Wall models in large eddy simulation of separated flow

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1. Motivation and objectives

The desire to perform large eddy simulation (LES) of wall-bounded turbulent flows at relatively high Reynolds numbers is typically confounded by the severe resolution requirements near the walls. The structure of the turbulent flow in a boundary layer can become very fine in the near-wall region, scaling as the distance from the wall, and the numerical grid required to resolve it, therefore increases dramatically with Reynolds number. In channel flow LES at moderate Reynolds number, for instance, about half of the grid points must be dedicated to the near-wall flow when it is resolved on a stretched mesh, and the time step is severely reduced by the CFL condition for the fine near-wall scales. The situation becomes even worse at higher Reynolds numbers. The finer near-wall scales also require the subgrid-scale (SGS) model in the LES to describe a larger share of the Reynolds stress than in the core of the flow; this may lead to substantial inaccuracies when standard SGS models based on isotropic models, like the popular Smagorinsky model, are employed in the near-wall region.

To perform LES of high Reynolds numbers, wall-bounded turbulent flow, one needs to remove the requirement of resolving the near-wall region by (a) simulating only the core region of the flow with approximate boundary conditions applied on the boundaries, or (b) simulating the entire domain, including the walls, with the near-wall forces appropriately modeled. Approach (a) has much in common with domain decomposition methods (see Baggett in this volume). Approach (b) has been employed by Deardorff (1970), Schumann (1975), Grötzbach (1987), Piomelli *et al.* (1989), and others (see reviews by Piomelli *et al.*, 1989; Bagwell *et al.*, 1993), who supplied boundary conditions for the flow components tangential to the walls in a channel based on the logarithmic law of the wall. Balaras *et al.* (1996) and Cabot (1995, 1996) also employed thin boundary layer equations to predict wall stress boundary conditions in attached channel and duct flow and in separated flow behind a step. While this strategy works adequately in predicting accurate mean flow statistics in attached flow, it fares more poorly in separating, reattaching, and recovering flow, in part because the assumptions used in modeling the wall (near-wall equilibrium conditions that give rise to the log law, or thin boundary layer approximations) break down.

The broad objective of this work is to develop a procedure, or set of procedures, for modeling the near-wall region in LES such that the numerical grids can be chosen independent of Reynolds number Re, based instead on the outer scales (determined, e.g., boundary layer thickness and flow geometry) or on the core turbulent integral length scales, which remain finite as $Re \to \infty$. This procedure should be general

