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Problems of astrophysical turbulent convection: A simple model of the Evershed effect

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The Evershed effect is an outflow in the penumbra of sunspots. It has been proposed that this flow is generated locally by the interaction of convection and an inclined magnetic field. The oscillatory instability of convection rolls in a low Prandtl number fluid gives rise to the generation of Reynolds and Maxwell stresses that cause a shear flow in the form of the Evershed effect.

1. Introduction

Since its discovery by John Evershed (1909) more then 100 years ago the strong outflow observed in the penumbra of sunspots has been a puzzling phenomenon. Numerous theoretical partially conflicting models have been proposed and the observations also have been controversial in many instances. Recently, computer capacities have become large enough that realistic simulations of flows in sunspot penumbrae can be attempted (Kitiashvili *et al.* 2009) and high-resolution observations from space crafts (Ichimoto 2009) have become available. These developments have encouraged the author to return to an old analytical model of the Evershed effect (Busse 1988) which appears to be useful for understanding the basic physics of the problem in light of new information that has become available.

The idea of the model is to use a minimum of ingredients that are sufficient to generate, without further assumptions, a mean flow such as the Evershed effect. For this reason, effects of compressibility will be neglected and the Boussinesq approximation for the description of thermal convection will be employed. The assumption of stress-free boundaries of the layer allows for simple solutions of the equations of motion in terms of trigonometric functions. Through this procedure, analytical expressions of nonlinear properties such as Reynolds and Maxwell stresses can be obtained from which the Evershed effect originates. A quantitative comparison with observations is not attempted.

The problem to be studied in this paper is also of interest from a general point of view. Turbulent states of fluids often exhibit coherent structures in the form of large-scale flows that are absent in the corresponding laminar state of the problem. Such flows often originate from mean Reynolds stresses generated by small-scale turbulence. Such stresses would vanish, of course, in the case of isotropic turbulence, but they may become finite in the case of anisotropic systems. An example is the mean shear generated by three-dimensional convection in a plane-horizontal fluid layer rotating about an axis that is inclined with respect to the vertical (Busse 1982, 1983; Hathaway & Somerville 1983). In this case no mean flow is generated close to the onset of convection because the two-dimensional convection rolls align themselves with the direction of the horizontal component of the rotation vector. The rolls thus do not differ from the rolls that would be found in the case of a purely vertical rotation vector. But as a transition to a three-

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dimensional form of convection occurs at higher values of the Rayleigh number, the anisotropy of the rotating layer manifests itself in the generation of a mean shear.

In the problem considered by Busse (1988) and extended in the present paper, the role of the rotation vector is replaced by the role of an imposed uniform inclined magnetic field. The inclination is essential in both cases because it breaks the symmetry in the horizontal directions and creates the horizontal anisotropy.

In the following section the mathematical problem is described. Solutions for finite amplitude convection rolls are given in section 3. The oscillatory instability is discussed in section 4 and its evolution at finite amplitudes is analyzed in terms of a perturbation approach in section 5. The paper ends with a discussion of the results in section 6.

2. Mathematical formulation of the problem

We consider an infinitely extended horizontal fluid layer of height d with the fixed temperatures T_1 and T_2 ($T_2 > T_1$) at the upper and lower boundary, respectively. The fluid layer is electrically conducting and is intersected by a homogeneous magnetic field with a flux density B_0 at an angle χ with respect to the horizontal. We adopt the Boussinesq approximation, i.e., all material properties are assumed to be constant except for the temperature dependence of the density, which is taken into account only in connection with the gravity term. Using as scales the length d; the time d^2/ν, where ν is the kinematic viscosity of the fluid; the temperature $(T_2-T_1)P/R$; and the magnetic flux density B_0 we obtain the dimensionless equations of motion for the velocity vector $\boldsymbol{u}$, the heat equation for the deviation Θ from the static temperature distribution, and the equation of magnetic induction for $\boldsymbol{B}$

$$(\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}) = -\nabla\pi + R\Theta\boldsymbol{k} + \nabla^2\boldsymbol{u} + \frac{Q\lambda}{\nu}\boldsymbol{B}\cdot\nabla\boldsymbol{B}, \tag{2.1a}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{2.1b}$$

