Towards a near-wall model for LES of a separated diffuser flow

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1. Motivation and objectives

Recently it was shown that LES is capable of predicting incipient separation in an asymmetric diffuser (Fatica *et al.*, 1997; Kaltenbach *et al.*, 1998). Despite the low Reynolds number of the flow ($Re_\tau = u_\tau \delta/\nu = 500$ in the inlet duct of height 2δ, $Re_b = U_{bulk}\delta/\nu = 9000$) the computational effort required to obtain good quantitative agreement with measurements is considerable due to the wide range of spatial and temporal scales encountered which necessitate the use of fine meshes as well as lengthy integration times. Proper prediction of the mean velocity profile and turbulence statistics of the incoming developed turbulent channel flow turned out to be challenging using LES in which the near-wall region was resolved. It is desirable to reduce the cost of the simulation both for the diffuser and for the time series of inlet planes by circumventing the need to resolve the fine scale turbulence in the near wall region.

The goal of the present study is to investigate the diffuser flow to see if the wall-model based LES method (Cabot 1995, 1996, 1997 and related contributions in this volume) can be applied. The statistics from a well resolved LES of the diffuser flow are used to study the near-wall zone in order to see (`i`) what are the relevant terms in the mean momentum balance and (`ii`) whether the turbulent shear stress in the near-wall layer can be predicted by an algebraic eddy-viscosity model. Based on the outcome of these *a priori* tests, we discuss a model formulation which treats the near wall region primarily in the RANS spirit with emphasis on accurate specification of the mean turbulent stresses.

2. Accomplishments

2.1 Near-wall momentum balance

Following Cabot (1996, 1997) we use data from a well resolved diffuser LES to identify important terms in the tangential-to-the-wall momentum balance. Here we focus on the mean flow statistics as defined through RANS and *not* on statistics for a control volume of typical size used in wall model LES as in Cabot (1995). The streamwise momentum balance in coordinates locally tangential and normal to the walls is integrated from $y = 0$ to y_0, yielding

$$\underbrace{\int_0^{y_0} \frac{\partial \overline{U}^2}{\partial x} dy}_{Adv_x} + \underbrace{\int_0^{y_0} \frac{\partial \overline{u'}^2}{\partial x} dy}_{adv_x} + \underbrace{\overline{U}\,\overline{V}|_{y_0}}_{Adv_y} + \underbrace{\overline{u'v'}|_{y_0}}_{adv_y} + \underbrace{\int_0^{y_0} \frac{\partial p}{\partial x} dy}_{PG} \underbrace{-\tau_{SGS}|_{y_0} - \nu \frac{\partial U}{\partial y}|_{y_0} + \tau_w}_{Visc} = 0\,.$$

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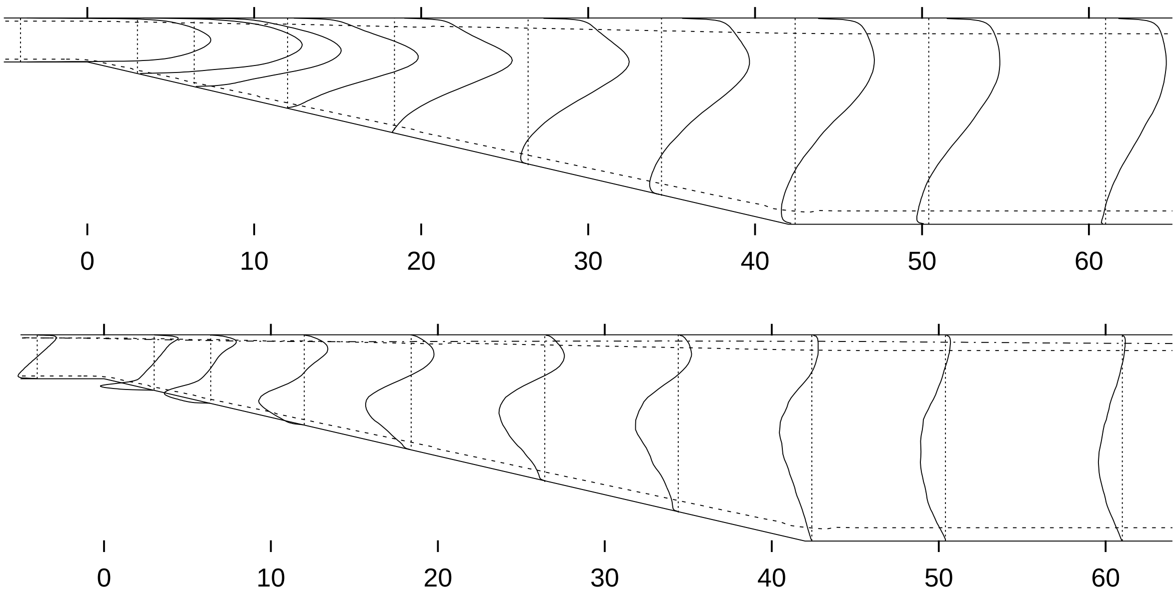

