Active control of turbulent channel flow

By P. Koumoutsakos

1. Motivations and objectives

The active control of turbulent flows is gaining recognition as a possible means for greatly improved performance of aerospace and marine vehicles. While passive devices have been used effectively in the past, active control strategies have the potential of allowing a significant improvement in the performance of future configurations.

Along with small and robust sensors and actuators, simple yet effective control algorithms, which are based on measurable flow quantities, are needed to make active feedback control of turbulence a reality. An algorithm for active feedback turbulent flow control (herein referred to as opposition control) was first introduced by Choi, Moin and Kim (1994). In the opposition control approach, the vertical motion of the turbulent flow near the wall is countered by an opposing blowing/suction distribution of velocity on the wall. Using this technique a 25% drag reduction was obtained by counteracting the velocity field sensed at $y^+ \approx 15$. Though the opposition control algorithm is simple and effective for viscous drag reduction, it has the substantial drawback that it requires measurements inside the flow domain. In order to alleviate this difficulty, Lee et al. (1997) employed a neural network to construct a simple feedback control algorithm using information only at the wall. Their methodology was shown to reduce skin friction by about 20%.

We outline here a novel feedback control algorithm using information that can be obtained at the wall. This framework relies on the identification of the near-wall structures via their induced wall vorticity flux. The present control scheme is based on the manipulation of the vorticity flux components, which can be obtained as a function of time by measuring the instantaneous pressure at the wall and calculating its gradient. An algorithm is presented which allows for the explicit calculations of the necessary control strengths. Application of the present control scheme to low Reynolds number turbulent channel flow produced drag reduction of up to 40% using wall information only. Moreover, it appears that using the present methodology open-loop control laws can be devised.

Further details of the present methodology and the results discussed herein can be found in Koumoutsakos (1997,1998) and Koumoutsakos et. al. (1997).

2. Accomplishments

The present scheme (Koumoutsakos, 1997) is based on the manipulation of the vorticity creation at a wall, using wall information only. The pressure field is sensed at the wall and its gradient (the wall vorticity flux) is calculated. Blowing/suction at the wall is the actuating mechanism and its strength is calculated explicitly by formulating the mechanism of vorticity generation at a no-slip wall.
In wall bounded flows, the tangential motion of fluid elements relative to the wall establishes velocity gradients. With the definition of vorticity ($\omega$) as the curl of velocity ($\omega = \nabla \times \mathbf{u}$), this may be equivalently described in terms of the vorticity that is acquired by the fluid elements near the wall. (Lighthill, 1963) envisioned the wall as a system of sources and sinks of vorticity.

We consider a cartesian coordinate system and flow over a flat wall identified with the $xz$ plane, normal to the $y$-axis. The vorticity flux vector is then expressed as:

$$\boldsymbol{\sigma} = -\left( \nu \frac{\partial \omega}{\partial y} \right)_w$$

For an incompressible viscous flow over a stationary wall, the vorticity flux is directly proportional to the pressure gradients, as the momentum equations reduce at the wall to (Panton 1984):

$$\nu \left( \frac{\partial \omega_x}{\partial y} \right)_w = \frac{1}{\rho} \left( \frac{\partial P}{\partial z} \right)_w, \quad -\nu \left( \frac{\partial \omega_y}{\partial y} \right)_w = \frac{1}{\rho} \left( \frac{\partial P}{\partial x} \right)_w$$

where $P$ is the pressure and $\omega_x$ and $\omega_y$ are the streamwise and spanwise vorticity components. Note that the flux of the wall normal vorticity, $\omega_y$, may be determined from the kinematic condition ($\nabla \cdot \omega = 0$).

### 2.1 Measurements of the wall vorticity flux

In order to assess the practical implications of the proposed algorithm, we outline some research efforts associated with measurements of the vorticity flux. Experimental measurements of the wall vorticity flux in a turbulent flow have been reported by Andreopoulos and Agui (1996). Their measurements demonstrated the significance of vorticity flux in describing near wall processes. They observed that fluid acquires or loses vorticity at the wall during rather violent events followed by periods of small fluctuations. Their experiments demonstrated that the major contributions to the vorticity flux come from the uncorrelated part of the pressure signals, at two adjacent locations, which contain a wide range of vortical scales. As the degree of correlation is smaller between the small scales, their contribution to the vorticity flux is more pronounced. This imposes a severe requirement on the spatial resolution of the pressure gradients/vorticity flux measurements. Practical applications (Moin & Bewley 1995) would require actuators and sensors with sizes in the order of 50$\mu$m and actuator frequencies of 1MHz. Recent advances in micro pressure sensor fabrication technology (Ho & Tai 1996) give us an opportunity to overcome these difficulties. Löfdahl et. al. (1996) presented measurements in a two-dimensional flat plate boundary layer with a resolution of eddies with wave numbers less than ten viscous units using microscopic silicon pressure transducers. It appears that using this new technology one may be able to describe in detail physical processes in terms of the wall vorticity and the wall vorticity flux.
2.2 Vorticity flux induced by blowing and suction at the wall

The role of the vorticity flux from oscillating walls as a mechanism for the control of unsteady separated flows was discussed by Wu et al. (1993). They concluded that wall oscillations can produce a mean vorticity flux that is partially responsible for phenomena of vortex flow control by waves. Gad-El-Hak (1990) has shown that the vorticity flux can be affected by wall transpiration as well as by wall-normal variation of the kinematic viscosity (ν) as a result of surface heating, film boiling, cavitation, sublimation, chemical reaction, wall injection of higher/lower viscosity fluid, or in the presence of shear thinning/thickening additive.

