Progress on LES of flow past a circular cylinder

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1. Motivation and objectives

The objective of the present research is to assess the usefulness of large-eddy simulation (LES) methodology for flows in complex geometries. Flow past a circular cylinder has been calculated using a central-difference based solver, and the results have been compared to those obtained by a solver that employs higher-order upwind biased schemes (Beaudan & Moin, 1994). This comparison allows us to assess the suitability of these schemes for LES in complex geometry flows.

2. Accomplishments

2.1 Numerical method

The solver used in the current work is based on the method developed by Choi *et al.* (1992). Previous simulations (Mittal, 1995) had shown that due to the relatively low accuracy of the second-order central difference scheme, the flow in the near wake of the circular cylinder was under-resolved in the spanwise directions, and as a result of this the downstream development of the flow was not simulated accurately. In order to increase the resolution in the spanwise direction, a Fourier discretization method was introduced, and this was found to be a cost effective alternative to increasing the number of grid points. The spanwise velocity is collocated at the pressure node, and dealiasing is performed using the $(2/3)^{rd}$ rule in order to stabilize the computations. Other significant changes to the code include a new line-zebra scheme for the iterative pressure Poisson solver, which incorporates full coupling across the branch cut resulting in significant acceleration of convergence. When used in conjunction with a multigrid scheme, acceptable reduction in residual can be achieved with less than 10 iterations for each spanwise wavenumber.

2.2 Flow past a circular cylinder

A C-mesh is used for the present simulation. The inflow, outflow, and far field boundaries are located at $19\mathcal{D}$, $17\mathcal{D}$, and $25\mathcal{D}$ respectively ($\mathcal{D}$ is the cylinder diameter). Uniform freestream velocity is prescribed at the inflow and far field boundaries, and a convective boundary condition is employed at the outflow boundary in order to smoothly convect the disturbances out of the computational domain. Previous simulations were carried out on domains that only extended to about $10\mathcal{D}$ from the cylinder in the vertical direction, and this resulted in significant streamwise acceleration of the flow at the edge of the wake region. In the current simulation the far field boundary is extended to about $25\mathcal{D}$, and this reduces the confinement effect of

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the boundaries to an acceptable level. The spanwise domain size of $\pi\mathcal{D}$ is chosen, which was found to be adequate for this flow by Beaudan & Moin. The simulation has been carried out on a 401×120×48 mesh with 140 points on the cylinder surface, 129 streamwise points along the wake centerline, 120 points in the wall normal direction, and 48 points along the spanwise direction. A non-dimensional time step size ($U_\infty \Delta t/\mathcal{D}$) of about 0.007 was used, which corresponds to a maximum CFL number of about 1.5. The solver takes about 70 seconds per time step on the CRAY C-90, and simulating one shedding cycle requires about 13 CPU hours. All the statistics for the current simulations have been averaged over about 12 shedding cycles. Furthermore, all of the results of Beaudan & Moin used here are from the simulation that employed the 5^{th}-order upwind biased scheme.

Some of the key wall and near wake statistics are summarized in Table 1 where $\overline{C_{p_b}}$, $\overline{C_D}$, $\overline{\theta_s}$, and St are the mean base pressure coefficient, mean drag coefficient, mean separation angle, and Strouhal number respectively. It can be seen that the wall statistics obtained from the current simulation are in good agreement with experiments and with the simulations of Beaudan & Moin (1994). Since the drag and base pressure coefficients depend strongly on the accurate prediction of near wake features like vortex rollup and formation of streamwise vortical structures, good prediction of these quantities implies that the development and evolution of the vortical structures in the near wake is being simulated reasonably accurately.

