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Vortices in protoplanetary disks and the formation of planetesimals

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The least understood step in the formation of planets is the creation of kilometer-size planetesimals from centimeter-size dust grains. It has been suggested that vortices within the protoplanetary disk may concentrate dust particles at their centers, which may enhance the dust density enough to trigger a gravitational instability to clumping. Our companion paper in this volume discusses the fluid dynamics of 3D vortices in a protoplanetary disk. Here, we present preliminary calculations of the motion of dust grains within such 3D vortices. We confirm that grains are focused toward the centers of vortices, and offer a simple physical picture as to why heavy particles are not centrifuged out.

1. Introduction: The planetesimal formation problem

A protostar forms when a dense region of the interstellar medium collapses due to its self-gravity (the Jeans instability). Due to conservation of angular momentum, matter cannot fall directly onto the central protostar, but spirals in, forming a protoplanetary accretion disk. It is within such dusty protoplanetary disks that protoplanets form. See Shu, Adams, & Lizano (1987) for a general review of star formation, and Lissauer (1993) for a general review of planet formation.

1.1. *Binary agglomeration versus gravitational instability*

In the earliest stages of planetesimal formation, micron-sized dust grains collide and combine to form larger particles – a process called binary agglomeration. However, it is unclear whether this mechanism can efficiently work once particles reach centimeter to meter sizes, since impact cratering and disruption become important. The mechanical and chemical processes involved in grain agglomeration are poorly understood for particles in this size regime. It would seem that two colliding "rocks" are just as likely, if not more likely, to break one another apart as opposed to combining to form a larger one (Weidenschilling 1984, Weidenschilling & Cuzzi 1993). Such slow growth for decimeter particles via binary agglomeration leads to a problem with the timescale associated with the formation of the giant planets. Rocky cores of several Earth masses must be formed in order to gravitationally capture sufficient gas to create the extensive atmospheres of the giant planets. However, this must be done in less than a million years, before the disk gas is dispersed via accretion, photoevaporation, stellar winds, or close stellar encounters (Hollenbach, Yorke, & Johnstone 1999).

An alternative theory is that if the protoplanetary disk is quiescent (that is, not turbulent), the dust grains can settle into a thin sub-layer that might be dense enough to be

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gravitationally unstable to clumping (Safronov 1960, Goldreich & Ward 1973). Whether the disk is turbulent or not is still an unresolved and greatly contested issue. Even if the dust could settle into a thin sub-layer at the midplane, Weidenschilling (1980) has argued that a Kelvin-Helmholtz instability would develop between the dust-dominated layer at the midplane (which orbits at the Keplerian velocity) and the gas-dominated regions above and below the midplane (which orbit at sub-Keplerian velocities due to partial support by the internal pressure gradient). This instability might generate turbulence that would "kick-up" the dust and inhibit the gravitational clumping (Champney, Dobrovolskis, & Cuzzi 1995, Cuzzi, Champney & Dobrovolskis 1993).

1.2. *Vortices in a protoplanetary disk*

Within the past five years, theorists have turned their attention to vortices within protoplanetary disks, and the role they might play in angular momentum transport, as well as the "seeding" of planet formation. Lovelace, *et al.* (1999) have found a linear instability for nonaxisymmetric Rossby waves in thin, nonmagnetized, Keplerian disks. They noted that in the nonlinear limit, such Rossby waves might break and coalesce to form vortices. Bracco, *et al.* (1998), using the incompressible "shallow-water" equations, have shown that long-lived, coherent, anticyclonic vortices form in a Keplerian disk that was initially seeded with a random perturbation field. They noted that smaller vortices merged to form larger vortices, reflecting the inverse cascade of energy from small to large scales that is characteristic of 2D turbulent flows. Godon & Livio (1999a, 1999b, 2000) have also studied the stability and lifetime of vortices in protoplanetary disks, and have shown that anticyclonic vortices can survive in the flow for hundreds of orbits. Barge & Sommeria (1995) and Tanga, *et al.* (1996) have proposed that vortices in a protoplanetary disk can capture dust grains and concentrate them in their centers. This would locally enhance the grain surface density which may trigger gravitational instability and form planetesimals.

The focusing of dust grains into protoplanetary disk vortices may seem quite surprising to those more familiar with laboratory flows in which heavy particles are typically centrifuged out of vortices on a short timescale. The key difference here is gravity. What follows is our physical picture for how vortices focus dust grains into their centers. The base flow in a cool, thin disk (with very weak radial pressure support) is Keplerian: $V_k = \sqrt{GM/r}$, and $\Omega_k = \sqrt{GM/r^3}$, where V_k is the linear (azimuthal) velocity, Ω_k is the angular velocity, r is the cylindrical radius from the center of the disk (where the protostar is) and M is the mass of the central protostar (Frank, King, & Raine, 1995). Physically, the Keplerian velocity is just the usual orbital velocity of any object in a circular orbit about a central gravitational source. The Keplerian shear is anticyclonic and of the same order of magnitude as the Keplerian angular velocity itself: $\sigma_k \equiv r d\Omega_k/dr = -\frac{3}{2}\Omega_k$. Marcus's extensive work with the Great Red Spot and other jovian vortices has shown that in order for a vortex in a shear to be long-lived, the vorticity of the vortex must be of the same sign as, and of at least the same order of magnitude as, the background shear (Marcus 1988, 1989, 1990, 1993, Marcus & Lee, 1994). Consider one such anticyclonic vortex in the disk orbiting the central protostar at a cylindrical radius r_0 from the center of the disk (see Fig. 1 & 2). The flow around the vortex on the side $r > r_0$ opposes the overall rotation of the disk, making the total angular velocity of the gas sub-Keplerian. For $r < r_0$, the anticyclonic vortex enhances the overall rotation of the disk, making the angular velocity super-Keplerian. (We have ignored the fact that the average gas flow in the disk is probably slightly sub-Keplerian due to partial support by an internal pressure gradient – our argument will be basically unchanged.)

