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Sensitivity of flame transfer functions of laminar flames

By **F. Duchaine**† and **T. Poinsot**†

The sensitivity of laminar premixed methane/air flames submitted to acoustic forcing is investigated using direct numerical simulations of two experimental burners. Five parameters are varied: (1) the flame speed s_L, (2) the expansion angle of the burnt gases α, (3) the inlet air temperature T_a, (4) the inlet duct temperature T_d, and (5) the combustor wall temperature T_w. Flame response is characterized by the delay of the flame transfer functions. Stationary flames are first computed and compared with experimental data. It is then shown that the numerical simulations reproduce the time delays of the flame transfer functions. The sensitivity analysis of the flame transfer function delay is performed using simple differentiation methods by changing one parameter only and measuring its effect. Results reveal the predominant role of the flame speed s_L as well as of the inlet duct temperature T_d.

Motivations and objectives

The prediction of acoustically coupled instabilities has become a major issue in combustion (Poinsot & Veynante 2005; Lieuwen & Yang 2005). Numerous authors have proposed approaches to predict the resonant modes between acoustics and combustion (Crighton *et al.* 1992; Culick 1994; Selle *et al.* 2004; Pankiewitz & Sattelmayer 2003). In all theories, a crucial ingredient is the flame transfer function (FTF) first introduced by (Crocco 1951, 1969) and (Tsien 1952). In its simplest form, the FTF $F(\omega)$ measures the response of the global unsteady reaction rate in the flame (q'/q) to an inlet velocity perturbation (u'/u):

$$F(\omega) = \frac{q'/q}{u'/u} \tag{0.1}$$

Although many of these studies were performed for complex geometry turbulent burners (Lieuwen 2003; Selle *et al.* 2004; Giauque *et al.* 2005; Hemchandra & Lieuwen 2007), they are usually limited and difficult to extrapolate to other regimes or other geometries because turbulent systems combine the difficulties of acoustic/flame coupling and turbulent flows. To isolate the mechanisms controlling FTF results, many groups have started investigating simpler laminar flames where the validity of acoustic/combustion theories can be tested in the absence of complex turbulent effects (Truffin & Poinsot 2005; Lieuwen & Neumeier 2002; Schuller *et al.* 2003). Studies dedicated to the FTF of laminar flames in multiple configurations (Noiray *et al.* 2008; Kornilov *et al.* 2009; Coats *et al.* 2010; Boudy *et al.* 2010) are now available, providing both experimental and numerical methods to obtain FTFs.

In all these configurations, the values obtained for the FTFs parameters are a gain n and a phase ϕ (or delay τ), which depend on the forcing frequency ω. These parameters are critical to predict the mode stability in acoustic solvers (Crocco 1951; Schuermans

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† IMF Toulouse, INP de Toulouse and CNRS, 31400 Toulouse , France

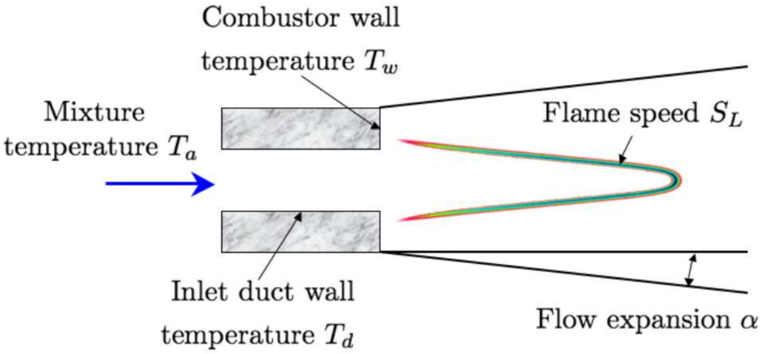

FIGURE 1. Parameters controlling the FTF of a laminar premixed flame

et al. 2006; Nicoud *et al.* 2007; Sensiau *et al.* 2009). Very small errors on the phase ϕ can lead to drastic changes in stability so that the question of uncertainties in measurement and simulation of FTF becomes an interesting issue. When computing the FTF of a flame for example, being able to evaluate the sensitivity of the results to modeling parameters is a critical question. For example, Kaess *et al.* (2008) computed the FTF of a laminar flame and concluded that an accurate computation was impossible without the knowledge of the temperature of the stabilizing plate.

