Progress in large-eddy simulation of trailing-edge turbulence and aeroacoustics

By Meng Wang

1. Motivation and objectives

Turbulent boundary layers near the trailing-edge of a lifting surface are known to generate intense, broadband noise through an aeroacoustic scattering mechanism (Ffowcs Williams & Hall 1970; Howe 1978). In addition, the fluctuating surface pressure (pseudo-sound) tends to excite structural vibrations and low frequency noise radiation (Blake 1986).

To numerically predict the trailing-edge noise requires that the noise-generating eddies over a wide range of length scales be adequately represented. This requirement cannot be met by the traditional computational fluid dynamics (CFD) methods based on Reynolds-averaged Navier-Stokes (RANS) equations or Euler equations. Large-eddy simulation (LES) techniques provide a promising tool for obtaining the unsteady surface-pressure fields and the near-field turbulence quantities. LES is best suited for computing the noise source at Reynolds numbers of engineering interest because it resolves only the energy-containing eddies, known to be significant contributors to noise radiation. The effect of small (subgrid) scale eddies on the large (resolved) scale motion is modeled, thus drastically reducing the computational cost as compared with direct numerical simulation (DNS). The latter approach, which attempts to resolve all the physical length scales, is limited to simple, relatively low Reynolds number flows even with today’s high performance computing capabilities.

In this project, we aim to develop numerical prediction methods for trailing-edge aeroacoustics using a combination of LES techniques and aeroacoustic theory based on Lighthill’s analogy (Lighthill 1952). With this approach, the instantaneous turbulent flow fields near the trailing-edge are obtained by means of LES. The space-time evolution of the surface pressure fluctuations, useful as forcing function for structural vibration models, is also computed directly. The simulation results allow the acoustic source functions, or the fluctuating Reynolds stress, to be evaluated. The radiated noise can then be computed from an integral-form solution to the Lighthill equation, along the line of Ffowcs Williams & Hall (1970). A second objective of the project is to study the physical mechanisms for the generation of sound and pseudosound. Besides the edge scattering effect, we are also interested in the roles played by pressure gradients and boundary-layer separation near a trailing edge.

The general framework and aeroacoustic formulation for the present project are outlined by Wang (1996). During the past year major effort has been devoted to the LES of the near-field, in order to evaluate the acoustic source functions and to assess the predictive capabilities of LES for surface pressure fluctuations.
Figure 1. Flow configuration and computational domain. The experimental measurement stations $B$-$G$ are located at $x/h =$ $-4.625, -3.125, -2.125, -1.625,$ $-1.125$, and $-0.625$, respectively.

2. Accomplishments

2.1 Flow configuration

The flow being simulated corresponds to the experiment conducted by Blake (1975). As shown in Fig. 1, a two-dimensional flat strut with a circular leading edge and an asymmetric, beveled trailing-edge of 25-degree tip-angle is placed in a uniform stream at zero-degree angle of attack. The strut has a chord to thickness ratio $C/h = 21.125$. The Reynolds number is based on free-stream velocity $U_\infty$, and the chord is $2.15 \times 10^6$. This flow is particularly interesting in that the asymmetric edge shape produces a separated flow on the low-pressure side and an attached boundary layer on the high-pressure side, thus creating complex shear-layer interactions in the vicinity of the trailing edge. The experimental data, including the mean and turbulent velocity magnitudes and the fluctuating surface pressure, are available for comparison with computational results.

2.2 Computational methodology

In order to reduce the computational cost while capturing the essential physical processes of interest, numerical simulations are conducted in computational domains that contain the aft section of the strut and the near wake, as illustrated in Fig. 1. Note that only the location of the inlet boundary is depicted exactly; the remaining three sides of the domain have been truncated for plotting clarity (see Table 1 for the actual domain sizes). The letters $B, C, D, E, F$, and $G$ indicate measurement stations in Blake’s experiment. They are located at $x/h =$ $-4.625, -3.125, -2.125, -1.625, -1.125$, and $-0.625$, respectively, in a Cartesian coordinate system originating from the trailing edge.