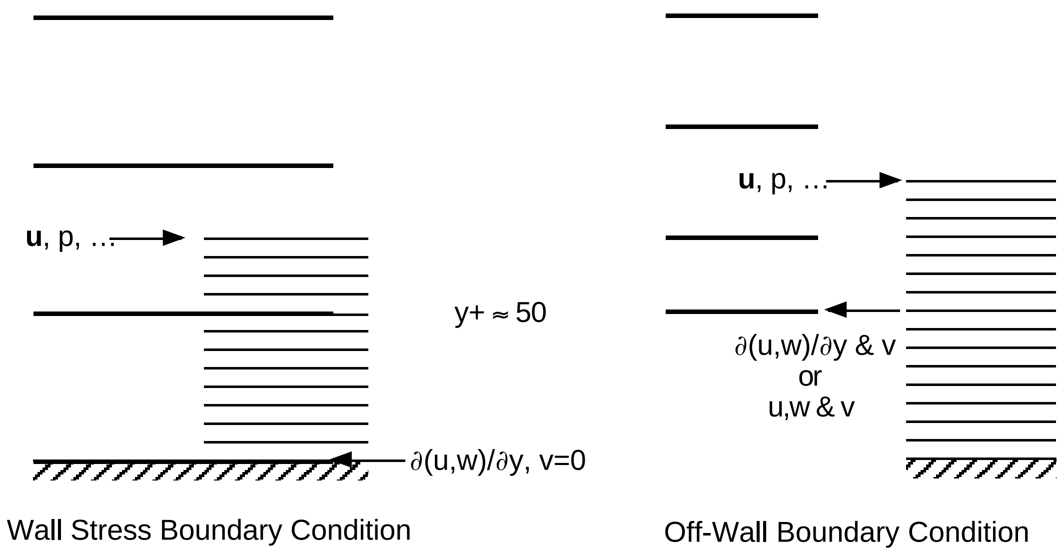

FIGURE 1. Wall stress boundary conditions are applied at the physical wall location, while off-wall boundary conditions are applied at a location away from the wall. Heavy lines represent the core flow's mesh, while the fine lines represent a fine near-wall grid which may be used by the wall model to generate boundary conditions (algebraic relations may also be used). The wall model extracts flow information from the core flow at an interior point and returns boundary conditions at the core flow's boundary. The distance between arrows illustrates the amount of overlap between the matching conditions.

enough to give accurate LES results in both attached and separated flow cases, and it should be substantially cheaper to use than LES with well resolved walls. There are several (interrelated) issues that need to be addressed on the way to developing successful wall modeling procedures. First, one needs to decide how accurate the LES results for the core flow need to be to deem the procedure a success. This is clearly subjective and depends on the tolerance of the particular flow problem, but at minimum it would be desirable to be able to predict mean flow speeds or mass flow to a few percent and other first-order wall quantities such as skin friction and pressure coefficient. The other physical and numerical issues concern making accurate (enough) wall models, patching together the near-wall and core flow solutions, and constructing consistent SGS models in the wall regions of flows:

(1) Is it more advantageous to apply wall boundary conditions (a) off of the wall, completely removing the wall from the LES, or (b) at the wall, keeping the wall in the LES? The difference between these types of boundary conditions is illustrated in Fig. 1. Numerical issues about gridding and stability arise in this case as well as the accuracy of the wall model used to supply the boundary conditions.

(2) What physical quantities need to be specified in the boundary conditions for

the core flow, and how accurate do they need to be? Velocity or velocity-gradient boundary conditions will be considered here since the velocity is the primitive variable in the simulation codes.

(3) What is the best near-wall model, or set of models, in terms of accuracy and cost, needed to describe a wide variety of physical flow conditions? Some form of the law of the wall may apply for attached boundary layers, but more general models are required, e.g., for separated flow. Thin boundary layer equations are inappropriate near flow separation and reattachment, and the cost of computing them is also generally Reynolds number dependent, which will become prohibitively expensive in many practical flows.

(4) What physical information from the core flow is needed for a particular wall model to specify accurate wall stress or off-wall boundary conditions? The location in the core flow where information is extracted can be a issue for stability and accuracy if it is too close to the numerical boundary and leads to spurious feedback effects.

(5) What modifications are needed in the SGS model of residual Reynolds stresses in the near-wall regions? The standard Smagorinsky SGS model, even when used with the dynamic procedure (Germano *et al.*, 1991), may predict inaccurate Reynolds stresses in the near-wall region, especially on very coarse meshes that effectively filter over large wall-normal variations in the flow. At what resolution does the SGS model give reliable results, and in turn, how close to the wall can one get with the LES? And how does one perform filtering near walls on coarse meshes, or alternatively, how does one model the implicit effects of this filtering properly?

The immediate goal of recent work (also see Baggett and Jiménez & Baggett in this volume) is to provide answers primarily to issues involving the proper type of boundary condition to supply the core flow and how to fit them consistently with the SGS model used in the LES. Here results from wall modeling experiments in two types of separated flows are discussed in relation to these issues. The shortcomings of wall stress models in the separated flow behind a backward-facing step (Akselvoll & Moin, 1995) noted by Cabot (1996) is reexamined briefly. Because one difficulty in this flow was the treatment of the corner behind the step, separated flow on a flat plate due to an induced adverse pressure gradient (Na & Moin, 1996) is being developed as a test bed for wall modeling without the geometric complications of the step.

