Interactions between freestream turbulence and boundary layers

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1. Motivation and objectives

The interaction between free-stream turbulence and boundary layers is one example out of many that involve different types of fluid motion in overlapping or adjacent regions of flow. We are concerned here with flows at high Reynolds and Peclet numbers, so that the effects on the interactions between these flows of molecular diffusion are small except close to the boundary [B] between them. In these complex configurations the overall flow is not generally dominated by a single mechanism; for example, perturbations do not grow everywhere at the same rate (Hunt & Carruthers, 1990), but in zones of limited extent with characteristic flow pattern such as thin shear layers, and on certain ranges of time and/or length scales, the flow can be dominated by specific mechanisms. These tend to be defined by only a few parameters. Interactions between the flow regions, say [F1] and [F2], are often dominated by such mechanisms in layers lying along the interface [B]. Some effects propagate into the interior of the regions by advection or wave motion (Fig. 1), while others act upon the boundary.

A large class of such flow problems that are of fundamental and practical interest are characterized by interactions between two distinct and weakly correlated turbulent velocity fields in adjacent regions. The turbulence in each region may have been generated by different kinds of instability, or they may simply differ in their statistics such as their integral length scales. Such interactions occur continually and randomly within turbulent flows and ionized fluids, for example, where small eddies impinge on large coherent structures or where the outer and inner parts of a turbulent boundary layer meet (Terry, Newman & Mattor 1992). In engineering these problems occur in the design of turbomachines. There, the flow approaching the rotating airfoil blade or centrifugal impeller contains turbulent eddies that are much larger than the small scale turbulence in the boundary layers on the solid moving parts. In order to determine the effects of this external turbulence on heat transfer or on the pressure distribution, it is necessary to understand how the intense small scale turbulence grows in the boundary layers that are initially laminar. This can occur at lower values of the Reynolds number than without external turbulence – the mechanism of 'bypass' transition. Is it caused by the external turbulence being simply advected into the growing boundary layers (an advected interaction AI), or, alternatively, does the external turbulence directly induce pressure and velocity

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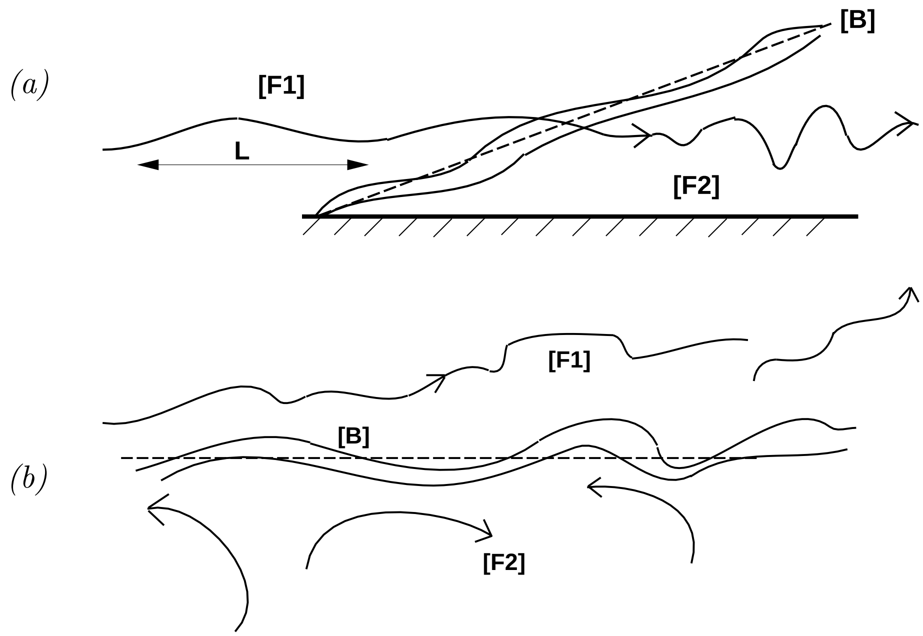

FIGURE 1. Schematic diagram of interactions between external region [F1], containing disturbances, and vortical region [F2], that may be turbulent. Regions separated by interface [B] of finite thickness whose mean position is indicated by a dashed line.

fluctuations in the shear profile of the boundary layer, which may be unstable? This external interaction (EI) mechanism may be very weak because of the tendency of a shear profile to be sheltered from external fluctuations. Experiments and numerical simulations for weak and moderate levels of freestream turbulence (e.g. Goldstein & Windrow 1998, Liu & Rodi 1991) cannot really discriminate between these competing mechanisms without a better theoretical framework, to which we are contributing in this study. Recent measurements by Thole & Bogard (1996) show quite different interactions when the external turbulence is strong relative to the turbulence in the boundary layer.