The simulations solve the spatially filtered, unsteady, incompressible Navier-Stokes equations in conjunction with the dynamic subgrid-scale model (Germano et al. 1991; Lilly 1992). The numerical code is an adaptation of the $C$-grid code described by Choi (1993) and Mittal (1996). Spatial discretization is achieved using second-order central differences in the streamwise and wall-normal directions.
and using Fourier collocation in the spanwise direction. A significant improvement has been made by implementing a phase-shift dealiasing strategy in the spanwise direction (Lund & Wray, private communication). Compared with the original method of dealiasing by padding, the new method saves 33% CPU time and memory. The time-advancement is of the fractional step type in combination with the Crank-Nicolson method for viscous terms and third order Runge-Kutta scheme for convective terms. The continuity constraint is imposed through a pressure Poisson equation solved at each Runge-Kutta sub-step using a multi-grid iterative procedure.

The boundary conditions at the inlet are obtained by the following procedure. First, an auxiliary RANS calculation is conducted in a C-grid domain enclosing the entire strut. The resulting mean velocities, accounting for the flow acceleration and circulation associated with a lifting surface, are used as the inflow profiles outside the boundary layers on both sides of the strut. Within the turbulent boundary layers the time series of inflow velocities are generated from two separate LES’s of flat-plate boundary layers with zero pressure gradient, using the method described by Lund, Wu & Squires (1996). The inflow-generation LES employs an identical mesh resolution as for the trailing-edge flow LES at the inlet and matches the local boundary layer properties, including the momentum thickness and Reynolds number, with those from the RANS simulation.

A no-slip condition is applied on the surface of the strut. The top and bottom boundaries are placed far away (∼ 20h for most simulations) from the strut to minimize the impact of the imposed velocities obtained from RANS calculations. At the downstream boundary the convective outflow condition (Pauley, Moin & Reynolds 1988) is applied to allow the vortical disturbances in the wake to leave the computational domain smoothly.

2.3 Simulations performed

A total of four simulations, summarized in Table 1, have been carried out to date, although only the last two will be described in detail. The first simulation was done on a very coarse grid in the course of code development and testing. More reasonable grid resolutions were employed in simulations 2 and 3, which differ only in the spanwise resolution. The resolution improvement in LES 3 results from a switch to the phase-shift dealiasing method mentioned previously, without increasing the computational cost. The two simulations (LES 2 and LES 3), however, showed insignificant differences in the velocity and mean pressure fields.

The newest simulation, LES 4, differs from LES 3 in two major aspects: the inflow conditions and the spatial resolution. Fig. 2 compares the inlet streamwise velocity profiles (normalized by free-stream velocity $U_\infty$) used in LES 3 and LES 4, obtained from RANS calculations using the $v^2-f$ turbulence model (Durbin 1995) and Menter’s (1993) SST $k-\omega$ model, respectively. In this figure the strut is located at $0 \leq y/h \leq 1$, and the two boundary layers are represented by the nearly horizontal lines. The two turbulence models produced a noticeable difference in the velocity overshoot (undershoot) outside the upper (lower) boundary layer. The inflow profiles for LES 4 are associated with a smaller mean circulation, which
Table 1. Domain size, grid size, and inflow \( Re_\theta \) for simulations performed.

<table>
<thead>
<tr>
<th>No.</th>
<th>Domain ((l_x \times l_y \times l_z))</th>
<th>Grid ((n_x \times n_y \times n_z))</th>
<th>Inlet ( Re_\theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(20.0h \times 82h \times 0.5h)</td>
<td>(576 \times 80 \times 32)</td>
<td>3660, 2860</td>
</tr>
<tr>
<td>2</td>
<td>(16.5h \times 41h \times 0.5h)</td>
<td>(1280 \times 88 \times 32)</td>
<td>3660, 2860</td>
</tr>
<tr>
<td>3</td>
<td>(16.5h \times 41h \times 0.5h)</td>
<td>(1280 \times 88 \times 48)</td>
<td>3660, 2860</td>
</tr>
<tr>
<td>4</td>
<td>(16.5h \times 41h \times 0.5h)</td>
<td>(1536 \times 96 \times 48)</td>
<td>3380, 2760</td>
</tr>
</tbody>
</table>

Figure 2. Mean streamwise velocity profiles at LES inlets, obtained from RANS calculations. —— LES 4; ---- LES 3.

is thought to promote trailing-edge separation (the corresponding RANS solution indeed has larger separation). The different inflow profiles also correspond to different Reynolds numbers \( Re_\theta \) based on momentum thickness and boundary layer edge velocity \( U_e \), as listed in Table 1.

