A new methodology for turbulence modelers using DNS database analysis

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1. Motivation and objectives

Many industrial applications in such fields as aeronautical, mechanical, thermal, and environmental engineering involve complex turbulent flows containing global separations and subsequent reattachment zones. Accurate prediction of this phenomena is very important because separations influence the whole fluid flow and may have an even bigger impact on surface heat transfer. In particular, reattaching flows are known to be responsible for large local variations of the local wall heat transfer coefficient as well as modifying the overall heat transfer. For incompressible, non-buoyant situations, the fluid mechanics have to be accurately predicted in order to have a good resolution of the temperature field.

Much previous work on such phenomena has used the case of the backward facing step. The sudden expansion of a channel flow indeed gives rise to a simple geometry, but one that contains complex aerodynamic features such as separation, a spreading shear layer, interaction of this shear layer with a wall, recirculation, and recovery of the reattached boundary layer. Moreover, a lot of databases are available from physical experiments to numerical simulations (DNS and LES); this makes for a good test case on which to validate turbulence models.

In past years, various two-equation turbulence models have been tested and compared with the available experiments. The principal feature of models using wall functions is an underprediction of the recirculation length by about 15%. In fact, recent simulations or experiments show an important departure of the velocity profiles from the law of the wall, not only in the recirculation, but also in the recovery region. A bad prediction of aerodynamics near the wall would have dramatic effects on predictions of heat transfer. The low-Reynolds number cases show strong anisotropies, not only in the boundary layer, but also in the spreading shear layer and in the recirculation; any linear two-equation model would fail to reproduce this important feature of the flow. For this reason, and with the aim of future studies of flows involving buoyancy, curvature, or rotation, we decided to use a Second Moment Closure (SMC) approach, which intrinsically can take into account these phenomena.

The closure of the Reynolds-stress equations consists partially in finding a model for the pressure-strain correlation which acts as a redistribution term between the Reynolds-stress components. Since the Launder, Reece and Rodi, 1975 (LRR) paper, a number of models have been proposed especially to take into account specific wall-behavior. However, all these models use a single-point closure and cannot actually represent the well-known non-local effects of the pressure-reflection that occurs near solid boundaries. In order to model these latter effects and to avoid the use
of two-point correlations—which are not viable for non-homogeneous turbulence—Durbin (1993) applied an elliptic operator on the pressure-strain correlations. This approach has been tested on various simple flows and with successful results. Moreover, it does not need the use of either wall functions, whose universality is more and more questionable, or damping functions, which often involve the "ill-behaved" distance to the wall and which can be highly non-linear and numerically stiff.

The aim of this study is to study this modeling methodology by using a SMC-model calibrated in fully-turbulent channel flow to compute the low-Reynolds backward-facing step for which a complete Direct Numerical Simulation (DNS) database is now available (Le, Moin & Kim, 1993). We will see the performance and some discrepancies of the model.

The present report also introduces a new \textit{a priori} test: it consists in freezing some variables for which DNS statistical fields are used and solving differential equations for the others. We think that this technique will be very helpful to turbulence modelers because it is usually difficult to analyze the solution of the whole system of highly coupled equations. Since all the variables are non-linearly coupled, it is difficult, and maybe impossible, to find where problems come from by just looking at the solution of the full computation. At the same time, simple algebraic substitution of DNS data into formulas for pressure-strain, followed by comparison to the pressure-strain data, gives no information on the mathematical and computational properties of the turbulence model. The present approach is better suited to testing analytical closures.

We will also present some ideas for improving the general model.

2. Accomplishments

2.1 Results of the basic model

The flow was computed using a modified version of INS2D, a finite difference code in generalized coordinates written at NASA Ames Research Center. The modifications involve subroutines to solve the turbulence model and the incorporation of Reynolds stress-gradientes into the mean flow solver.

