Near-wall models in large eddy simulations of flow behind a backward-facing step

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1. Motivation & objectives

Accurate large eddy simulation (LES) of a wall-bounded flow generally requires a near-wall resolution comparable to that in direct numerical simulation (DNS). As much as 50% of the total grid points — and computational cost — are expended in the near-wall regions in a typical simulation. This limits LES to fairly low Reynolds numbers on current computers. To perform practical flow applications at realistically high Reynolds numbers, such as flow over an airfoil, it is desirable to replace very thin, near-wall regions in the LES with easily and inexpensively computed wall models to specify the near-wall boundary conditions.

This approach is expected to be feasible in simple flows with well developed boundary layers where local equilibrium conditions are expected to hold and known, empirical law-of-the-wall scalings can be exploited. Cabot (1995) found that LES of channel flow, with wall stresses predicted by either an instantaneous log law or by integration of simple boundary layer equations, produced accurate low-order statistics. Piomelli *et al.* (1989) used modified log law models for the wall stress in channel flow. Using the same models, Arnal & Friedrich (1993) obtained mean flow statistics in high Reynolds number flow over a backward-facing step in fair agreement with experimental measurements. Balaras *et al.* (1996) used simple boundary layer equations to successfully simulate channel flow and flow in a square duct.

In practical applications the flow is usually more complicated and can feature large adverse pressure gradients and extensive regions of separation, reattachment, and recovery. Such is the case in the flow over a backward-facing step. The objective of this work is to study the applicability of the simple near-wall models, similar to those used in channel flow, to the flow over a step, even though equilibrium conditions will not be valid in the reattachment region and turbulent models may be invalid in the separated region. If these wall models fail to give satisfactory results, different near-wall modeling schemes that can handle a wider range of flow conditions will need to be formulated and tested; the tests performed here may provide some insight into what physical ingredients these more general wall models need to incorporate.

2. Accomplishments

In this study of the flow over a backward-facing step, only the bottom wall behind the step was modeled, since this is where the most complex flow behavior occurs (Fig. 1). Differential boundary layer equations were used as the basis of the near-wall model in a thin "sublayer" region, in the hope that they would be flexible enough to treat local flow and pressure variations in a more accurate manner than

FIGURE 1. Sketch of the flow over a step (*not to scale*). The cross-hatched region along the lower wall behind the step is replaced by a near-wall model.

algebraic relations. Results from LES with wall models were compared with results from LES with resolved walls at moderately high Reynolds numbers by Akselvoll & Moin (1995), referred to hereafter as "the full LES". The results were also compared with experimental measurements for nearly the same flow configuration by Adams *et al.* (1984) and Vogel & Eaton (1985).

The flow has a Reynolds number Re_h (based on the inlet centerline velocity U_c and step height h) of 28,000, and the outlet-to-inlet expansion ratio is 1.25. The flow is separated from $x/h \approx 2$–7 (where x is the streamwise location past the step). Reattachment occurs in the mean at $x_r \approx 7h$, beyond which the flow recovers (almost) to a standard boundary layer by the time it reaches the outlet at $x \approx 20h$.

2.1 Near-wall momentum balance

A preliminary look at the complexity of the flow behind the step is provided by calculating the balance of terms in the streamwise momentum equation in the near-wall region. The time- and span-averaged flow field from the full LES is used for this purpose. The streamwise component of the Navier-Stokes equation is integrated from the wall to a height $y = 0.08h$ (corresponding to $y^+ \approx 60$ in wall units near the outlet). This gives

$$\underbrace{-\int_0^y (\partial\langle u^2\rangle/\partial x)\,dy}_{1} - \underbrace{\langle uv\rangle}_{2} - \underbrace{\int_0^y (\partial P/\partial x)\,dy}_{3} + \underbrace{(\nu\partial U/\partial y - \tau_w)}_{4} + \int_0^y (\nu\partial^2 U/\partial x^2)\,dy \approx 0\ , \tag{1}$$

where $\langle\cdots\rangle$ denotes the average, and where $U = \langle u\rangle$ and $P = \langle p\rangle$. The streamwise wall stress, $\tau_w = \nu\partial U/\partial y$ at $y = 0$, is the term that one is most interested in predicting from the wall model. The first term is dominated by the mean velocity