FIGURE 1. Profiles of mean streamwise velocity component (*top*) and shear stress $\overline{uv}$ (*bottom*) at stations $x/\delta = -4, 3, 6.4, 12, 18.4, 26.4, 34.4, 42.4, 50.4, 61$. The dashed lines mark the border of the near wall zone defined to be 7% of the local duct height, whereas the dash-dotted line corresponds to a distance of 70 wall units.

The choice of a meaningful wall distance y_0, which defines the zone in which the horizontal grid resolution is too coarse for LES to yield correct turbulent stresses, is not obvious in the present case. For the inlet duct a definition of y_0 in terms of a characteristic distance based on wall units is appropriate. As the flow separates along the inclined wall, this definition is no longer valid, and we have to find an alternate characteristic length scale.

Inside the inlet duct we define the near-wall zone as the wall-parallel layer in a distance of 70 wall units. For a developed turbulent channel flow, the peak values of the turbulence intensities and turbulent production lie inside this zone.

Figure 1 reveals that the flow inside the expansion is asymmetric with separation occurring on the inclined wall. Since the flow remains attached along the flat wall, the distance of the near wall zone could in principle be chosen based on the local value of u_τ and a thickness of 70 wall units. However, it would be wrong to conclude that this zone is of equal importance all along the flat wall. This can be seen from the position of maximum shear stress $|\overline{uv}|_{max}$, which moves out of the near wall zone into the core flow as the flow decelerates inside of the expansion (see Fig. 1).

Since a u_τ-based definition of a near-wall zone is ill-posed along the inclined wall, we use an alternate definition which simply states that y_0 corresponds to 7% of the local duct height. This definition is equivalent to a distance of 70 wall units along a considerable part of the flat wall (see Fig. 1). A significant part of the duct area lies inside of the near-wall zones, which now account for 14% of the total cross-section. This is a rather atypical property of the present flow, being entirely due to the low Reynolds number. However, for the purpose of meaningful testing of the wall-model LES approach, it is essential that the zone where the flow is being modeled is thick enough to allow significant reduction of horizontal spacings. As a guideline one can use the following rule: since the horizontal spacing defines the size of the smallest turbulent scales to be resolved, the thickness of the near-wall zone should correspond roughly to the average horizontal spacing.

Because of its atypical thickness, the near-wall zone carries considerable parts of the total mass and momentum fluxes. For this reason we consider it useful to separate the issue of defining the near-wall zone thickness y_0 from the question of adequate numerical approximation of the flow inside this zone. The momentum balance reveals that, unlike turbulent channel flow where wall models have principally been investigated, particularly strong contributions are provided by the mean flow advection terms Adv_x and Adv_y (see Fig. 2). This means that the wall-normal spacing of the mesh has to be reasonably fine to allow meaningful representation of mean flow profiles $\overline{U}(y)$ and $\overline{V}(y)$ for $y < y_0$. In case the near-wall solution is computed on a separate grid, representation of the mean flow inside this zone is essentially a 2D-problem. Note that this is different from the approach of Cabot (1997) where *each* grid cell in the x, z-plane has a locally refined mesh in the wall-normal direction for computing instantaneous values of $u(y)$ and $v(y)$ in the near-wall layer.

For the flow under investigation, no drastic computational savings are achieved by coarsening the mesh in the y-direction. Designing a mesh with the first off-wall line at $y^+ = 10$ is a fair compromise between mesh coarsening and proper representation of the mean flow solution near the wall. This mesh design has consequences for the model approach as outlined in section 2.2.

Figure 2 reveals the important role of mean flow advection for the near wall momentum balance. Turbulent shear stress and viscous stress roughly balance each other in the inlet duct. Near the rounded corner of the diffuser entrance, the magnitude of mean flow advection terms and of the pressure gradient term is substantially larger than in the rest of the domain, emphasizing the need for proper wall-normal resolution in order to represent $\overline{U}$ and $\overline{V}$. Along both walls, the term related to the streamwise turbulence intensity, adv_x, is of little significance except for a small region near the inlet. In the diffuser rear section, along the flat wall both mean momentum flux Adv_y and turbulent shear stress adv_y are equally important whereas along the inclined wall Adv_y has little significance. Our conclusion from this evaluation is the following: in certain regions of the diffuser flow, the near wall momentum balance depends not only on proper prediction (modeling) of the turbulent shear stress $\overline{uv}$, but also on accurate representation of the mean flow and the associated vertical flux of horizontal momentum, $\overline{U}\,\overline{V}|_{y_0}$.