A non-uniform grid of 120 \times 120 cells, refined near the walls and around the corner of the step, was used to cover the region \(x/h = -3\) to 35, \(x = 0\) being the location of the sudden expansion and \(h\) being the step height. Our solution has been checked for grid-independence; a twice finer grid in both directions gave indistinguishable results. Inlet values for the mean velocities, Reynolds stresses, and dissipation were taken from the DNS database. The elliptic relaxation procedure of Durbin (1993) has been combined with the Speziale, Sarkar, Gatski (SSG 1991) pressure strain model in the ‘neutral’ formulation of Laurence \textit{et al.} (1995). The resulting equations have been calibrated using DNS and experimental data for both channel flow and zero pressure gradient boundary layers. That model was directly applied to the backstep flow without further modification. All the equations and constants used for this computation can be found in the appendix of Parneix \textit{et al.} (1996).
Fig. 1 shows the predicted streamlines compared to the DNS data. One can observe that the reattachment length is very well predicted and a secondary bubble is found. The flow seems to have a correct behavior near the reattachment point: it does not show the anomalous streamline pattern that has been found in other computations (Hanjalic 1996). However, the size of this corner bubble is much smaller in the computation than in the data. This defect seems to be linked to an underprediction of the maximum friction coefficient in the backflow (cf Fig. 2a). By looking to the profiles of the streamwise mean velocity (cf Fig. 2b), one can observe that the overall features of the mean flow is reproduced, but the intensity of the backflow is missed by a factor of 2, which could be problematic for the prediction of near-wall heat transfer. Note that this specific problem seems to be common to every existing Second Moment Closure model, whatever near-wall model is used (low-Reynolds model or wall function). Note also that, as with all existing turbulence models including eddy viscosity models, the recovery after reattachment is too slow. The friction coefficient distribution shows similarities (too slow near-wall flow) both upstream and downstream the reattachment point, but we don’t know yet if these two problems (too slow backflow and too slow recovery) are linked.

Figures 3 to 5 present profiles of the computed normal component of the mean velocity $V$ and statistics (Reynolds stresses $u_iu_j$, turbulent kinetic energy $k$, and dissipation of turbulence $\varepsilon$) compared to the DNS data. The $V$-profiles at $x/h = 0$
and 2 confirm the distribution of the friction coefficient: the intensity of the two bubbles is severely underestimated. By looking at the profiles before reattachment, one can observe that $V$ is also underestimated in the shear layer by about 15%. In fact, all the problems can be linked together by noting that more entrainment in the shear layer would lead to a more intense main recirculation (and maybe also a more rapid recovery), leading to a bigger and more intense secondary bubble. However, a modification of the backflow would also change the pressure distribution and thereby influence the velocity distribution in the shear layer.

**Figure 3.** (a) $V$ profiles (b) $u^2$ profiles (legends similar to Fig. 2b).

**Figure 4.** (a) $v^2$ profiles (b) $-uv$ profiles (legends similar to Fig. 2b).

The statistics seem to be reasonably well represented by the model. However, $u^2$ shows a too slow recovery: the peak near the wall has not yet appeared at $x/h = 10$. Moreover, the model overestimates the gradients $\partial k/\partial y$ and $\partial u^2/\partial y$ at $x/h = 2$. The level of turbulence is too high at this important location, but $k$ and $u^2$ do not enter directly into the momentum equations and should have a secondary
effect. Indeed, the main active terms in the turbulent force are $(-\partial \overline{uv} / \partial y)$ for the $U$-equation and $(-\partial \overline{v^2} / \partial y)$ for the $V$-equation (cf. Figs. 10 and 11). Figure 4b shows an almost perfect distribution of $\overline{uv}$, although a slight underestimation of the gradient $(-\partial \overline{uv} / \partial y)$ exists in the shear layer. The primary fault is that $\overline{v^2}$ is underestimated everywhere, especially in the shear layer where $\partial \overline{v^2} / \partial y$ is badly predicted. In fact, if one looks at the results of the following a priori tests (cf. Figs. 10 and 11), the conclusion will be exactly in the opposite direction (i.e. the problem comes from the $uv$-equation instead of the $\overline{v^2}$-equation). This is because, in the full computation, the gradients of mean flow are badly estimated in some regions, which alters the production term of $uv$ and $\overline{v^2}$. So, without the method presented below for decoupling the model solution from the mean flow solutions, the problem cannot be tracked down.