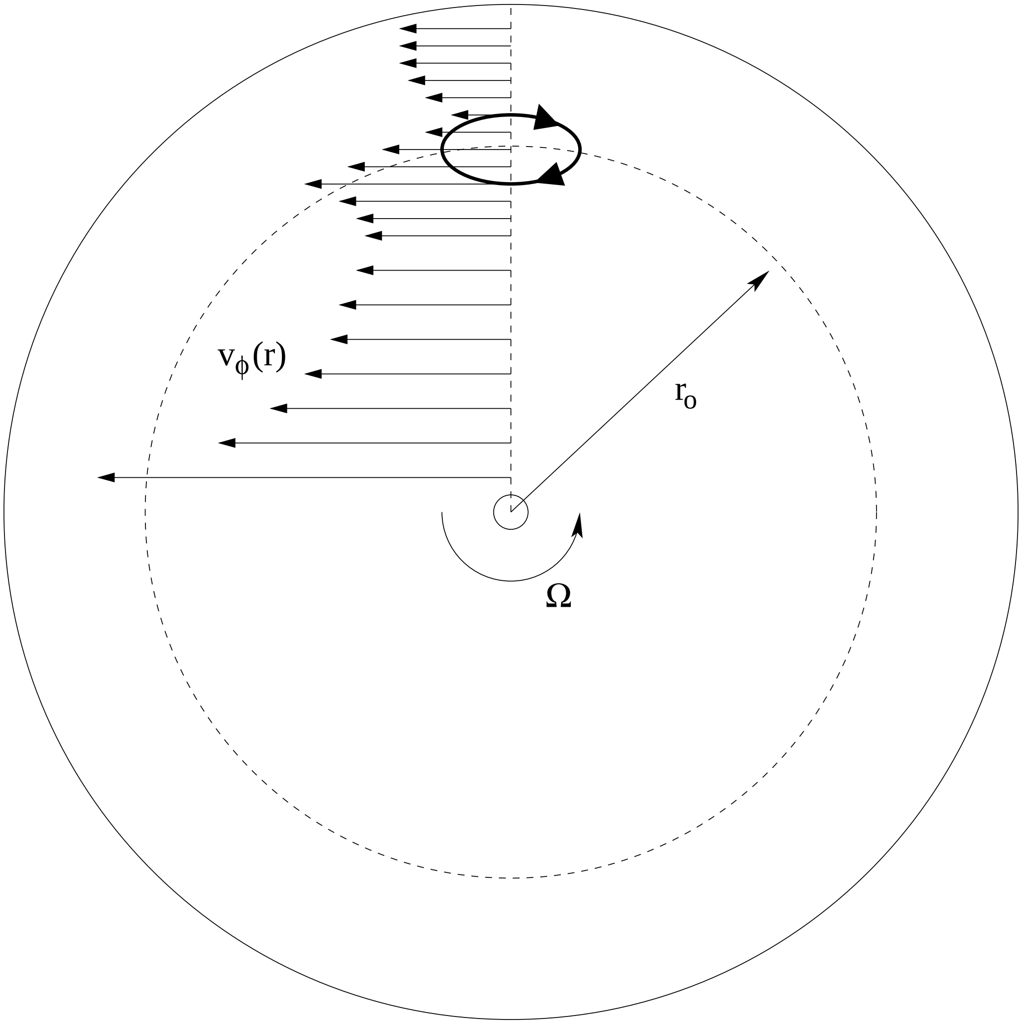

FIGURE 1. Keplerian disk with anticyclone. Anticyclone is located at cylindrical radius r_0 from the center of the disk, where the protostar is located. Arrows indicate the total azimuthal velocity in the inertial frame (mean velocity of the Keplerian disk plus that due to the anticyclone). The mean azimuthal velocity (without vortices) is Keplerian: $V_k = \sqrt{GM/r}$, where M is the mass of the central protostar. Note that the anticyclone increases the azimuthal velocity within the disk for $r < r_0$ and decreases the azimuthal velocity for $r > r_0$.

