Turbulent spots inside the wall layer of the zero-pressure-gradient turbulent flat-plate boundary layer

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Deciphering the constitutive vortex element structure in the near-wall region of the zero-pressure-gradient turbulent flat-plate boundary layer has remained a central fundamental fluid mechanics theme for more than six decades. Here, we present evidence for the existence and plurality of turbulent spot structures in the fully turbulent flat-plate boundary layer. This evidence is extracted from a spatially developing direct numerical simulation, which carries the inlet Blasius boundary layer through a bypass transition, arriving at the canonical turbulent boundary layer state over a moderate Reynolds number range. The turbulent spots form around the legs of hairpin packets and then grow to become a mixture of hairpin vortices and random vortex filaments. Although structurally analogous to the transitional-turbulent spots, these turbulent-turbulent spots are not transported from the far-upstream transition but instead are generated locally in the chaotic fully turbulent environment. Viscous sublayer streaks are strongly indented and often segmented by the turbulent-turbulent spots. A three-dimensional spatial connectivity-based approach is employed to systematically identify the turbulent spots and compute distributions of their length scales, time scales, and swirling strength intensities. We also compute the period of turbulent-turbulent spot detection measured on one-dimensional slices through the identified three-dimensional structures at constant wall-normal heights and analyze the similarities and differences with the boundary layer bursting period reported in previous hot-wire experiments.

1. Introduction

A zero-pressure-gradient smooth flat-plate boundary layer is the simplest external flow and thus serves as the idealized limiting case and calibration benchmark of atmospheric and oceanic planetary boundary layers, as well as aeronautical, maritime, and automotive boundary layer flows. One central theme in fundamental fluid mechanics research has been the search for the basic constitutive organized vortex structure in the turbulent flat-plate boundary layer, particularly inside the inner layer less than 100 viscous units away from the plate, where most of the turbulence kinetic energy is produced. The issue is an important one because it tests our intellectual ability to identify the elemental vortex structure in even the simplest wall-bounded external flow. From a practical perspective, a well understood instantaneous boundary layer vortex structure is the prerequisite in the design of physics-based active flow control strategies for viscous drag reduction. Near-wall boundary layer turbulence modeling and theory become more meaningful and powerful when they faithfully interpret the genuine, rather than perceived, picture of the inner layer dynamics. The chance for acquiring such a genuine picture becomes promising

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with the construction of a high-fidelity spatially developing turbulent flat-plate boundary layer database, either experimental or numerical, with sufficient temporal resolution and duration, and with sufficient three-dimensional spatial resolution and extent.

Apparent consensus seems to have been reached that the inner region of the turbulent flat-plate boundary layer is populated by randomly distributed quasi-streamwise vortices as well as elongated high- and low-momentum streaks. These streaks participate in a self-sustaining bursting cycle including streak generation, lift-up, oscillation, breakdown, and streak regeneration (Robinson 1991; Adrian et al. 2000). Evidence for this theory was obtained using mostly two-dimensional and coarse-resolution smoke or hydrogen bubble visualization techniques (Kline et al. 1967). Large-scale vortex motion is inferred from the observed two-dimensional streak patterns. While the geometric characteristics of the streaks (spacing and width) have been firmly established (Robinson 1991), existing information on the dynamic aspects of the streaks and their interaction with inner layer vortex structure is far from clear. Given the coarse resolution and indirectness of the evidence and the degree of inference involved, there is room to question whether the above streak-centered cycle is actually the most elementary turbulence regeneration process in the flat-plate boundary layer, or perhaps it is merely part of the upper level symptoms of a more fundamental underlying process. Notwithstanding these uncertainties, theories have been developed based on the aforementioned evidence and on additional hypotheses to construct a framework to explain the mechanism of inner layer turbulence regeneration (Hussain 1983). However, these theories are mostly developed using an internal fully-developed channel flow rather than an external spatially developing flat-plate boundary layer. Carpenter et al. (2007) aimed specifically at the zero-pressure-gradient flat-plate boundary layer, but also made the unphysical assumption of a zero-growth parallel boundary layer without freestream entrainment into the wall layer. As in previous theories, Carpenter et al. (2007) hypothesized that quasi-streamwise vortices produce near-wall streaks, which in turn interact non-linearly with traveling plane waves analogous to the Tollmien-Schlichting type. They further assumed that the non-linear interaction results in oblique waves growing algebraically, eventually resulting in streak breakdown. New quasi-streamwise vortices for the next bursting cycle were assumed to be produced by further interactions between the plane wave and the oblique wave. It is clear that a high-fidelity flat-plate boundary layer database with very fine resolution details on near-wall vortex dynamics and with sufficient spatial and temporal coverage ranges would help provide a firmer foundation for flat-plate wall layer turbulence theory and modeling.