	Central Diff.	Upwind Biased	Experiments
$\overline{\mathbf{C_{P_b}}}$	−0.93	−0.95	−0.9±0.05⋆
$\overline{\mathbf{C_D}}$	1.0	1.0	0.98±0.05⋆
$\overline{\theta_\mathbf{s}}$	86.9^o	85.8^o	$85^o \pm 2^o$†
St	0.207	0.203	0.215±0.005‡
mean bubble length	$1.4\mathcal{D}$	$1.36\mathcal{D}$	$1.33\mathcal{D} \pm 0.3\mathcal{D}$‡

Table 1. Wall Statistics. ⋆ Norberg, 1987; † Son and Hanratty, 1969; ‡ Cardell, 1993.

Figure 1a shows the mean streamwise velocity profiles at four different streamwise stations in the near wake. In this and subsequent figures, profiles obtained from simulations of Beaudan & Moin (1994) and experiments are also plotted whenever available. Furthermore, the profiles have been suitably shifted along the $y-$axis to fit multiple profiles in one plot. It is found that the streamwise velocity profiles obtained from the current simulations are in reasonable agreement with the experiments. Furthermore, the profiles are found to match closely with Beaudan & Moin (1994). Figure 1b shows the mean vertical velocity profiles at three streamwise locations in the recovery region. It is found that the agreement with the experiments at the first two locations is in general not good, and both simulations over-predict the peak vertical velocity. However, there is a good match with the profiles of of Beaudan & Moin (1994). Also at $x/\mathcal{D} = 3.0$, both simulations match the experimental