The streamwise resolution improvement in LES 4 occurs mainly along the upper surface, on which 640 grid points are nonuniformly distributed, compared with 448 points for LES 3. This reduces the maximum grid spacing in wall units at the location of skin-friction peak (cf. Fig. 7) from \( \Delta x_{max}^+ \approx 105 \) in LES 3 to \( \Delta x_{max}^+ \approx 60 \). Along the lower surface 512 and 448 points are used in LES 4 and LES 3, with \( \Delta x_{max}^+ \approx 74 \) and 62, respectively. In both cases, 192 points are distributed along the wake line (branch cut). The wall-normal resolution is increased slightly, although the grid spacing for the first layer of cells adjacent to the surface remains unchanged at \( \Delta y^+ \approx 2 \). In the spanwise direction, the same number of points with uniform spacing are used in both simulations. \( \Delta z_{max}^+ \) is approximately 55 at the skin friction...
peak and is substantially smaller elsewhere.

The CPU time requirement to advance the simulation one flow-through time, \textit{i.e.}, to follow a fluid element to traverse the streamwise domain length, is approximately 150 single-processor hours on CRAY C90 for LES 3, and 200 hours for LES 4. At least two to three flow-through times are required to eliminate the initial transients and collect converged statistics.

2.4 Results

Figure 3a depicts contours of the instantaneous streamwise velocity $u/U_\infty$ at a given spanwise location. The mean streamwise velocity $(U/U_\infty)$ contours, obtained by averaging over the homogeneous spanwise direction and time, are plotted in Fig. 3b. The results of LES 4 are used for both figures. It is observed that the numerically simulated fields exhibit realistic turbulence structures and a small separated zone near the trailing edge. The two shear layers, arising from the separated boundary layer on the upper side and the attached boundary on the lower side, interact in the near wake region to shed unsteady structures downstream.

In Fig. 4, the magnitude of the mean velocity $\bar{U} = (\bar{U}^2 + \bar{V}^2)^{1/2}$, normalized by its value at the boundary-layer edge $\bar{U}_e$, is plotted as a function of vertical distance from the upper surface at streamwise stations (from left to right) $C-G$ defined in Fig. 1. The solid and dashed lines are based on LES 4 and LES 3,
Figure 4. Profiles of the normalized mean velocity magnitude as a function of vertical distance from the upper surface, at streamwise stations (from left to right) C, D, E, F, and G. —— LES 4; ——— LES 3; • Blake's experiment. Individual profiles are separated by a horizontal offset of 1 with the corresponding zero lines located at 0, 1, ..., 4.

respectively, and the symbols represent Blake’s experimental data. Good agreement with the experimental results is obtained at station C and all the upstream locations. However, significant deviations occur at stations D and E, where the experimental profiles are less full in the near-wall region. Further downstream, at stations F and G, the discrepancy diminishes, and the computed profiles, particularly those from LES 4, compare well again with the experimental results. Between the two simulations, LES 4, which has a smaller mean circulation and better grid resolution, provides better agreement with the experiment.

