Numerical Methods — overview

Large-eddy simulation (LES) of turbulent flows has become an attractive tool for prediction of complex engineering flows. Tackling such applications has become possible due to advances in numerical methods, computer hardware, and subgrid-scale (SGS) models. These advances have also rendered LES affordable, and therefore viable for industrial applications. The main reason for the higher accuracy of LES compared with Reynolds-averaged Navier-Stokes (RANS) is the resolution of a portion of the turbulence spectrum, as opposed to phenomenological modeling of the entire turbulence stresses in RANS. It is therefore important to minimize the distortion of the resolved turbulence structures by numerical artifacts. For example, a common practice in traditional computational fluid dynamics, which has been adopted in some LES, is the introduction of numerical dissipation to counter nonlinear numerical instabilities, to capture shock waves, and to achieve robustness of the computations. However, numerical dissipation artificially dampens the turbulent eddies that one presumably intended to resolve in the first place by choosing LES over RANS. Other important requirements for LES are higher grid quality, suitable for the resolution of turbulent eddies, and compatibility of the SGS model with the numerical method. These issues were the focus of the five projects by the Numerical Methods group.

In one of the most complex applications of LES to date, Lehmkuhl *et al.* simulated flow over realistic aircraft configurations at high angles of attack. Finite-element and finite-volume codes were used. The only common feature of the methods was the use of minimal numerical dissipation. Both calculations used the same equilibrium wall model and SGS model. The agreement of the global forces and pressure distributions with the experiments at various angles of attack, including at maximum lift and post stall, was remarkably good. Another remarkable outcome was that the calculations were completed in 1 to 3 days on 2000 CPUs, demonstrating that LES has now "arrived" and is ready for exploitation in a myriad industrial applications.

In applications with highly complex configurations, the use of hybrid unstructured mesh topologies is often necessary. Domino *et al.* assessed the suitability of heterogeneous hybrid meshes for turbulence simulations in a benchmark low-Reynolds-number channel flow. The mesh was intentionally chosen not to be symmetric across the channel, and the symmetry of the flow statistics was used as a measure of the quality of the results. The mean velocity profile was predicted with reasonable accuracy. However, as expected, given the strong link between the SGS model and the grid, turbulent intensities were asymmetric across the channel, with the symmetry increasing with mesh refinement.

Discontinuous Galerkin (DG) methods have received some attention for numerical simulation of turbulent flows owing to their parallel scalability and potential for higher-order numerical accuracy. The variational multiscale approach (VMS) originally proposed for the finite-element method is naturally extended to DG. Naddei *et al.* analyzed the spectral properties of energy transfer in the DG-VMS approach and defined the large-scale field in LES based on truncation of polynomial expansions in DG. In an application to the Taylor-Green vortex, these authors showed that, with sufficient resolution, the energy transfer spectrum achieved the desired shape, consisting of concentration of energy transfer to subgrid scales from the smallest resolved scales. Murman & Frontin showed that SGS modeling approaches should take into account the numerical dissipation (or stabilization schemes) used in DG methods. They showed that an entropy-conservative scheme, together with an appropriate SGS model, results in better predictions of isotropic turbulence and channel flow than entropy-stabilized schemes with or without SGS models.

Finally, Lv *et al.* developed a novel wall model for DG-based LES of turbulent boundary layers. The model is based on augmenting the DG expansion in the cell adjacent to the wall by an additional basis function that incorporates the logarithmic law of the wall. The results from LES of high-Reynolds-number turbulent channel flow and the NASA transonic bump are very encouraging. Similar to Murman & Frontin, Lv *et al.* noted the necessity of including explicit SGS model in DG, as well as the inadequacy of numerical stabilization in DG as an SGS model.

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