Similar problems arise on a range of larger scales in meteorology (e.g. Collier *et al.* 1994). For these types of complex flow, practical models are needed; one approach is to make simplifying assumptions about the nature of the interactions and broadly classify them as: (i) superposition (S) of flows in overlapping regions so that interactions can be ignored, (ii) exclusion of flows, or flow processes, in certain regions because a particular mechanism is dominant, especially near the boundary [B]; or (iii) significant interactions (AI and EI) between the flows in the adjoining regions, in which new phenomena or mechanisms may arise.

Theoretical analysis of the appropriate vorticity dynamics and some new direct numerical simulations is the method we use to study these interactions. The theoretical approach is different from but complementary to that based on the hydrodynamic stability theory for small disturbances (Jacobs & Durbin 1998). Both approaches demonstrate how a shear layer can block certain kinds of external disturbances so that the flow inside the layer is 'sheltered' from them. If the boundaries of the shear layer are highly contorted, then the interactions are different, and it is possible for the weak mean vorticity of the outer part of the shear layer to be distorted and dispersed into the freestream – the process of 'vortex stripping' (Legras & Dritschel, 1993).

2. Analysis of external perturbations to boundary layers

2.1 Long length scale, low amplitude perturbations, and shear sheltering

Our object here is to analyze the external interactions (EI) between perturbations $\mathbf{u}^{[\infty]}(\mathbf{x}, t)$ in the freestream, where the streamwise mean velocity is $\overline{u} = U_\infty$ and the mean velocity profile $U(y)$ in the adjacent boundary layer over a rigid surface at $y = 0$. We are not considering the advected interactions (AI) of the perturbations as they enter the growing layer; in fact, we assume here that the layer has constant thickness h. Thus

$$U(y) = U_\infty \widetilde{U}(\widetilde{y}), \text{ where } \widetilde{y} = y/h$$

and

$$\widetilde{U} \to 1 \text{ as } \widetilde{y} \to \infty, \ \widetilde{U} = 0 \text{ at } \ \widetilde{y} = 0. \tag{2.1}$$

We consider a relatively weak 2-dimensional fluctuation with magnitude $u_0 << U_\infty$, with a length scale L, and that moves with a velocity c in the freestream. In order to obtain analytic solutions and demonstrate the key processes, we assume that $L >> h$; this approximation is relevant to many experiments and practical configurations (see Fig. 2). Because of their long length scale, any of these external perturbations interacting with a turbulent boundary layer effectively interact only with the smoothly varying mean profile. So any initial boundary layer fluctuations are ignored here but not in §2.2.

Thus in the freestream, as $y/L \to \infty$, the total velocity field $\mathbf{u}^*$ is given by $\mathbf{u}^* = \mathbf{u} + \overline{\mathbf{u}}$, where the perturbation field is expressed in moving coordinates as $\mathbf{u} = \mathbf{u}^{[\infty]} = u_0 \mathbf{f}(\widehat{x}, \widehat{y})$, where

$$\widehat{x} = \frac{(x - ct)}{L}, \ \widehat{y} = \frac{y}{L}, \text{ and } \mathbf{f} = (f_x, f_y, 0). \tag{2.2}$$

Either the maximum value $f_x \approx 1$, or if it is random its rms value $f_x^{'} \approx 1$, so that u_0 indicates the magnitude of the freestream disturbance. We assume that

$$u_o << U_\infty. \tag{2.3}$$

We now consider how $\mathbf{u}$ changes above and within the layer as it is advected downwind. Previous studies by Grosch & Salwen (1978) and Jacobs & Durbin (1998) have considered small disturbances, where $\mathbf{f}$ is periodic in x and y, that travel at the same speed as the mean flow, i.e. $c = U_\infty$. They showed that as $Re(= hU_\infty/\nu) \to \infty$,