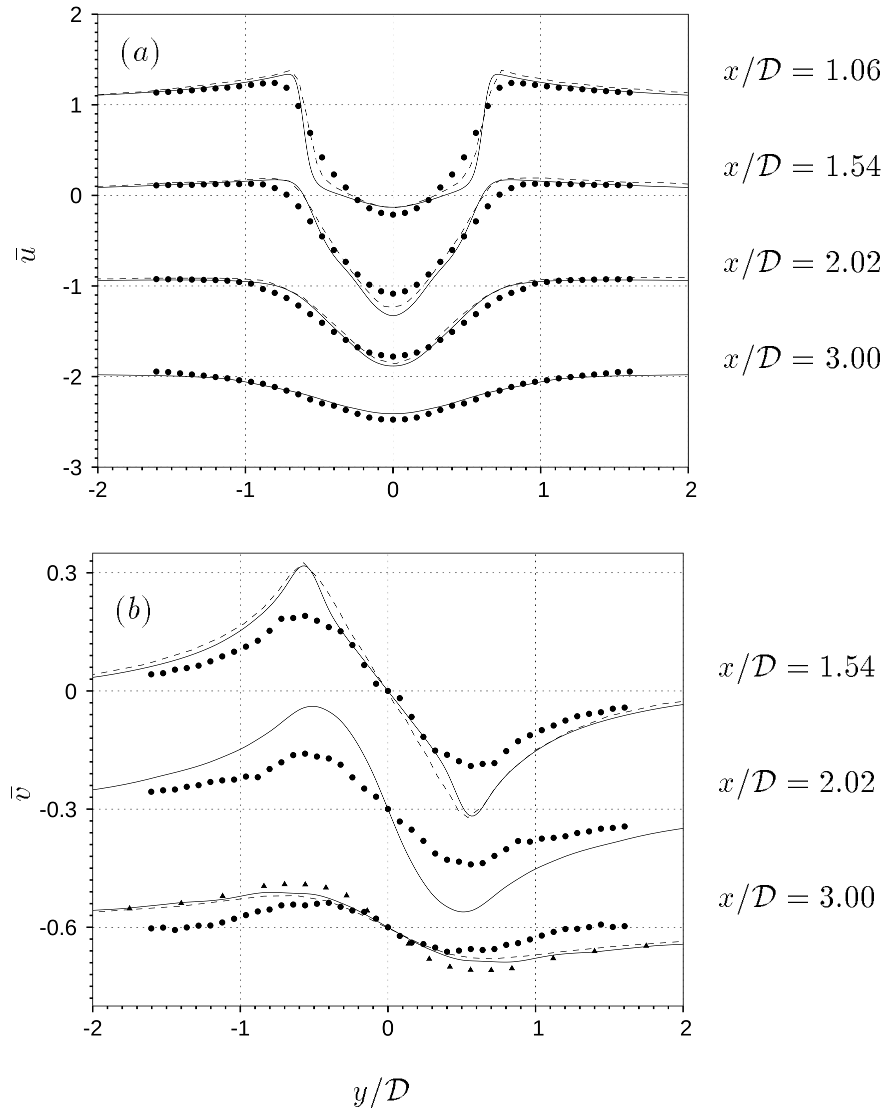

FIGURE 1. Mean velocity profiles in the near wake region. —— Current simulation, ---- Beaudan & Moin, • Lourenco & Shih, ▲ Ong & Wallace. (a) Streamwise velocity (b) Vertical velocity.

profiles of Ong & Wallace (1993) quite well. Beaudan & Moin (1994) noted significant symmetry errors in the vertical velocity profiles of Lourenco & Shih (1993), and this could possibly account for the disagreement between the simulations and their experiment.

From the above comparison of mean velocity profiles, we find that the current simulation produces results which compare reasonably well in the near wake region with the experiments and with the simulations of Beaudan & Moin (1994). The bubble length seems to be better predicted by the simulations of Beaudan & Moin (1994). This is probably due to the fact that for the current simulation, the streamwise grid spacing in this region is roughly the same as that used by Beaudan & Moin (1992), which results in a relatively lower resolution due to the lower order method used here. However, the mean velocity profiles from the two simulations are in reasonable agreement with each other, and we therefore expect that differences in the downstream evolution of the flow in the two simulations will be solely due to differences in the in-plane resolution, and thus comparison of the statistics in the downstream portion of the wake should allow us to compare the performance of the different schemes.

A comparison shows that the streamwise grid spacing in the current simulation is $20-30\%$ better between $x/\mathcal{D}=4$ and 7 than the simulations of Beaudan & Moin. However, at $x/\mathcal{D}=10.0$ both simulations have roughly the same streamwise grid spacing. The difference in the streamwise grid spacing cannot be avoided since the grid cannot be stretched in the streamwise direction as fast in the central difference simulation as was done in the upwind-biased simulations (Mittal, 1995). It should be pointed out that