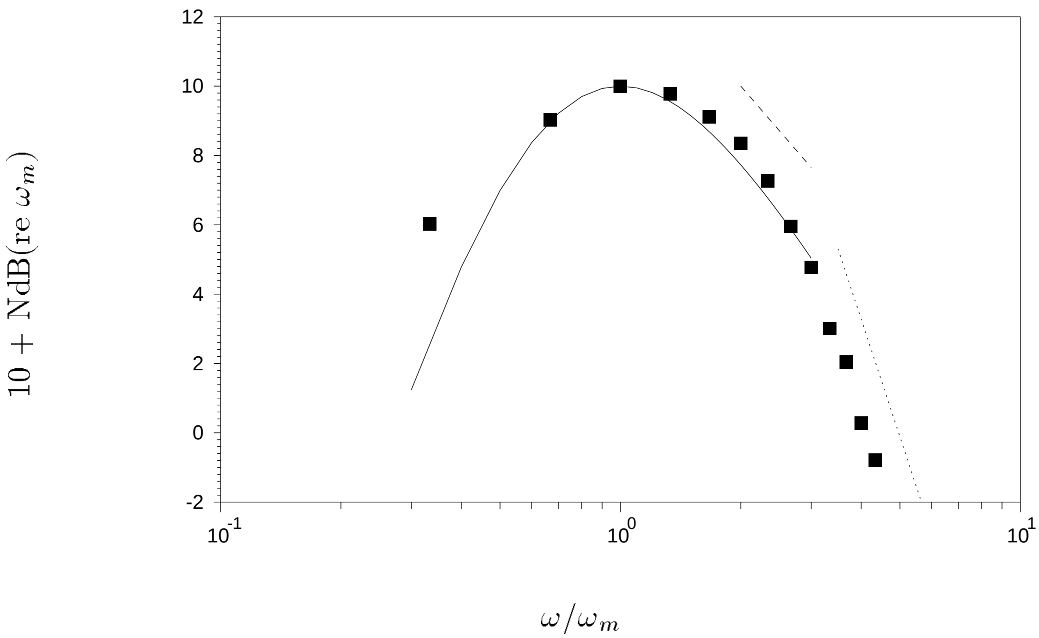

FIGURE 1. Acoustic energy spectrum in isotropic turbulence. Sarkar & Hussaini (1993) ■ : ——— $(\omega/\omega_m)^4/(1+2(\omega/\omega_m)^2)^3$: - - - - $\omega^{-4/3}$: ········ $\omega^{-7/2}$

becomes

$$\alpha_P^{(1)} = 1.80(T_1 - 1)S_T^4. \tag{25}$$

When the flatness factor, $T_1 = 3$, as discussed above, and the reference Strouhal number, $S_T = 1$, we find the Proudman constant, $\alpha_P^{(1)} = 3.6$. The available (DNS) databases gave values of S_T between 1 and 1.25. The temporal covariance was checked between the (DNS) space-time covariance results of Dubois (1993) and the far-field acoustic spectra obtained by Sarkar and Hussaini (1993).