Sustained efforts have been made to study properties and dynamics of transitional turbulent spots in the natural or bypass transitions of the zero-pressure-gradient flat-plate boundary layer. The hope is that information extracted from the transitional turbulent spots during the inception of boundary layer turbulence may shed light on the wall layer dynamics of turbulence regeneration in the fully turbulent boundary layer. Park et al. (2012) compared statistics, including vorticity, dissipation rate, and quadrant-based momentum and heat fluxes, conditionally sampled from transitional turbulent spots with those from the downstream fully turbulent region. They found the compared statistics were quite similar in both flow regions, which seems to suggest, albeit indirectly, that there might exist a certain degree of connection between the inception of boundary layer turbulence and the near-wall region of a fully turbulent boundary layer. Until now, there has been no direct evidence to structurally connect transitional turbulent spots with the inner layer dynamics in the fully turbulent boundary layer.
In this work, we identify and analyze the turbulent spots using a direct numerical simulation (DNS) of a spatially developing zero-pressure-gradient smooth flat-plate boundary layer. The flow evolves from laminar to turbulent over a moderate Reynolds number range through a well-controlled clean transition process. Visual examination of the data reveals that the viscous sublayer streaks are strongly indented and often segmented by concentrated small-scale vortices across the buffer layer region. These inner-layer concentrated vortices are defined by regions of the flow with high values of swirling strength (Adrian et al. 2000) and are used to identify turbulent-turbulent spots in the buffer region. Characteristics of the spots are quantified using conditional sampling techniques. The presence of turbulent-turbulent spots within the near-wall region of the boundary layer bridges the studies on the late stage boundary layer transition (inception and growth of transitional turbulent spots) and on the near-wall dynamics for fully developed turbulence.

2. Numerical experiments and data analysis

2.1. Computational setup

The results presented here are from a DNS of an incompressible zero-pressure-gradient smooth flat-plate boundary layer beneath a continuous flow of nearly isotropic free-stream turbulence (FST). The mesh size is 16384 × 500 × 512, corresponding to a domain size of 21562θ₀ × 2250θ₀ × 843.75θ₀ where θ₀ is the inlet momentum thickness. The momentum thickness Reynolds number, Re_θ, covers a range from 80 to 3000. From the inlet to the exit, the FST decays from 3% to 0.8% and the boundary layer thickness, δ, increases from 7.175θ₀ to 312θ₀. Re_τ reaches 1003 at the exit. The grid resolutions are 3.5 < Δx⁺ < 5.5 and 4.5 < Δz⁺ < 7, where + denotes wall-units defined in terms of the kinematic viscosity ν and the friction velocity u_τ. At Re_θ = 2900, using the local Kolmogorov length scale (η = (ν³/ε)¹/₄), 2 < Δx/η < 3 for y/δ < 0.1, 0.5 < Δx/η < 2 in the outer region, and 0.4 < Δy/η < 2 throughout the layer. Temperature is included in the DNS at unit molecular Prandtl number. The non-dimensional temperature, φ, is 1 at the wall and 0 in the upper boundary as in Wu & Moin (2010). At the inlet, φ is prescribed using the Blasius profile without any fluctuations. The time step of the DNS is Δt = 1.125θ₀/U_f.