a comparison of the modified wavenumber (Beaudan & Moin, 1994) for the schemes suggests that roughly twice the number of grid points are needed for a second-order central difference scheme to match the resolution of these higher-order upwind biased schemes at low-wavenumbers. Thus, the smaller grid spacing is required in the current simulations in order to adequately resolve the energy containing scales in the wake.

In Fig. 2 we have plotted the one-dimensional frequency spectra, E_{11}, at three locations in the downstream region of the wake. Spectra from both simulations and experiment (Ong & Wallace, 1996) are plotted together for comparison. The streamwise grid spacing limits the highest frequency that can be locally resolved in the simulation, and this corresponds to the implicit "grid-filter". The vertical lines in the plots indicate the grid cutoffs for the two simulations. The experimental spectra shows about half a decade of inertial range extending from about $\omega/\omega_{st}=2$ to 7. Figure 2a clearly shows that the spectra from the current simulation matches the experimental spectra much better than the simulation of Beaudan & Moin (1994). A closer look at the spectra at the three locations obtained from the upwind-biased simulation of Beaudan & Moin (1994) shows that only the energy in the lower 20-25% of the resolved wavenumbers matches with the experiment. On the other hand, in the current simulation the damping at the higher wavenumbers is not as severe, and spectra in the lower 40-50% of the resolved wavenumber range matches well with the experiment. The marginal performance of the upwind-biased schemes in the downstream wake region was attributed to the dominance of numerical dissipation. Thus, given the fact that the spectra for the current simulation shows better agreement with the experiment than Beaudan & Moin, it is reasonable to expect that the turbulence statistics obtained from current simulation will also be better predicted in the current simulation.