Let us now consider the motion of dust grains in the disk. If there were no forces acting on the grains other than gravity from the central protostar, the grains would naturally follow circular, Keplerian orbits. However, drag between the grains and the gas can alter such orbits in non-intuitive ways. First, let's consider a grain in a Keplerian orbit with $r > r_0$. When it approaches the vortex (or more precisely, as the vortex approaches the dust grain since the grain is on an outer, slower orbit), the ambient flow will be sub-Keplerian. Since the dust grain is going faster than the gas, any drag by the gas will cause an azimuthal deceleration of the grain. This decrease of the grain's angular momentum does not cause its azimuthal velocity to decrease, but instead, the grain moves inward

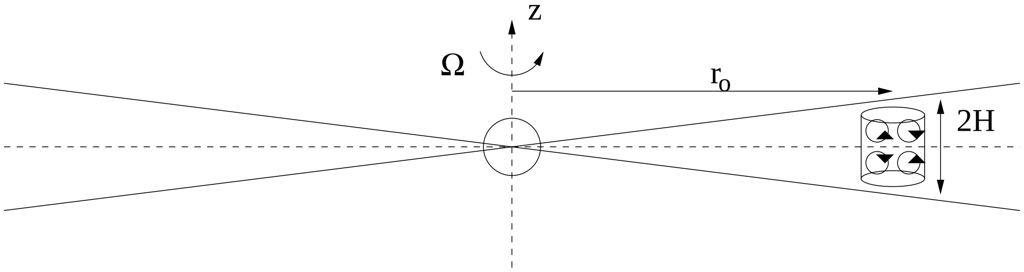

FIGURE 2. Side view of Keplerian disk with anticyclone. The disk is believed to be "flared", with scale height H increasing with radius. The anticyclone fills the disk in the vertical direction.

toward the protostar, which is also the direction towards the vortex center. On the other hand, when a grain orbiting at a radius less than r_0 encounters the anticyclone, the ambient fluid flow is super-Keplerian and drag accelerates the grain azimuthally, pushing it radially outwards and towards the radius of the vortex center. Thus, any type of fluid drag causes grains to be deflected towards the location of the anticyclone. For similar, but slightly more complex reasons, dust grains are also attracted (under some conditions) azimuthally towards the vortex center.

Drag is not the only way that vortices can trap dust grains. We know that the Great Red Spot on Jupiter sustains itself against dissipation by capturing small vortices and "consuming" their vorticity. We propose to study this possibility for vortices in a protoplanetary disk. We suspect that a long-lived vortex will be in a dynamic balance between growth via mergers with smaller vortices and dissipation via Rossby wave radiation. We hypothesize that as a large vortex consumes smaller vortices, Rossby waves will carry away excess angular momentum, keeping the area of the large vortex nearly constant, but that these waves will not drive out dust grains. In other words, the large vortex consumes the dust, but not the area, of the smaller vortices, and hence the grain density would increase.

2. Equations of motion

2.1. $Ro \approx 1$, *3d vortices*

The reader is directed to our companion paper in this volume (Barranco, Marcus, & Umurhan, 2000) that discusses the details of finding vortex solutions within a protoplanetary accretion disk. Here, we would just like to highlight some key assumptions and scalings that make our work significantly different from that of others who are studying vortices in the context of planetesimal formation.

As previously discussed, the Keplerian shear in the disk is anticyclonic and of the same order of magnitude as the Keplerian angular velocity itself: $\sigma_k \equiv r d\Omega_k/dr = -\frac{3}{2}\Omega_k$. Since the vorticity associated with an anticyclone must be of the same order as the background shear if the vortex is to be long-lived, then the relative vorticity associated with the vortex is of the same order as the "planetary vorticity" of the disk itself: $[\omega] \sim \sigma_k \sim \Omega_k$. (Here, we use square brackets to indicate order of magnitude of the bracketed quantity.) This immediately implies that the Rossby number for vortices in a Keplerian shear is of order unity: $Ro = [\omega]/2\Omega_k \sim 1$.

Another key assumption is that long-lived vortices should be subsonic; otherwise shocks would develop that would quickly dissipate the vortex motion. The characteristic velocity

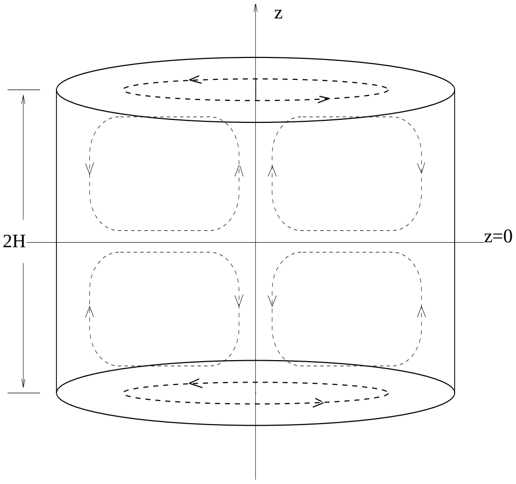

FIGURE 3. Schematic representation of flow within a 3D vortex in a protoplanetary disk. Note that the vertical velocity vanishes in the midplane of the disk.

of the vortex motion is: $[v] \sim (dV_k/dr)L_r \sim (L_r/r)V_k$, where L_r is the characteristic radial extent of a vortex. It can easily be shown that hydrostatic balance in the vertical direction implies $c_s/V_k \sim H/r$, where c_s is the sound speed, and H is the scale height of the disk (Frank, King, & Raine 1985). Hence, $[v] \sim (L_r/H)c_s$, or $L_r/H \sim [v]/c_s$. Thus, in order to have subsonic vortices, their horizontal extent must be less than the thickness of the disk, and the vortices are 3-D, not 2-D.

