A comparative study of wall models for LES of turbulent separated flow

By P. S. Iyer†, G. I. Park AND M. R. Malik¶

Large Eddy Simulation (LES) coupled with near-wall models is becoming popular for applications involving high Reynolds number flows. Reduced cost and reasonably good accuracy make wall-modeled LES (WMLES) an attractive option for complex turbulent flows. While different wall models yield good predictions for attached flows, separated flows serve as more challenging test cases. We assess the performance of the equilibrium and non-equilibrium wall models for two test cases: (1) Transonic axisymmetric flow past a bump corresponding to experiments of Bachalo & Johnson (1986), and (2) Subsonic flow over the NACA4412 airfoil at near-stall conditions (\(\alpha = 12^\circ\)) corresponding to experiments of Wadcock (1987). The equilibrium wall model used by Bodart & Larsson (2012) and the non-equilibrium wall model developed by Park & Moin (2014) are evaluated by comparisons with available experimental data. A no-slip (no wall model) simulation on the same WMLES grids has also been performed to show the effect of the wall model boundary condition on the flow for the given grid resolution. Although reasonable agreement is observed with experiment overall, the reasons for discrepancies are examined.

1. Introduction

High-fidelity simulation methodologies are required for an improved understanding of flows in complex geometries. In the field of aeronautics, better understanding of flow around an airplane can lead to improvements in design and drag reduction. While Reynolds-Averaged Navier-Stokes (RANS)-based methods are currently widely used in industry due to their affordable cost, higher-fidelity LES-based methods are becoming more feasible owing to increasing computing power and resources. However, the resources required by wall-resolved LES (WRLES) for high \(Re\) flows is still demanding, making wall-modeled LES (WMLES) more attractive and feasible in the near future.

Wall models can be constructed with different levels of complexity by making suitable approximations to the near wall RANS-type equations. Cabot & Moin (2000) and Piomelli & Balaras (2002) discuss various wall models commonly used in WMLES simulations. However, there are few studies that make a detailed comparison of different wall models for turbulent flows involving separation. Cabot & Moin (2000) compared the performance of equilibrium and non-equilibrium wall models for the flow past a backward-facing step. While all the wall models used in the study predicted the mean velocity profiles reasonably well, differences were apparent in the variation of the wall skin friction coefficient.

WMLES of turbulent flows involving separation were previously studied by Bodart & Larsson (2012) and Bodart et al. (2013) for a multi-element airfoil, Balakumar et al. (2014) for the periodic hill flow, and Park (2016) and Iyer & Malik (2016) for the NASA

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hump configuration, among others. The transonic axisymmetric flow past a bump and the near-stall airfoil configurations are challenging test cases owing to additional complexities such as predicting the correct shock location, capturing the boundary layer transition on the upper surface of the airfoil and obtaining smooth flow at the airfoil trailing edge. These test cases are relevant to practical applications such as airplane wings and hence a detailed investigation of the WMLES results for these flows can yield insights for simulating more complex configurations. This paper is organized as follows: The details of the flow solver and wall model are briefly discussed in Section 2; the flow conditions and computational grid details are discussed in Section 3; and the results, including comparison to experiment, are presented in Section 4. A brief summary concludes the paper.

2. Numerical details

The compressible CharLES† solver used in the simulations solves the compressible Navier-Stokes equations on unstructured grids by using a cell-centered finite volume methodology. An explicit third-order Runge-Kutta scheme is used for time advancement. The constant coefficient Vreman or dynamic Smagorinsky model was used to model the sub-grid terms. The solver uses an ENO-based scheme for shock capturing. Further discretization and implementation details can be found in Park & Moin (2016).

Two wall models are used in this study. The equilibrium wall model solves the simplified boundary layer equations in the vicinity of the wall using information from the LES as the outer boundary condition. A RANS eddy viscosity based on the mixing-length model is used to model the unclosed terms in the wall model equations. The equations reduce to a system of coupled ordinary differential equations (ODEs) that need to be solved at each time step on the wall boundary faces. Further details of the wall model can be found in Bodart & Larsson (2012). The non-equilibrium model developed by Park & Moin (2014) solves the full 3D RANS equations at each time step with a mixing length-type model for the unclosed terms. The key ingredient of this model is that the RANS eddy viscosity/conductivity models only the unresolved portion of the Reynolds stress/turbulent heat flux. Since the boundary conditions from LES for the wall model RANS equations are unsteady, a portion of the Reynolds stress and turbulent heat flux is resolved by the wall model. The additional cost of solving the wall model equations is roughly 10–30% for the equilibrium wall model and 100–150% for the non-equilibrium wall model. Also, a full 3D wall model grid is necessary for the non-equilibrium wall model whereas no wall parallel connectivity is required for the equilibrium model, considerably simplifying the problem setup for complex geometries.