2.2 Model prediction for shear stress in the near-wall region

A widely used model for the near-wall zone consists of prescribing values for instantaneous wall stresses $\tau_{w,xy}$ and $\tau_{w,zy}$ based on information from the interior and assuming that the logarithmic law of the wall is valid for the mean flow across the mesh cell adjacent to the wall. This approach works under the assumption that the last grid cell extends well into the logarithmic region, the first off-wall grid line y_N then being near $y_N^+ \approx 70$. In the control volume adjacent to the wall, the wall

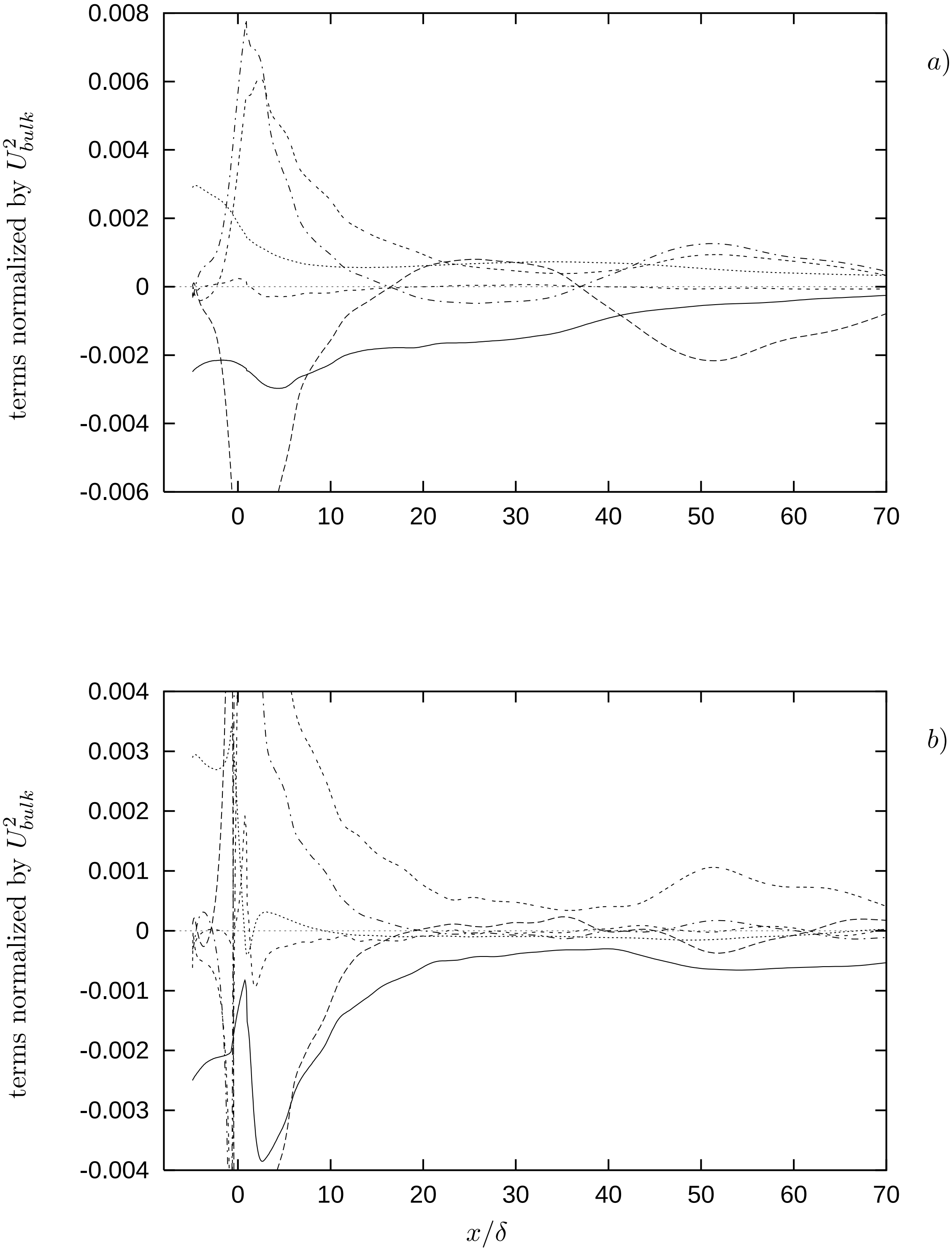

FIGURE 2. Terms of the near wall momentum balance evaluated along flat wall (a) and inclined wall (b) of the diffuser. Line code: Adv_x – – – – , adv_x - - - - , Adv_y —·— , adv_y ——— , PG --- --- , $Visc$ ········ .