The three dimensionality, as well as the fact the vortex flow is of order unity Rossby number, has been neglected by all previous researchers (Adams & Watkins 1995, Bracco, *et al.* 1998, Sheehan, *et al.* 1999, Godon & Livio 1999a, 1999b, 2000). The quasi-geostrophic and "shallow-water" sets of equations are not appropriate for the study of vortices in a protoplanetary disk, and one must develop a new set from a rigorous asymptotic analysis. Again, the reader is referred to our companion paper for more details. In this article, we are concerned with the motion of dust particles in and around 3D vortices. Figure 3 shows a schematic of the type of 3D vortices we believe exist in protoplanetary disks. For this preliminary study, we have assumed that the horizontal component of the gas velocity is due to an elliptical patch of constant vorticity embedded within a Keplerian shear flow (Moore & Saffman, 1971). The vertical component of the gas velocity is an approximate analytical fit for the vertical velocity of 3-D vortices in numerical simulations of Taylor-Couette flow.

2.2. *Lagrangian tracking of particles*

Now we consider the motion of individual grains of dust in and around a vortex in a protoplanetary disk. Consider a vortex whose center is located at a cylindrical radius r_0 from the protostar. Henceforth, we work in a rotating frame so that the center of the vortex is stationary. The angular velocity of the rotating frame, with respect to the inertial frame of "fixed stars", is $\Omega_0 = \sqrt{GM/r_0^3}$. We will also "Cartesianize" the domain of interest: let x be the (negative) azimuthal direction ($-\phi \to x$), y be the radial direction ($r = r_0 + y$, so that $y = 0$ at center of vortex), and z be the height above the midplane. The forces acting on a grain are gravity from the central protostar, Coriolis and centrifugal forces, and frictional drag due to the relative velocity between the gas and dust particles. The equations of motion for the grains are:

$$\ddot{x} = 2\Omega_0 \dot{y} - \frac{1}{t_s}(\dot{x} - V_x^{gas}), \tag{2.1}$$

$$\ddot{y} = -\frac{GM}{(r_0 + y)^2} + \Omega_0^2 (r_0 + y) - 2\Omega_0 \dot{x} - \frac{1}{t_s}(\dot{y} - V_y^{gas}), \tag{2.2}$$

$$\ddot{z} = -\frac{GMz}{(r_0 + y)^3} - \frac{1}{t_s}(\dot{z} - V_y^{gas}), \tag{2.3}$$

where $V_{x,y,z}^{gas}$ is the velocity of the gas in the rotating frame, and t_s is the stopping time, i.e., is the e-folding time for the particle to come to rest.

The exact form for the stopping time depends on the size and shape of the dust grains as well as on the physical conditions within the protoplanetary nebula. The Reynolds number for the flow around a grain is of order unity: $Re = [v]d/\nu \sim 1$, where we have taken the characteristic velocity in a vortex to be bounded by the sound speed $[v] \sim c_s \sim 1$ km/s, the diameter of a grain is of order $d \sim 10$ cm, and the kinematic viscosity for the gas is of order $\nu \sim 10^6$ cm^2/s. In fact, this is an overestimate for the the Reynolds number since the velocity we used in computing the Reynolds number should actually be the *differential* velocity between the grain and the gas, which is typically much less than the velocity of the gas itself. Thus, for this preliminary study, we assume that the flow around grains is approximated by Stokes flow:

$$t_s = \frac{1}{36} \frac{\rho_{grain}}{\rho_{gas}} \frac{d_{grain}^2}{\nu} \sim 10^4 \mathrm{s} \sim 0.001 \times T_{orb}, \tag{2.4}$$

where ρ_{grain} is the density of an individual dust grain, of order a few g/cm^3, $\rho_{gas} \sim 10^{-9}$ g/cm^3 is the density of gas in the disk, d_{grain} is the diameter of the dust grains, of order a few decimeters, and T_{orb} is the orbital period of the vortex around the protostar, of order a year at 1 AU (one astronomical unit, equal to the distance between the Earth and the Sun, 1.5×10^{13} cm). It turns out that the mean free path of the gas molecules within the nebula is of the same order as the size of the grains, so the Stokes flow assumption may not be an entirely valid one. In this regime, the interaction between the grains and the gas particles is a problem of kinetic theory, not fluid or continuum mechanics. However, we don't expect that the qualitative behavior of the motion of the dust grains will depend strongly on the exact nature of the drag. This will be explored in more detail in future work.

3. Preliminary results

3.1. *The settling of particles in the midplane*

Figure 4 shows the 3D evolution of an ensemble of randomly placed grains in and around a 3D vortex in a protoplanetary disk. For clarity, the vortex flow itself is not shown. The vortex center is located at a distance 1 AU away from the protostar. Regarding the scale, one unit on the axes corresponds to approximately 0.25 AU. Note how quickly grains settle into the midplane of the disk. The timescale for the settling is a few stopping times. This was expected as we have not yet included any turbulence within the disk. Although the vortex does have a vertical component of velocity, it is zero within the midplane. Most particles have settled in the midplane long before they encounter the vortex, and thus are not excited by the vertical velocity of the vortex. We expect these results to be fundamentally altered when we explicitly include the effects of turbulence.