FIGURE 2. Streamwise momentum balance terms from Eqs. (1) and (2) in the region near the bottom wall behind a step from Akselvoll & Moin's (1995) $Re_h = 28000$ LES: —·— streamwise advection (term 1); ——— wall-normal advection (term 2); – – – streamwise pressure gradient (term 3); ━━━ viscous stress (term 4). Term 2 is also decomposed into ········ mean momentum flux (term 2a) and - - - - Reynolds stress (term 2b).

component ($\langle u^2 \rangle \approx U^2$) and the lattermost viscous term is completely negligible. Further, the wall-normal momentum flux $\langle uv \rangle$ can be decomposed into a mean part (UV, where $V = \langle v \rangle$) and a Reynolds stress part ($\langle u'v' \rangle$, which includes the contribution from the subgrid-scale model). This gives

$$-\int_0^y (\partial U^2/\partial x)\, dy - \underbrace{UV}_{2a} - \underbrace{\langle u'v' \rangle}_{2b} - \int_0^y (\partial P/\partial x)\, dy + (\nu \partial U/\partial y - \tau_w) \approx 0 \; . \qquad (2)$$

These terms are shown in Fig. 2. It is seen that the viscous stress roughly balances the Reynolds stress only far downstream in outlet region ($x/h \approx 20$), where the flow begins to resemble a well developed, zero-pressure-gradient boundary layer. In the separated region ($x/h \approx$ 2–7), however, the large advection and adverse pressure gradient terms are very important in the momentum balance. One might therefore suspect *ab initio* that wall models based simply on stress balance would not perform well in this flow.

The presence of relatively large Reynolds stresses in the separated region seems to contradict the common notion that this is an inherently two-dimensional, laminar roller. Although it is possible that the Reynolds stress here is not a measure of turbulence so much as a measure of spanwise oscillations and streamwise movement of the unsteady roller structure, it is evident from the LES results of Le & Moin (1993) that a large amount of turbulence is transported to the reattachment and separated regions from the overlying shear layer (also see the article by Parneix & Durbin in this volume).

2.2 Large eddy simulations with near-wall boundary layer equations

The boundary layer equations are derived from the Navier-Stokes equations under the assumption that, in the very thin wall region, the horizontal (x and z) length scales are much greater than the wall-normal (y) scales, and that y derivatives are much greater than x, z derivatives. For this reason viscous terms involving horizontal gradients are neglected, and the wall-normal pressure gradient is assumed to be negligible. These resulting equations for the horizontal velocity components, in which $\widetilde{\ }$ denotes a space and/or time filter, are

$$\frac{\partial \widetilde{u}_\ell}{\partial t} + \frac{\partial (\widetilde{u_\ell u_j})}{\partial x_j} = -\frac{\partial \widetilde{p}}{\partial x_\ell} + \nu \frac{\partial^2 \widetilde{u}_\ell}{\partial y^2} \,, \quad \ell = 1, 3 \,; \tag{3}$$

the wall-normal velocity component v is found from the continuity equation,

$$\widetilde{v} = -\int_0^y \left(\frac{\partial \widetilde{u}}{\partial x} + \frac{\partial \widetilde{w}}{\partial z} \right) dy \,. \tag{4}$$

The standard procedure is to integrate these equations on a fine wall-normal mesh with a fixed horizontal pressure gradient that is set at the base of the overlying interior flow. The boundary conditions for the horizontal velocity components are that they vanish at the wall and match the corresponding interior velocity components at the top of the sublayer. The matching horizontal velocity and pressure gradient from the interior flow are filtered to the same resolution as sublayer calculation. Note that there is no pressure solution required in the sublayer, which simplifies the calculation considerably. The wall-normal derivatives of $\widetilde{u}_\ell$ at the walls are computed from the sublayer solution to give the wall stress components, $\widetilde{\tau}_{w\ell} = \nu \partial \widetilde{u}_\ell / \partial y$, which are used as boundary conditions by the interior flow solution.