These results are for low Reynolds numbers and low Mach numbers, and there are doubts as to their applicability to higher Reynolds numbers and Mach numbers. The low Reynolds numbers of the (DNS) data precludes the existence of an inertial subrange and there is less than a decade of separation between wavenumbers in the energy-containing and Taylor microscale ranges of eddies. The peak frequency of the radiated noise is at a frequency slightly higher than that of the energy containing eddies. This suggests that the dominant eddies responsible for the generation of sound are slightly smaller than those in the energy containing range. This is consistent with the deductions of Lighthill and Proudman. At high Reynolds numbers all simplified models of turbulence along with dimensional analysis suggest it is the eddies of scales close to the energy containing range which are responsible for the bulk of the sound generation. In a recent paper Zhou and Rubinstein (1995) consider the noise radiated from the turbulent inertial subrange and find that the temporal correlations derived by Lilley (1994) are consistant with the sweeping hypothesis of Kraichnan (1964), and Praskovsky *et al.* (1993), involving a nonlocal property of the energy containing eddies. Zhou *et al.* deduce that the noise power generated at high Reynolds numbers should have a spectral decay of $\omega^{-4/3}$. The current low Reynolds number database as shown in Fig. 1 suggests the decay law is of order ω^{-2} over a wide range of frequencies before falling exponentially in the dissipation range, although near the energy containing range the spectral decay does follow the $(\omega/\omega_m)^{-4/3}$ law. Zhou *et al.* also show, at high Reynolds numbers, the straining hypothesis would lead to a spectrum of radiated noise, in the inertial subrange of $(\omega/\omega_m)^{-7/2}$. If we compare these results with the output from the DNS data, noting an inertial subrange barely exists at these low Reynolds numbers, we find from Fig. 1 this law could only exist at much higher wavenumbers. However Zhou *et al.* show the assumptions made by Proudman (1952) lead to results for the acoustic power output consistant with the straining hypothesis, whereas the assumptions made by Lilley (1994) are more consistant with the sweeping hypothesis.

In addition Zhou *et al.* (1995) have examined a large databank of high Reynolds number atmospheric and windtunnel turbulence data at around the peak and higher wavenumbers to derive values of the incompressible fourth-order space time covariance and so find values for the Proudman constant using the formulas derived by Lilley (1994) and discussed above. Although this data is largely for anisotropic turbulence it is regarded as a useful guide to the Reynolds number dependence of the integral properties of isotropic turbulence which govern noise generation and its acoustic power. The calculated value of the Proudman constant obtained by Zhou *et al.* (1995) is within the range found by Lilley (1994), based on the databases described above, suggesting there is only a weak dependence on Reynolds number.

The contribution $p_s^{(2)}$ can be combined with $p_s^{(1)}$ and their combined contribution is similar to that when enthalpy fluctuations are absent. In the evaluation of $p_s^{(3)}$ we need the value of the fourth-order covariance $<(u_x h')_A (u_x h')_B>$. If we assume Gaussian statistics and impose Millionshtchikov's hypothesis, and noting that in isotropic turbulence $<u_x h'>$ is zero in incompressible flow,

$$<(u_x h')_A (u_x h')_B - <u_x h'>^2> \approx <(u_x)_A (u_x)_B><(h')_A (h')_B> . \tag{26}$$

On the assumption that the non-dimensional correlation function for the enthalpy fluctuations is equal to $f(r,\tau)$, then the acoustic power spectral density arising from $p_s^{(3)}$, is similar to that arising from $p_s^{(1)}$ and $p_s^{(2)}$. We find that

$$p_s^{(3)} = \frac{4\sqrt{2}}{\pi} \frac{\rho_0^2 u^6 S_T^4}{\rho_\infty c_\infty^3 L} \frac{<(h')^2>}{h_\infty^2} \frac{(\gamma_0 - 1)^2}{(\gamma_\infty - 1)^2}. \tag{27}$$

Our final values for the two terms in the contributions to the acoustic power output are

$$p_s = \alpha_P \frac{\rho_0^2}{\rho_\infty} \frac{u^8}{c_\infty^5 L} + \alpha_H \frac{\rho_0^2}{\rho_\infty} \frac{u^6}{c_\infty^3 L} \tag{28}$$

where

$$\alpha_P = \frac{4\sqrt{2}}{\pi} (T_1 - 1) S_T^4 \left(1 + \frac{3(\gamma_0 - 1)^2}{4}\right) \tag{29}$$

and

$$\alpha_H = \frac{4\sqrt{2}}{\pi}\left(\frac{\gamma_0 - 1}{\gamma_\infty - 1}\right)^2 S_T^4 \frac{< (h')^2 >}{h_\infty^2}. \tag{30}$$

We find that the term involving the enthalpy fluctuations generates acoustic power proportional to u^6 and hence dominates over the u^8 contribution at low Mach numbers. These results show that, typically, the dipole contribution equals the quadrupole contribution when $M_T = 0.28$.