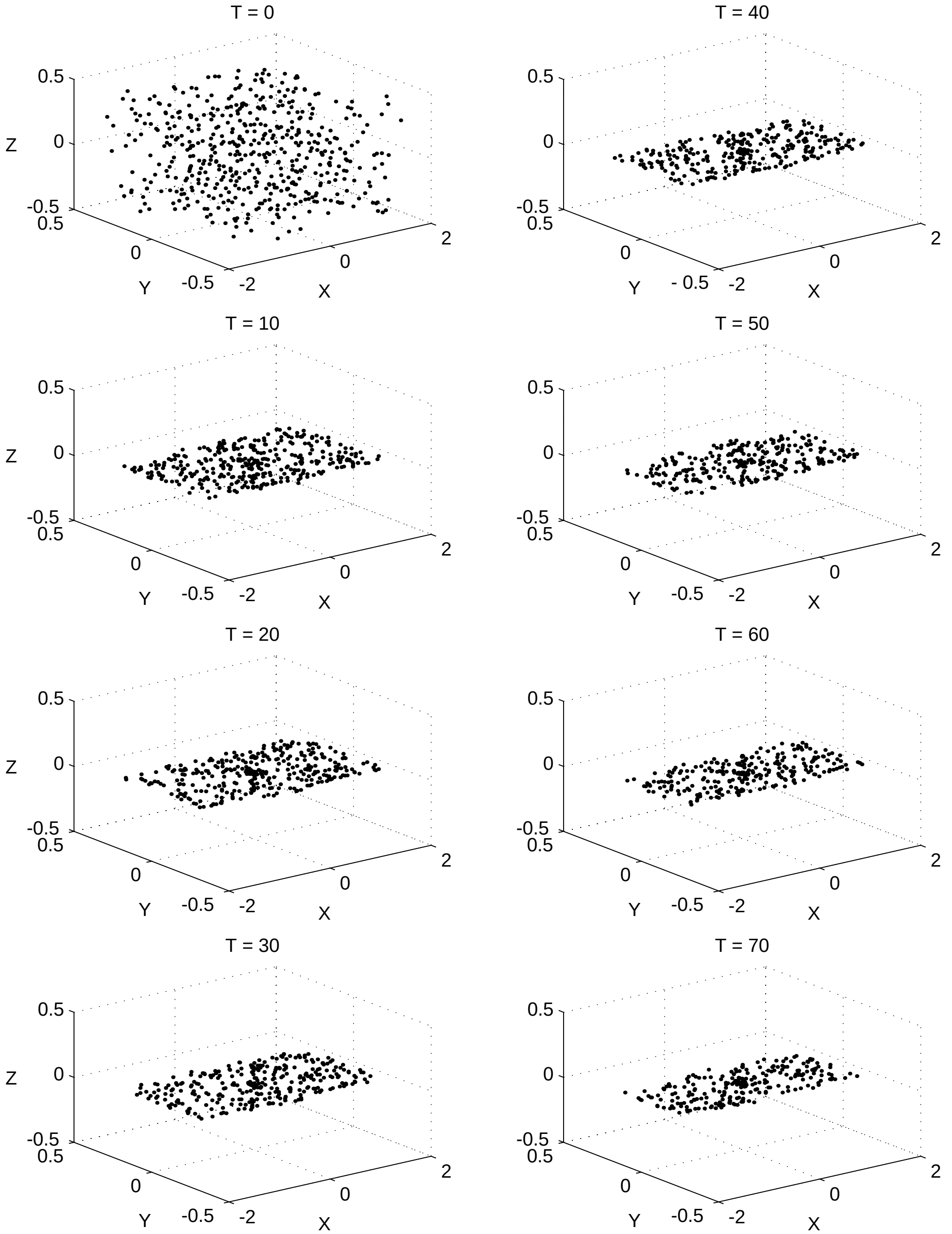

FIGURE 4. 3D plots of the evolution of an ensemble of grains in and around a vortex in a protoplanetary disk. For clarity, the vortex flow is not shown. The center of the vortex is located 1 AU from the protostar. One unit on the axes corresponds to approximately 0.25 AU. The x-direction is the azimuthal direction, the y-direction is the radial direction, and the z-direction is the height above the midplane. Timestep between each snapshot is 10 orbital periods.