The term $\widetilde{u_\ell u_j}$ must be modeled, just as in the filtered equations for LES or Reynolds-averaged Navier-Stokes (RANS) equations. For the sublayer, near-wall eddy viscosity models based on the law-of-the-wall have been generally used, similar to RANS models (cf. Menter, 1991):

$$\widetilde{u_\ell v} \sim \widetilde{u}_\ell \widetilde{v} - \nu_t \frac{\partial \widetilde{u}_\ell}{\partial y} \,, \quad \nu_t = \kappa y u_s D^2 \,, \quad D = 1 - \exp(-y u_d / \nu A) \,, \tag{5}$$

where κ is the von Kármán constant (≈ 0.4), u_s is a velocity scale, and D is an *ad hoc* damping function for the viscous layer, where u_d is another velocity scale and $A = 19$ is a damping constant. In a standard law-of-the-wall model, the model velocity scales are just the friction velocity, $u_s = u_d = u_\tau \equiv (\widetilde{\tau}_{w1}^2 + \widetilde{\tau}_{w3}^2)^{1/4}$. In the Johnson-King model (Johnson & Coakley, 1990), u_s and u_d are different melds of u_τ and the square root of the maximum Reynolds stress; this model gives good results in flows with mild adverse pressure gradients and separation (Menter, 1991). No models are used for the horizontal momentum flux in the sublayer, viz.,

$$\frac{\partial \widetilde{u_\ell u}}{\partial x} \sim \frac{\partial \widetilde{u}_\ell \widetilde{u}}{\partial x} \,, \quad \frac{\partial \widetilde{u_\ell w}}{\partial z} \sim \frac{\partial \widetilde{u}_\ell \widetilde{w}}{\partial z} \,; \tag{6}$$

FIGURE 3. Wall-normal grid behind the step in the full LES and the LES with the wall model sublayer (cross-hatched region) embedded in the lower half of the wall cell.

adding the corresponding eddy viscosity models for these terms has not been found to give significantly different statistics. The boundary layer equations for the near-wall sublayer become

$$\frac{\partial \widetilde{u}_\ell}{\partial t} + \frac{\partial(\widetilde{u}_\ell \widetilde{u}_j)}{\partial x_j} + \frac{\partial \widetilde{p}}{\partial x_\ell} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \widetilde{u}_\ell}{\partial y}\right] , \quad \ell = 1, 3 . \tag{7}$$

Because the Reynolds stress model is based on RANS models, which assume an ensemble average (or an equivalently broad filter in space and/or time), the sublayer is computed on a horizontal mesh that is twice as coarse as the interior mesh; the matching velocities and pressure gradients are further filtered in time with a running time average, which is exponentially damped with an e-folding time of h/U_c.

Equation (7) is used all the way into the corner behind the step, although at some point the neglected streamwise gradients must become important. As will become evident, this improper treatment may lead to some aberrant flow behavior in the corner.

The LES used in this study is identical to the $Re = 28000$ backstep simulation of Akselvoll & Moin (1995), except for the removal of grid points from the interior LES domain along the bottom wall behind the step. The wall-normal stretching in the original grid behind the lower half of the step was also removed to prevent numerical instability. The original and modified wall-normal meshes are compared in Fig. 3. The LES uses a second-order central finite difference scheme on a staggered grid with a compact, third-order Runge-Kutta time advancement and fractional step method for the pressure solution (Le & Moin, 1991). The original (x, y, z) grid

TABLE 1. Wall model cases

case	$\nabla\cdot\widetilde{\boldsymbol{u}}\widetilde{\boldsymbol{u}}$	$(\nabla\widetilde{p})_m$	κ	u_s, u_d
LW0	no	no	0.4	u_τ
LW0p	no	yes	0.4	u_τ
LW1A	$(\text{yes})_m$	yes	0.4	u_τ
LW1	yes	yes	0.4	u_τ
DLW1	yes	yes	0–0.2*	u_τ
JK1	yes	yes	0.4	$\{u_\tau, u_m\}$

* "dynamic"

behind the step used a $146 \times 33 \times 96$ mesh on a $20h \times h \times 3h$ domain. The LES with the sublayer uses a $146 \times 23 \times 96$ interior mesh on the same domain; the sublayer itself uses a $74 \times 33 \times 48$ mesh between the wall up to the first off-wall nodes of the horizontal velocities at $y \approx 0.04h$.