$$P(\partial_t\Theta + \boldsymbol{u}\cdot\nabla\Theta) = R\boldsymbol{u}\cdot\boldsymbol{k} + \nabla^2\Theta, \tag{2.1c}$$

$$\partial_t\boldsymbol{B} + \boldsymbol{u}\cdot\nabla\boldsymbol{B} = \boldsymbol{B}\cdot\nabla\boldsymbol{u} + \frac{\lambda}{\nu}\nabla^2\boldsymbol{B}, \tag{2.1d}$$

$$\nabla\cdot\boldsymbol{B} = 0, \tag{2.1e}$$

where ∂_t denotes the partial derivative with respect to time t and where all terms in the equation of motion that can be written as gradients have been combined into $\nabla\pi$. The dimensionless parameters, the Rayleigh number R, the Chandrasekhar number Q, and the Prandtl number P are given by

$$R = \frac{\gamma g(T_2-T_1)d^3}{\nu\kappa}, \qquad Q = B_0^2 d^2/\rho_0\mu\lambda\nu, \qquad P = \frac{\nu}{\kappa}, \tag{2.2}$$

where γ is the coefficient of thermal expansion, g is gravity, and κ and λ are the thermal and magnetic diffusivities, respectively. We use a cartesian system of coordinates with the z-coordinate and the unit vector $\boldsymbol{k}$ in the direction opposite to gravity and the x-coordinate in the direction of the horizontal component of the imposed magnetic field. We use stress-free boundary conditions for the velocity field and require that Θ vanishes at the upper and lower boundaries,

$$u_z = \partial^2_{zz}u_z = \Theta = 0 \qquad \text{at} \quad z = \pm\frac{1}{2}. \tag{2.3}$$

In order to eliminate the continuity Eqs. (2.1b) and (2.1e), we introduce the general representations for the solenoidal vector fields $\boldsymbol{u}$ and $\boldsymbol{B}$,

$$\boldsymbol{u} = \overline{\boldsymbol{u}} + \nabla \times (\nabla \times \boldsymbol{k}\phi) + \nabla \times \boldsymbol{k}\psi \equiv \overline{\boldsymbol{u}} + \boldsymbol{\delta}\phi + \boldsymbol{\eta}\psi, \tag{2.4a}$$

$$\boldsymbol{B} = \boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi + [\overline{\boldsymbol{b}} + \nabla \times (\nabla \times \boldsymbol{k}h) + \nabla \times \boldsymbol{k}g]\frac{\nu}{\lambda}$$
$$\equiv \boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi + [\overline{\boldsymbol{b}} + \boldsymbol{\delta}h + \boldsymbol{\eta}g]\frac{\nu}{\lambda}, \tag{2.4b}$$

where overbars in $\overline{\boldsymbol{u}}$ and $\overline{\boldsymbol{b}}$ denote the average over the x, $y-$plane of $\boldsymbol{u}$ and of the modification $\boldsymbol{b} = \overline{\boldsymbol{b}} + \boldsymbol{\delta}h + \boldsymbol{\eta}g$ of the imposed magnetic field. The functions ϕ, h and ψ, g describing the poloidal and toroidal components of the velocity and of the magnetic field, respectively, are uniquely defined if the conditions $\overline{\phi} = \overline{h} = \overline{\psi} = \overline{g} = 0$ are imposed. After the application of the differential operators $\boldsymbol{\delta}$ and $\boldsymbol{\eta}$ onto Eqs. (2.1a) and (2.1d) we arrive at the following equations for ϕ and ψ and for h and g,

$$\nabla^4\Delta_2\phi + \Delta_2\Theta + Q(\boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi)\cdot\nabla\nabla^2\Delta_2 h = \frac{\partial}{\partial t}\nabla^2\Delta_2\phi + \boldsymbol{\delta}\cdot(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}, \tag{2.5a}$$

$$\nabla^2\Delta_2\psi + Q(\boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi)\cdot\nabla\Delta_2 g = \frac{\partial}{\partial t}\Delta_2\psi + \boldsymbol{\eta}\cdot(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}, \tag{2.5b}$$