Profiles of ε scaled by (u_τ²/ν) as a function of the wall-normal distance are shown in Figure 1 to prove the quality of the DNS. Over 1000 < Re_θ < 3000, the profiles collapse in the inner layer, suggesting that the turbulent boundary layer is in statistical equilibrium. The ε profiles also agree well with the approximation often used in experiments where ε_{iso} = 15ν(∂u/∂x)² (Balint et al. 1991).

2.2. Data analysis

After reaching a statistically steady state, the velocity, pressure, and temperature fields were saved every 100 Δt for a period of 30000 Δt. Time-history data of the velocity and temperature fields were also stored along four selected spanwise line bundles at every time step for 34000 Δt. The four lines are located at Re_θ = 670 and 2536 and at two wall-normal positions, y/δ = 0.05 and 0.5. Each bundle contains the selected line and its four immediate neighboring lines to allow for the calculation of spatial derivatives.

Three-dimensional fields of the swirling strength, λ_ci, are used to identify and study the spot structures both qualitatively and quantitatively. The turbulent spots are then defined as regions of high swirling strength. For both the qualitative and quantitative analyses, the λ_ci fields are located in the fully turbulent region from 2100 < Re_θ <
2860 and over the full spanwise and wall-normal domain. Isosurfaces of intense $\lambda_{ci}$ and temperature $\phi$ at varying threshold values are used to examine the formation of the turbulent-turbulent spot structures and their interaction with the viscous sublayer streaks in Sections 3.1 and 3.2 of this report. To obtain a more quantitative description, a spatial connectivity-based approach is employed in Section 3.3 to identify turbulent-turbulent spot structures in the three-dimensional $\lambda_{ci}$ fields.

Following Del Alamo et al. (2006) and Lozano-Duran et al. (2012), we identify turbulent spots as 6-connected regions in space satisfying the condition

$$\lambda_{ci} > \alpha \cdot \lambda'_{ci},$$

where $\lambda'_{ci}$ is the standard deviation of $\lambda_{ci}$ over the entire domain, and $\alpha$ is a thresholding parameter determined by the percolation diagram in Figure 2(a). Figure 2(b) shows an example of an individual three-dimensional turbulent spot structure for the nominal threshold chosen, $\alpha = 2.2$. The results presented here were also computed with $\alpha$ values as high as 3.5 and the overall trends remained the same. After all the three-dimensional structures in each of the $\lambda_{ci}$ fields have been identified, one can easily compute statistics like distributions of spatial and temporal length scales and average intensity of $\lambda_{ci}$ within the spots, and these results are presented in Section 3.3.

3. Results

3.1. Formation and structure of turbulent-turbulent spots

In the present section, we visually analyze the general structure of the turbulent spots in the fully turbulent region of the boundary layer. Figure 3(a) shows isosurfaces of $\lambda_{ci}$ and reveals turbulent spot structures similar to those present in the transitional region extensively studied in Park et al. (2012). However, the turbulent-turbulent spots look much more disorganized than their transitional counterparts, and their structure is closer to a collection of random vortex filaments.
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Figure 2. (a) Percolation diagram: Ratio of the largest identified object’s volume to the volume of all identified objects (solid line); ratio of the number of objects identified at a given threshold to the maximum number of objects identified over all considered thresholds (dashed line), chosen threshold level, \( \alpha = 2.2 \) (dotted line). (b) Example of an identified turbulent spot structure in the three-dimensional \( \lambda_{ci} \) data colored by the wall-normal distance.

Figure 3. (a) \( \lambda_{ci} \) isosurface showing several turbulent-turbulent spots; (b) infant turbulent-turbulent spot: forms around the leg of a hairpin vortex; (c) young turbulent-turbulent spot: longer and noisier structure; (d) mature turbulent-turbulent spot: larger collection of hairpins and random vortex filaments.