Finally, the dissipation, $\varepsilon$, is very well simulated except in the near-wall region where one can notice an important underestimation, which is consistent with the fault found in the prediction of $\partial k / \partial y$. Nevertheless, similar comments may be made for $\varepsilon$: a bad production term alters the overall analysis; the present a priori method is needed to truly assess the model differential equations.

With all the equations coupled, it is difficult, maybe impossible, to know where the problem comes from by just looking to the solution of the full computation. The analysis of the full solution might suggest that one has to improve both the $\overline{v^2}$ and $\varepsilon$ equations. However, the following study will prove that these equations behave sufficiently well, and that the $uv$-equation is the main problem. It should be pointed out that every other low-Reynolds SMC model or SMC computation with the logarithmic law of the wall underpredicts the backflow intensity. In order to obtain a precise understanding of this and to go further in the study of the DNS database, we generated a new technology for carefully testing each equation of the model.

2.2 A new a priori technique for analyzing and improving turbulence models

The standard way of analyzing a DNS database consists in using the full DNS

![Figure 5](image-url). (a) $k$ profiles (b) $\varepsilon$ profiles (legends similar to Fig. 2b).
data to compute the distribution of some important variables like the turbulent Reynolds number $Re_t$, in order to understand the main physical features of the flow and to get some new ideas for modeling. Figure 6, showing $Re_t$ and the budget of the U-momentum equation in the middle of the recirculation ($x/h = 4$), presents an example of such a study. In their experiment, Jovic and Driver (1995) found that the minimum of $C_f$ follows a ‘laminar-like’ law: $C_f = -0.19 (Re_h^{-0.5})$ for Reynolds numbers between 5,000 and 50,000. However, $Re_t$, which represents about ten times the ratio between the turbulent viscosity and the molecular viscosity (in the $k-\varepsilon$ model), is in the range $200 - 800$ in the whole domain, including the bubble (of course, it goes down to 0 at the wall). If one is still not sure of the completely turbulent feature of this recirculation, it becomes obvious by looking at the U-momentum budget: at station $x/h = 4$, the Reynolds stress gradients predominate the viscous force in the whole bubble except very near the wall, far below the maximum of the reverse flow.

![Figure 6](image)

**Figure 6.** Direct analysis of the DNS database (a) $Re_t$ profiles at locations: $x/h = 2$ (○); 4 (■); 6 (∆); 8 (∆); and 10 (×). (b) U-momentum budget at station $x/h = 4$ (middle of the recirculation): ○: convection; ■: turbulent force; ◆: viscous force; △: pressure force.

It is possible to go further in the investigation of a DNS database. Rodi & Mansour (1993) directly used DNS data for testing turbulence models and looking for improvements. The idea was to introduce the DNS ‘exact’ data into the modeled equations and to analyze the differences with the corresponding ‘exact’ DNS terms. In particular, they showed that it was possible to find some efficient damping functions for improving the behavior of a $k-\varepsilon$ model near the wall. One important problem with such a technique is that, even if a ‘perfect’ equation is found for every term of the global budgets (which means that the modeled equation fits perfectly with the DNS data), the general convergence of the global system has not been included in the study, and the resulting model can be numerically unstable. Moreover, such terms as dissipation or transport of Reynolds stresses are not well enough resolved, even by recent DNS, for an accurate and complete analysis term by term.
In order to avoid these problems, a new set of \textit{a priori} tests has been generated. These consist in solving the full differential equations of each individual variable one by one, while the others are taken directly from the DNS database. This kind of computation allows a finer analysis of the true effects of terms like pressure-strain or transport models; it also permits more confidence in the numerical stability of eventual improved model terms. Moreover, an overall comparison between the full simulation and \textit{a priori} tests explains why an analysis using only the full computation may entail erroneous conclusions. For example, the results presented in the previous section could lead one to believe that the model has some problems in both $v^2$ and $\varepsilon$ equations ($\varepsilon$ is often regarded as the weakest point of turbulence models), whereas the \textit{a priori} tests show that discrepancies are mainly located in the $uv$-equation. To our knowledge, this idea was first applied by Hanjalic (1994) for 1-D, fully developed channel flow. Although he did not discuss the method, his Fig. 13 was computed using the Reynolds stresses $\overline{u_iu_j}$ and the mean streamwise velocity $U$ from the DNS of a channel flow, solving the $\varepsilon$-equation. Hanjalic then tested different models. His tests were not extended to the other variables. The present report presents the first 2-D use of this technique. We have used the RANS code to obtain the solutions by this \textit{a priori} technique.