normal derivative of the shear stress reduces to

$$\frac{\partial \tau_{xy}}{\partial y} \approx \frac{\tau_{xy}|_N - \tau_{xy}|_S}{\Delta_y} = \frac{-uv|_N + \nu dU/dy|_N + \tau_{SGS}|_N - \tau_{w,xy}}{\Delta_y} \quad ,$$

where the subscript N refers to the location of the first off-wall grid line. The prediction of the resulting force in the streamwise momentum balance depends on the ability of the simulation to capture the correct stresses $\tau_{xy}|_N$.

In cases where the wall-normal grid-spacing is much finer than the distance y_0 — for reasons outlined in the previous section — specification of τ_w only is likely to fail because the grid-scale turbulence (together with SGS stresses) cannot provide the correct stress τ_{xy} at the first off-wall grid line. (An example for this type of failure is given in section 2.3.) Adapting a near-wall model to this specific situation requires that additional "supporting shear stresses" are supplied inside the cells belonging to the near-wall zone. Inside this region the wall-parallel spacings Δx and Δz will be too coarse to support the correct near-wall turbulence structure, and as a result the resolved stresses $\overline{uv}$ will be too low, or the near-wall turbulence structures will be artificially amplified and distorted to provide the correct resolved stresses (Baggett, this volume). There is evidence (Baggett *et al.* 1997, Jiménez & Moser, 1998) that simple SGS models are *not* able to cope with this problem by supplying the missing part to the total shear stress. This has to do with the fact that inside the near-wall region the SGS model would have to carry the *major* part of the shear stress — a situation for which commonly used models are not designed.

Spalart *et al.* (1998) have proposed to modify the subgrid-scale eddy-viscosity in such a way that the missing shear stress (in the mean sense) is supplied by adding a RANS-type contribution to the SGS eddy viscosity inside the near-wall zone; see also Baggett (this volume). Another way of achieving this goal is to add a body force (possibly restricted to the wall-normal direction) to stimulate resolved-scale motion inside the near-wall zone in order to achieve the desired distribution of mean shear stress. This type of scheme has recently been applied successfully in numerical experiments related to delaying boundary layer separation (Driller, 1998).

We see two advantages for the proposed treatment of the near-wall zone for the flow under investigation: (i) a fine wall-normal resolution is desirable for accurate representation of the mean flow and the associated momentum flux $\overline{U}\,\overline{V}$ as outlined in the previous section; (ii) the formulation avoids a sharp interface between near-wall zone and "core" flow, thereby possibly improving the prediction for the core flow since the "supporting stresses" can gradually fade out with increasing distance from the wall.

A central question in this context is whether a good prediction of the near-wall mean shear stress $\overline{\tau}_{xy}$ is possible using a RANS formulation inside the near wall-layer which uses the running time-average of the core flow from the LES solution (cf. Lund *et al.*, 1998) as a boundary condition away from the wall. For this purpose we have started to compare the near-wall stress distribution from the fine-grid diffuser LES with model predictions. As a preliminary step in this direction, we compared the Johnson-King eddy viscosity model (abbreviated as JK model) since it is known