However, these works do not provide us with an explicit formulation for the actuator strength necessary to induce a desired vorticity flux at the wall. This may be achieved by considering the generation of vorticity at the wall as a fractional step algorithm (Lighthill 1963).

At each time step (δt) the no-slip boundary condition can be rendered equivalent to a vorticity flux boundary condition (Koumoutsakos, Leonard, & Pepin 1994) which is materialized in successive substeps. During the first substep we consider the inviscid evolution of the vorticity field in the presence of solid boundaries. The no-through flow boundary condition is enforced via the introduction of a vortex sheet γ(s) along the surface (s) of the body. The vortex sheet is equivalent to a spurious slip velocity on the boundary that needs to be eliminated in order to enforce the no-slip boundary condition. This is achieved at the next substep of the algorithm as the vortex sheet enters diffusively into the flow field. When γ is eliminated from the body surface in the interval [t, t + δt], the circulation (Γ) of the flow field would be modified according to:

\[ \int \gamma(s) \, ds = \int_t^{t+\delta t} \frac{d\Gamma}{dt} \, dt \]  

(1)

On the other hand, Kelvin’s theorem states that the rate of change of circulation induced to the fluid elements due to the presence of the body is:

\[ \frac{d\Gamma}{dt} = \nu \int \frac{\partial \omega}{\partial n}(s) \, ds \]  

(2)

If we consider this vorticity flux to be constant over the small interval of time (δt), we will have:

\[ \nu \frac{\partial \omega}{\partial n}(s) = -\gamma(s)/\delta t \]  

(3)

This constitutes then a Neumann type vorticity boundary condition for the vorticity field equivalent to the no-slip boundary condition (Koumoutsakos, Leonard, & Pepin 1994).

This formulation helps us determine the vorticity flux induced by a set of actuators such as ideal sources/sinks located at the wall. Without loss of generality we consider a two-dimensional flow over a flat wall and a system of sources/sinks of strength q_j that are distributed uniformly over a panel of size d_j, centered at
locations $x'_i, j = 1, 2, 3, \ldots N$. When the sources/sinks are switched on, the induced tangential velocity at point $x_i$ on the wall and the corresponding vorticity flux can be determined as:

$$\nu \delta t \frac{\partial \omega}{\partial y}(x_i) = \sum_{j=1}^{N} \frac{q_i}{2\pi} \int_{-d_i/2}^{d_i/2} ds \frac{d}{x - s}$$

where $x = x_i - x'_j$. The methodology outlined herein may be formulated for a variety of actuators, such as wall acceleration, deformation, etc.

3. An active control strategy

For the purposes of our control scheme we consider a series of vorticity flux (or equivalently pressure gradient) sensors on the wall at locations $x_i, i = 1, 2, 3, \ldots M$. Using the formulas described above we can explicitly determine the actuator strengths necessary to achieve a desired vorticity flux profile at the wall at a time instant, $k$, by solving the linear set of equations:

$$Bu_k + X_{k-1} = D_k$$

where $D_k = (\frac{\partial \omega^k}{\partial y}(x_1), \frac{\partial \omega^k}{\partial y}(x_2), \ldots, \frac{\partial \omega^k}{\partial y}(x_M))$ is an $M \times 1$ vector of the desired vorticity flux at the sensor locations, $X_{k-1} = (\frac{\partial \omega^{k-1}}{\partial y}(x_1), \frac{\partial \omega^{k-1}}{\partial y}(x_2), \ldots, \frac{\partial \omega^{k-1}}{\partial y}(x_M))$ is an $M \times 1$ vector of the measured vorticity flux at the sensor locations and $u_k = (q_1^k(x'_1), q_2^k(x'_2), \ldots, q_N^k(x'_N))$ is an $N \times 1$ vector of source strengths at the actuator locations, $B$ is an $M \times N$ matrix whose elements $B_{ij}$ are determined by evaluating the integrals in Eq. 4. The unknown source/sink strengths are determined by solving the system in Eq. 5. If the relative locations of the sensors and actuators remain constant, matrix $B$ need be inverted only once, thus minimizing the computational cost of the method.

We may distinguish between in-phase control (implying enhancement of the wall vorticity flux) by selecting $D_k = 2X_{k-1}$ and out-of-phase control (implying cancellation of the induced vorticity flux) by selecting $D_k = 0$.