2. Accomplishments

2.1 Near-wall momentum balance in the flow behind a step

Various models based on the law of the wall, using either instantaneous log laws or boundary layer equations, were employed by Cabot (1996) to provide wall stress boundary conditions on the coarsely resolved bottom wall behind the backward-facing step. In all instances, including the use of no model at all (in which the wall stress in underpredicted by a factor of 2–3), the main separation bubble is observed to accelerate in a deeper pressure low than observed in the LES with resolved walls

(Akselvoll & Moin, 1995). The backward flow penetrates all the way to the step, washing out secondary recirculation features in the corner. On the other hand, the reattachment length and the wall stresses for the attached flow at the outlet were generally well predicted. It was noted that the backward flow has a jet-like structure near the wall, which is unresolved on the coarse mesh, making models based on the law of the wall particularly suspect.

To test the hypothesis that it was the poorly predicted wall stress that caused the aberrant flow behavior, the correct mean wall stress was supplied from Akselvoll & Moin's LES, with the instantaneous values made proportional to the tangential velocity above the wall sublayer (similar to the way Schumann, 1975, and Grötzbach, 1987, applied wall stresses in channel flow). This had surprisingly little effect on the flow development of the separation bubble compared with other wall models. In a related test, the "exact" wall stresses were recorded for a (step-to-outlet) flow-through time using Akselvoll & Moin's code and were fed as wall boundary conditions to the poorly resolved case using the same initial field. Again, an acceleration of the separation bubble toward the step was noted. Note that extending this test to longer times may not be very meaningful because the flow structure in the poorly resolved case may deviate significantly from the resolved case and the wall stresses can no longer be considered exact. Both of these tests suggest that other factors than poor wall stress models are at play here since the anomalous flow behavior occurs even when "good" wall stresses are applied.

One possibility is that the Reynolds stresses being predicted by the SGS model are too inaccurate on the coarse grid near the wall. In this LES, only horizontal (plane) filtering is used in the dynamic procedure with an isotropic (Smagorinsky) base model. Because the grid is very coarse in the wall-normal direction ($\Delta y^+ \approx 40$ at the outlet), the implied grid filter spans large variations in the variables, making the SGS model responsible for a larger fraction of the Reynolds stresses. The mean correction to the streamwise advection term due to wall-normal filtering from the wall at $y = 0$ to $y = y_\ell$,

$$-\frac{\partial}{\partial x_j}\left(\frac{1}{y_\ell}\int_0^{y_\ell} u_j u\, dy - \frac{1}{y_\ell}\int_0^{y_\ell} u_j\, dy\, \frac{1}{y_\ell}\int_0^{y_\ell} u\, dy\right) , \tag{1}$$

averaged over a flow-through time, is shown in Fig. 2 in comparison with the correction to the wall friction. The advection correction is comparable to the friction correction in the separated region ($x/h \approx 2$–7), but it becomes more negligible downstream in the attached region ($x/h > 7$). The similarity in shape of the two terms is interesting and suggests that there may be fairly simple ways to model the correction to the advection term. When both of these forcings were applied in the wall cells of the coarsely resolved LES, the separation bubble accelerated less toward the step, but the trend was still evident. Perhaps the flow readjusted itself to a different state with this forcing, or other errors due to the low order of the numerical scheme cause substantial differences for on the coarse mesh. In any case, the general conclusion that can be drawn from Fig. 2 is that wall-normal filtering must be taken into account on coarse near-wall grids either through explicit