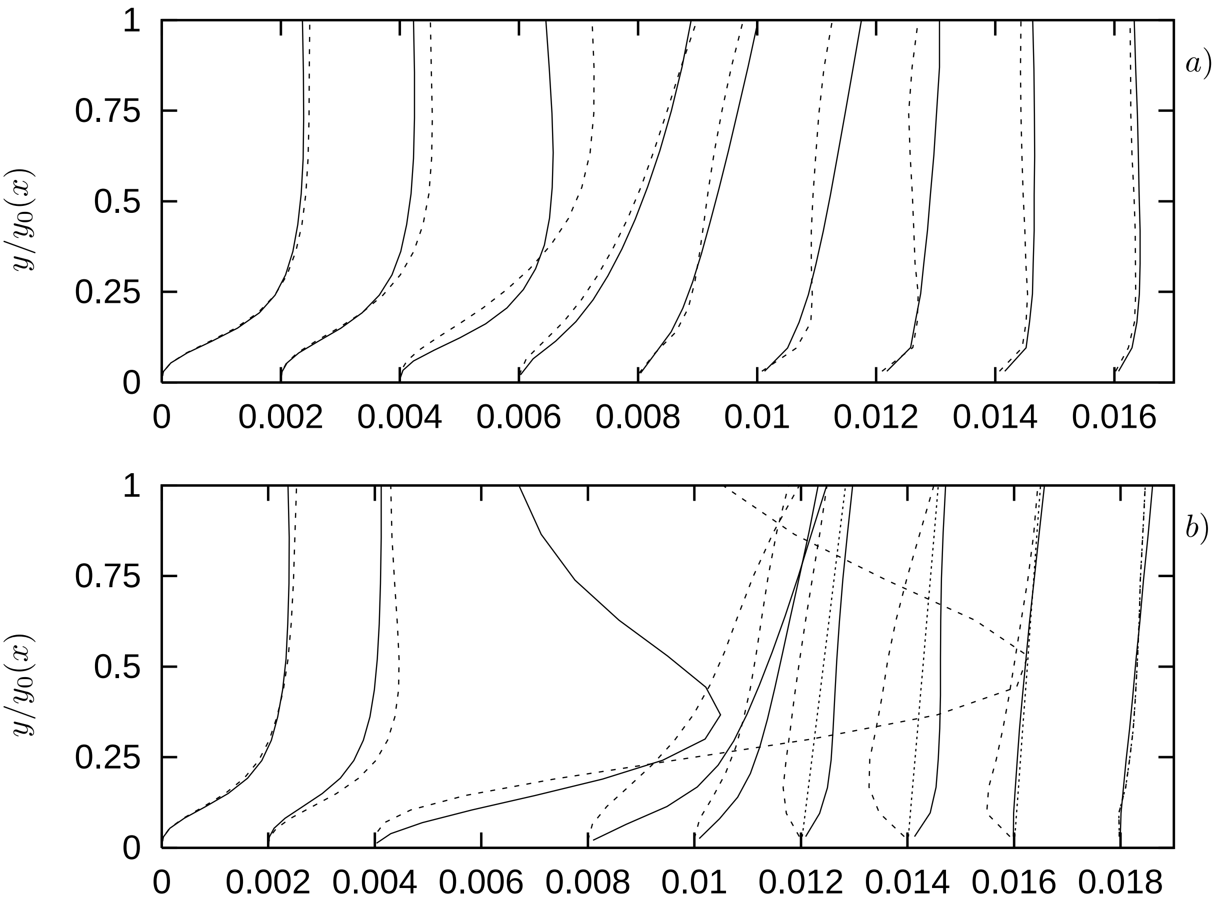

FIGURE 3. Comparison of turbulent shear stress $|\overline{uv} + \tau_{SGS}|$ from fine-grid LES —— with prediction from the Johnson-King model $\nu_t dU/dy$ ---- and a linear extrapolation $\overline{uv}_{max} y/y_{max}$ ········ at stations $x/\delta = -4.1$, -1.0, 1.0, 6.3, 14.2, 26.4, 38.1, 45.6, 65.8 along flat (a) and inclined wall (b). The distance from the wall has been normalized with the thickness y_0 of the near-wall zone.

to perform well in "mildly" separated flows. We use the formulation of Johnson & Coakley (1990) as outlined in Cabot (1995). The essential idea of this model is to use a blend of u_τ and $u_m = \sqrt{-\overline{uv}_{max}}$ as the velocity scale for the eddy viscosity. For the comparison we use the model constants $A = 19$ and $\kappa = 0.4$. Figure 1 shows that downstream of the inlet duct the peak of $\overline{uv}$-profiles lies outside of the near-wall region, thus u_m could in principle be determined from the running time-average of the core flow.

Figure 3 reveals that the JK model gives a satisfactory prediction of shear stresses in a considerable fraction of the diffuser domain. The model performs well along the entire flat wall where the flow remains attached. However, along the inclined wall we find serious deficiencies of the model as soon as the flow enters the expansion. A striking feature of the flow along the inclined wall is the drastic increase in turbulent shear stress close to $x/\delta = 1$. There, the JK model gives the right trend but overpredicts maximum stresses by a factor of two. Ahead of the location of mean separation and early into the separated flow region, the JK model underpredicts the stresses. It fails completely in the region where backflow occurs because of the inability of an eddy-viscosity approach to cope with "countergradient" momentum transfer, which occurs close to the wall inside of the separated flow region. There, a simple linear extrapolation of the $\overline{uv}$-profile from the location y_m of maximum turbulent stress down to the wall is superior but can only be regarded as a rough estimate for the shear stress in the near-wall region. Note that the near-wall stresses are of less importance in the separated region since their magnitude is small when compared to the maximum shear stress in the core flow.