$$\nabla^2\Delta_2 h = -(\boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi)\cdot\nabla\Delta_2\phi, \tag{2.5c}$$

$$\nabla^2\Delta_2 g = -(\boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi)\cdot\nabla\Delta_2\psi, \tag{2.5d}$$

where we have neglected all terms multiplied by ν/λ because we assume the limit $\nu << \lambda$. We return to this assumption later in this paper. Equations for the mean flow $\overline{\boldsymbol{u}}$ and the mean distortion of the magnetic field $\overline{\boldsymbol{b}}$ are obtained by averaging the x- and y-components of Eqs. (2.1a) and (2.1d) over the $x-y$ plane,

$$\frac{\partial^2}{\partial z^2}\overline{\boldsymbol{u}} = \frac{\partial}{\partial t}\overline{\boldsymbol{u}} - \frac{\partial}{\partial z}\overline{\Delta_2\phi\left(\nabla_2\frac{\partial}{\partial z}\phi + \boldsymbol{\eta}\psi\right)} - Q\sin\chi\boldsymbol{k}\cdot\nabla\overline{\boldsymbol{b}}, \tag{2.6a}$$

$$\frac{\partial^2}{\partial z^2}\overline{\boldsymbol{b}} = -\sin\chi\boldsymbol{k}\cdot\nabla\overline{\boldsymbol{u}}. \tag{2.6b}$$

Because there is no z-component of the mean flow, the average of the z-component of Eq. (2.1a) determines the z-derivative of the mean pressure.

Eq. (2.1c) can now be written in the form

$$\nabla^2\Theta + R\Delta_2\phi = (\boldsymbol{\delta}\phi + \boldsymbol{\eta}\psi + \overline{\boldsymbol{u}})\cdot\nabla\Theta + \frac{\partial}{\partial t}\Theta. \tag{2.7}$$

In writing Eqs. (2.6a) and (2.6b) we have introduced the horizontal gradient, $\nabla_2 \equiv \nabla - \boldsymbol{k}\boldsymbol{k}\cdot\nabla$, and the horizontal Laplacian, $\Delta_2 \equiv \nabla_2\cdot\nabla_2$. In line with (2.3), Eqs. (2.5) must be solved subject to the boundary conditions

$$\phi = \frac{\partial^2}{\partial z^2}\phi = \frac{\partial}{\partial z}\psi = \Theta = g = 0 \quad \text{at} \quad z = \pm\frac{1}{2}. \tag{2.8}$$

There is no need to specify a boundary condition for h because $\nabla^2\Delta_2 h$ in Eq. (2.5a) can be replaced by $-(\boldsymbol{i}\cos\chi + \boldsymbol{k}\sin\chi)\cdot\nabla\Delta_2\phi$ according to Eq. (2.5c).

It is well known that the solution of Eqs. (2.5) and (2.7) corresponding to the lowest value of R is independent of the x-coordinate, i.e. it assumes the form of steady rolls aligned with the horizontal component of the imposed magnetic field. The solution satisfying the boundary conditions (2.8) can be written in the form

$$\phi_0 = A\cos\alpha y\cos\pi z + O(A^3P^2), \tag{2.9a}$$

$$\Theta_0 = [(\pi^2+\alpha^2)^2 + Q\pi^2\sin^2\chi]\left(\phi_0 + PA^2\alpha^2\sin 2\pi z/8\pi\right) + O(A^3P^2), \tag{2.9b}$$

$$R = [(\pi^2+\alpha^2)^2 + Q\pi^2\sin^2\chi]\left((\pi^2+\alpha^2)/\alpha^2 + P^2A^2\alpha^2/8\right) + ... \tag{2.9c}$$

The term independent of the amplitude A in the expression for R gives the critical value of the Rayleigh number after it has been minimized with respect to the wavenumber α. For large values of Q, the critical wavenumber α_c approaches the value $\pi(Q\sin^2\chi/2)^{1/4}$. The fields $\psi, g, \overline{\boldsymbol{u}}$ and $\overline{\boldsymbol{b}}$ vanish for the solution (2.9).