Moreover, the present technique gives us the flexibility to adapt the actuator strengths to specific constraints. In the present calculations the requirement of zero net mass flux

$$\sum_{j=1}^{N} q_j = 0$$

is easily incorporated in the above scheme by appropriately adjusting matrix $B$. A square, invertible matrix is always possible by accordingly modifying the number of sensors and actuators. The simplicity of the present scheme allows for a number of different placements of sensors and actuators. Here we chose the locations of sensors and actuators to be collocated. Physically this may be understood as an advantageous situation as the sensors are able to detect the vorticity field induced by the actuators, which allows the control scheme to suitably compensate for it.
4. Control of turbulent channel flow

Simulations of the model problem of vortex dipole-wall interactions (Koumoutsakos, 1997) have revealed that the present control scheme can alter drastically these interactions. In-phase control results in the “trapping” of the primary vortices by the enhanced secondary vorticity field. The out-of phase control has resulted in the absorption of the impinging dipole and the establishment of small oscillating vortical structures over the wall. These structures are maintained by the present algorithm as the system constantly reacts to the production of the vorticity induced by the actuators.
Figure 2. Contour plots of actuator strengths. Black (blowing) and white (suction) color coding is used for the plots.

Figure 3. Skin friction drag on the bottom wall of the controlled and uncontrolled flow.

We present here some results from the application of this vorticity-flux control algorithm on a low Reynolds number turbulent channel flow ($Re_\tau = 200$).

The numerical method (Le, Moin and Kim 1997) is a fractional step algorithm in primitive variables ($u$ - $P$), using central finite differences for spatial discretization and a third order Runge-Kutta time advancement scheme. The channel dimensions
are $2\pi, 2.10, 2.0$ for the streamwise, spanwise, and wall normal direction. Simulations were carried out with a grid resolution of $N_x \times N_z \times N_y = 128 \times 64 \times 128$. A cosine spacing was employed for the grid points in the wall-normal direction. The non-dimensional discretization is: $\Delta x^+ \approx 12$, $\Delta z^+ \approx 8$, $\Delta y^+ \approx 0.1 - 7$.

Control is applied only on the bottom wall with a collocated arrangement of sensors and actuators. In this arrangement the rows of sensors and actuators are located at alternating streamwise grid locations on the bottom wall. Their strength is determined using a technique similar to the two-dimensional techniques already described.

In the present scheme for three-dimensional flows, the ‘desired’ and the measured vorticity flux may be related as:

$$\begin{pmatrix} \nu \frac{\partial \omega_x}{\partial y} \\ \nu \frac{\partial \omega_z}{\partial y} \end{pmatrix}_{\text{control}} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \nu \frac{\partial \omega_x}{\partial y} \\ \nu \frac{\partial \omega_z}{\partial y} \end{pmatrix}_{\text{measured}}$$

(6)

The coefficients $a, b, c, d$ can be chosen a-priori, and they may be constant or spatially varying. The parameter space can be optimized for drag reduction/increase.

We have conducted several sets of simulations, varying locally and globally the coefficients $a, b, c, d$. Most of our simulations have been conducted with the set of parameters $a = b = c = 0$ and $d = \pm 1$, which is equivalent to considering In/Out of phase control of the spanwise vorticity flux.

In particular, we present here some results from the out-of phase control. After a short transient state a drastic modification of the inner wall structure is observed. The streaks are eliminated, and highly spanwise correlated patterns are established for the spanwise vorticity flux, the shear stresses (Fig. 1), and the actuator strengths (Fig. 2). These structures and further flow visualizations (Koumoutsakos, 1998) suggest the formation of unsteady spanwise vortical “rollers” in the inner layer of the wall. These spanwise vortical rollers result in the formation of positive and negative shear stresses at the wall. The spanwise correlation of the near wall structures persist till about $y^+ = 15$, beyond which the influence of the wall is not discernible in the flow field. Moreover, the regularity in the resulting actuator strengths (Fig. 2) suggest that it is possible to devise open loop control laws using the present methodology (Koumoutsakos, 1998).

The elimination of streaks and the disruption of the near wall processes by the establishment of the particular vortical “rollers” resulted in skin friction drag reduction in the order of 40% (Fig. 3).

5. Conclusions

A new feedback control algorithm (Koumoutsakos, 1997) based on the manipulation of vorticity creation at the wall was outlined. In this scheme the vorticity flux is sensed at the wall via the measurement of wall pressure. A simple control strategy allows calculation of the strength of wall transpiration to achieve a desired wall vorticity flux. Implementation of the vorticity flux feedback control algorithm in the simulation of a low Reynolds number turbulent channel flow shows a drastic modification of the near wall vortical structures and indicates high skin friction drag.
reduction (≈ 40%). The results of the present simulations suggest that it is possible to devise open-loop control laws using the present methodology. Work is underway to implement the proposed strategy in the control of unsteady separated bluff body flows.

6. Acknowledgments

This work has greatly benefited from numerous discussions with Bill Cabot, Georges-Henri Cottet, Karim Shariff, and Alan Wray. I wish to acknowledge my exposure to the problem of turbulent flow control by Tom Bewley, Nagi Mansour, and Parviz Moin. Computer time was provided by the NASA Ames supercomputing center.

REFERENCES


Hornung, H. 1990 *Sources of Vorticity, Ae 232. Class Notes, California Institute of Technology.*


Koumoutsakos, P. 1998 A new method for the active control of turbulent channel flow. *Submitted for publication.*