In this paper, the FTFs of laminar premixed flames were computed using DNS to evaluate the influence of five critical input parameters (Fig. 1): (1) the flame speed s_L, (2) the shape of the domain characterized by its expansion angle α, (3) the inlet air temperature T_a, (4) the inlet duct temperature T_d, and (5) the combustor wall temperature T_w.

All these parameters have a direct effect on the FTF delay τ (or phase ϕ). The flame speed s_L obviously controls the flame length and the delay to react to velocity changes. The shape of the domain determines the expansion of the burnt gases and the flow velocity, thereby also changing the FTF delay, here it is supposed to have a conical shape of angle α. Many experiments (and computations) are designed to perfectly match periodic arrays of flame (Noiray *et al.* 2008; Kornilov *et al.* 2009) where α should be zero. In practice however, these flames are only partially confined, the gases produced by each individual flame can expand both in the axial and transverse directions. This can be accounted for in the DNS by using an expanding computation domain. Values of α up to ten degrees are commonly observed experimentally. The inlet air temperature T_a affects both the gas velocity and the flame speed, whereas the inlet duct temperature T_d will change the temperature and velocity profiles at the burner inlet. The combustor lateral walls temperature T_w determine the lift-off of the flame and can also control the FTF delay.

The objective of this work is to determine the sensitivity of the FTF to these five parameters. This identification is done using simple differentiation methods with a linear approximation of the derivatives (i.e. changing only one parameter and measuring its effect on the FTF delay). The exercise is performed on two recent laminar flame experiments at atmospheric pressure (Fig. 2) for which extensive sets of experimental results are available. The experiment of Boudy (Fig. 2 left) corresponds to #49 flames stabilized on a perforated plate while the configuration of Kornilov (Fig. 2 right) corresponds to an array of slot flames. Both flames use methane as fuel. The equivalence ratio for the Boudy experiment is 1.03, whereas it is 0.8 for the Kornilov cases used here.

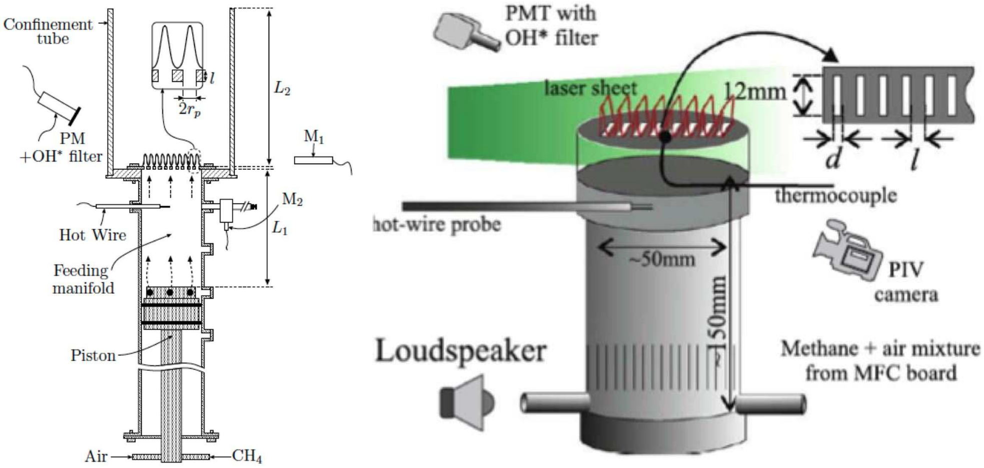

FIGURE 2. The two laminar flame experiments computed in this work. Left: the experiment of (Boudy *et al.* 2010). Right: the experiment of (Kornilov *et al.* 2009).