A first \textit{a priori} test might concern the mean flow. As soon as the Reynolds stresses are interpolated from the DNS and are assumed to be exact, there is no longer any modelization in the momentum equations. A full RANS solution of the momentum and continuity equations would give ‘perfect’ mean flow profiles. Unfortunately, the DNS have been conducted with a too short channel (20 step heights downstream of the step) to allow our steady-state computations to converge. In fact, it is well known that, for the kind of outlet boundary conditions formerly used (constant pressure), a channel length of at least 30 step heights is needed to avoid reflections from the outlet boundary, making the computation unstable. However, the mean momentum equation demands that this particular computation will reproduce the DNS mean flow, so there is nothing to be learned from it.

In order to describe in detail the methodology we used to test turbulence models, we will focus first on the $k$-equation:

$$\frac{\partial k}{\partial t} + U \cdot \nabla k = P - \varepsilon + \nabla \cdot (\nu \nabla k) + D_T$$

(1)

$P$ is the rate of production of turbulent kinetic energy: $P = -\overline{u_iu_j} \partial_j U_i$, $D_T$ is the transport term: $D_T = -\partial_k (\kappa u_k)$. All the other variables (i.e. $U$, $V$, $\varepsilon$, and $\overline{u_iu_j}$) are fixed by DNS data in the present test. Thus, $(P - \varepsilon)$ is a fixed source term for the $k$-equation. The solution of (1) basically will evaluate the efficiency of the model for the transport term $D_T$ (the only term which needs to be modeled). Two models are currently used in SMC computations:

- the Daly-Harlow model (1970) with $0.20 \leq C_{D_H} \leq 0.25$:

$$D_T^{DH} = \frac{\partial}{\partial x_k} (C_{D_H}^T \overline{u_k u_i} \frac{\partial k}{\partial x_i})$$

(2)
- the Hanjalic-Launer model (1972) with $C_{\mu}^{HL} = 0.11$:

$$D_{T}^{HL} = \frac{\partial}{\partial x_k}(C_{\mu}^{HL}Tu_k u_l \frac{\partial k}{\partial x_l}) + \frac{\partial}{\partial x_k}(C_{\mu}^{HL}Tu_l u_l \frac{\partial u_k}{\partial x_l})$$  \hspace{1cm} (3)

$T$ being the time scale.

**Figure 7.** A priori test of the k-equation. (a) $k$-budget at $x/h = 4$. •: production; ×: dissipation; □: convection; △: transport; ◆: diffusion; ······: a priori computation. (b) k-profiles. Legend similar to Fig. 2b; ······: DH model ($C_{\mu}^{DH} = 0.2$); ······: DH model ($C_{\mu}^{DH} = 0.25$).

In the full computation presented in the previous section, we used the DH model with $C_{\mu}^{DH} = 0.20$. We tested this model by the new a priori technique. The budgets (Fig. 7a) look perfect, but this does not mean that the model is accurate because $(P - \varepsilon)$ is fixed; convection is quite small, so at convergence any transport model would balance all the other terms and fall close to the DNS profiles. However, the resulting $k$-profiles, shown in Fig. 7b, could come out wrong. The figure emphasizes the effectiveness of the transport model. Surprisingly, the behavior of the DH model is excellent in this case (backstep at low Reynolds number), an increase of $C_{\mu}^{DH}$ up to 0.25 giving even better results. In fact, the only discrepancy that has been found concerns the secondary recirculation ($x/h = 0$ and 2) where a severe overprediction has been obtained. This is not improved by modifying the $C_{\mu}^{DH}$ constant. The more elaborated HL-type model improves the behavior in this region but performs poorly after reattachment (see profiles at $x/h = 6$ to 10, Fig. 9a).