3. Problem description and computational details

3.1. Axisymmetric transonic bump

Transonic flow past a bump is simulated at conditions corresponding to the experiments of Bachalo & Johnson (1986). This configuration is a test case for the NASA Revolutionary Computational Aerosciences (RCA) challenge under the Transformational Tools and Technologies (TTT) project, and is also part of the NASA Langley Turbulence Modeling Resource website‡. The bump is placed over a cylindrical pipe and the entire geometry is

† Cascade Technologies, Webpage: http://www.cascadetechnologies.com
‡ Link: http://turbmodels.larc.nasa.gov/axibump_val.html
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Figure 1. The computational grid in the vicinity of the bump is shown in the figure at the center of the bump (left) and near the trailing edge of the bump where the flow separates (right).

<table>
<thead>
<tr>
<th>Region</th>
<th>$\Delta x/c$, $\Delta y_w/c$, $\Delta z_w/c$</th>
<th>$\Delta x^+/$, $\Delta y_w^+/$, $\Delta z_w^+$</th>
<th>$\delta/c$</th>
<th>$N_x/\delta$, $N_y/\delta$, $N_z/\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/c = 0.5$</td>
<td>$4 \times 10^{-3}$, $6.3 \times 10^{-4}$, $3.2 \times 10^{-3}$</td>
<td>354, 56, 283</td>
<td>0.07</td>
<td>18, 32, 22</td>
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<tr>
<td>$x/c = 0.5$</td>
<td>$3 \times 10^{-3}$, $3 \times 10^{-4}$, $3.2 \times 10^{-3}$</td>
<td>364, 37, 390</td>
<td>0.05</td>
<td>16, 32, 16</td>
</tr>
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</table>

Table 1. Grid spacings used in the axisymmetric transonic bump simulations. Here, $\delta$ is the local boundary layer thickness and is estimated from the simulation results. The superscript ‘$+$’ denotes the viscous wall spacing.

Figure 2. Instantaneous iso-contours of the $Q$-criterion is shown to depict the vortical features for the transonic axisymmetric flow past a bump.

axisymmetric. The oncoming freestream Mach number ($M_\infty$) is 0.875 and the Reynolds number based on the bump chord length ($Re_c$) is 2.763 million. The bump extends between $x/c = 0$ and 1. The leading edge of the cylinder is located at $x/c = -3$ in the experiments, at which the inflow of the computational domain is placed. Freestream con-
Figure 3. The computational grid for the flow past NACA4412 airfoil is shown in the figure. The region close to leading edge is refined in the wall-normal direction to capture the transition of the flow (left), and the grid resolution is shown near the separation bubble (right).

Table 2. Grid spacings used in the NACA4412 simulations. The local boundary layer thickness ($\delta$) and wall skin friction coefficient ($C_f$) used to compute the viscous spacings are taken from the experiments of Wadcock (1987).

<table>
<thead>
<tr>
<th>Region $x/c$</th>
<th>$\Delta x/c$, $\Delta y_w/c$, $\Delta z_w/c$</th>
<th>$\Delta x^+$, $\Delta y_w^+$, $\Delta z_w^+$</th>
<th>$\delta/c$</th>
<th>$N_x/\delta$, $N_y/\delta$, $N_z/\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x/c = 0.2$</td>
<td>$2 \times 10^{-3}$, $2.5 \times 10^{-4}$, $1 \times 10^{-3}$</td>
<td>132, 16, 66</td>
<td>0.01</td>
<td>5, 20, 10</td>
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<td>$x/c = 0.6$</td>
<td>$2 \times 10^{-3}$, $1 \times 10^{-3}$, $1 \times 10^{-3}$</td>
<td>100, 50, 50</td>
<td>0.02</td>
<td>10, 20, 20</td>
</tr>
</tbody>
</table>