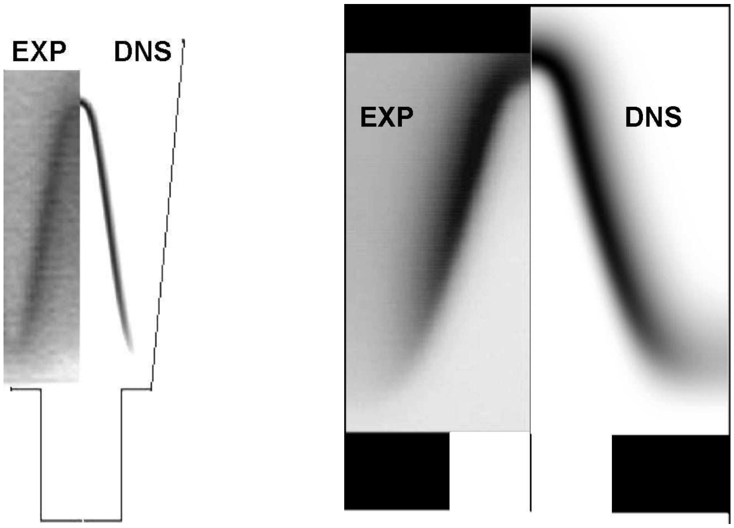

FIGURE 3. Comparison of experiment (left) and DNS (right) result for the steady flames. Left: experiment of (Boudy *et al.* 2010). Right: experiment of (Kornilov *et al.* 2009).

Methodology

For both configurations, a single steady laminar flame is first computed using DNS and a two-step chemical scheme (2S-CM2) for methane/air combustion (Selle *et al.* 2004). Computational domains are displayed in Fig. 3 for specific parameters listed in Tables 1 and 2. The computation of the conical flames of Boudy is performed with a two dimensional axisymmetric mesh and the case of Kornilov in two dimensions.

Figure 3 displays the heat release fields obtained by DNS for the steady Boudy and Kornilov flames and compares them with experimental results. For these cases, both flames are slightly lifted from the anchoring plate and their height matches experimental data reasonably well. Precise validations of DNS results have been done on both steady flames but are not shown here.

s_L (flame speed, cm/s)	35.8
α (domain expansion, degrees)	5
T_a (inlet air temperature, K)	293
T_d (duct wall temperature, K)	293
T_w (combustor lateral wall temperature, K)	430

TABLE 1. Parameters for the baseline computations of the Boudy configuration.

s_L (flame speed, cm/s)	27.4
α (domain expansion, degrees)	0
T_a (inlet air temperature, K)	293
T_d (duct wall temperature, K)	373
T_w (combustor lateral wall temperature, K)	373

TABLE 2. Parameters for the baseline computations of the Kornilov configuration.

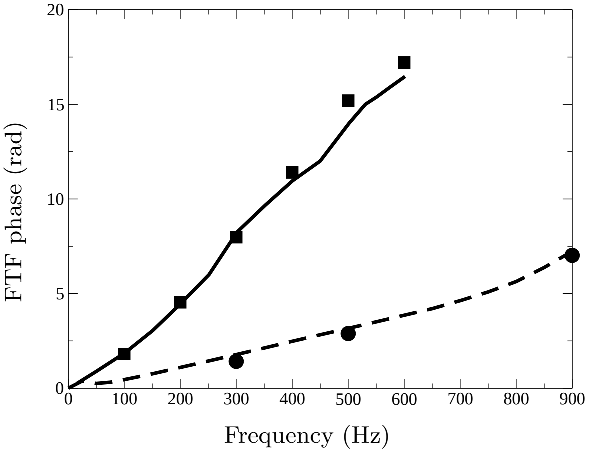

FIGURE 4. Flame transfer functions results. Comparison of experimental and DNS results. Solid line: Kornilov experiment, filled squares: Kornilov DNS. Dashed line: Boudy experiment, circles: Boudy DNS.

FTFs

An example of FTFs results is given in Fig. 4 for both configurations using the parameters given in Tables 1 and 2. For these sets of parameters, the agreement between experiments and DNS is excellent. Note however that results can be much worse for other sets of parameters, as shown in the next sections, by the high sensitivity of delays to input parameters.

Sensitivity	Units	Kornilov	Boudy
$\frac{\partial \phi}{\partial s_L}$	(rad/cm/s)	-68.2	-7.1
$\frac{\partial \phi}{\partial \alpha}$	(rad/degrees)	0.09	0.03
$\frac{\partial \phi}{\partial T_a}$	(rad/K)	-0.05	-0.01
$\frac{\partial \phi}{\partial T_d}$	(rad/K)	-0.02	-0.004
$\frac{\partial \phi}{\partial T_w}$	(rad/K)	-0.001	-0.0003

TABLE 3. Sensitivity of the phase ϕ of the FTF versus computations parameters.