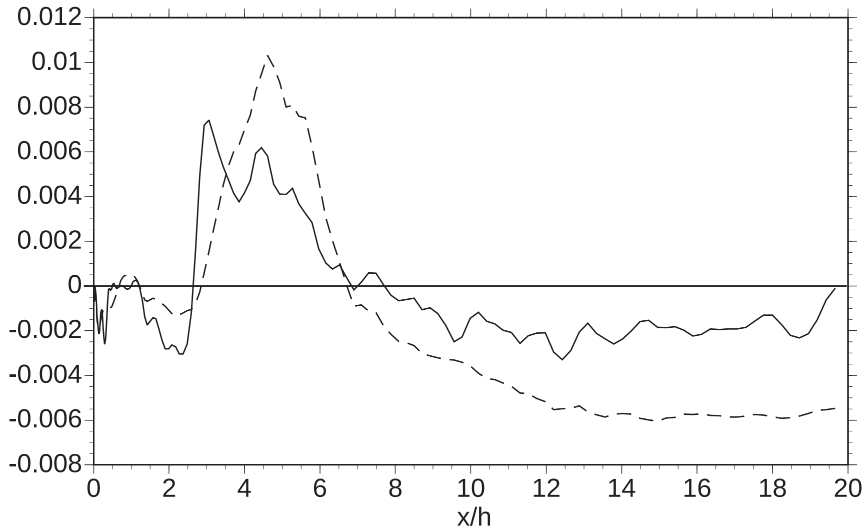

FIGURE 2. The mean correction to the advection term (———) in the streamwise momentum equation due to wall-normal filtering (eq. [1]), and the mean correction to the molecular friction (----) due to underresolution of the wall gradient in the flow along the wall behind a step, computed from a well resolved LES and averaged over span and a flow-through time.

test-filtering in the dynamic procedure or by explicitly modeling the effect of the near-wall inhomogeneity with additional stress or forcing terms. Filtering normal to boundaries is difficult to apply in concept and in practice, and it is unlikely to provide a reliable estimate of near-wall stresses anyway if the models are based on isotropic turbulence. More general SGS models are probably needed in wall regions, e.g., based on Reynolds-averaged Navier-Stokes (RANS) models (Bradshaw, personal communication).

2.2 Separated flow in an adverse pressure gradient

Weak separation in this boundary layer flow over a flat plate is produced by an adverse pressure gradient induced with strong blowing and sucking on the top boundary (Na & Moin, 1996). The inlet boundary layer flow has a Reynolds number of 300 based in momentum thickness and 500 based on displacement thickness (δ). The DNS computes the $357 \times 64 \times 50\delta$ (streamwise, wall-normal, spanwise) domain on a $512 \times 192 \times 128$ mesh stretched in the wall-normal direction. The simulation code uses second-order central finite differences on a staggered grid with third-order Runge-Kutta time advancement and a fractional step method for the pressure.

Preliminary tests of off-wall boundary conditions were performed with the core DNS flow by removing the mesh below $y/\delta \approx 2$; this corresponds to $y^+ \approx 50$ at the inlet, or 1/3 of the total mesh points, and it cuts through the middle of the separation bubble. Simulations were limited to about a quarter of a flow-through time because of their great expense. Horizontal velocities or their wall-normal gradients were specified at the new lower boundary. The wall-normal velocity needed to be specified (rather than its gradient) in order to fix the global mass balance in the numerical scheme. A simple stress balance model with a mixing length eddy viscosity was used to represent the horizontal velocity components in the near-wall sublayer at each horizontal position on the lower boundary:

$$\frac{\partial u_\ell}{\partial t} = \frac{\partial}{\partial y}(\nu + \nu_T)\frac{\partial u_\ell}{\partial y} , \quad \nu_T = \kappa y u_\tau D^2 , \quad D = 1 - \exp(-y u_\tau / \nu A_+) , \qquad (2)$$

for $\ell = 1, 3$, where ν and ν_T are the molecular and eddy viscosity, κ is the von Kármán constant, y is the distance from the wall, u_τ is the friction velocity, and D is a wall damping function using the constant $A_+ = 17$. This model gives a meld between a log law and a viscous law, which works adequately for attached flow. The wall-normal velocity is given by continuity:

$$u_2 = -\int_0^y \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z} \right) dy' \ . \qquad (3a)$$

The horizontal velocity in the sublayer is set to zero at the wall and is matched to the core flow at a height somewhat above the lower boundary of the core flow (as in Fig. 1) at $y_m/\delta \approx 3$ ($y_m^+ \approx 75$ at the inlet). This overlap was found to be necessary to avoid an unstable feedback between the boundary and matching conditions; a more precise description is needed for the minimum amount of overlap required. Even this overlap was not sufficient when the wall-normal velocity was computed by integrating down from the matching point:

$$u_2 = v(y_m) + \int_y^{y_m} \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z} \right) dy' \ . \qquad (3b)$$