2.2 The acoustic power from a heated jet

The physical process of noise generation in the mixing region of a jet is assumed similar to that in isotropic turbulence. However the turbulence is now anisotropic and inhomogeneous and is dependent on the mean rate of strain. Its Reynolds stress tensor contains both shear and normal stress components. Nevertheless with respect to the principal axes of stress only the direct stresses act. The sum of these enables us to find the local values of the average kinetic energy of the turbulence. The turbulence intensity is assumed proportional to the velocity difference across the shear layer. In the fully developed mixing region of a jet, the turbulent intensity depends on the velocity difference between the center-line velocity of the jet, which decays with downstream distance, and the external velocity. The integral scale of the turbulence is assumed to be proportional to the local width of the mixing region based on the vorticity thickness where the mean flow growth is governed by entrainment and the mean shear. The intense turbulence is found to exist near the center of the mixing region. The turbulence is intermittent, but a useful model is to assume the average properties of the turbulence are approximately uniform over the mean vorticity thickness of the jet and zero outside. The average convection speed, V_c, of the main energy-containing eddies in a turbulent mixing region over a wide range of different gases, velocities, and temperatures can be obtained from the work of Papamoshou and Roshko (1988). For the mixing region of an unheated jet near the nozzle exit, V_c is about $0.58V_j$. With these properties we may assume the turbulence is quasi-isotropic having a mean convection speed, V_c. In the model used here we have neglected the orientation of the principal axes of strain to the mean convection direction and its effect on the noise directivity.

The radiated noise to the far-field of a mixing region is estimated based on the hypothesis that the fourth order space-retarded time covariance has similar properties in shear flow turbulence as in isotropic turbulence, apart from changes in the scales of length and velocity. The local reference turbulent velocity, based on the local kinetic energy, and a local reference integral length scale, corresponding to the scale of the energy containing eddies, are defined at each section of the mixing region or jet. The spectrum of turbulence is assumed to be similar to that of isotropic turbulence but with the frequency of the peak energy, ω_m, proportional to the mean velocity gradient. The turbulent Strouhal number, S_T, in the case of the mixing region, is of order 1.7 when based on the values used for the peak frequency and the reference velocity and length scales.

In the jet mixing region it is assumed that since the turbulent Mach number is small we may neglect the effect of density fluctuations on the noise generated even in the case of the heated jet. We further assume the mean density to be a constant across the mixing region at any station downstream of the nozzle exit with a value based on the density at the position where the local mean velocity is equal to the mean convection speed. The mean flow is assumed to be self-preserving and the mean density, temperature, stagnation enthalpy, and velocity profiles are calculated throughout the flow using a simple eddy viscosity model. In this model the equations of momentum and total enthalpy are similar and hence the mean velocity is a linear function of the mean total enthalpy. The reference density, compatible with the convection speed, is then determined at each downstream station. The effects of turbulence convection can be applied using the Lighthill-Ffowcs William (1963) theory of convective amplification.

2.2.1 The noise power from heated jets and comparison with experimental data

The total acoustic power radiated from a circular heated jet can be evaluated from the results for isotropic turbulence with the modifications discussed above to allow for the effects of anisotropy, mean density variation, and convection. The present theory does not address the acoustic power from supersonic jets when shock waves are present and the 'mixing region' noise is augmented by shock-cell noise and 'screech'.