A number of different variations of boundary layer Eqs. (7), with continuity (4) and eddy viscosity model (5), were used in the LES, a sample of which are presented here (summarized in Table 1). The simplest law-of-the-wall, stress-balance model, LW0, neglects advection and pressure gradient terms of the left-hand side of (7) altogether, and, with $u_s = u_d = u_\tau$ and $\kappa = 0.4$, gives a meld between the standard log law and viscous layer. Model LW0p includes only the pressure gradient fixed at the matching height, and LW1A included both advection and pressure gradient terms fixed at the matching height. Model LW1 includes all the terms on the left-hand side of (7), with the advection terms computed internally in the sublayer using the same integration scheme as the interior mesh. Model JK1 is the same as LW1 but uses the inner Johnson-King model (Menter, 1991) to calculate u_s and u_d; the maximal shear stress used to evaluate the velocity scales is found from the interior and sublayer flow field behind the step, which significantly increases the expense of the calculation. It was found that the JK1 model predicted eddy viscosity values 2–3 times larger than the law-of-the-wall model. Because some of the Reynolds stress is retained in the advection terms on the left-hand side of (7) on the sublayer mesh, any RANS model for eddy viscosity is likely to overpredict the amount of Reynolds stress in the sublayer; thus another model, DLW1, was used in which the coefficient κ was estimated "dynamically" by matching the Reynolds stress $(\widetilde{uv} - \widetilde{u}\,\widetilde{v})$ in the interior flow just above the sublayer to the law-of-the-wall model. As expected, this generally results in values of κ substantially lower than 0.4, as seen in Fig. 4.

The LES results show that the flow behind the step is much more sensitive to the wall model than was the case in channel flow, especially in the separated region. The steady-state wall stress predicted by the different models is shown in Fig. 5 and the pressure coefficient is shown in Fig. 6. Experimental results for a similar flow configuration are also shown for comparison. The LW0 model underpredicts the magnitude of the wall stress in the recovery region ($x/h \approx$ 7–15) as well as in the separated region ($x/h \approx$ 2–7) compared with the full LES results. Note, however, that the experimental wall stress is also about half the magnitude of the full LES in the separated region. Similar results have been found in other

FIGURE 4. The near-wall eddy viscosity coefficient κ (averaged in z and over a short time) determined by a fit to the stress above the sublayer (———) is used in the DLW1 case, and its standard RANS value, the von Kármán constant (········) is used in the LW1 case.

high Reynolds number experiments (Driver & Seegmiller, 1985); the LES results of Akselvoll & Moin (1995), in fact, resemble low Reynolds results (Le & Moin, 1993). The cause of this discrepancy is not known. But it obviously leads to confusion in attempting to gauge the performance of these wall model results. The JK1 model clearly overpredicts the wall stress everywhere due to its excessively large eddy viscosity. The LW1 case does a reasonable job in the recovery and outflow region, but overpredicts the wall stress in the separated region with excessively rapid backflow and low pressure (Fig. 6). There is little difference between DLW1 and LW1 cases, even though the eddy viscosity is more than halved (Fig. 4) in the former case; the magnitude of the wall stress is slightly less in the separated region, but the separated region spreads farther out from the step with reattachment moving from $x_r/h \approx 7.0$–7.2. In fact, when the eddy viscosity is set to zero, there is little change in the results from the DLW1 case, indicating that the large pressure gradient and advection terms dominate the balances in the sublayer equations in these cases.

The corner regions in LES using any of the wall models all compare poorly with the full LES (although the comparison is somewhat better with the experimental results). Instead of forming a well developed corner eddy, the flow rushes straight into the corner and up the step. Large fluctuations in pressure and velocities are often observed in the corner. It is likely that the haphazard treatment of the corner region in the near-wall model, both in terms of the governing equations and grid resolution, may lead to the ill behavior of the flow. Also unanswered at this time is

FIGURE 6. The pressure coefficient $C_p = 2(p_w - p_{\text{ref}})/U_c^2$ along the bottom wall behind the step for the full LES (———) and LES with wall models: cases – – – LW0, ——— LW1, ········ DLW1, and —·— JK1; compared with Adams *et al.*'s (1984) experimental data (∘ ∘ ∘).

FIGURE 7. Mean streamwise velocity profile in the middle of the separated region for the full LES (━━━) and LES with wall models: cases – – – LW0, ——— LW1, and —·— JK1; compared with Adams *et al.*'s (1984) experimental data (∘ ∘ ∘).

whether the corner eddy fails to form because of the excessive speed of the backflow into the corner, or if it is the absence of the corner eddy (due to deficiencies in the wall model implementation) that allows the strong backflow to develop in the first place.