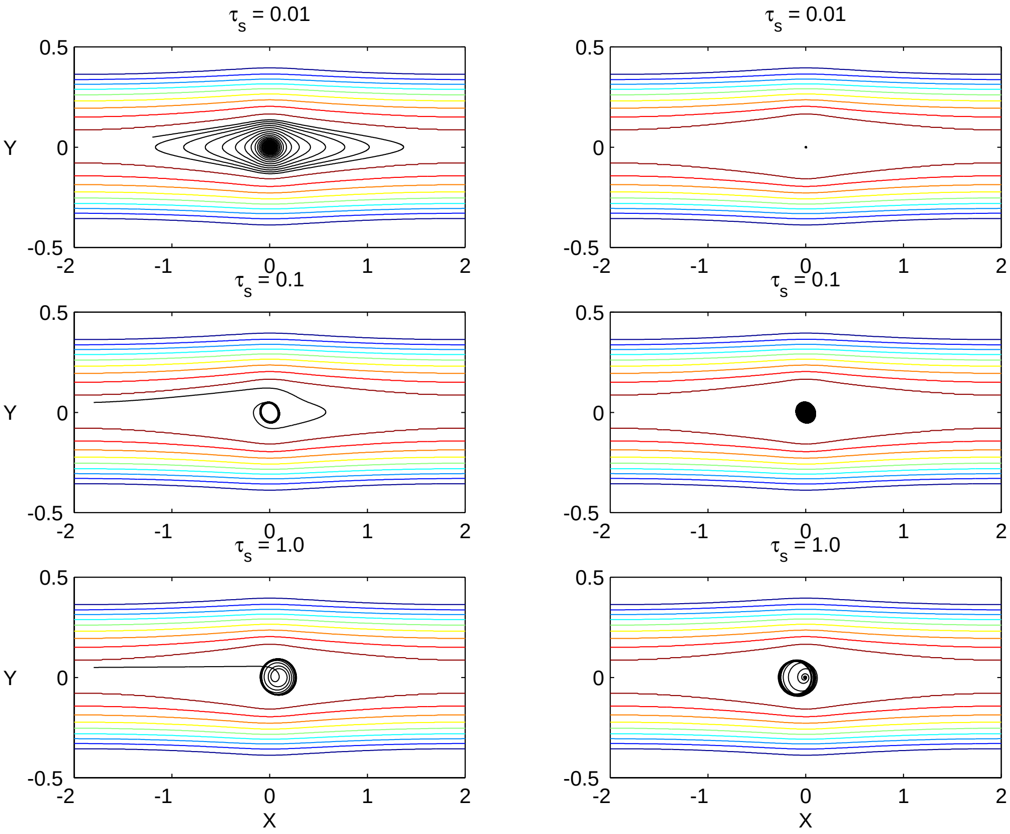

Figure 5. Trajectories (projected into the midplane) of individual grains. The gray lines of various shades indicate the streamlines of the gas flow around the vortex. The vortex itself is not shown for clarity. Solid black lines are the trajectories of individual grains. $\tau_s \equiv t_s/T_{orb}$ is the stopping time normalized by the orbital period. From the top down, $\tau_s = 0.01$, 0.1, 1.0. First column shows trajectories of grains that were started outside the vortex on Keplerian orbits. Second column shows trajectories of grains started at the center of the vortex.

Turbulence will "kick up" the grains, preventing them from settling into a thin layer about the midplane. Particles that encounter the vortex out of the midplane will be excited by the vortex's vertical velocity, further stirring up the particle motion.

3.2. *The spiraling of particles to the centers of vortices*

Figure 5 shows the trajectories (projected into the midplane) of individual grains. The vortex location and scale are the same as that described in the previous section. Here, we vary the stopping time, now normalized by the orbital period (1 year at 1 AU for a solar mass protostar): $\tau_s \equiv t_s/T_{orb}$. A shorter stopping time corresponds to smaller particles, which quickly react to the gas flow. We expect that the trajectories of these smaller particles will closely follow the streamlines of the gas. Longer stopping times correspond to larger particles, which, because of their inertia, take longer to adjust to the gas flow.

In the first column of figures in Fig. 5, the grains start outside the vortex on Keplerian orbits around the protostar. Note that the lighter particle immediately reacts to the presence of the vortex, closely following the closed streamlines, yet slowly spiraling into