This evaluation shows that the JK model has some potential for use in predicting the near-wall shear stresses. Still, certain regions of the diffuser exhibit stress distributions near the wall which require a more advanced RANS model. However, before other models should be tested, it is desirable to find out whether specification of the proper mean shear stress distribution near the wall is sufficient for the core flow to behave as in the fine grid LES.

2.3 Inflow boundary conditions

One of the main findings of the original LES of the diffuser flow with resolved near-wall regions is that proper prediction of the flow inside the expansion depends crucially on the quality of the flow in the inlet duct. There exist two methods for creating unsteady Dirichlet boundary conditions to be specified at the inlet plane of a wall-model based LES of the diffuser: (`i`) extracting planes from an independent channel simulation, which by itself uses a near-wall model and therefore matches the diffuser grid at the inlet; (`ii`) interpolate available inflow from a fine-grid channel flow database onto a coarser mesh.

Here we focus on the second approach primarily because we want to be as close as possible to the conditions of the fine-grid LES which are in good agreement with experiments. The underlying idea of this method is that, by using data from a finer resolved case, the structure of the incoming "turbulence" is preserved over a considerable distance downstream of the inlet plane since the flow does not immediately feel the adverse effect of having a coarsened mesh.

To test how well this strategy would work in the diffuser, we interpolated the available time series of instantaneous u, v, w-slices onto a mesh which was considerably coarser in the y- and z-directions. The mean flow profile is well preserved, but we find that omitting half the number of modes in the spanwise direction causes a significant drop in the peak values of the turbulence intensity and shear stress profiles. Ideally, the SGS-model should make up for the missing stresses, but *a posteriori* tests show that it cannot do so.

We have used the filtered inflow database to simulate a short stretch of the inlet duct using $64 \times 40 \times 64$ cells. The streamwise spacing was either $\Delta x = 0.06\delta$ ($\Delta x^+ = 30$) as in the original fine grid LES or $\Delta x = 0.2\delta$ ($\Delta x^+ = 100$). We present results from four cases, two for each grid consisting of a run without a wall-model and a simulation with an instantaneous wall-stress boundary condition which guarantees that the mean flow experiences the correct wall-stress corresponding to the pressure drop of the fine-grid channel flow LES. In the latter cases, the instantaneous wall