Figure 3 shows velocity profiles at three selected locations in this region. We observe that the streamwise and vertical velocity profiles obtained from both simulations agree reasonably well with the experiment. Figure 4 shows the Reynolds stress profiles at these locations. The comparison in Fig. 4a indicates enhanced level of streamwise normal stress at the first two streamwise locations. The simulation of Beaudan & Moin (1994) predicts the peak streamwise normal stress at $x/\mathcal{D}=4.0$

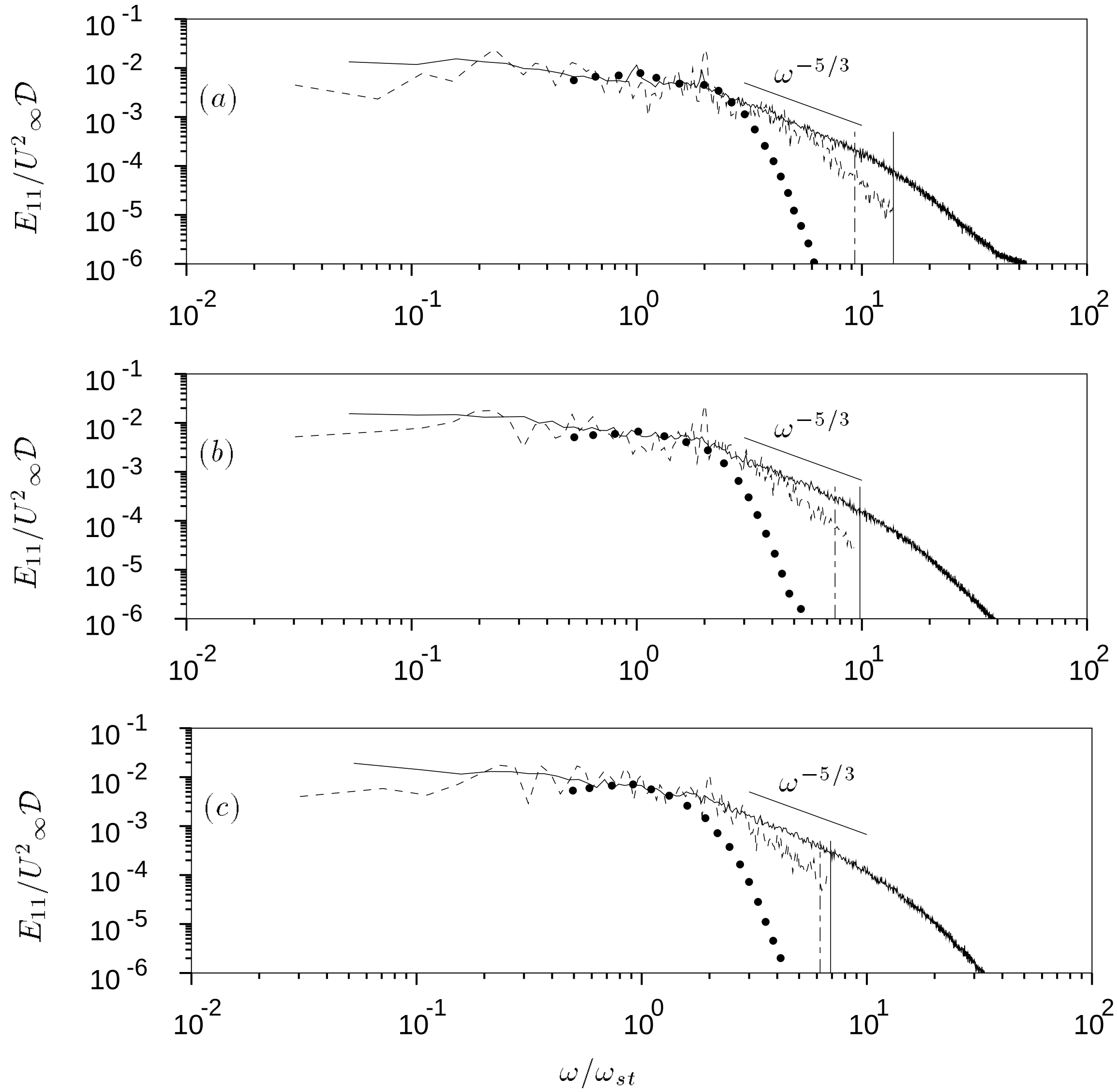

FIGURE 2. One dimensional spectra E_{11} along the wake centerline. ——— Ong & Wallace, ---- Current simulation, • Beaudan & Moin. Grid cutoffs are shown by vertical lines: ——— Current simulation, —–— Beaudan & Moin. (a) $x/\mathcal{D} = 5.00$ (b)$x/\mathcal{D} = 7.00$ (c) $x/\mathcal{D} = 10.00$.

quite well and the current simulation over-predicts the peak streamwise normal stress. At $x/\mathcal{D} = 7.0$ the simulation of Beaudan & Moin (1994) under-predicts the peak streamwise normal stress significantly, whereas the current simulation shows better agreement in both the magnitude of the peak stress and shape of the stress profile. At $x/\mathcal{D} = 10.0$ streamwise stress profiles from both the simulations match quite well and both under-predict the experimental stress level significantly. Since streamwise Reynolds stress at the wake centerline is directly related to the area underneath the curves shown in Fig. 4, it is somewhat surprising that the current simulation does not predict a streamwise stress level which is significantly higher