Another interesting a priori test that has been performed concerns the dissipation $\varepsilon$, which is usually considered as the weak point of any turbulence model. In fact, the generation of the $\varepsilon$-equation relies mainly on intuition, so most of the shortcomings of turbulence models were thought to be in this equation — and most of the modifications of turbulence models were done to it. In this study, we used the ‘primitive’ equation, derived by Hanjalic & Launder (1972) with only two small modifications: $\varepsilon/k$ has been replaced by the inverse of the time scale $1/T$ (Durbin 1993), and an extra-production term was included into $C_{\varepsilon_1}$ for taking into account
the wall effect (Parnéix et al. 1996). We fixed all the other variables (i.e. $U$, $V$, $k$, $\bar{u}_i\bar{u}_j$) and solved this equation. Surprisingly, the results are very good, especially in the recirculation and recovery regions (Fig. 8). In these conditions, it is difficult to believe that only a modification of the $\varepsilon$-equation will cure all the problems, and especially the underprediction of the backflow and the recovery. However, one could argue that the reattachment length is sensitive to the difference $(C_{\varepsilon_2} - C_{\varepsilon_1})$, but this strong dependency seems to lie in the shear layer where the dependence of the growth of the shear layer on $(C_{\varepsilon_2} - C_{\varepsilon_1})$ is well known. Concerning the behavior near the wall, the equation presented above seems to be more than sufficient for this case.

Since we now think that $\varepsilon$ is no longer responsible for the backflow and recovery discrepancy, we come back to other modeled terms; first to the transport model. We saw that both DH and HL models perform very well in the shear layer but overpredict the turbulence in the near-wall region, which could affect either backflow or recovery. In these conditions, we tried to improve these predictions by generating a new model. One has then to revisit the third-moment equation:

$$
D_i \bar{u}_i \bar{u}_j \bar{u}_k = - \bar{u}_i \bar{u}_j \bar{u}_k \partial_i U_k - \bar{u}_i \bar{u}_k \partial_j U_i - \bar{u}_k \bar{u}_j \partial_i U_k \\
+ \bar{u}_i \bar{u}_j \partial_k U_i + \bar{u}_j \bar{u}_k \partial_i U_j + \bar{u}_i \bar{u}_k \partial_j U_i \\
- \partial_k \bar{u}_i \bar{u}_j \bar{u}_k \bar{u}_l \\
+ \Pi_{ijk} + D_{ijk} - \varepsilon_{ijk}
$$

The three last terms (pressure-deformation, viscous diffusion, and dissipation) are usually grouped together in a relaxation term, suggested by the corresponding Rotta approximation for the pressure-deformation in the second-moment equations:

$$
\Pi_{ijk} + D_{ijk} - \varepsilon_{ijk} = - \frac{1}{C_\varepsilon T} \bar{u}_i \bar{u}_j \bar{u}_k
$$

**Figure 8.** A priori test of the $\varepsilon$-equation (legend similar to Fig. 2b).
By recourse to a Gaussianity assumption, the quadruple correlations are approximated in terms of second order correlation:

\[ u_i u_j u_k u_l = u_i u_j u_k u_l + u_i u_k u_j u_l + u_i u_l u_j u_k \]  \hspace{1cm} (6)

By neglecting the convection and production terms, Hanjalic & Launder came up with their algebraic expression of triple correlations (see above). Daly & Harlow only retained the terms involving \( \partial \bar{u}_i \bar{u}_j \) and approximated the other terms by modifying the constant. Note that their expression of triple moments itself is not invariant under permutation of the three indices, but the remaining transport model (cf. above) preserves the symmetry on two indices. If one no longer neglects the production terms (note that these latter show the proper slope at the wall, contrary to the DH and HL models), a linear system is then obtained that can be solved analytically. Unfortunately, the derived expression of triple correlations in terms of second moment and gradients of mean now becomes messy and unfeasible for practical applications. When only \( \partial_y U \) is retained (it has been checked to be the most important mean gradient everywhere in the domain), the correction \( D_{T}^{cor} \) (in addition to the HL model) in the \( k \)-equation is still too messy, but one main term can be emphasized:

\[ D_{T}^{cor} = \partial_x ((C_s T)^2 [\partial_y U] S_{i2}) + \cdots \]  \hspace{1cm} (7)

with \( S_{i2} = \partial \bar{u}_i \partial \bar{u}_i \partial \bar{u}_i \partial \bar{u}_i \). By doing the same operation as Daly & Harlow (keeping the terms involving \( \partial \bar{u}_i \bar{u}_j \), i.e., \( \partial \bar{u}_i \bar{u}_j \) for the \( k \)-equation), we derived the following expression for the correction of the DH transport model:

\[ (D_T)_{ij} = (D_T^{DH})_{ij} + \frac{\partial}{\partial x_k} \left\{ \alpha (C_s T)^2 \frac{\partial U_k}{\partial x_n} \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_l} \right\} \]  \hspace{1cm} (8)

which becomes in the \( k \)-equation:

\[ D_T = D_T^{DH} + \frac{\partial}{\partial x_k} \left\{ \alpha (C_s T)^2 \frac{\partial U_k}{\partial x_n} \frac{\partial k}{\partial x_l} \right\} \]  \hspace{1cm} (9)

\( \alpha = 0 \) leads to the classical DH model. Figure 9 presents the results we obtained with \( \alpha = -0.8 \) and \( C_s = 0.25 \). An important improvement in the prediction of \( k \) in the small bubble region \((0 \leq x/h \leq 2)\) can be seen without significant modification of the profiles in the rest of the domain (see the difference between the classical DH model in Fig. 7b and the modified DH model in Fig. 9a). Unfortunately, when this new model was implemented in a computation, only 10% of the discrepancy concerning the friction coefficient minimum was found to have been cured (Fig. 9b). In fact, the flow field is really only improved around \( x/h = 2 \), where the improvement was also seen to be greatest during the \( a \) \( \text{priori} \) test. The backflow and the recovery do not show any specific modification.

In conclusion, these results lead us to think that both \( k \) and \( \varepsilon \) equations seem to represent quite well the physics of the problem (at least in this case!) and that the
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Figure 9. A new transport model. (a) A priori $k$-profiles: legends similar to Fig. 2b for symbols; — : HL model; —— : modified DH model. (b) Friction coefficient: legends similar to Fig. 2a; —— : full computation with modified DH model.

backflow and recovery problems should come from other equations of the model, i.e. the Reynolds-stress equations.

We repeated the same kind of a priori tests for each $\bar{u}_i \bar{u}_j$-equation, coupled to its associated elliptic operator for the pressure-strain correlation. We have already explained the interest of doing such tests: indeed the $\bar{u}\bar{v}$-profiles obtained through the full computation seem to be almost perfect whereas the a priori test (with $\bar{u}^2$, $\bar{v}^2$, $U$, $V$, $k$, and $\varepsilon$ fixed to their DNS value) shows an overprediction of $-\bar{u}\bar{v}$ almost everywhere. We tried to be even more pragmatic by computing those a priori turbulent stresses that will directly effect the mean flow. So, $\bar{u}^2$ (resp. $\bar{u}\bar{v}$ and $\bar{v}^2$) have been solved giving the a priori turbulent force $-\partial \bar{u}^2 / \partial x$ (resp. $-\partial \bar{u}\bar{v} / \partial x$, $-\partial \bar{u}\bar{v} / \partial y$ and $-\partial \bar{v}^2 / \partial y$).