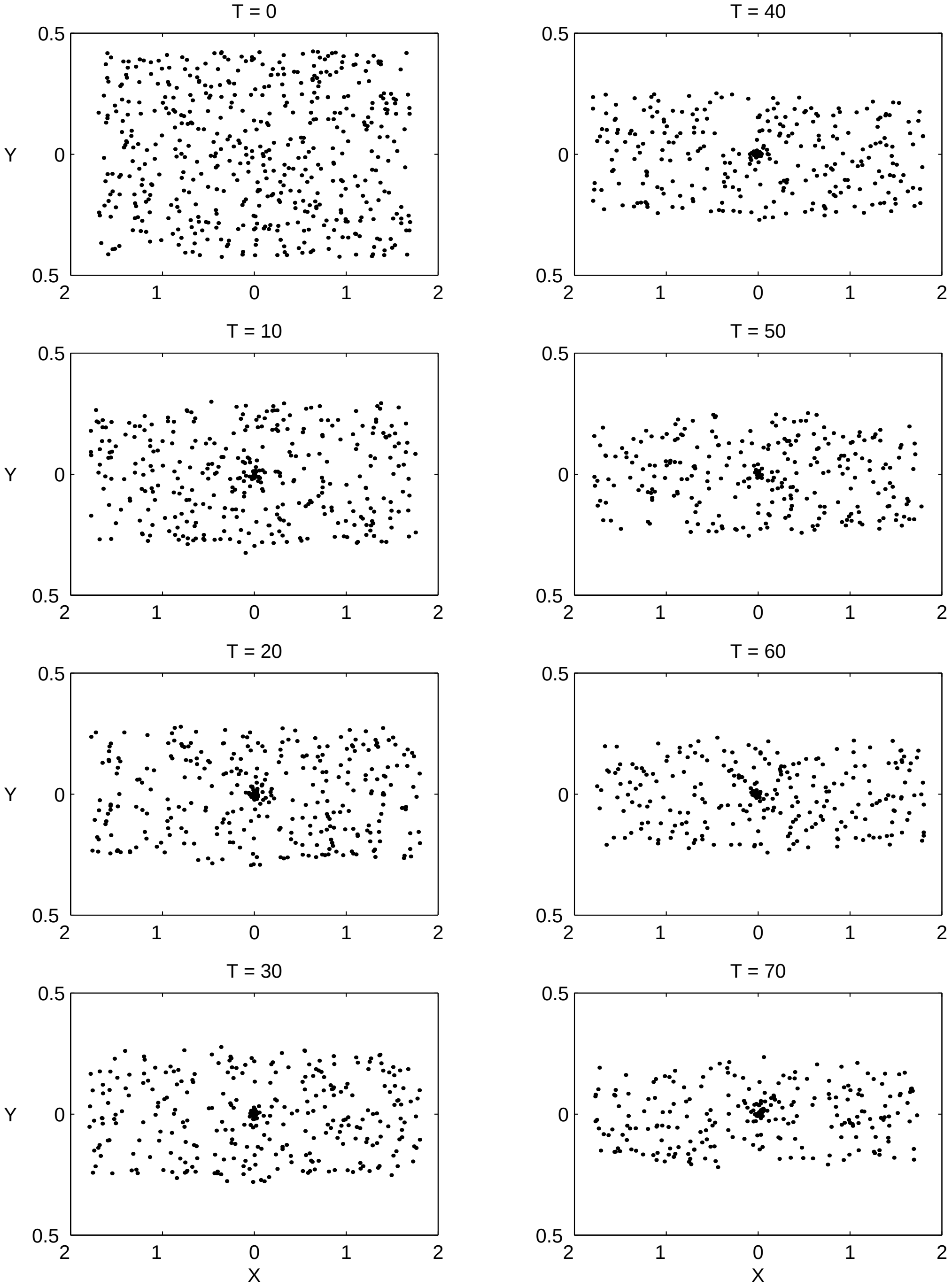

FIGURE 6. Same as Fig. 4, but projected into the midplane of the disk. For clarity, the vortex flow is not shown. The center of the vortex is located 1 AU from the protostar. One unit on the axes corresponds to approximately 0.25 AU. The x-direction is the azimuthal direction, the y-direction is the radial direction, and the z-direction is the height above the midplane. Timestep between each snapshot is 10 orbital periods.

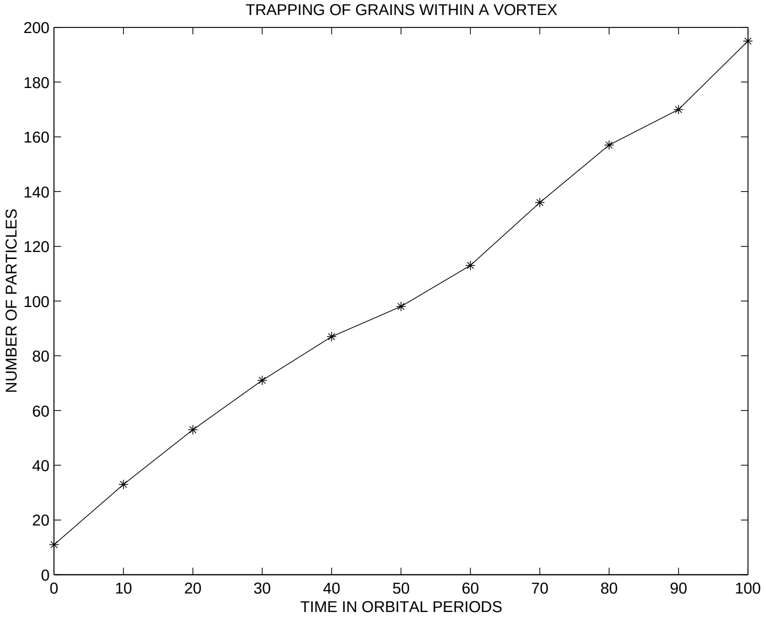

Figure 7. Number of particles trapped within vortex as a function of time. The vortex boundary is defined by the boundary of the patch of constant vorticity.

the center. The heavier particles follow their Keplerian orbits, not reacting to the vortex until they get very close.

Also notice that whereas the lightest particle eventually spirals in very deep into the center of the vortex, the heavier particles settle down onto orbits around the center of the vortex. The longer the stopping time, the larger the radius of the final orbit. We wanted to test this further by starting the grains an infinitesimal distance away from the center of the vortex (see the second column of Fig 5). The lightest particle remains at the center of the vortex; this is a stable point. The heavier particles are seen to spiral out, eventually settling into orbits around the vortex center. In fact, the final orbit for these particles is the same whether they start outside or inside the vortex. In the future, we would like to further explore the exact nature of these "attractors".

Figure 6 shows the 2D projection into midplane of the same data shown in Fig. 4. One can start to see the concentration of grains within the vortex. Figure 7 is a much clearer illustration of this phenomenon. The number of grains within the vortex boundary (i.e. the boundary of the elliptical patch of constant vorticity) is plotted as a function of time (expressed in orbital periods). We observe that the density of grains inside the vortex increases by a factor of roughly 20 in 100 orbital periods, consistent with the previous results of Barge & Sommeria (1995) and Tanga, *et al.* (1996).