There was no substantial difference using the horizontal velocity or its gradient as the boundary condition for the core flow. The largest effect was due to the low Reynolds stress at the boundary when Eq. (3a) was used instead of (3b); this led to a noticeable acceleration of the flow over short runs. The use of (3b), however, led to long-term instabilities at the boundary. Equation (3b) also generally yields non-zero wall transpiration. The magnitude of the wall stress in the separated region is very small both in the DNS and the wall model case, and the backflow there did not change appreciably. Sufficiently accurate Reynolds stresses at off-wall boundaries appear to be needed to obtain good core flow results. We are currently investigating how much structural information is actually required by the core flow.

Because the DNS runs are very expensive to perform on vector supercomputers, a less expensive LES version is currently being evaluated on grids with 7 and 20 times fewer grid points than in the DNS; this code will also eventually be converted to a parallel architecture. The stability of this simulation has been found to be sensitive to grid spacing and stretching in the region where the blowing from the top meets the stream as it rides over the separation bubble. Without SGS or wall models, the flow is found to separate noticeably farther downstream than in the DNS, but reattachment is at approximately the same location. The LES being tested uses the dynamic procedure with explicit spatial volume filtering and a mixed base model for the residual Reynolds stress (inspired by Bardina *et al.*, 1980, and Piomelli *et al.*, 1988) comprising a "scale-similar" or "Leonard term", which effectively deconvolves the low wavenumber part of the field for broad filters, and the usual dissipative Smagorinsky part:

$$\boldsymbol{\tau} \sim \overline{\overline{\mathbf{u}}\,\overline{\mathbf{u}}} - \overline{\overline{\mathbf{u}}}\,\overline{\overline{\mathbf{u}}} - 2\nu_t \overline{\mathbf{S}} \ , \tag{4}$$

where the overbar denotes the volume filter for the resolved field and ν_t is the eddy viscosity determined by the dynamic procedure. The dynamic procedure formulation of Vreman *et al.* (1994) is used, although other formulations are possible (e.g., Zang *et al.*, 1993). The mixed model has the nice property of accounting for the large scale features of the flow in the residual stress (in this case, the blowing and sucking at the top boundary) that otherwise would lead to erroneous estimates of the eddy viscosity, which is only meant to account for small-scale energy transfer. (Alternatively, one could apply the high-order accurate filters developed by Vasilyev in this volume to avoid the spurious residuals.)

Both wall stress and off-wall boundary conditions have been implemented in the LES code and will be tested in the future. A matter of particular concern is performing explicit filtering near boundaries. The problem is that one can only resolve a boundary to within the filter width, which can span several grid points with test filtering in the dynamic procedure. This is not so critical for the top (blowing and sucking), inlet, and outlet boundaries, where one can extrapolate values from the interior without doing much damage. The main concern is near the lower boundary on coarse meshes. When the flow is resolved near the wall, wall-normal filtering has little effect, and one can appeal in any case to known asymptotic behavior. It may also be possible to extrapolate values near the off-wall boundary with sufficient accuracy, but this will need to be carefully tested. When the coarse LES mesh extends all the way to wall, it becomes very difficult to estimate the near-wall SGS residual stresses through test filtering, and no simple extrapolation or interpolation may work. In this case, a new or supplemental near-wall model for the residual Reynolds stress may be required. Another possibility would be to contract the wall-normal test filter toward the grid filter level as one approaches the wall; however, it is not known how badly the test signal will degrade near the wall or if there are other significant commutation errors introduced by such a procedure.