Mean streamwise velocity profiles appear to be rather insensitive to the different wall stresses predicted by the models. In general, the agreement between LES with wall models and the full LES is quite good, which in turn are in good agreement with experiments (see Akselvoll & Moin, 1995). The largest difference occurs in the separated region, where the LW1 model gives a noticeably larger backflow at $x/h = 4.5$ than in the full LES (Fig. 7). (All of the LES results in fact show a substantially larger backflow than the experiment.) Surprisingly, the mean flows for LW0 and JK1, which show the greatest deviation from the full LES in terms of wall stress, have the best overall agreement in term of mean streamwise velocity above the wall layer, even in the separated region.

The mean reattachment point of $x_r/h \approx 7.0$ is found for most of the wall model cases presented so far, in fair agreement with the full LES and experiments, due to a proper cancellation of (or in the case of LW0, a fortuitous absence of) terms in the sublayer. The simulation with a wall model that neglected the advection terms but retained the pressure gradient in (7) (case LW0p in Table 1) gave $x_r/h \approx 8.5$, and another that fixed both the pressure gradient and advection terms from the overlying interior flow (case LW1A in Table 1) gave $x_r/h \approx 7.5$; the wall stress

FIGURE 8. The friction coefficient on the bottom wall behind the step for the full LES (━━━) and LES with wall models: cases — — — LW0, ········ LW0p, —·— LW1A, and ——— LW1; compared with Vogel & Eaton's (1985) experimental data (∘ ∘ ∘).

for these cases are shown in Fig. 8. These results suggests that the separation and reattachment regions are very sensitive to the near-wall balance between pressure gradients and advection terms in the sublayer.

3. Future plans

The cause of the sometimes poor results with the wall model behind the step needs to be ascertained. This is especially true of the separated and corner regions: Flows with boundary layer wall models develop overly rapid backflow (along with lower pressure) in the separated region than in the full LES, and they lack a well developed corner eddy. In broad terms, the culprit could be deficiencies in implementation of the wall model and/or in the equations representing the sublayer region. Many of these issues, outlined below, will be addressed in the future.

Implementation issues. Sensitivity to the sublayer mesh grid — in both horizontal and wall-normal directions — should be established. In the wall-normal direction, a strategy of overlapping the sublayer and interior zones is being explored, which will allow more flexibility in matching the two solutions. It may be proven to be more important to match things like the wall-normal momentum or stress than the streamwise velocity. The corner region requires special treatment, and it will need to be replaced with a different grid meshing and governing equations, or a different "corner" model (as yet undetermined). In principle, the wall model should be applied along all walls, including the corner, the face of the step, and around the step's edge. A consistent treatment of the whole "near-step" problem will therefore be explored. A realistic cost appraisal of these methods will also then be possible. Wall models may also be implemented in an alternative flow, the diffuser, which features separation and adverse pressure gradients without the severe step geometry; both LES and experimental data exist for this flow as well.

Modeling issues. Modifications need to be made to the governing equations for the near-wall sublayer since the boundary layer equations are known to be inappropriate for the reattachment region where wall-normal scales are comparable to horizontal scales. The eddy viscosity model for the unresolved sublayer stress, based on RANS models, needs improvement. It has been tuned for well developed turbulent boundary layers, but is inappropriate for the nonequilibrium conditions behind the step in the separated and reattachment regions where turbulence is largely transported from high shear layers above. Alternative models that better describe this situation need to be formulated and tested; they can perhaps be merged smoothly to the standard stress model depending on flow conditions. The ability of the outer flow to communicate to the sublayer which type of flow to model may need to be developed in terms of global criteria rather than simple, local diagnostics. The *ad hoc* damping function in the law-of-the-wall eddy viscosity model (5) needs to be replaced with a more physical near-wall condition, perhaps based on the wall-normal velocity. The eddy viscosity model of the Reynolds stress in the separated region will likely need to be augmented or replaced altogether.

The discrepancy between the full LES and experimental results in the flow over the backward-facing step at high Reynolds numbers, particularly in the separated region, makes comparisons with LES with wall models rather ambiguous. For testing models that are intended to handle the separated region correctly, it may be better to use lower Reynolds number DNS and LES results (Le & Moin, 1993; Akselvoll & Moin, 1995), which agree very well with experiments.

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