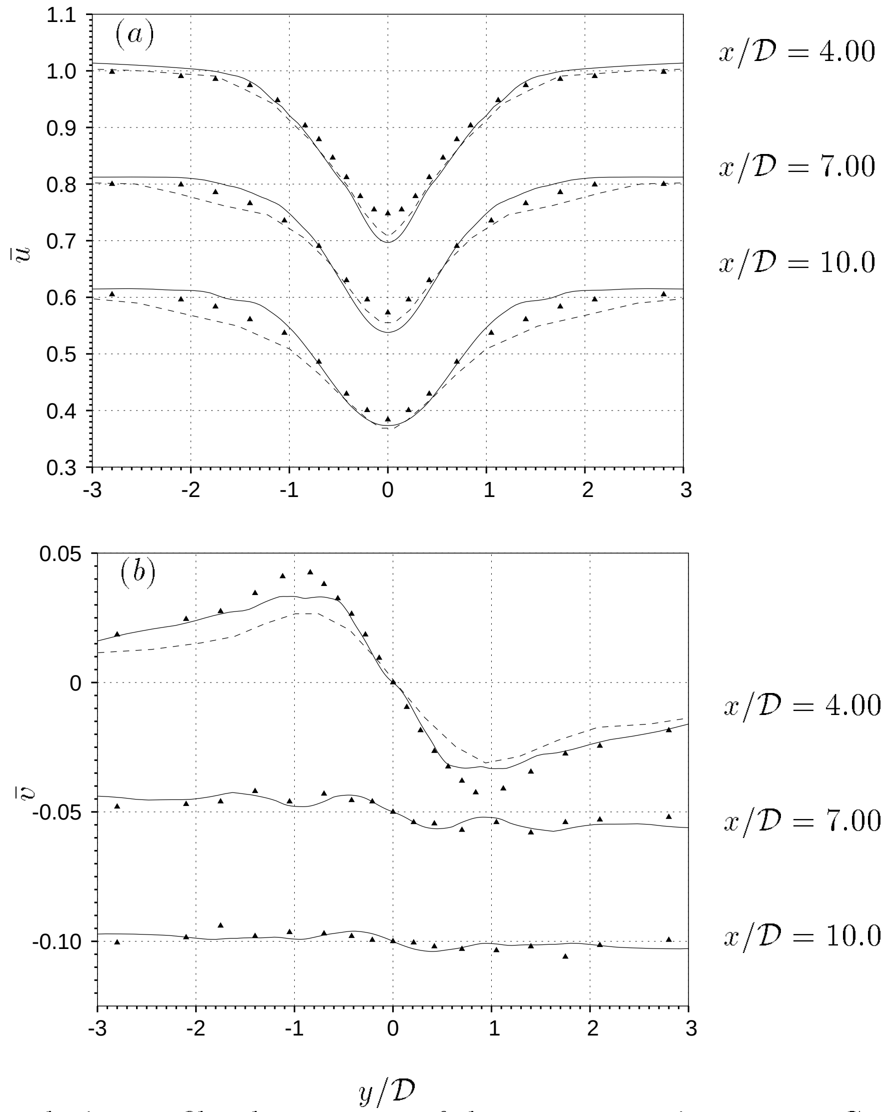

FIGURE 3. Mean velocity profiles downstream of the recovery region. ——— Current simulation, ---- Beaudan & Moin, ▲ Ong & Wallace. (a) Mean streamwise velocity (b) Mean vertical velocity.

than the simulation of Beaudan & Moin. However, this can be explained by noting that for this flow most of the contribution to the Reynolds stress come from fluctuations in a narrow frequency band extending from about $0.5\omega_{st}$ to $3.0\omega_{st}$, and in this frequency band the energy in both the simulations is comparable. Thus, even though the simulations of Beaudan & Moin exhibit significant damping of the higher frequencies, this does not have a significant impact on the low-order turbulence statistics.

Figure 4b shows the vertical normal stress profiles at these three locations. Again, a slightly enhanced level of stress is observed at $x/\mathcal{D} = 4.0$; however, overall the predictions from the two simulations at the first two locations are quite similar. At $x/\mathcal{D} = 10.0$ the two simulations predict roughly the same peak stress level; however,