3. Future plans

Large eddy simulations of the separated boundary layer on a flat plate will be performed on meshes with coarse and fine near-wall meshes; we will then try to reproduce the statistics of the LES in which the wall is well resolved using both off-wall models and wall stress models. The main focus will be in the following areas: (1) SGS modeling: An accurate way will be developed to predict residual Reynolds stresses from the SGS model on coarse near-wall meshes, which will be implemented in the separated boundary layer simulations and perhaps in the simulation of flow over a step. As a guide DNS fields for these flows (Na & Moin, 1996; Le *et al.*, 1997)

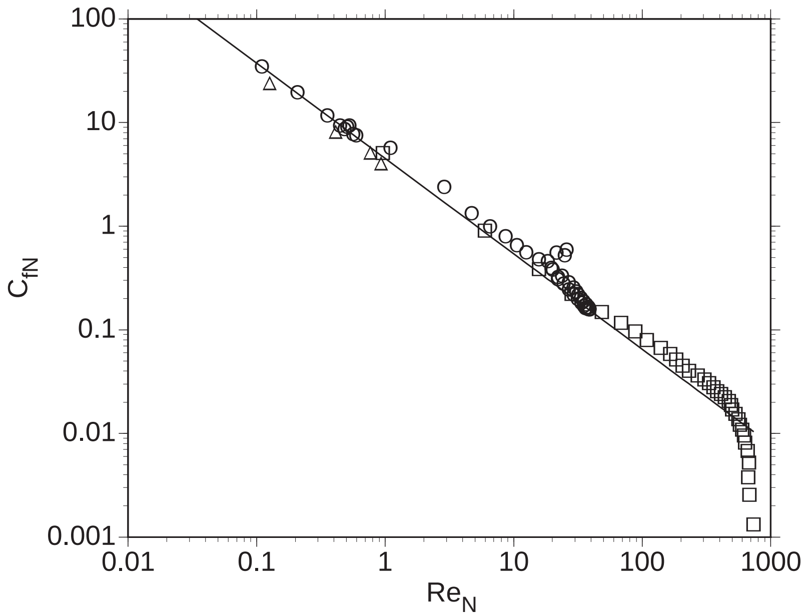

FIGURE 3. Scaling of the magnitude of the skin friction with backflow Reynolds number (see Eq. [5]) in recirculating regions behind a step in Akselvoll & Moin's (1995) $Re_h = 28000$ LES: ▫ primary, ∘ secondary, and ▵ tertiary recirculation regions; ——— Le *et al.'s* (1997) fit from a DNS at $Re_h = 5100$. The fall-off at the lower right occurs for points near the head of the main separation bubble at $x/h \approx 2$.

will be filtered to give residual stresses. (2) Wall modeling: General wall stress and off-wall boundary conditions will be developed for a variety of flow conditions. One approach will be to attempt to merge various scalings that have been developed for different flows, such as wall jets and reattachment points as well as attached flow, into a useful package. For instance, a scaling of the skin friction in separated regions based on the peak backflow speed U_N and its distance from the wall N (Le *et al.*, 1997),

$$C_{fN} = 2|\tau_w|/U_N^2 \approx 4.5 Re_N^{-0.92} \ , \quad Re_N = U_N N/\nu \ , \tag{5}$$

holds roughly in the LES of flow behind a step (§2.1), as shown in Fig. 3. Part of the problem will be devising criteria to sense what flow regime needs to be treated from conditions in the turbulent core flow. It is not yet clear if we must resort to this cataloging approach for each type of flow (with some sort of continuous patching), or if it is possible that the solution of a RANS-like set of differential equations with sufficient physical input from the core flow will prove effective. The best RANS model to consider is probably Durbin's *V2F* model (Durbin, 1991; Parneix in this volume), which does not require ad hoc wall damping functions. We will address the issue of interfacing this model with the LES of the core flow (cf. Carati in this volume) and perform tests with it to determine the performance and cost effectiveness of the approach. (3) Implementation: The degree of overlap in the patching procedure needs to be quantified into a useful prescription (for a given wall model) such that the distance of overlap allows time for the signal at the boundary to become sufficiently decorrelated with the input signal to the wall model to avoid excessive feedback. Through an examination of DNS databases we also intend to determine the proper grid spacing and location of the wall boundaries for which LES can be expected to perform accurately.

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