Turbulent-turbulent spots are found to persist through changes in the threshold. For low values of the threshold, the turbulent-turbulent spots are discernible but surrounded and shrouded by disorganized, weaker vortex structures. As the threshold level is progressively increased, the weaker structures gradually fade away and the turbulent-turbulent spots become more distinct. For very large values of the threshold, the last remaining structures are the cores of the turbulent-turbulent spots, supporting the idea that they are markers of a physical phenomenon rather than a threshold-dependent visual artifact.
Figure 4. (a) Isosurface of 80% of the maximum wall temperature over $2500 < \text{Re}_\theta < 2600$ colored by height with white representing $y^+ = 10$. (b) Temperature isosurface below isosurface of $\lambda_{ci}$ colored by height where dark color corresponds to about $y^+ < 20$ and light color corresponds to about $y^+ = 60$.

Our visual study of $\lambda_{ci}$ also shows that the turbulent-turbulent spots are not convected downstream from the transitional region, but are indeed generated locally in the fully turbulent region. An example is shown in Figures 3(b-d), where one turbulent-turbulent spot is tracked downstream through its formation process, which turns out to be reasonably similar to the formation process of transitional turbulent spots.

3.2. Streaks and turbulent-turbulent spots

Visual analysis also reveals an interesting interaction between the viscous sublayer streaks, which have received central attention in turbulent boundary layer structure research, and the turbulent-turbulent spots. We visualize the sublayer streaks using isosurfaces of temperature. These follow similar patterns to those observed in experiments using dye or smoke released from the plate. A ridge in the isosurface of $\phi$ provides a three-dimensional view of a low-momentum streak. Figure 4(a) shows the isosurface of $\phi = 0.8$ (where $\phi$ is the non-dimensional wall temperature as defined in Section 2.1) over the range $2500 < \text{Re}_\theta < 2600$, where the bright white color indicates $y^+ = 10$. Here, the viscous sublayer is seen to be populated by streaks, some of which exhibit the well-known meandering behavior. However, many of the streaks are indented and even terminated in regions where concentrated indentations are present. When an isosurface of the swirling strength is overlain on the temperature isosurface (Figure 4(b)), the locations of indentations and terminations of the sublayer streaks match with the locations of the turbulent-turbulent spots. This agreement supports the idea that there is some relationship between the turbulent-turbulent spots and the disruptions in the streaks.

3.3. Length scales

We now present a quantitative description of the spatial length scales of the turbulent-turbulent spot structures identified using the three-dimensional spatial connectivity approach described in Section 2.2.

The streamwise, wall-normal, and spanwise lengths of the spots are measured by the edges of the bounding boxes containing all the points identified as part of an individual object. The distributions of lengths are presented in Figure 5. Turbulent-turbulent
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Figure 5. (a) Streamwise (solid line), wall-normal (dashed line), and spanwise (dotted line) length scales of the turbulent spot structures identified in the three-dimensional $\lambda_{ci}$ fields; (b) joint probability density function (PDF) contours of the ratio of the actual volume of each individual structure and the volume of its bounding box, $V_s/V_b$, and the actual volume of the structure, $V_s$. The three contours are 5% (innermost curve), 10% (middle curve), and 50% (outermost curve) of the maximum value of the joint PDF.

spots have comparable spanwise and wall-normal length scales but are elongated in the streamwise direction (Figure 5(a)). This is reasonable since the mean velocity profile in the boundary layer probably stretches the spots in the streamwise direction. As seen in Figure 5(b), smaller spots tend to fill most of their bounding boxes, whereas larger spots fill less of their bounding boxes. This suggests that the spots start out as isotropic structures but, as they grow, their shapes become more complex. It is also interesting that wall-attached structures, defined as those whose lowest point is within $y^+ < 5$, represent 80% of the volume of all the identified spot structures, which is consistent with Townsend’s attached-eddy hypothesis (Townsend 1976).