3. The oscillatory instability of convection rolls

The steady solution (2.9) becomes unstable as R is increased beyond the critical value R_c for onset of convection. Among the instabilities that induce a transition to a three-dimensional form of convection, the oscillatory instability predominates at low Prandtl numbers. Following the analysis of Busse (1972) in the case where $Q = 0$, we superimpose disturbances of the form

$$\tilde{\phi} = (\tilde{\phi}_0 + b\tilde{\phi}_1 + b^2\tilde{\phi}_2 + \dots)\exp\{ibx + \sigma t\}, \tag{3.1a}$$

$$\sigma = \sigma_0 + b\sigma_1 + b^2\sigma_2 + \dots \tag{3.1b}$$

where the quantities $\tilde{\phi}_n$ are functions of y and z only. The expressions for $\tilde{\psi}_n, \tilde{g}_n, \tilde{\Theta}_n$, and $\tilde{h}_n$ are analogous to that for $\tilde{\phi}_n$. The expansion in powers of the wavenumber b has been introduced because the oscillatory instability assumes relatively small values of b at low values of the Prandtl number P on which our analysis is focussed. In fact, the oscillatory instability can be regarded as a modification of the shift mode, which describes a neutral translation of the convection roll pattern in the y-direction, i.e.,

$$\tilde{\phi}_0 = -\frac{1}{\alpha A}\frac{\partial\phi_0}{\partial y}, \quad \tilde{\Theta}_0 = -\frac{1}{\alpha A}\frac{\partial\Theta_0}{\partial y}, \quad \tilde{h}_0 = -\frac{1}{\alpha A}\frac{\partial h_0}{\partial y}, \tag{3.2a}$$

$$\sigma_0 = 0. \tag{3.2b}$$

Another property that contributes to the instability is the presence of a horizontal motion independent of z and of y given by $\tilde{\psi}_0 = const.$, which is thus barely damped by viscous dissipation. These properties remain unchanged as Q is increased from zero if electrically insulating boundaries are assumed,

$$\frac{\partial\tilde{h}}{\partial z} = \frac{\partial\tilde{h}^e}{\partial z}, \qquad \tilde{g} = 0 \ \text{ at } \ z = \pm\frac{1}{2}, \tag{3.3}$$

where $\tilde{h}^e$ denotes the potential magnetic field outside the conducting layer which must be matched to the poloidal field inside. In lowest order these conditions lead to the result

$$\tilde{h}_0 = -\frac{\pi\sin\chi\sin\alpha y}{\pi^2+\alpha^2}\left(\sin\pi z - \frac{\sinh\alpha z}{\cosh\frac{\alpha}{2} + \sinh\frac{\alpha}{2}}\right), \qquad \tilde{g}_0 = (\frac{1}{8} - \frac{z^2}{2})ib\cos\chi\tilde{\psi}_0. \tag{3.4}$$

The analysis in the higher orders b^n follows the procedure described by Busse (1972). σ_1 is purely imaginary and determines the frequency of the oscillatory instability, whereas a positive σ_2 is obtained for sufficiently large values of A. Because there is no space here to describe the complex analysis in detail, we refer to the numerical analysis of Busse and Clever (1990) where the critical Rayleigh number R_c and the Rayleigh number R_{II}

for onset of the oscillatory instability have been computed for various values of Q, χ, and P. While the critical value R_c increases strongly with $Q \sin\chi$, the difference $R_c - R_{II}$ depends only weakly on Q at low values of P.