(a)

(b)

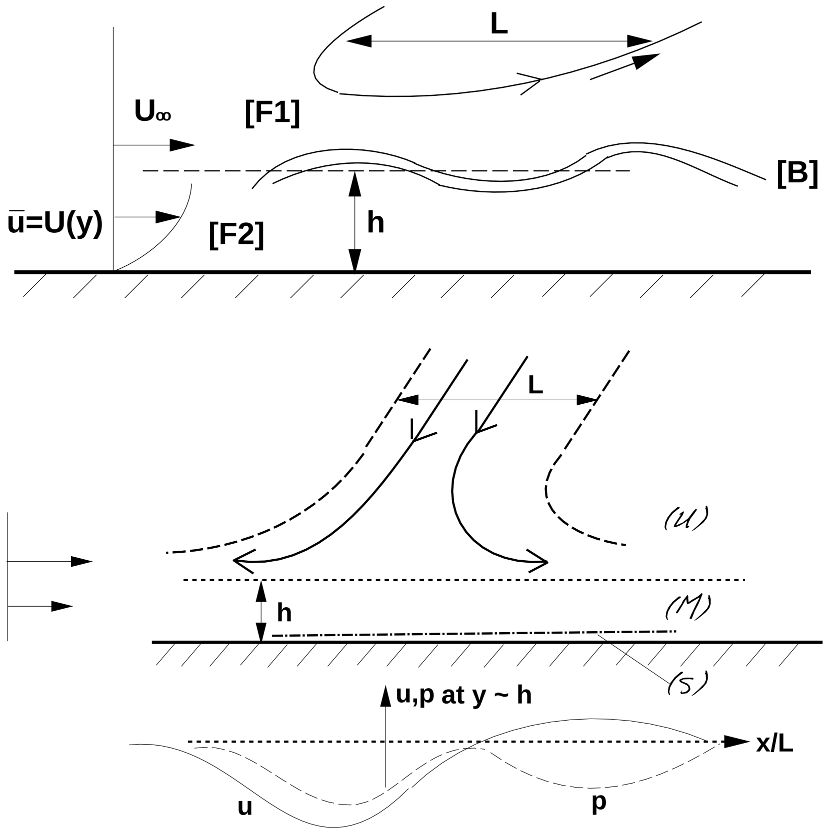

FIGURE 2. External interactions between a boundary layer flow in [F2] and small amplitude disturbances traveling with the freestream speed U_∞ in [F1]; *(a)* schematic diagram with scales showing the flow zones $\{\mathcal{U}\}$ in [F1] and $\{\mathcal{M}\}$, $\{\mathcal{S}\}$ in [F2]; *(b)* the perturbation streamlines in a moving wake traveling outside the boundary layer (after Hodson 1985) and profiles in the streamwise direction of the perturbation velocity u and pressure p at the top of the middle $\{\mathcal{M}\}$ zone.

external disturbances are damped within the boundary layer. If only linear disturbances are considered, they are exponentially small, below a penetration distance δ of order

$$h(hRe/L)^{-1/3}$$

so that as h/L decreases, δ/h increases. In the rest of this study we assume Re is very large, and we ignore such viscous effects except where they are very large in a thin zone, denoted as S, at the surface. These results demonstrate the principle of shear sheltering for linear disturbances when $c = U_\infty$. What happens if these constraints are relaxed? The experimental and numerical evidence is that some penetration can occur.

Consider the problem of a mathematically 'compact' moving disturbance such that $|\mathbf{f}| \to 0$ as $|\widehat{x}| \to \infty$. This could be the wake of a body moving across the stream ahead of the plate (Hodson 1985, Liu & Rodi 1991); in that case $f_x, f_y < 0$. Since $f_y \neq 0$ on $y = 0$, the external disturbances impact on the boundary layer and the plate. This creates a perturbation velocity $\Delta\mathbf{u}$, which is analyzed in different zones corresponding to different mechanisms, namely: upper $\{\mathcal{U}\}$, where $y > h$; middle $\{\mathcal{M}\}$, where $h < y < h_s$; and surface $\{\mathcal{S}\}$ with depth h_s, where $h_s > y > 0$. As in other rapid distortion problems, the changes to the initial or freestream disturbances are linear over a travel time $T = x/U_\infty$ less than T_d, provided that T_d is much less than the time scale of the disturbance $T_L \sim L/u_0$. In the zone $\{\mathcal{U}\}$ above the boundary layer where the only vorticity is that of the disturbance, this vorticity field is simply advected by the mean flow and is not distorted by the changes to the perturbation velocity near the plate (Hunt & Graham 1978). This implies that the perturbation velocity field is the sum of the initial freestream field and an irrotational field, i.e.,