Sensitivity results

The computations described in the previous sections were repeated for multiple values of the parameters of Tables 1 and 2 to obtain values of the sensitivity of the delay (expressed here as a phase ϕ) versus each parameter. Results are given in Table 3 at a fixed forcing frequency of 500 Hz.

If typical error margins are known for each parameter, Table 3 allows a diagnostic of what is important and what is not in these simulations. For flame speeds (s_L), typical errors Δs_L are of the order of 2 cm/s even with the best present chemical schemes. The expansion angles (α) of flame domains can be approximately measured from experiments and are of the order of 4 degrees with an error margin $\Delta\alpha$ of the order of 1 degree. The gas inlet temperature is generally controlled accurately, typically within 2 K. The duct temperature T_d is very difficult to evaluate and errors of the order of 50 K must be expected. Similarly the wall temperature T_w is usually known within a 50 K margin.

Gathering these uncertainties with the sensitivities of Table 3 leads to Table 4, which reveals that absolute errors are all larger for the Kornilov experiment than for the Boudy case. Nevertheless, as the values of the delay are smaller for the Boudy case (Fig. 4), the relative errors are of the same order for the two cases. In other words, both experiments are almost equally sensitive to input parameters. Moreover, certain parameters such as the combustor wall temperature T_w (which control the flame lift-off) have almost no influence in both cases, and it is not worth spending time trying to determine them. Flow expansion (α) also has a limited effect. The air temperature has a rather high sensitivity (Table 3) but it is usually well known, leading to small absolute errors. On the other hand, two parameters play a significant role:

- The flame speed s_L has a direct effect on the delay. Unfortunately, this is typically a quantity that is not well known and difficult to specify with precision. In the present computation, s_L is not specified directly because finite rate chemistry is used, but it is a direct function of the preexponential constants used in the chemistry description. The main problem here is that experiments do not allow flame speed measurements within a 2 cm/s range for hydrocarbon flames so that it is difficult to adjust kinetic models for DNS.
- The duct wall temperature T_d induces significant errors on the FTF phase ϕ because it is difficult to evaluate precisely. For both cases (Kornilov and Boudy) it is essential to know the wall temperature of the inlet duct to predict the phase correctly. Moreover

Absolute error (rad)	Kornilov	Boudy
Due to errors on s_L	-1.36	-0.14
Due to errors on α	0.36	0.12
Due to errors on T_a	-0.10	-0.03
Due to errors on T_d	-0.96	-0.18
Due to errors on T_w	-0.007	-0.02

Relative error (%)	Kornilov	Boudy
Due to errors on s_L	-9.0	-4.9
Due to errors on α	2.4	4.2
Due to errors on T_a	-0.7	-0.9
Due to errors on T_d	-6.4	-6.3
Due to errors on T_w	-0.4	-0.6

Table 4. Absolute and relative errors on the phase ϕ of the FTF versus computations parameters.

for the present cases, the duct temperature was assumed to be the same everywhere, in practice, it could also vary with spatial position.

The importance of the duct wall temperature comes from multiple facts. The premixed gas passing through the inlet duct is heated significantly by the hot walls. The gas velocity increases (because of the reduced density) and the local flame velocity also increases. These two factors modify the velocity field and the response of the flame to pulsations.

Conclusions

Two laminar premixed flames (Boudy *et al.* 2010; Kornilov *et al.* 2009) have been used to compute FTF (flame transfer function) and evaluate the sensitivity of the phase of the FTF to five inlet parameters: (1) the flame speed, (2) the shape of the domain characterized by its expansion angle, (3) the inlet air temperature, (4) the inlet duct temperature, and (5) the combustor wall temperature. Results show that the most important parameters controlling the delay of the FTF are the flame speed and the temperature of the inlet duct. A direct implication of this result is that coupled computations of flame and heat transfer through the stabilization plate are needed to obtain T_d and be able to predict FTFs in such configurations.

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REFERENCES

Boudy, F., Durox, D., Schuller, T., Jomaas, G. & Candel, S. 2010 Describing function analysis of limit cycles in a multiple flame combutor. In *ASME Turbo expo* (ed. GT2010-22372). Glasgow, UK.

Coats, C., Chang, Z. & Williams, P. 2010 Excitation of thermoacoustic oscillations by small premixed flames. *Combust. Flame* **157**, 1037–1051.