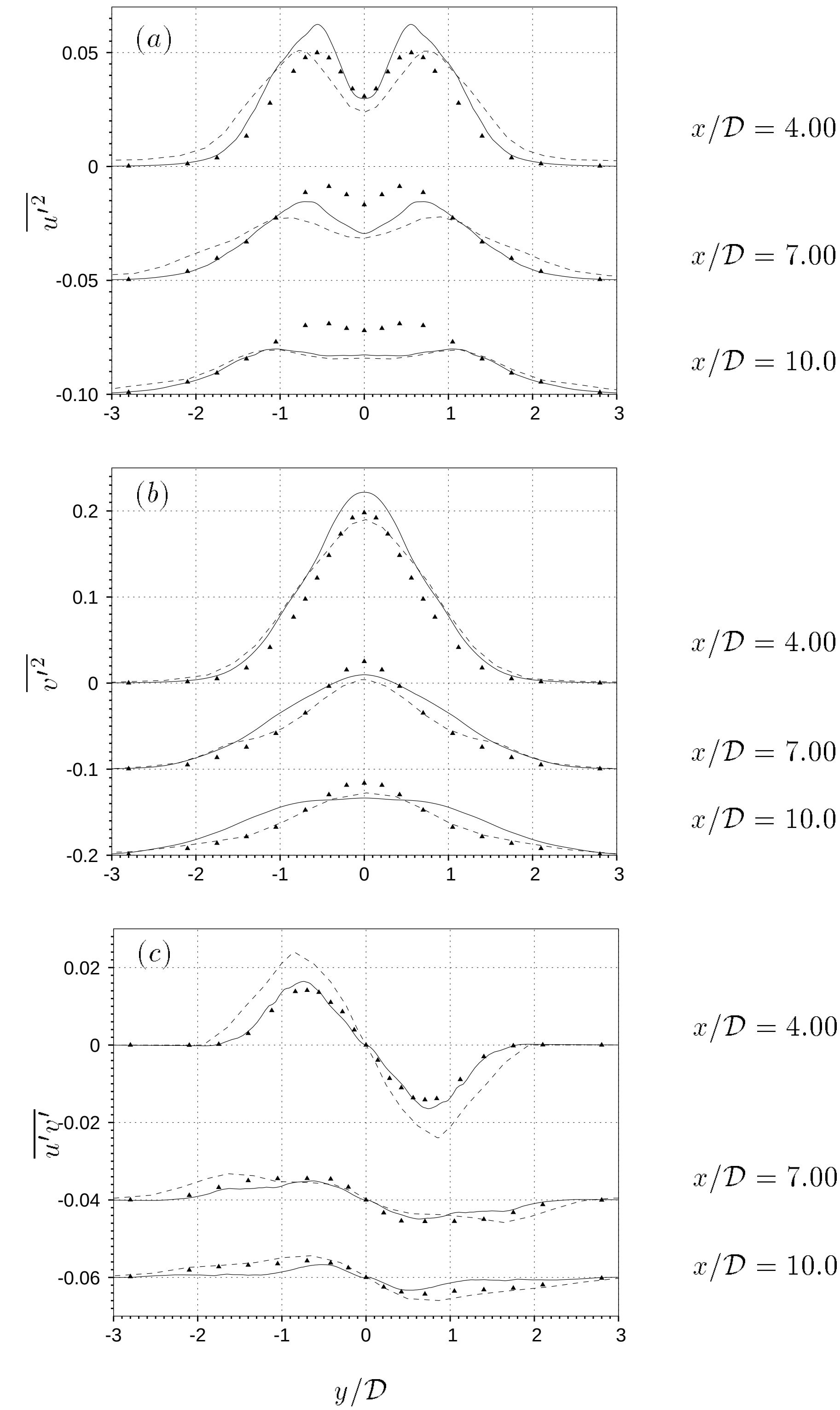

FIGURE 4. Reynolds stress profiles downstream of the recovery region. ——— Current simulation, - - - - Beaudan & Moin, ▲ Ong & Wallace. (a) Streamwise Reynolds normal stress (b) Vertical Reynolds normal stress. (c) Reynolds shear stress.

the shape of the experimental profile matches the profile of Beaudan & Moin (1994) better than it does for the current simulation. Figure 4c shows the Reynolds shear stress profiles at these three locations. It is observed that the current simulation shows better agreement with the experiments than Beaudan & Moin (1994) at $x/\mathcal{D} = 4.0$. At the two other downstream locations, predictions from both the simulation are comparable and in reasonable agreement with the experiments.

Thus, it is found that in the downstream portion of the wake where the grid is relatively coarse, the numerical dissipation inherent in the higher-order upwind-biased scheme removes energy from roughly three-quarters of the resolved wavenumber range. In the central-difference simulation, since there is no numerical dissipation, the smaller scales are more energetic, and we find that the spectra agrees well with the experiment up to about half of the resolved wavenumber range. However, the enhanced energy in the small scales has no significant effect on the low order statistics, and mean velocity and Reynolds stress profiles in this region obtained from the two simulations are comparable. This is due to the fact that most of the stress contribution comes from fluctuations whose frequency is centered in a narrow band around the shedding frequency, and change in the energy of the small scales does have any significant effect on the magnitude of the Reynolds stresses. It should be pointed out that in applications such as flow generated noise and reactive flows, small scales play a crucial role, and it is therefore critical to retain the energy in these scales. In such applications energy conservative schemes would be clearly preferable over upwind schemes.

In addition, we find that with about a 20-30% smaller grid spacing, the second-order central difference scheme gives results that are comparable to those obtained by the high-order upwind biased schemes. The higher-order upwind based solver is more expensive on a per-point basis than the second-order central difference solver, and this partially offsets the additional cost of the increased resolution required by the second-order method. A drawback of the second-order central scheme is that the simulations are sensitive to numerical aspects such as grid discontinuities and outflow boundary conditions, and thus grids and boundary conditions have to be designed with extreme care.

In future work we plan to use kinetic energy conserving central difference schemes whenever possible. Work is continuing on the development of higher order central difference schemes with the expectation that these will allow for accurate simulation results on meshes coarser than currently required for the second order scheme.

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