Instead of pursuing the general analysis we focus here on the generation of a mean flow that becomes possible as the amplitude $\tilde{A}$ of the growing disturbances Eq. (3.1a) becomes finite. For this purpose the equations for $\tilde{\phi}_1, \tilde{\Theta}_1$ and $\tilde{h}_1$ must be solved,

$$\nabla^4\Delta_2\tilde{\phi}_1 - \Delta_2\tilde{\Theta}_1 + Q\sin\chi\frac{\partial}{\partial z}\nabla^2\Delta_2\tilde{h}_1 = iQ\cos\chi\sin\chi\frac{\partial}{\partial z}\Delta_2\tilde{\phi}_0, \tag{3.5a}$$

$$\nabla^2\tilde{\Theta}_1 - R\Delta_2\tilde{\phi}_1 = 0, \tag{3.5b}$$

$$\nabla^2\Delta_2\tilde{h}_1 + \sin\chi\frac{\partial}{\partial z}\Delta_2\tilde{\phi}_1 = -i\cos\chi\Delta_2\tilde{\phi}_0. \tag{3.5c}$$

To simplify the analysis, terms proportional to A have been neglected in Eqs. (3.5a, b, c). Those terms will generate additional contributions to the solutions of $\tilde{\phi}_1$ and $\tilde{\Theta}_1$ of different symmetry, but they will not change qualitatively the following derivations. After eliminating $\tilde{\Theta}_1$ and $\tilde{h}_1$ from Eqs. (3.5a, b, c) and using expression (3.2a) for $\tilde{\phi}_0$ we obtain the following equation for $\tilde{\phi}_1$,

$$\left(\nabla^6 - Q\sin^2\chi\frac{\partial^2}{\partial z^2}\nabla^2 - R\Delta_2\right)\tilde{\phi}_1 = iQ\sin 2\chi(\pi^2+\alpha^2)\pi\sin\alpha y\sin\pi z. \tag{3.6}$$

The solution of this equation satisfying the boundary conditions can be written in the form

$$\tilde{\phi}_1 = i\sin\alpha y\sin 2\chi\sum_{n=1}^{\infty}\frac{8n}{4n^2-1}D_n\sin 2n\pi z. \tag{3.7}$$

where the coefficients D_n are given by

$$D_n = Q(-1)^n(\pi^2+\alpha^2)\Big((4n^2\pi^2+\alpha^2)^3 + Q(4n^2\pi^2+\alpha^2)4n^2\pi^2\sin^2\chi - R\alpha^2\Big)^{-1}. \tag{3.8}$$

In the following the antisymmetry of expression (3.7) with respect to the mid-plane, $z = 0$, and the positivity of the coefficients D_n will be the most important properties. For later use we also derive the solution for $\tilde{h}_1$,

$$\tilde{h}_1 = i\sin\alpha y\sin 2\chi\sum_{n=1}^{\infty}\frac{16n^2\pi D_n}{(4n^2-1)(\alpha^2+4n^2\pi^2)}\left(\cos 2n\pi z - \frac{\cosh\alpha z}{\alpha(\cosh\frac{\alpha}{2}+\sinh\frac{\alpha}{2})}\right)$$
$$+ i\cos\chi\frac{\sin\alpha y}{\pi^2+\alpha^2}\left(\cos\pi z - \frac{\cosh\alpha z}{\alpha(\cosh\frac{\alpha}{2}+\sinh\frac{\alpha}{2})}\right), \tag{3.9a}$$

which satisfies the boundary condition (3.3).

4. Reynolds and Maxwell stresses of oscillatory convection

The straight convection rolls described by expressions (2.9a, b, c) are qualitatively identical to convection rolls in the absence of a magnetic field. As the convection pattern becomes three-dimensional with growing amplitude $\tilde{A}$ of the disturbances $\tilde{\phi}, \tilde{\psi}, \tilde{\Theta}, \tilde{h}$, and $\tilde{g}$, finite Reynolds stresses develop that tend to generate a shear flow $\overline{\boldsymbol{u}} = \boldsymbol{i}U(z)$ in the x-direction. Connected with this shear is a mean distortion of the magnetic field, $\overline{\boldsymbol{b}} = B_x$. No mean flows or mean magnetic fields in other directions are generated. The interaction between the basic steady convection rolls and the oscillatory disturbances does not give rise to any mean effects. The Reynolds stresses are a nonlinear effect of the disturbances and are thus proportional to $(\tilde{A})^2$ in lowest order. In addition, we want to consider the effect of Maxwell stresses. These turn out to be of the order ν/λ and thus might be negligible according to our previous assumption. But because they are also proportional to Q, it is of interest to consider the terms of first order in $Q\nu/\lambda$.