Figure 5 compares the computational and experimental profiles of the “turbulence intensity”, or the normalized rms velocity fluctuations as measured by a single hot-wire thermal anemometer system, at streamwise stations (from left to right) B, D, E, F, and G. In terms of the mean and fluctuating velocity components in the $x$-$y$ plane, the fluctuating velocity measured by a single wire is approximately

$$\bar{u}' \approx \frac{U}{(U^2 + V^2)^{1/2}}u' + \frac{V}{(U^2 + V^2)^{1/2}}v'.$$

(1)

$\bar{u}' \approx u'$ in an attached boundary layer where $V \ll U$. The agreement between the LES and the experimental results is fairly good except in the near-wall region and at the last two stations. One notices that the experimental intensity profiles consistently miss the near-wall peaks known to exist in turbulent boundary layers,
Figure 5. Profiles of the rms velocity fluctuations defined in (1) as a function of vertical distance from the upper surface, at streamwise stations (from left to right) \( B, D, E, F, \) and \( G \). \( \text{--- LES 4; ----- LES 3; • Blake's experiment. Individual profiles are separated by a horizontal offset of 0.15 with the corresponding zero lines located at 0, 0.15, ..., 0.60.} \)

suggesting a possible lack of spatial resolution or high-frequency response as the probe approaches the wall. The large discrepancy observed in the separated region (stations \( F \) and \( G \)) may be caused by both simulation and measurement errors. In general, hot-wire readings become increasingly difficult to interpret if the rms turbulence intensity exceeds 30\% of the local mean velocity (Bradshaw 1971). This is seen to be the case in the separation bubble where the mean velocity is very small (cf. Fig. 4). It should also be pointed out that the LES results represent the resolved portion of velocity fluctuations only. No attempt was made to account for the contributions from the subgrid scale stresses.

The dimensionless mean pressure \( (= C_p/2) \) and the local skin-friction coefficient are depicted in Figs. 6 and 7, respectively, as functions of \( x \). Both simulations show unsatisfactory comparisons with the experimental \( C_p \) data, although LES 4 represents a clear improvement over LES 3. The improvement arises from the smaller circulation and the larger separation zone near the trailing edge. The latter can be observed from the \( C_f \) curves for the upper surface (cf. Fig. 7), where the solid curve representing LES 4 exhibits a longer portion of negative skin friction.

Comparisons have also been made between the boundary-layer properties predicted numerically and experimentally. Figures 8 and 9 show the streamwise distributions of displacement thickness and momentum thickness, respectively. The experimental values, represented by the solid circles, are given at (from left to right)
Figure 6. Mean wall pressure distribution near the trailing edge. 

- LES 4; 
- LES 3; 
- Blake’s experiment.

Figure 7. Distribution of the local skin friction coefficient near the trailing edge. 

- LES 4; 
- LES 3.
Figure 8. Distribution of boundary layer displacement thickness near the trailing edge. — LES 4; —— LES 3; • Blake’s experiment (from left to right: stations B-G).

Figure 9. Distribution of boundary layer momentum thickness near the trailing edge. — LES 4; —— LES 3; • Blake’s experiment (from left to right: stations B-G).
Figure 10. Time history of surface pressure fluctuations from LES 4, at streamwise stations (from bottom to top) B, C, D, E, F, G, and T.E. (trailing edge), at a fixed spanwise coordinate. Individual curves are separated by a vertical offset of 0.05 with the corresponding zero lines located at 0.05, 0.10, ..., 0.35.

stations B-G. The displacement thickness predicted by LES is in general agreement with the experimental data except at station D. The momentum thickness is also predicted well except at the last two stations. The poor agreement at these stations is unexpected, given that the numerical and experimental mean profiles agree well in Fig. 4.

Temporal variations of wall-pressure fluctuations are exemplified in Fig. 10. The signals are obtained from LES 4 for stations (from bottom to top) B-G and the trailing edge, at a fixed spanwise location. At stations B-E the pressure signals consist of predominantly high frequency fluctuations associated with small scale eddies in the attached turbulent boundary layer. The oscillation amplitude is decreased in the favorable pressure gradient region (station C) and increased in the adverse pressure gradient region (stations D and E). After the boundary layer is separated (stations F and G), the high frequency content is diminished, and the surface pressure is characterized by lower-frequency and higher-amplitude oscillations caused by the unsteady separation. The high-frequency content reappears at the trailing edge, owing to the contribution from the attached turbulent boundary layer on the lower side.