The contributions to the acoustic power are integrated over the complete volume of the flow. A large number of flow parameters must however be specified. These include the jet exit Mach number and temperature, the flight Mach number, and the corresponding convection Mach number. Also required is the corresponding mean jet exit density and enthalpy ratios, the length of the potential core, the growth of the jet in the initial mixing region and far downstream, the mean turbulent intensity and its law of decay, and the ratio of the integral turbulence scale to the local jet width. All these parameters are functions of the jet exit Mach number and the ratio of the jet to flight Mach numbers. For the hot jet we require the mean square of the enthalpy fluctuations. Due to the near linear increase in the turbulent integral length scales in the jet mixing region with downstream distance, we find the dominant frequency of the noise generation decreases inversely proportional to distance from the nozzle exit. Thus the radiated noise spectrum reflects more the peak energy contributions in the local noise spectra than the contributions from all frequencies in the local spectra. A consequence is the radiated noise spectrum of a jet increases as ω^2 before the peak frequency and then falls as ω^{-2}. The proof of this simplification in the pattern of the noise generation from a jet rests in the detailed comparison between the far-field noise polar correlation measurements made by Fisher *et al.* (1977) on model and full-scale jet engines and the corresponding predictions made by Lilley (1991), using Lighthill's acoustic analogy with a jet noise model similar to that described above.

Comparison of the present results with experimental data is also shown in Figs. 2 and 3. The results show the correct trends for the heated jet at low Mach numbers and the changes in the acoustic power in the upper end of the subsonic jet Mach number range and at supersonic speeds for the fully expanded jet. In these figures A_j and V_j are respectively the jet exit area and speed. $M_j = V_j/c_\infty$ is the so-called

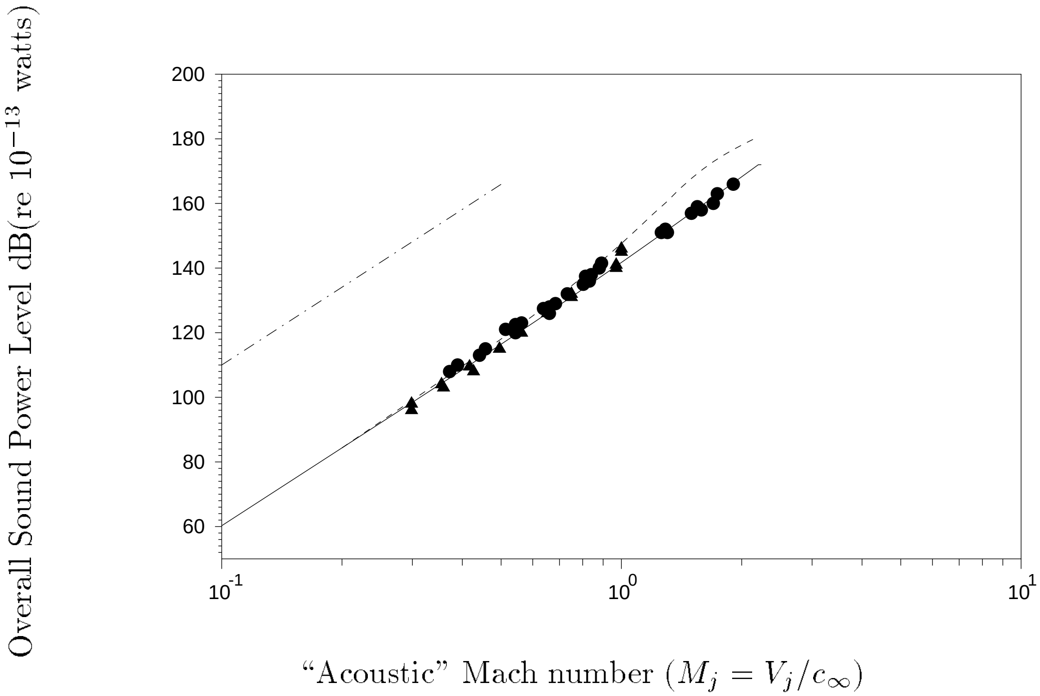

FIGURE 2. Overall noise power for cold jets. $A_j = 0.000507m^2$. ---- neglects refraction: —— Lighthill-Ffowcs Williams with refraction: —·— V_j^8: ▴ Lush (1971): • Olsen *et al.* (1973)

"acoustic" Mach number.