According to Eqs. (2.6a, b), the equations for U and B_x are given by

$$\frac{\partial^2}{\partial z^2}U = \frac{\partial}{\partial t}U - \tilde{A}^2\frac{\partial}{\partial z}\overline{\Delta_2\tilde{\phi}\frac{\partial^2}{\partial x\partial z}\tilde{\phi}} - Q\sin\chi\frac{\partial}{\partial z}B_x + \frac{\nu}{\lambda}\frac{\partial}{\partial z}M_1, \tag{4.1a}$$

$$\frac{\partial}{\partial z}B_x = -\sin\chi U + \frac{\nu}{\lambda}M_2, \tag{4.1b}$$

where the Maxwell stresses M_1 and the corresponding induction terms M_2 have been added, although terms with $\tilde{\psi}$ and $\tilde{g}$ have been neglected because they do not contribute to the average. We have used the integrated form of the equation of induction since it can be presumed there is no mean electric field in the x-direction. Because the horizontal average is time independent and because we are interested in the steady state we neglect the time derivative in the following. The expressions M_1 and M_2 are given by

$$M_1 = \tilde{A}^2Q\overline{\Delta_2\tilde{h}\frac{\partial^2}{\partial x\partial z}\tilde{h}} = \tilde{A}^2Q\left(\overline{\Delta_2\tilde{h}_0\frac{\partial^2}{\partial x\partial z}\tilde{h}_1} + \overline{\Delta_2\tilde{h}_1\frac{\partial^2}{\partial x\partial z}\tilde{h}_0}\right) + \ldots, \tag{4.2a}$$

$$\begin{aligned} M_2 &= \tilde{A}^2\left(\overline{\Delta_2\tilde{\phi}\frac{\partial^2}{\partial x\partial z}\tilde{h} - \Delta_2\tilde{h}\frac{\partial^2}{\partial x\partial z}\tilde{\phi}}\right) \\ &= \tilde{A}^2\left(\overline{\Delta_2\tilde{\phi}_0\frac{\partial^2}{\partial x\partial z}\tilde{h}_1 - \Delta_2\tilde{h}_0\frac{\partial^2}{\partial x\partial z}\tilde{\phi}_1} + \overline{\Delta_2\tilde{\phi}_1\frac{\partial^2}{\partial x\partial z}\tilde{h}_0 - \Delta_2\tilde{h}_1\frac{\partial^2}{\partial x\partial z}\tilde{\phi}_0}\right) + .. \end{aligned} \tag{4.2b}$$

In evaluating Reynolds and Maxwell stresses it must be taken into account that only the real parts of expressions (3.1a, b) for the disturbances have physical meaning. Accordingly we obtain after elimination of B_x from Eqs. (4.1a, b)

$$\begin{aligned} \frac{\partial^2}{\partial z^2}U - Q\sin^2\chi U =& \tilde{A}^2b^2\alpha^2\sin 2\chi\sum_{n=1}^{\infty}nD_n\left(\sin(2n+1)\pi z + \sin(2n-1)\pi z\right) \\ &+ \frac{\nu}{\lambda}\left(\frac{\partial}{\partial z}M_1 - Q\sin\chi M_2\right). \end{aligned} \tag{4.3}$$

The solution of this equation in the limit $\frac{\nu}{\lambda} = 0$ is given by

$$U = -\tilde{A}^2b^2\alpha^2\sin 2\chi\sum_{n=1}^{\infty}nD_n\left(\frac{\sin(2n+1)\pi z}{(2n+1)^2\pi^2 + Q\sin^2\chi} + \frac{\sin(2n-1)\pi z}{(2n-1)^2\pi^2 + Q\sin^2\chi}\right). \tag{4.4}$$

This result shows that U is antisymmetric with respect to the mid-plane $z = 0$, and it is positive for $z > 0$ for positive χ. The terms proportional to $\frac{\nu}{\lambda}$ give rise to contributions that are symmetric with respect to the plane $z = 0$ as can be seen from the inspection of the expressions (4.2a, b). All of these contributions vanish when integrated over z such that boundary condition $\tilde{B_x} = 0$ at $z = \pm 1/2$ can be satisfied.