Figure 11. A priori turbulent force acting in the $V$-momentum equation (legends similar to Fig. 10). (a) resolution of the $\bar{u}\bar{v}$-equation with $\bar{u}^2$, $\bar{v}^2$, $k$, $\varepsilon$, $U$ and $V$ fixed, (b) resolution of the $\bar{v}^2$-equation with $\bar{u}^2$, $\bar{u}\bar{v}$, $k$, $\varepsilon$, $U$ and $V$ fixed.
We kept the same scaling in each figure (10a, 10b, 11a, 11b) in order to be able to compare the effective action of each stress on the momentum budget. At every location (except maybe near the corner of the step), the streamwise gradients are negligible. The turbulent force mainly results in $-\partial \overline{u^2}/\partial x$ in the $U$-equation and $-\partial \overline{v^2}/\partial y$ in the $V$-equation. One conclusion is that $\overline{u^2}$ has a secondary effect on the mean flow. By looking at the normal gradients ($\partial /\partial y$), one can notice a peculiar behavior at the corner of the step. The DNS trends are not at all reproduced in this region but this deficiency seems to stay local and to have little influence on the rest of the domain; the kink we can see at location $x/h = 0.1$ has not been transported further ($x/h \geq 2$). Nevertheless, a more careful study should definitely be done in the corner area.

With the exception of this problem, $\overline{v^2}$ is accurately predicted (cf. Fig. 11b), the only discrepancy can be found near the wall beyond the reattachment point at $x/h = 4$ and 6 (slight underprediction of the gradient), but this should not affect directly the mean flow because, at this location, the flow is nearly parallel to the wall ($V$ is basically equal to 0). However, locations $x/h = 4$, 6, and 8 of Fig. 10b show an overprediction of the turbulent force acting in the $U$-momentum by a factor of 2 around the reattachment point in the region where lies most of the backflow ($y/h \leq 0.15$). In this area, $U$ is negative and the turbulent force (which is one of the main terms in the $U$-momentum budget, see Fig. 6b) acts to slow down the flow, thus the overprediction by a factor of 2 should explain the severe underprediction of backflow we obtained with the full SMC computation. Let us note that the model seems also to be deficient in the recovery region (overprediction of $-\overline{\partial \overline{u^2}}/\partial y$) but this defect appears in a thinner region; moreover, $U$ is positive here and the turbulent stress is then acting as a positive force. In these conditions, a decrease of $-\partial \overline{u^2}/\partial y$ should slow down the flow in a region where the flow itself has already
been too slow. The origin of the recovery problem still has to be found.

3. Future plans

A full second moment closure computation has been carried out for evaluating the turbulent flow over a backward-facing step at low Reynolds number \((Re = 5,100)\). The model, including elliptic relaxation of pressure-strain to take into account the non-local effects of pressure near to walls, has been calibrated solely with channel flow and zero pressure gradient turbulent boundary layer data at various Reynolds numbers; it has been directly applied to the backstep without any modification. The results show a very good prediction of the recirculation length but an underprediction of the backflow by a factor of 2. The recovery has been seen also to be too slow.

An analysis of the corresponding DNS database proved that the main bubble is definitely turbulent even at this low Reynolds number. A new technique of a priori testing with DNS data has been developed; it consists, basically, in evaluating the accuracy of each equation of the model by solving one variable while fixing all the others to their DNS values. It came out that both Daly-Harlow transport model for \(k\), and the \(\varepsilon\)-equation do surprisingly well, contrary to what is generally thought in the literature.

Regarding the Reynolds stresses, \(\overline{u_i u_j}\), \(\overline{u^2}\) has a secondary effect on the mean flow because the streamwise gradients occurring in the turbulent force \((-\partial \overline{u^2}/\partial x\) and \(-\partial \overline{u\nu}/\partial x\) are negligible compared to the normal gradients \((-\partial \overline{u\nu}/\partial y\) and \(-\partial \overline{v^2}/\partial y\). In fact, the main problem seems to come from the \(\overline{u\nu}\)-equation, which gives an overprediction of the turbulent force by a factor of 2 in the backflow region.

In the future, we would like to focus on the \(\overline{u\nu}\)-equation in order to find the model term that is deficient and to propose an improved model.

Improving the model with the help of this new testing technique, and getting the right distribution of friction coefficient in the backflow region, will best prove the efficiency of this novel method for turbulence modelers.

In the meantime, we plan to apply the idea of elliptic relaxation on other configurations, including both 2D and 3D geometries. Moreover, at this time, the global numerical stability of Re-stress modeling is still an issue, and some improvements in this domain are still needed.

REFERENCES