4. Future work

There are many unresolved issues regarding planetesimal formation in vortices, specifically with regard to turbulence in the protoplanetary disk. Until now, most have focused only on whether 2D *laminar* vortices can capture dust in a 2D *laminar* disk. Even if a vortex can capture dust grains, it has not yet been demonstrated that this triggers gravitational instability, given that disk turbulence prevents the dust grains from settling. All previous research on disk vortices has been 2D, and therefore unable to even consider the vertical settling of dust grains within a vortex. Our research will focus on 3D vortices, and we will be well-positioned to tackle these issues. Specifically, we want to examine whether vortices laminarize the flow in their interiors (the way laboratory vortices do), shielding the captured dust from the turbulence and allowing the grains to settle into a dense enough layer that becomes gravitationally unstable. We also want to examine the effect of the vertical velocity within the vortex on the accumulation of grains.

REFERENCES

Adams, F. C. & Watkins, R. 1995 Vortices in circumstellar disks. *ApJ.* **451**, 314-327.

Barge, P. & Sommeria, J. 1995 Did planet formation begin inside persistent gaseous vortices? *A&A.* **295**, L1-4.

Bracco, A., Provenzale, A., Spiegel, E. A. & Yecko, P. 1998 Spotted disks. In *Proceedings of the Conference on Quasars and Accretion Disks* (M. Abramowicz, Ed.), Cambridge University Press, Cambridge.

Champney, J. M., Dobrovolskis, A. R. & Cuzzi, J. N. 1995 A numerical turbulence model for multiphase flows in the protoplanetary nebula. *Phys. Fluids.* **7**, 1703-1711.

Cuzzi, J. N., Dobrovolskis, A. R. & Champney, J. M. 1993 Particle-gas dynamics in the midplane of a protoplanetary nebula. *Icarus.* **106**, 102-134.

Frank, J., King, A. & Raine, D. 1985 *Accretion Power in Astrophysics.* Cambridge University Press, Cambridge.

Godon, P. & Livio, M. 1999 On the nonlinear hydrodynamic stability of thin Keplerian disks. *ApJ.* **521**, 319-327.

Godon, P. & Livio, M. 1999 Vortices in protoplanetary disks. *ApJ.* **523**, 350-356.

Godon, P. & Livio, M. 2000 The formation and role of vortices in protoplanetary disks. *ApJ*, **537**, 396-404.

Goldreich, P. & Ward, W. R. 1973 The formation of planetesimals. *ApJ.* **183**, 1051-1061.

Hollenbach, D., Yorke, H. W. & Johnstone, D. 2000 Disk dispersal around young stars. In *Protostars and Planets IV*, in press.

Lissauer, J. J. 1993 Planet formation. *ARAA.* **31**, 129-174.

Lovelace, R. V. E., Li, H., Colgate, S. A. & Nelson, A. F. 1999 Rossby wave instability of Keplerian accretion disks. *ApJ.* **513**, 805-810.

Marcus, P. S. 1988 Numerical simulations of the Great Red Spot of Jupiter. *Nature.* **331**, 693-696.

Marcus, P. S. 1989 Vortex dynamics. *Physics Today.* S40-41.

Marcus, P. S. 1990 Vortex dynamics in a shearing zonal flow. *J. Fluid Mech.* **215**, 393-430.

Marcus, P. S. 1993 Jupiter's Great Red Spot and other vortices. *ARAA.* **31**, 523-573.

Marcus, P. S. & Lee, C. 1994 Jupiter's Great Red Spot and zonal winds as a self-consistent one-layer quasi-geostrophic flow. *Chaos.* **4**, 269-288.

Moore, D. W. & Saffman, P. G. 1971 Structure of a line vortex in an imposed strain. In *Aircraft Wake Turbulence and its Detection.* (J. Olsen, A. Goldberg, & N. Rogers, Eds.), Plenum, New York.

Safranov, V. S. 1960 *Ann. Astrophys.* **23**, 901-904.

Sheehan, D. P., Davis, S. S., Cuzzi, J. N. & Estberg, G. N. 1999 Rossby wave propagation and generation in the protoplanetary nebula. *Icarus.* **142**, 238-248.

Shu, F. H., Adams, F. C. & Lizano, S. 1987 Star formation in molecular clouds: Observation and theory. *ARAA.* **25**, 23-81.

Tanga, P., Babiano, A., Dubrulle, B. & Provenzale, A. 1996 Forming planetesimals in vortices. *Icarus.* **121**, 158-170.

Weidenschilling, S. J. 1980 Dust to planetesimals: Settling and coagulation in the solar nebula. *Icarus.* **44**, 172-189.

Weidenschilling, S. J. 1984 Evolution of grains in a turbulent solar nebula. *Icarus.* **60**, 555-567.

Weidenschilling, S. J. & Cuzzi, J. N. 1993 Formation of planetesimals in the solar nebula. In *Protostars and Planets III* (E. H. Levy and J.I. Lunine, Eds.), University of Arizona Press, Tucson.