Crighton, D. G., Dowling, A. P., Williams, J. E. F., Heckl, M. & Leppington, F. 1992 *Modern methods in analytical acoustics*. New-York: Springer Verlag.

Crocco, L. 1951 Aspects of combustion instability in liquid propellant rocket motors. part i. *J. Am. Rocket Soc.* **21**, 163–178.

Crocco, L. 1969 Research on combustion instability in liquid propellant rockets. In *12th Symp. (Int.) on Combustion*, pp. 85–99. The Combustion Institute, Pittsburgh.

Culick, F. E. C. 1994 Some recent results for nonlinear acoustic in combustion chambers. *AIAA J.* **32** (1), 146–169.

Giauque, A., Selle, L., Poinsot, T., Buechner, H., Kaufmann, P. & Krebs, W. 2005 System identification of a large-scale swirled partially premixed combustor using LES and measurements. *J. Turb.* **6** (21), 1–20.

Hemchandra, S. & Lieuwen, T. 2007 Response of turbulent premixed flames to harmonic acoustic forcing. *Proc. Combust. Inst.* **31**, 1427–1434.

Kaess, R., Polifke, W., Poinsot, T., Noiray, N., Durox, D., Schuller, T. & Candel, S. 2008 Cfd-based mapping of the thermo-acoustic stability of a laminar premix burner. In *Proc. of the Summer Program* , pp. 289–302. Center for Turbulence Research, NASA AMES, Stanford University, USA.

Kornilov, V., Rook, R., ten Thije Boonkkamp, J. & de Goey, L. 2009 Experimental and numerical investigation of the acoustic response of multi-slit bunsen burners. *Combust. Flame* pp. 1957–1970.

Lieuwen, T. 2003 Modeling premixed combustion-acoustic wave interactions: A review. *J. Prop. Power* **19** (5), 765–781.

Lieuwen, T. & Neumeier, Y. 2002 Nonlinear pressure-heat release transfer function measurements in a premixed combustor. *Proc. Combust. Inst.* **29**, 99–105.

Lieuwen, T. & Yang, V. 2005 Combustion instabilities in gas turbine engines. operational experience, fundamental mechanisms and modeling. In *Prog. in Astronautics and Aeronautics AIAA*, , vol. 210.

Nicoud, F., Benoit, L. & Sensiau, C. 2007 Acoustic modes in combustors with complex impedances and multidimensional active flames. *AIAA J.* **45**, 426–441.

Noiray, N., andT. Schuller, D. D. & Candel, S. 2008 A unified framework for nonlinear combustion instability analysis based on the flame describing function a unified framework for nonlinear combustion instability analysis based on the flame describing function. *J. Fluid Mech.* **615**, 139–167.

Pankiewitz, C. & Sattelmayer, T. 2003 Time domain simulation of combustion instabilities in annular combustors. *ASME J. Eng. Gas Turbines Power* **125** (3), 677–685.

Poinsot, T. & Veynante, D. 2005 *Theoretical and Numerical Combustion*. R.T. Edwards, 2nd edition.

Schuermans, B., Paschereit, C. & Monkiewitz, P. 2006 Non-linear combustion instabilities in annular gas-turbine combustors. In *44th AIAA Aerospace Sciences Meeting and Exhibit, AIAA paper 2006-0549*.

Schuller, T., Durox, D. & Candel, S. 2003 A unified model for the prediction of laminar flame transfer functions: comparisons between conical and v-flames dynamics. *Combust. Flame* **134**, 21–34.

Selle, L., Lartigue, G., Poinsot, T., Koch, R., Schildmacher, K.-U., Krebs, W., Prade, B., Kaufmann, P. & Veynante, D. 2004 Compressible large-eddy simulation of turbulent combustion in complex geometry on unstructured meshes. *Combust. Flame* **137** (4), 489–505.

Sensiau, C., Nicoud, F. & Poinsot, T. 2009 A tool to study azimuthal and spinning modes in annular combustors. *Int. J. Aeroacoustics* **8** (1), 57–68.

Truffin, K. & Poinsot, T. 2005 Comparison and extension of methods for acoustic identification of burners. *Combust. Flame* **142** (4), 388–400.

Tsien, H. S. 1952 The transfer functions of rocket nozzles. *J. Am. Rocket Soc.* (May-June), 139–143.