An easily observable influence of flow-acoustic interaction occurs at high frequencies, where the sound waves propagating through the flow at small angles to the flow direction are refracted by the flow, resulting in a near zone of silence in the high frequencies close to the jet boundary as shown in Fig. 4. The present results shown in Figs. 2 and 3 include the elementary effects of refraction. The theory of flow-acoustic interaction, which embraces the effects of refraction, is discussed in Goldstein (1978), and in the discussion on the detailed DNS calculations of Colonius *et al.* (1995) on the vortex pairing phenomenon in mixing layers. An important consequence of the phenomenon of flow-acoustic interaction is the result that the far-field observer "hears what is seen".

3. Future plans

The present paper concerns the noise power per unit volume from near incompressible isotropic turbulence based on the fourth-order space-retarded time covariance of T_{ij}. These results are extended analytically to the case of heated turbulence on the assumption that for turbulent Mach numbers, based on the root mean square value of the turbulent velocity and the ambient speed of sound, less than 0.3, the effects of density fluctuations in the turbulence on the noise generated can be neglected. A hypothesis is then introduced whereby the non-dimensional form of the isotropic fourth-order space-retarded time covariance of T_{ij} is used as an input to compute the noise power from a heated circular jet at subsonic and supersonic

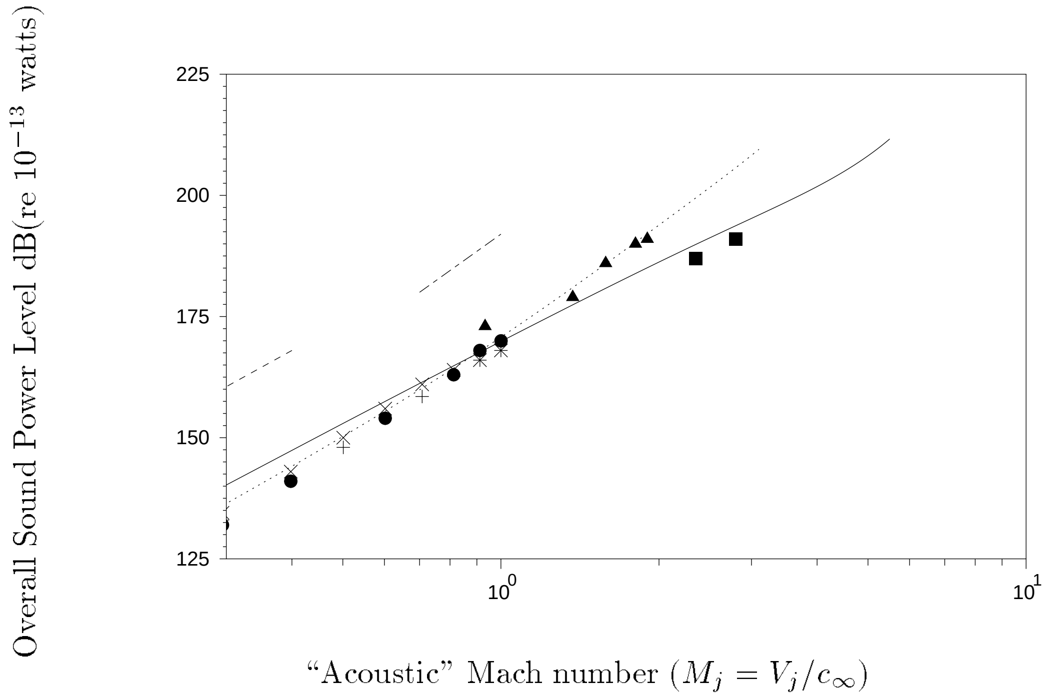

FIGURE 3. Overall noise power for hot jets of various enthalpy ratios h_{sjet}/h_∞. $A_j = 1.0m^2$.: Exps. Hoch *et al.* (1973): × 1.2: + 1.4: ● 1.7: Tanna (1977) ▲ 2.0: ■ 6.25: Theory ········ 2.0: ——— 6.25. - - - - V_j^6: — — — V_j^8