$$\mathbf{u} = \mathbf{u}^{[\infty]} + \Delta\mathbf{u}, \text{ where } \mathbf{u} = (u, v), \text{ and } (\Delta u, \Delta v) = \nabla\phi. \tag{2.4a}$$

To satisfy continuity

$$\nabla^2\phi = 0. \tag{2.4b}$$

Since the scale of the freestream perturbation $\mathbf{u}^{[\infty]}$ is large compared to the boundary layer depth h, the boundary condition on $\Delta\mathbf{u}$ near the plate is that

$$\text{as } y/L \to 0, \qquad \Delta v = \frac{\partial\phi}{\partial y} = -u_0 f_y. \tag{2.4c}$$

In the free stream as $y/L \to \infty$, $\Delta\mathbf{u} = |\nabla\phi| \to 0$. This linear calculation implies that $\Delta\mathbf{u}$ and ϕ are also functions of $\widehat{x}$ and $\widehat{y}$ and are not varying in time as they move downstream.

Note that further downstream where $T > T_L$, the impingement of the free stream perturbations onto the plate leads to significant distortion of their vorticity, typically rolling up into vortex tubes near the surface (Hodson 1985, Perot & Moin 1995)

In the middle layer $\{\mathcal{M}\}$, the equation for the vertical velocity perturbation v is essentially the long wave Rayleigh equation for small perturbations to a shear flow (Drazin & Reid 1980). It can be expressed in coordinates moving at the speed of the disturbance c as

$$\frac{\partial^2\widehat{v}}{\partial\widehat{y}^2} - \widehat{v}\left(\frac{d^2U}{d\widehat{y}^2}/(U-c)\right) = 0, \tag{2.5}$$

where $\widehat{v}(\widehat{x},\widehat{y}) = v(x,y,t)$, and, for consistency, $\widehat{u}(\widehat{x},\widehat{y}) = u(x,y,t) = -\int \frac{\partial v}{\partial y} dx$. To solve (2.5) it is convenient to write $\overline{U} = U(\widehat{y}) - c$, noting that $\widehat{y} = \widetilde{y}h/L$ and that $d^2U/d\widehat{y}^2 = d^2\overline{U}/d\widehat{y}^2 \sim U_\infty L^2/h^2$. If $c = U_\infty$, then $\widetilde{U} \approx 0$ at the top of the layer, and $\overline{U} \approx -c$ at the plate ($\widehat{y} = 0$).

The solution for $(\widehat{u},\widehat{v})$ can be expressed as a separated variable (Lighthill-Stewartson) solution so that

$$\widehat{u}(\widehat{x},\widehat{y}) = A(\widehat{x})\frac{d\overline{U}}{d\widehat{y}} + \frac{B(\widehat{x})Z(\widehat{y})}{U_\infty}, \tag{2.6}$$

where

$$\widehat{Z}(\widehat{y}) = \left[(d\overline{U}/d\widehat{y})\int_{\widehat{y}_1}^{\widehat{y}} \overline{U}^{-2}(\widehat{y}^+)d\widehat{y}^+ + \overline{U}^{-1}(\widehat{y})\right]U_\infty$$

and $A(\widehat{x})$, $B(\widehat{x})$ and $\widehat{y}_1$ are determined by satisfying the velocity and pressure conditions. In order that $v = 0$ on $y = 0$, and since $\overline{U}(0) \neq 0$, it follows that, $A(\widehat{x}) = 0$, and $\widehat{y}_1 = 0$.