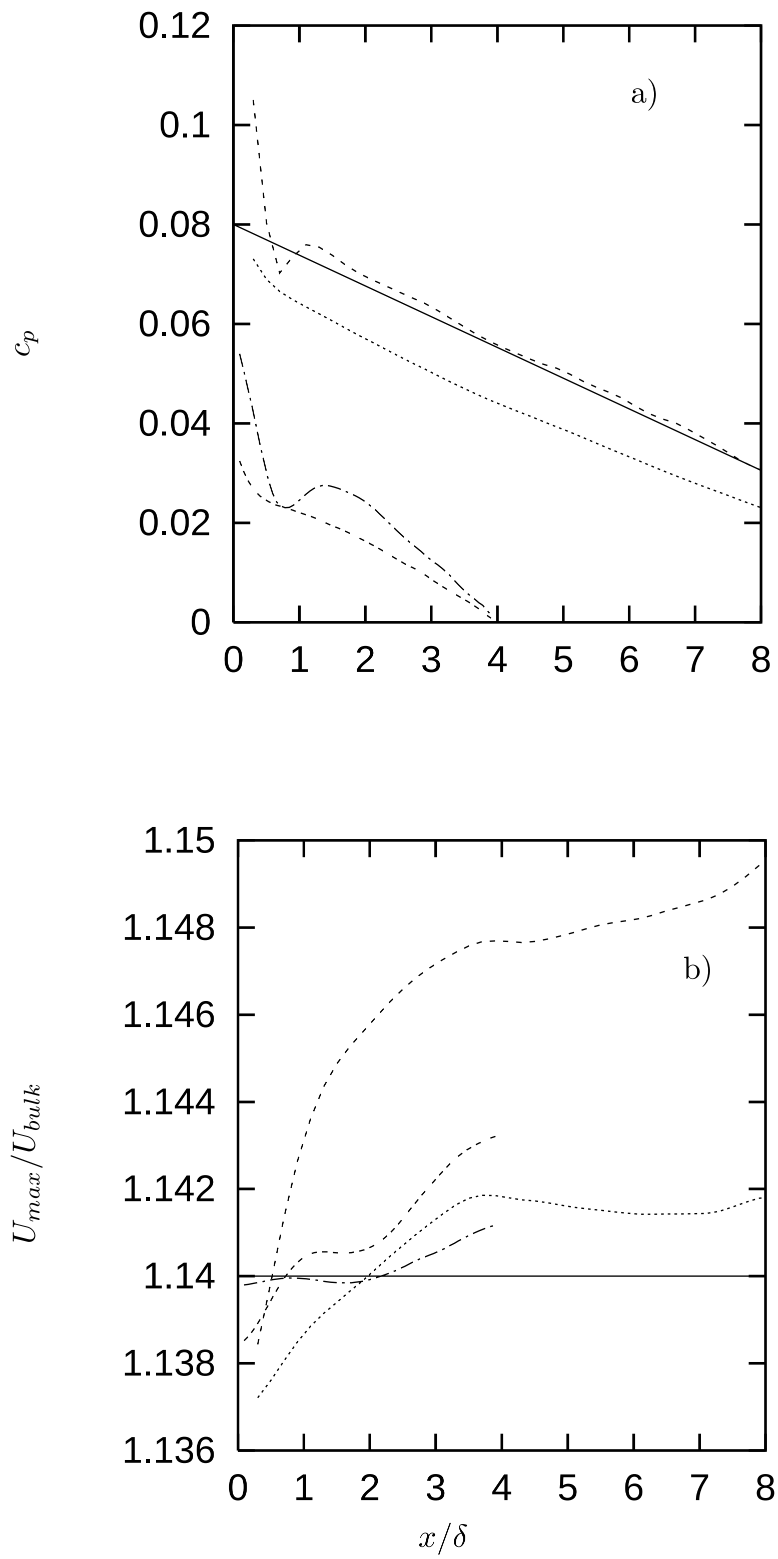

FIGURE 4. Streamwise development of pressure drop (a) and velocity ratio (b) in channel flow using filtered inflow data. Line coding: fine grid LES (target) ——— , CNM ········ , CWM – – – – , FNM --- --- , FWM —·— .