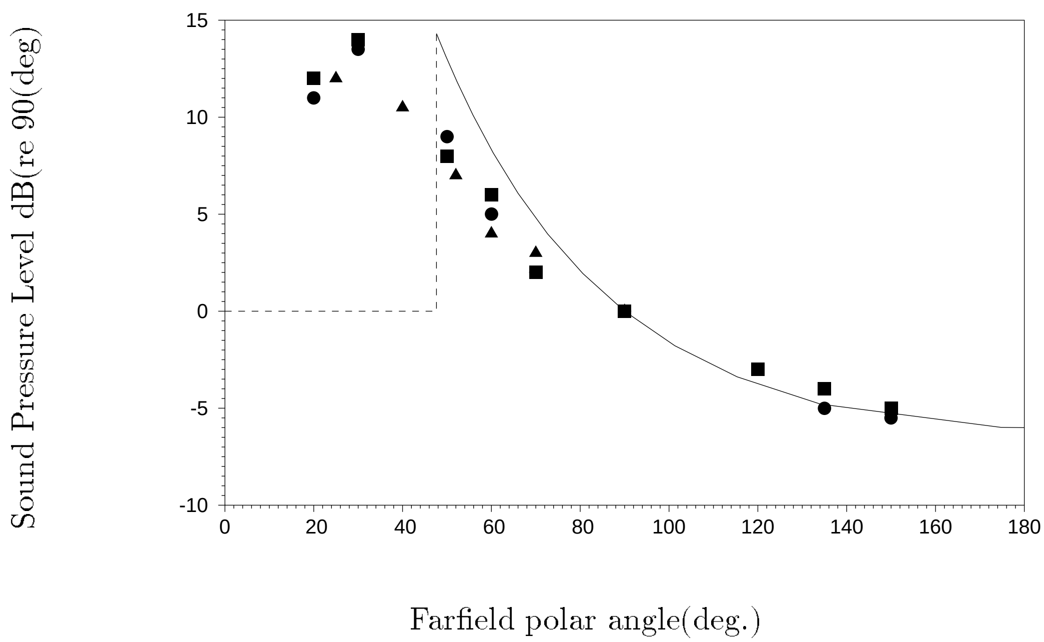

FIGURE 4. Directivity of jet noise. Exps. Lassiter & Hubbard (1952) ■ : Westley & Lilley ○ : Fitzpatrick & Lee △ : Lighthill-Ffowcs Williams with refraction ——— : Zone of silence - - - -

speeds, when the jet is fully expanded and no shocks are present. The results are compared with subsonic and supersonic noise measurements covering a wide range of Mach numbers and jet to ambient temperature ratios. Fair agreement is obtained, but of greater importance is the fact that the trends in noise power prediction for the heated jet based on Lighthill's theory, but including the effects of refraction, is verified by this comparison with experiment. Without the input from the DNS database, this work would not have been possible.

Future work should include new evaluations of the fourth- order space retarded time covariance of T_{ij} in heated isotropic turbulence, in compressible mixing regions, and in jets at subsonic and supersonic speeds. Current DNS and LES mixing region databases could be used as a start for these evaluations, but further work, using LES, is needed to generate the corresponding data for the jet. To find the changes with jet Mach number and temperature on the total volume and amplitude of the noise producing acoustic sources, it will be necessary to use two-equation RANS calculations of the compressible circular jet covering a wide range of velocity and temperature differences between the jet and the uniform external medium. There is also need to extend the present work on flow-acoustic interaction to include its effects at higher Reynolds numbers on the turbulent jet over a range of jet Mach numbers and temperatures.

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