3.2. NACA4412 at near stall

Flow over an NACA4412 airfoil at near-stall conditions is studied at conditions based on the experiments of Wadcock (1987). The Reynolds number based on the chord length ($Re_c$) is 1.64 million; the freestream Mach number ($M_\infty$) is 0.2; and the angle of attack ($\alpha$) is 12°. A similar flow configuration is part of the NASA Langley Turbulence Modeling Resource website† at $\alpha = 13.87^\circ$ corresponding to experiments of Coles & Wadcock (1979). While the experiment of Wadcock (1987) had sawtooth-shaped elements to trip the boundary layer to turbulence at $x/c \approx 0.023$, we allow the flow to naturally transition

† Link: http://turbmodels.larc.nasa.gov/naca4412sep_val.html
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Figure 4. Instantaneous iso-contours of the $Q$-criterion is shown to depict the vortical features for the NACA4412 flow.

in the current simulations based on numerical disturbances generated by the computational grid and solver numerics. We closely follow Park & Moin (2014) in the problem setup, boundary conditions and other computational details. For the Park & Moin (2014) grid, simulations run with and without the wall model produced reasonable agreement with experiment. Hence, a coarser grid was generated to study the effect of the two wall models. The coarse grid had to be adapted to increase the wall-normal grid resolution close to the leading edge of the airfoil to obtain satisfactory results as will be discussed later. The grid closer to the leading edge and near the separation bubble is shown in Figure 3. Note the improved wall-normal grid resolution closer to the leading edge.

The LES grid contained around 13 million points in the leading edge-adapted grid. The size of the computational domain is $(L_x/c, L_y/c, L_z/c) = (9, 2.37, 0.1)$. The grid spacings used are listed in Table 2. The minimum viscous wall spacing $(\Delta y^+_w)$ is around 50 at $x/c = 0.6$, which is 5 times the spacing used by Park & Moin (2014). Hence, we expect the wall model to affect the flow to a greater extent for the current grid. For illustrative purposes, the instantaneous vortical features are shown in Figure 4 from the Park & Moin (2014) grid. It can be observed from the figure that the flow naturally undergoes transition to turbulence close to the leading edge and separates at around $x/c \approx 0.8$, consistent with experiment. The non-equilibrium wall model and no-wall model results are run using the Dynamic Smagorinsky Model for the LES subgrid terms, while the equilibrium wall model results are run using the Vreman model. We expect the wall model to have a greater influence on the results than the LES model, although this needs to be verified. The effect of the LES model will be assessed in a future study.

4. Results

The results from WMLES simulations using the equilibrium wall model (EQWM), non-equilibrium wall model (NEQWM) and without a wall model (NOSLIP) are discussed. Quantitative comparisons with available experimental data are used to assess the effect of the wall model.

4.1. Axisymmetric transonic bump

The incoming transonic turbulent boundary layer accelerates as it passes over the bump, produces a shock, and separates downstream of the shock. The flow separates at around $x/c = 0.7$ and reattaches at $x/c = 1.1$ in the experiments of Bachalo & Johnson (1986).
Figure 5. Mean streamwise velocity contours with streamlines depicting the separation bubble (left) and instantaneous density gradient contours depicting the shock (right) for the axisymmetric transonic bump case. Results from EQWM (top), NEQWM (center) and NOSLIP (bottom) are shown. The mean separation and reattachment locations are around 0.7 and 1.1 in the experiments of Bachalo & Johnson (1986).

Figure 6. Mean wall pressure variation is compared to experiment for the axisymmetric transonic bump flow. Legend: — EQWM, –– NEQWM, ·· NOSLIP and ■ Bachalo & Johnson (1986) experiment.

The mean streamwise velocity contours with streamlines depicting the separation bubble, along with instantaneous density gradient contours to depict the shock, are shown in Figure 5 for the EQWM, NEQWM and NOSLIP simulations. The shock is clearly captured in all the simulations. The size of the separation bubble noticeably differs between the three simulations, with the EQWM being in closest agreement with experiment. The NEQWM, which incorporates non-equilibrium effects, should ideally model the separated flow better, which is contrary to the present results. A finer grid calculation with increased resolution in the accelerating boundary layer over the bump and in the vicinity
of the shock will be needed to ascertain whether the same trend is observed in the finer grid before exploring possible reasons for the anomaly.