Figure 11 depicts the wall-pressure frequency spectra

\[ \phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} < p^{i}(t)p^{i}(t+\tau) > e^{i\omega \tau} d\tau \]  

(2)

calculated from LES 4 for stations C, E and G. The ensemble average \(< >\) is
Figure 11. Frequency spectra of wall pressure fluctuations compared with Blake's experimental measurements at selected streamwise stations. The lines are from LES 4 (station C; station E; station G), and the symbols are from Blake’s experiment (● station C; ● station E; ■ station G).

replaced by time- and spanwise averages. Outer variables $U_\infty$, $\delta^*$, and the dynamic pressure $q_\infty = \rho U_\infty^2/2$ are used to scale the data for comparison with Blake’s experimental measurements. The calculated spectra agree relatively well with the experimental data at stations C and E, although they fall off more quickly at the high frequency end due to limited grid resolution. The high frequency content corresponds to fine spatial structures not resolved on the simulation grid. In the separated region (station G), the LES is seen to significantly overpredict the pressure spectra. The surface pressure frequency-spectra from the coarser grid simulation LES 3, not shown here, show similar agreement with the experiment in a somewhat narrower frequency range. It should be mentioned that the pressure signals plotted here have not completely converged to the statistically stationary state, as suggested by the slight upward drift of some curves in Fig. 10. As a result, the pressure spectra, particularly at the low frequency end, will be subject to small corrections as the simulation continues.

2.5 Discussion and summary

The preliminary LES results described above are encouraging in terms of quantitative predictions of the trailing-edge velocity fields and surface pressure fluctuations. However, significant discrepancies still exist between the computed quantities and those measured experimentally at certain measurement stations. Factors that may have contributed to these discrepancies include the inflow velocity conditions,
spatial resolution, and computational domain size. In addition, experimental errors may have also played a role.

The inflow velocity profiles constitute a major uncertainty for the present LES since they are not available from Blake's experiment. The experimental measurements are limited to the upper-side of the strut, and even there the available data are insufficient for boundary condition specification. As a result, we had to resort to RANS calculations to provide the inflow mean velocities, thus severely compromising the accuracy of the LES. The inflow profiles are directly related to the circulation, which affects the entire flow field including the trailing edge region.

Two major simulations, LES 3 and LES 4, with different inflow profiles and streamwise resolutions were described in this report. The one with smaller mean circulation and better resolution (LES 4) is shown to generate a larger separated region and mean velocity profiles in better agreement with the experimental data. Likewise, the pressure coefficient obtained from LES 4 represents a better approximation to the experimental data although the pressure rise from the suction peak to the trailing edge is still exaggerated significantly. Thus, the circulation associated with the experiment must be smaller than that in either simulation. In principle, one could estimate the circulation based on the lift or the surface integral of the static pressure. This is, however, not feasible because no measurement data were given on the lower surface.

The rms velocity fluctuations from the experiment and both simulations are in general agreement on the flat strut section and the first two stations on the descending ramp, except in the near-wall region where the experiment fails to record the peak. The cause for the large disparity at the last two stations needs to be investigated. Unfortunately, individual components of velocity and Reynolds stress are not available from the experiment, which impedes a more rigorous validation or diagnosis of the computational solutions.

The surface pressure frequency spectra reported here are rather preliminary. We are in the process of validating their statistical convergence as more simulation data become available.

3. Future plans

First, the near-field LES needs to be further validated and the discrepancies with the experimental data reconciled. The effect of inflow conditions and the mean circulation on the edge-flow behavior will be investigated, and more grid refinement studies are to be carried out. Other possible artifacts that may affect the computational solutions such as the computational domain size (particularly in the spanwise direction) should be examined. A careful evaluation of the experimental accuracy is also necessary.

Once a reliable near-field solution is established, we will conduct detailed studies of the structure of wall pressure fluctuations and scattering by the edge. Cross-correlation and spectral analyses will be conducted to investigate the unsteady surface pressure generation and scattering mechanisms. The radiated far-field noise will be calculated following the acoustic analogy formulation with a hard-wall Green's function, as outlined in Wang (1996).
LES of trailing-edge flow

REFERENCES