The average intensities of the spots, measured as the mean $\lambda_{ci}$ in units of $U_\infty/\theta_0$ over all the constituent points of the object, are investigated in Figure 6. The distribution of intensities peaks at values close to the threshold and decays with a long potential tail. Figure 6(b) tests a possible correlation between the intensity and size of the spots. However, even if larger turbulent-turbulent spots are slightly more likely to have a higher average swirling strength intensity, the average swirling strength intensity does not depend much on the size of the spot. Hence, throughout their lifetimes, the spots maintain roughly the same average swirling strength intensity.

3.4. Time scales

The average streamwise time period between the turbulent-turbulent spots is of interest because it provides a link to the previous hot-wire experiments that investigated the streak lift-up and bursting theory (Rao et al. 1971; Kim et al. 1971). However, studying these three-dimensional structures with one-dimensional time histories obtained from hot-wire experiments is a difficult task. In this last section, we reduce the three-dimensional data to two and one dimensions and evaluate the accuracy of one-dimensional experiments in extracting the full three-dimensional information. We have quantified the differences for $y/\delta = 0.05$ and $Re_\theta = 2536$.

A first limitation of working with lower dimensional data becomes evident in Figure 7. The coherent three-dimensional turbulent spots are fragmented when cut at a constant
wall-normal distance, making the individual vortices that make up the spot appear disconnected in space. As a consequence, time periods are artificially smaller than the ones computed from full three-dimensional data. This was a known problem also encountered in the earlier bursting experiments (Bogard & Tiederman 1986). Each burst had multiple ejections, and researchers had to decide whether a set of ejections belonged to the same burst or to different bursts. The preferred method for this was setting a maximum time between ejections beyond which the ejections were considered to be from different bursts. However, the time period results using this approach appear to be somewhat dependent on this maximum time parameter.

We use the full three-dimensional fields available from the present DNS to compute the spatial streamwise separation between the turbulent spots by mimicking the one-dimensional experiments, that is, by taking one-dimensional slices of the three-dimensional data at $y/\delta = 0.05$. The main difference with respect to the experiments comes from the fact that we still have the three-dimensional information, and disjointed vortices can be correctly grouped into their corresponding three-dimensional spot struc-
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Figure 8. Streamwise time period, scaled in outer units, between identified structures from one-dimensional slices at \( y/\delta = 0.05 \) at each spanwise location in the three-dimensional data fields with identified structures grouped (solid line), one-dimensional slices at \( y/\delta = 0.05 \) at each spanwise location in the thresholded but not grouped spatial three-dimensional data (dotted line), and un-grouped time-history data at \( y/\delta = 0.05 \) and \( \text{Re}_\theta = 2536 \) at each spanwise location (dashed line).

Turbulent spots, defined as concentrations of vortices with high swirling strength, have been found in the fully turbulent region of a high-quality zero-pressure-gradient smooth flat-plate boundary layer DNS. The spots are locally generated and structurally analogous to the well-studied transitional turbulent spots, giving a link between the dynamics of transition and the generation of turbulence in the fully turbulent flat-plate boundary layer.

4. Conclusions

Turbulent spots, defined as concentrations of vortices with high swirling strength, have been found in the fully turbulent region of a high-quality zero-pressure-gradient smooth flat-plate boundary layer DNS. The spots are locally generated and structurally analogous to the well-studied transitional turbulent spots, giving a link between the dynamics of transition and the generation of turbulence in the fully turbulent flat-plate boundary layer.
boundary layer. The turbulent-turbulent spots are found to strongly indent and segment the viscous sublayer streaks. Applying a spatial connectivity-based approach to identify the spot structures in the full three-dimensional swirling strength fields reveals that the turbulent-turbulent spots are primarily wall-attached structures elongated in the streamwise direction that begin as isotropic structures and grow into more complex shapes. The average streamwise time period between the identified three-dimensional turbulent-turbulent spot structures agrees reasonably well with the average streamwise time period measured in experimental bursting phenomenon hot-wire studies, suggesting the two phenomena are closely related. Overall, this work raises questions about the validity of existing turbulence regeneration theories for the flat-plate boundary layer and suggests that there is a direct relationship between the generation of turbulence in the transitional region and in the fully turbulent region.

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REFERENCES