To leading order $p^{[\mathcal{M}]}$ does not vary with y in $\{\mathcal{M}\}$ and is, therefore, equal to its value $p^{[\mathcal{U}]}(x)$ at the bottom of the upper layer $\{\mathcal{U}\}$ above the boundary layer. Thus

$$B(\widehat{x}) = -p^{[\mathcal{M}]}(\widehat{x}) = -p^{[\mathcal{U}]}(\widehat{x}, \widehat{y} \to 0) \tag{2.10}$$

Note that v and p at these two levels at the bottom of $\{\mathcal{U}\}$ and the top of part of $\{\mathcal{M}\}$ match each other.

$$\widehat{v} \sim (h/L)^2 u_0^2/U_\infty. \tag{2.11}$$

Thus in $\{\mathcal{M}\}$, $\hat{v}$ becomes very much less than u_0. Therefore the blocking boundary condition for v in the upper zone $\{\mathcal{U}\}$ is applicable at the level $y \sim h$. For $y \leq h_\ell$, the streamwise velocity is given in terms of $p^{[\mathcal{M}]}$ by

$$u = p^{[\mathcal{M}]}(\widehat{x}) Z(\widehat{y})/U_\infty \tag{2.12}$$

Thus in $\{\mathcal{M}\}$ $\hat{u} \sim (u_0^2/U_\infty)$ and is much less than in $\{\mathcal{U}\}$. But $\hat{u} >> \hat{v}$ if $h/L << 1$. At the top of $\{\mathcal{M}\}$, non-linear or viscous processes determine the smooth transition between these layers. An approximate form for u that is finite and continuous across the critical layer at $y \sim h$ and is asymptotically correct when $L >> y > h$ and when $y << h$ is

$$u(\widehat{x}, \widehat{y}) = \frac{-p^{[\mathcal{M}]}(\widehat{x})}{U_\infty/Z(\widehat{y}) + \lambda(\widehat{x})} \quad \text{where } \lambda = -p^{[\mathcal{M}]}/(u(\widehat{x}, \widehat{y} \to 0)) \text{ in } \{\mathcal{U}\}. \tag{2.13}$$

To illustrate these effects of the blocking of the external normal velocity $v^{[\infty]}$ by the vorticity in the boundary layer, when $c = U_\infty$, and the sheltering of the flow within the layer, we consider an example of a small but finite amplitude freestream perturbation that moves with the freestream and is of such a form that the pressure perturbation far above the plate is exactly zero. We take the practical example of a weak jet or wake such that $u = -(\cos\alpha, \sin\alpha) u_0 \widehat{f}$ where $\widehat{f} = 1/(1 + \widehat{x}^2)$. This corresponds to a traveling wake impacting on the boundary layer (Hodson 1985) if $\pi/2 > \alpha > 0$ or an atmospheric downburst if $\pi > \alpha > \pi/2$.

Then for $T < T_d$, at the bottom of the zone $\{\mathcal{U}\}$ just above the boundary layer the solution to (2.4) shows that $\Delta u(\widehat{x}, \widehat{y} = 0) = \lambda_u(\widehat{x}) u_0$, where

$$\lambda_u = \widehat{x} \sin\alpha/(1 + \widehat{x}^2), \tag{2.14}$$

so that for the wake the streamwise velocity perturbation just above the boundary layer consists of the negative freestream perturbation and a forward jet on the leading side of the perturbation and a negative one on the trailing edge. See Fig. 2b.

The results (2.10) and (2.12) show that to first order the velocity fluctuations in the boundary layer ($y < h$) are zero, but to second order are finite and depend on the pressure perturbation p_s in $\{\mathcal{U}\}$, where

$$p^{[\mathcal{M}]} = -(1/2)(u_\infty + \Delta u(y/L \to 0))^2. \tag{2.15a}$$

In this example

$$p^{[\mathcal{M}]}(x) = -\frac{1}{2}u_0^2\left[(-\cos\alpha + \widehat{x}\sin\alpha)/(1+\widehat{x}^2)\right]^2 . \tag{2.15b}$$

Note that the form of $u(x)$ in $\{\mathcal{M}\}$, derived from (2.12) differs from that in $\{\mathcal{U}\}$, being negative and having two minima. In the surface zone $\{\mathcal{S}\}$, viscous effects induce velocity profiles with inflexion points and may trigger instability there.