The mean wall pressure coefficient ($C_p$) variation with the streamwise co-ordinate ($x/c$) is shown in Figure 6. The shock location is correctly captured in all the simulations, although it is not as sharp as in the experiment, indicating the need for better grid resolution around the shock. The length of the separation bubble is best captured by EQWM. Profiles of mean horizontal velocity ($\overline{\Pi}/u_\infty$), resolved turbulent kinetic energy ($k = 0.5\overline{u'w' + v'v'}/u_\infty^2$), and Reynolds stress ($\overline{u'v'}/u_\infty^2$) from the simulations and experiment are shown in Figure 7 at $x/c = -0.25$ (upstream of bump), 0.688 (near shock), 0.938 (center of separation bubble) and 1.125 (downstream of reattachment point). Overall, the agreement with experiment is reasonable. Upstream of the bump, the results from the EQWM and NEQWM are identical since the non-equilibrium effects are negligible. The profiles on and downstream of the bump show differences between the three simulations with the EQWM agreeing better closer to the wall, and the NEQWM agreeing better away from the wall. A finer grid calculation is necessary to draw conclusions regarding the performance of the wall models for this flow.
The simulations were performed using the grid (coarser) described in Section 3.2 and the grid (finer) from Park & Moin (2014). The results from the two grids for the mean wall pressure coefficient ($C_p$) are compared to experiment in Figure 8. Both EQWM and NEQWM results match well with experiment. The no-wall model (NOSLIP) simulation also produces good agreement with experiment for the finer grid, but fails to predict the separation region for the current grid, indicating that the wall model has a greater impact on the flow for the latter grid. It was also observed that the grid closer to the leading edge should have sufficient wall-normal resolution to capture laminar-turbulent transition of the boundary layer. Figure 9 shows the instantaneous streamwise velocity along with mean streamlines for the coarse grid with and without adaptation of the grid in the wall-normal direction (as discussed in Figure 3) until $x/c = 0.3$. The grid without leading-edge adaptation fails to capture the separation bubble, showing the importance of resolving the upstream boundary layer to capture transition.

The mean streamwise velocity ($\overline{u_p}$), resolved normal ($\overline{u_p' u_n'}$) and shear ($\overline{u_p' u_n}$) component of Reynolds stress profiles are shown for the coarse grid (with wall-normal grid adaptation near the leading edge) in Figure 10. Here, $u_p$ and $u_n$ are aligned with the local tangential and normal directions of the surface for the locations on the airfoil while they are aligned with the freestream and wall-normal directions in the wake. The profiles are shown at $x/c = 0.529$ (attached turbulent boundary layer) and 0.952 (within the separation bubble) on the airfoil, and $x_w/c = 0.007$ and 0.282 downstream of the airfoil trailing edge ($x_w = 0$ at the airfoil trailing edge). Reasonable agreement is observed with
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Figure 10. Profiles of time-averaged wall-parallel velocity ($u_p$), normal ($u_p^nu_p^n$) and shear ($u_p^nu_p^s$) component of Reynolds stress are compared to the experiment. Legend: — EQWM, – – NEQWM, – · · NOSLIP and – – Wadcock (1987) experiment.

experiment in both the mean velocity and Reynolds stresses except at $x/c = 0.952$, where the stresses predicted by the simulations deviate significantly. The mean velocity profile at $x/c = 0.952$ indicates that the simulations severely underpredict the magnitude of reverse flow near the surface, possibly explaining the differences in the turbulent stresses. Overall, the difference between EQWM and NEQWM predictions for this flow appear to be only marginal for the quantities currently reported, with the NEQWM results agreeing better with experiment.

5. Summary

Wall-modeled Large Eddy Simulation was performed for two challenging turbulent flows involving separation. The transonic axisymmetric flow past a bump and subsonic flow past the NACA4412 airfoil at near-stall conditions were simulated. Overall, reasonable agreement with experiment was observed for both the equilibrium and non-equilibrium wall models. For the transonic flow past a bump, the equilibrium wall model agreed better with experiment, whereas the non-equilibrium model yielded marginally better results for the flow past NACA4412 at near-stall conditions. Further analysis is
Iyer et al. needed to assess the merits and demerits of the two wall models for turbulent separated flow.

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REFERENCES