A physical interpretation for the property that the Reynolds stress exhibits the same sign as χ can be given on the basis of the magnetic analogue of the Taylor-Proudman-Theorem (see, for instance, Chandrasekhar 1961). Any circulation in the x, z-plane will

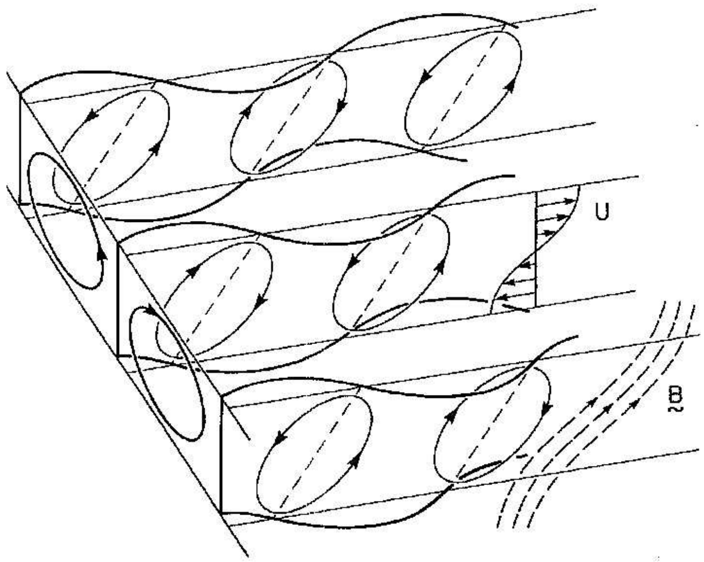

FIGURE 1. Sketch of the shear flow U generated by oscillating convection rolls in the presence of an imposed inclined magnetic field

be elongated in the direction of the imposed magnetic field. It thus carries positive x-momentum upward and negative x-momentum downward thereby creating the shear flow (4.4). Figure 1 illustrates this process. The action of the Reynolds stress is balanced by the viscous stress exerted by the shear and by the Maxwell stress of the mean magnetic field.

5. Discussion

The Evershed effect has been a well known phenomenon in solar physics for more than 100 years and several theoretical models have been proposed for its explanation. In his review of sunspots Thomas (1981) noted that there are two schools of thought. Meyer and Schmidt (1968) created the "siphon"-model in which the Evershed flow is driven by a pressure difference. For more recent developments in the spirit of this model we refer to Schlichenmaier *et al*. (1998). The latter paper also deals with the nature of the bright and dark filaments in the penumbra. The second school of thought goes back to Danielson (1961), who derived a penumbra model based on radially oriented convection rolls. Galloway (1975) used this model and outlined a theory of the Evershed effect in which the flow is driven by the Lorentz force originating from the curvature of the magnetic field lines in the dark filaments. A detailed analysis based on this suggestion has not been carried out, however. The self-consistent theory of Busse (1988) shares with the proposal by Galloway that the generation of the Evershed flow occurs locally. The main addition to the theory of Busse (1988) given in the present paper is the derivation of the Evershed flow for a different magnetic boundary condition and the elucidation of the role of Maxwell stresses.

It has already been emphasized that the model presented in this paper is too idealized for a quantitative comparison with observations. An unrealistic feature of the model is the limit of small λ/ν. Without this assumption the accumulation of magnetic flux at the boundaries along the lines of flow convergence can not be described, which could explain the increased strength of the Evershed flow in the dark regions between the bright filaments. Another effect that could contribute to the stronger flow in the dark regions arises from the property that the inclination χ of the magnetic field plays a dual role. The stabilizing effect of the magnetic field, as expressed by the term $Q \sin^2 \chi$ in the denominators of expression (4.4) and of D_n, increases in proportion to $\sin^2 \chi$, whereas the destabilizing effect in the numerator of expression (4.4) grows in proportion to $Q \sin \chi$ for small χ. Thus a stronger Evershed effect must be expected in the dark filaments where the inclination of the magnetic field is observed to be smaller than in the bright filaments.

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