These results change as the travel time T increases so as to become comparable with T_L, when the vorticity of the impacting disturbance is significantly distorted. In addition, if the boundary layer is laminar, instabilities tend to be stimulated and modulated by the traveling disturbance above the layer, as recent direct numerical simulations demonstrate (Wu *et al.* 1998). The experimental flow studied by Liu & Rodi (1991) corresponds to that of our example, and the results in the early stages of the interaction are very similar to these theoretical results. Both the DNS and experiments demonstrate the sharp difference between the form and magnitude of the fluctuations in the zones $\{\mathcal{U}\}$ and $\{\mathcal{M}\}$, and both show that the instabilities are initiated very near the surface.

If the disturbances travel at speeds c significantly different from the freestream speed, as occurs in atmospheric downbursts, shear sheltering does not occur. Indeed the surface may be quite large, and their form may be strikingly different from those generated in normal conditions (Collier et al. 1994).

In terms of the concepts of interacting flows proposed in the introduction, these flows demonstrate the phenomena of exclusion (X) in some circumstances and essentially superposition (S) in others, depending largely on the parameter c/U and to a lesser extent on the amplitude u_0/U.

2.2 Finite amplitude perturbations and vortex stripping

In our previous analysis it was assumed that across the bounding interface [B] between the external region [F1] and the vortical region [F2], the vorticity $\vec{\omega}^{[2]}$ in the latter decreases abruptly to a much lower level in [F1]. However, in many vortical regions there is a gradual decrease in the magnitude of $\vec{\omega}^{[2]}$ from characteristic value ω_C in the core to a significantly lower value ω_B near the interface [B] where it is comparable with or smaller than the strain rate in the external region [F1]. Following Legras & Dritschel (1993), we review here the mechanisms for how in these flows external perturbations in [F1] cause large distortions and displacements of the interface [B] over distances of order h, the length scale of [F2]. These are associated with changes to the vorticity field in [F2] that are overwhelming in the outer part and small though significant in the core. Such interactions, involving a different type of inhomogeneity in [F2], plays a critical role in the formation and persistence of large scale vortical motions in the atmosphere and oceans and in the structure of shear flows with high levels of external turbulence.

We consider the interaction between a compact vortical region and a coplanar straining motion $U(x)$ in the external region [F1], having a length scale L that is large compared with h and a characteristic strain rate U_∞/L, see Fig. 3. We make the following assumptions for simplicity: in the core part of [F2], whose length scale

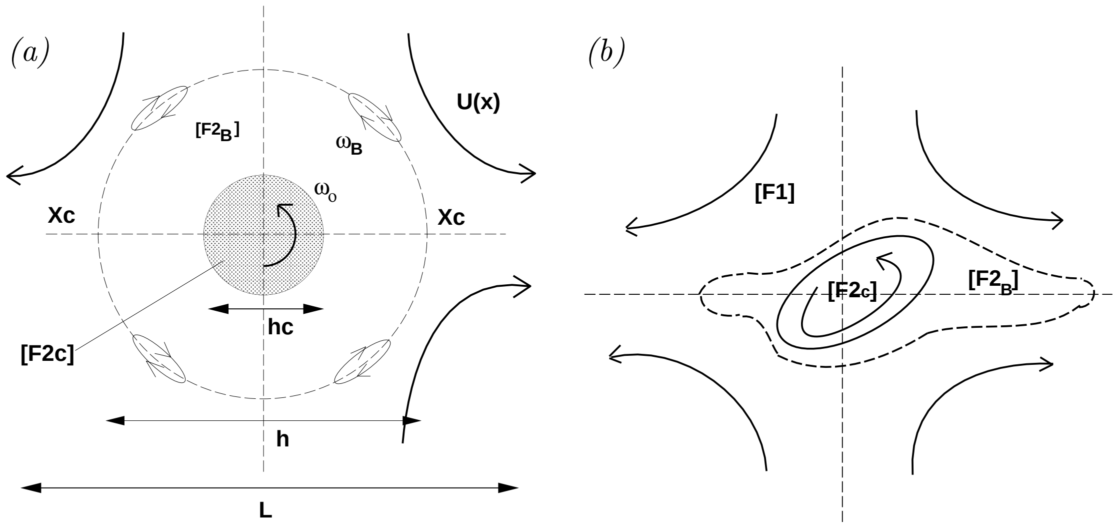

FIGURE 3. Schematic diagram of the mechanism for how external straining motion in [F1] remove, by 'vortex stripping', the low vorticity flow in the outer part $[F2_o]$ of the vortical region, while the inner core [F2c] is only slightly distorted; *(a)* showing the vortex sheet surrounding [F2] when the straining motion is initiated and the convergence points X_c where viscous diffusion leads to detrainment of vorticity.