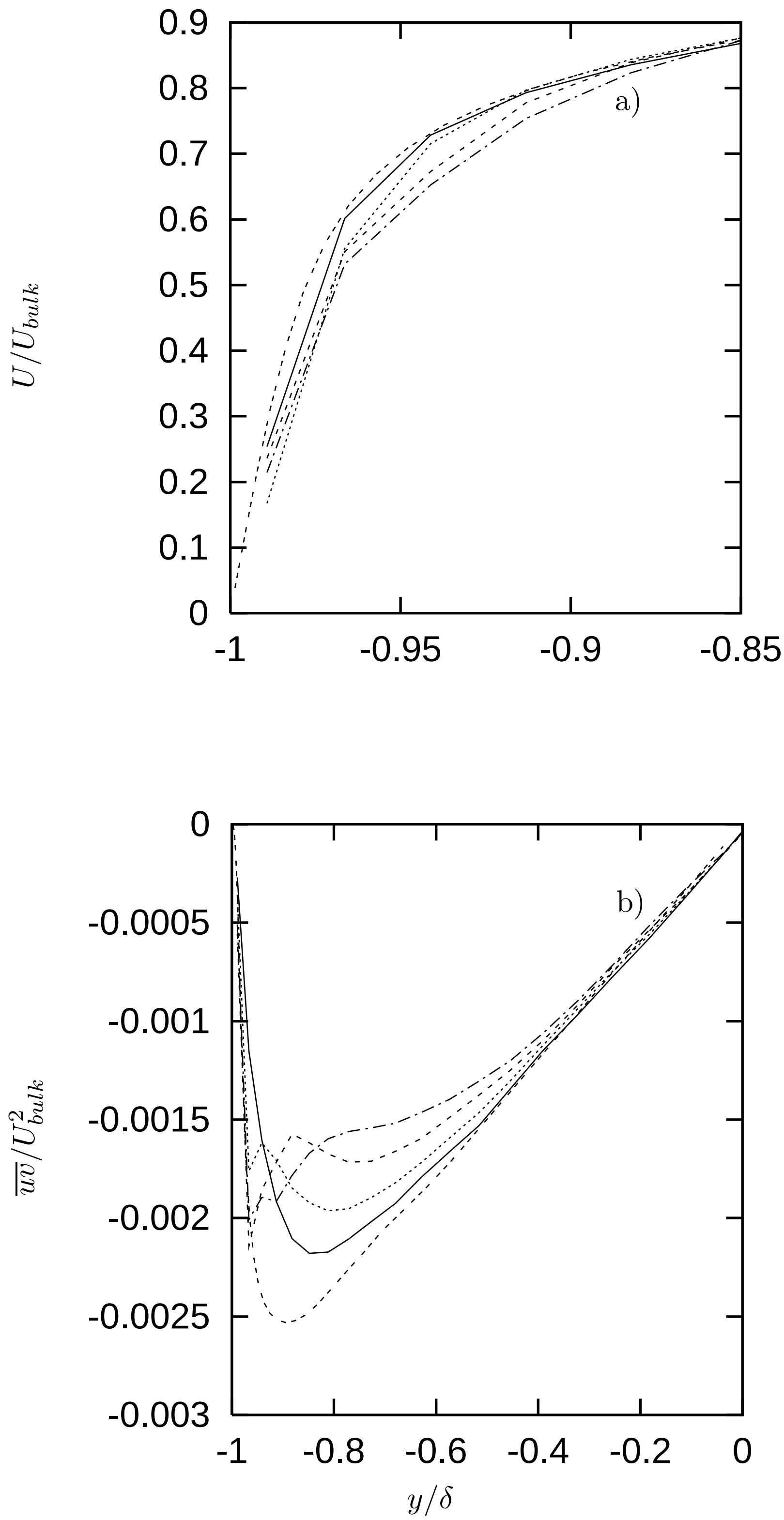

FIGURE 5. Profiles of $\overline{U}(y)$ (a) and $\overline{uv}$ (b) from case FWM at distances $x/\delta = 0.2$ ——— , 0.8 ········ , 2. – – – – , 3.4 —·— . Profiles from the fine-grid LES are marked as - - - - - - .

stress was constructed using the instantaneous horizontal velocity at the first off-wall position (cf. Wu & Squires, 1998). The four cases are denoted CNM, CWM, FNM, FWM with C, F denoting *coarse* and *fine* grids, respectively, and with NM, WM denoting *no model* or *wall model*, respectively.

Figure 4 compares c_p curves and the development of the mean flow profile shape. All the simulations experienced an unnatural pressure drop close to the inflow plane, which we contribute to changes in the mean profile shape and the associated momentum fluxes. Surprisingly, cases with the correct average wall stress experience the largest drop for which we do not yet have an explanation. Case CWM is the only one which — after considerable readjustments — reaches the correct pressure drop corresponding to the prescribed wall stress. As a result of underprediction of the wall-stress in case CNM, the pressure gradient is lower than the target value. Because the mean flow decelerates near the wall (which is caused by a deficit of shear stress inside the near-wall layer), the core flow speeds up, leading to an increase of the centerline to bulk velocity ratio. Figure 5 shows the corresponding profile changes for case FWM. Coarse grid cases CWM and CNM show larger profile shape deficits than cases FWM and FNM. The coarse (C) cases experience an additional problem which has to do with the staggered variable configuration at the inlet plane: since the inflow database was created with u extracted half of a fine-grid mesh cell downstream of v and w, it is incorrect to feed these data into a domain which has considerably coarser streamwise spacing. It should be investigated whether this problem can be alleviated by invoking Taylor's hypothesis to shift u-slices to a position which is consistent with the coarse mesh staggered variable configuration.

Our conclusion from this test is that for the present configuration supplying the correct wall-stress is not sufficient in order to predict the mean flow profile with the required accuracy. Additional modifications are required to guarantee that the flow experiences the correct total turbulent shear stress inside the near-wall zone in order to reproduce the correct flow inside the inlet duct. It is useless to attempt a full diffuser simulation with a wall-model before this problem is solved.

3. Future plans

At this stage it is desirable to know whether the proposed model concept, i.e. guaranteeing that the flow in the near-wall zone experiences the correct turbulent shear stress in the time-averaged sense, is sufficient for the core flow to render the correct results. For this purpose we plan a simulation where the exact average stresses from the fine-grid diffuser LES are used as target values for the near-wall zone of a coarse LES. Some details about the best way to prescribe or excite the "supporting stresses" have to be sorted out first. If the outcome of this test is satisfactory, we will proceed by coupling the LES with a RANS-based prediction method for the near-wall shear stress distribution.

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