is $h_C(<< h)$ and is denoted by $[F2_C]$, the initial vorticity is assumed to be much greater than the external strain rate so that $\omega_C >> U_\infty/L$, and in the larger outer part of the region, $[F2_B]$, the initial vorticity is much smaller and is of order ω_B where $\omega_B \leq U_\infty/L$.

The evolution of this flow can be analyzed by inviscid vortex dynamics following G. I. Taylor (See Batchelor 1970) and the theoretical and experimental methods of Rottman *et al.* (1987).

Imagine that the boundary [B] is rigid up to the time $t = 0$ (which means that the external flow passes round the vortex) and is then dissolved (or consider the flow to be generated by a rapidly growing instability); then a vortex sheet is generated around [B]. This vorticity distribution induces the fluid in the interface to follow the direction of the streamlines of the flow in [F1] but does not travel at the same speed. (This is analogous to how, when a cylinder of fluid is suddenly introduced into a cross flow, it distorts itself into a vortex pair and moves downstream at about half the speed of the flow).

The form of [B] as it moves depends on the relative strengths in the outer part of [F2] of the strain rates induced by the external flow and by the core vorticity, indicated by the parameter

$$\Sigma_B = (U_0/L)/(\omega_C h/h_C) \sim (U_0/L)/\omega_B.$$

If $\Sigma_B > 1$, much of the fluid and the vorticity in $[F2_0]$ is swept away in two vortices, leaving a trail behind them back to the core vortex. But if $\Sigma_B < 1$, the fluid and outer vorticity are carried round the core vortex in the form of an elliptical ring.

There is a sharp transition between these two outcomes as Σ_B increases as a result of the formation of singular (zero velocity) position on [B].

Note that, although the core vortex is strong enough that it is only slightly deformed, it is rotated by a finite angle until it reaches a position of equilibrium where it induces a velocity field that is opposite to that of the strain field.

This simple example demonstrates how weaker vorticity can be 'stripped' from the outer region of a vortical region by an external straining flow. Legras & Dritschel (1993) have quantified this process for different types of rotational and irrotational straining motion, and shapes, and orientations of the vortical regions. They find results that are consistent with observations of the changing shape of the the polar vortex and its accompanying 'ozone hole'.

The effect of finite amplitude external perturbations 'stripping' away the weak vorticity at the outer edge of shear layers has been demonstrated in two earlier laboratory studies. Hancock & Bradshaw (1990) measured the interactions between large scale freestream turbulence with rms velocity u_o and length scale L_x and the outer, low vorticity ($\sim \omega_B$) part (or 'wake') of a turbulent boundary layer whose depth is h. Their results show that when $u_0/L_x > \omega_B \sim u_*/h$ (or $u_0 > u_*$ the friction velocity or rms turbulence in the boundary layer), the mean vorticity ω_o in the outer part is stripped away and the thickness and structure of the boundary layer is reduced to that of the higher shear logarithmic region. For lower values of the external turbulence, there was no structural change. Rottman *et al.* (1987) obtained similar results when they measured how the the outer shear region of a gravity current was stripped away by external turbulence when $u_0/L_x > \omega_0$.

This model problem also shows how when vorticity is 'shed' from the boundary of a vortical region, it tends to develop into coherent patches of vorticity even in flows where the two regions are not coplanar as in jets in cross flow Coelho & Hunt 1989). These may have significant dynamical back effects on the region [F2] it 'left behind' and may transport matter and heat away from [F2].

In real rather than model complex flows, the vortical regions have finite gradients of vorticity, evolve on finite time scales, and, at their interface with the external flow, viscous diffusion of vorticity is part of the process of detrainment or shedding of vorticity. We have considered the first two of these idealizations; what about the third?

Vorticity tends to diffuse from a fluid interface around 'convergence' points, denoted by X_c in Fig. 3a, where the flow parallel to the surface converges and streamlines move into the exterior region [F1] from near the surface. Once a vortical region [F2] has developed into a steady form, if it is located in a unidirectional external flow U, the vorticity that diffuses from X_c can be advected away from [B]. Because of the converging flow, this detrained vorticity tends to be confined to 'wakes' whose width is small compared with h, as is observed below rising vortex rings (Turner 1963). Therefore, in such a flow over most of the exterior side of the interface [B], there is little shed vorticity so that the large scale interactions and the dynamics determining the response of [F2] to external perturbations is essentially inviscid, as we have assumed. In support of this hypothesis, one notes that in the above example of a deformed vortical region, the detrainment of vorticity by unsteady vortex induced motions is very similar to that produced in a slowly changing flow with viscous diffusion, as is also found in many other flows (e.g. Dritschel 1990).

3. DNS study

A direct simulation of transition induced by turbulent wakes incident on a laminar boundary layer has been performed as part of the ASCI/CITS program at Stanford. The ideas discussed in the previous section have been applied to that study. Details can be found in Wu *et al.* (1998).

4. Implications and future work

The analysis in §2 of external fluctuations, with long streamwise length scale, traveling with the flow has shown how they are distorted by the mean shear of the boundary layer so as to be blocked above the layer and to be diminished within the layer. This interaction is not covered by the receptivity theory of Goldstein & Wundrow (1998), which is relevant to the disturbances advected into the layer and inducing long wave Klebanoff mode instabilities there. The transition phenomena simulated here do not have a ready theoretical explanation – this requires a more detailed look at the simulated flow fields (for example, the form and the effective Reynolds number of the inflected profile induced below the traveling disturbance) and perhaps more simulations with different initial conditions. Nevertheless, it became clear that the even the reduced level of velocity perturbations induced by the external unsteady wakes was sufficient to trigger transition, depending on the amplitude of the free stream fluctuations caused by the wake eddies. For low amplitudes the types of instability induced by infinitesimal disturbances were simulated; but as their amplitude became large enough, the instabilities could grow to the non-linear stage within the time of passing of the finite length external disturbance – a quantitative estimate of this threshold is desirable. Once this threshold was reached, the transition process did not change when the frequency of the external disturbances (i.e., its average level but not its peak) was increased. This suggests a saturation level was reached that is consistent with non-linear, dynamical systems concepts (Reddy *et al.* 1998).

To discuss the continuing effect in our simulations of the external disturbances once the boundary layer had become fully turbulent, it is helpful to relate them to previous studies of the interactions between external turbulence and turbulent boundary layers. These can be categorized into three groups. When 'weak' external turbulent eddies have a scale L that is of the order of the thickness h of the boundary layer and are less energetic than those of the boundary layer (i.e., $u_o < u_*$), they are swept round by the swirling movements of the large eddies at the edge of the boundary layer and entrained; their energy adds slightly to that of the turbulence in the boundary layer. But if their scale is large, they are essentially blocked by the mean shear.

When the eddies are of moderate strength (i.e., $u_o > u_*$), where $L \sim h$, the vortex stripping process can operate, and, although this disperses the vorticity upwards, this effectively means that the outer vorticity is negligible, and the thickness of the boundary layer is reduced to a new level h/t where the mean shear is comparable to the external strain i.e., $du/dy \sim u_o/L$. This implies that the structure of the layer is changed to one where there is no outer 'wake' element, and only a 'log' law profile extends to the outer edge of the layer. As explained in §2.2, this is consistent with previous turbulence experiments. The simulations show that as the average vale of u_o is raised by increasing the frequency of the wake passing, the same trend in the profile is observed. This explanation needs to be tested in studies of the eddy structure in the outer region, for example, using interface sampling methods.

When the external eddies are much stronger than those of the undisturbed layer, (i.e. $u_* << u_o \sim U_\infty$), then its structure becomes more like that of a shear free turbulent boundary layer with the downdrafts and updrafts of the external eddies dominating the structure of the turbulence near the plate, including the surface shear. Thole & Bogard (1995) suggest that theoretical models and simulations (Perot & Moin 1995) for this limiting case are appropriate approximations when $u_o/U_\infty \sim 0.25$.

The next challenge is to investigate which of these results can be modeled with Reynolds averaged statistical equations. Some of the first attempts were reviewed by Pironneau *et al.* (1992).

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