Optimization of trailing vortex destruction by evolution strategies

By G.-H. Cottet†, I. Sbalzarini‡, S. Müller‡ AND P. Koumoutsakos‡

We apply evolution strategies to optimize the instability growth of several pairs of vortices which model the wake of airplanes in landing configuration. For the case of two pairs, the evolution strategy finds a set of optimal parameters strikingly similar to those found in the linear stability analysis of Crouch (1997). The case of four pairs is also considered and leads to a larger distortion of the tip vortex.

1. Introduction

Trailing vortices are naturally shed by airplanes. They result in a strong down-wash which extends for several miles behind the plane and poses a hazard to following aircraft, in particular, at take-off and landing. Several previous studies propose to alleviate the hazard by introducing perturbations to trigger instabilities, and ultimately, break up the vortices (Bilanin & Widnall 1973, Crow & Bate 1997).

Most of these studies have focused on exciting the Crow instability, which operates on a single pair of counter-rotating vortices and has a wavelength much larger than the vortex core size. Unfortunately, however, for realistic perturbation amplitudes (those which would not cause large unsteady forces on the plane) excitation of the Crow instability would lead to vortex destruction at large distances behind the plane that exceed current FAA separation rules for aircraft in IFR conditions.

Recent studies (Crouch 1997, Rennich & Lele 1998) have considered instabilities unique to several pairs of vortices which model aircraft wakes in landing configuration (Spalart 1998, see Fig. 1). Some of these vortices quickly merge, but others persist for long times. At a distance of several spans behind a typical airplane, three persistent vortex pairs can generally be observed, originating at the tips of the wings, the outboard flaps, and the fuselage (respectively numbered 50, 52, and 55 in Fig. 1). Crouch (1997) has studied the linear stability of two pairs of corrotating vortices (tip and outboard flap, 50 and 52 in Fig. 1). He identified several instability modes depending on the angle, wavelength, and amplitudes of the perturbations that are imparted to each pair. The modes are summarized in Fig. 2. Roughly speaking, a long wave instability (top sketch in Fig. 2), similar to the Crow instability, takes place when the two pairs are excited in a symmetric fashion. An instability with a wavelength shorter than for the Crow instability (but still much longer than the core size) can also result (bottom sketch). The most efficient instability (middle sketch) arises when the eigenmodes are non-orthogonal leading to transient growth rates exceeding the maximum eigenvalue. This instability mechanism produces long waves which, when the outboard vortices are initially unperturbed, grow at a rate several times larger than the Crow instability for a single vortex pair.

† LMC-IMAG, University Joseph Fourier, Grenoble, France
‡ Institute of Computational Sciences, ETH Zürich, Switzerland

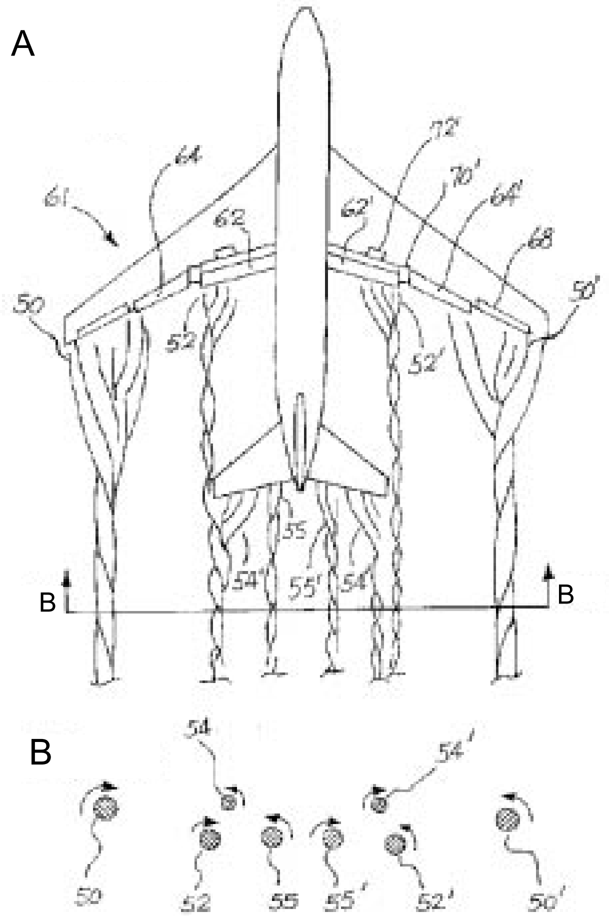

FIGURE 1. Sketch of vortex system shed by an airplane (Courtesy of J. Crouch, 2000). B is a cross section of A as shown.

Based on the analysis of Crouch (1997), Crouch & Spalart (2000) propose a strategy for breaking up the vortices that relies on appropriate cycling of control surfaces. Their experiments and numerical simulations indicate that this strategy could reduce separation rules.

Alternatively, Rennich & Lele (1998) have studied the system of vortex pairs with opposite signs, corresponding to inboard and outboard flaps. Vortex filament and direct numerical simulations (DNS) indicate in this case also large amplification rates for certain values of vortex separation times, which could also loosen the current mandatory separations.

Despite the fact that the points of view adopted in these works differ in several respects, in particular in the way the instability growth is measured, they have in common the ability to provide us with a better understanding of the mechanisms by which the cooperative instabilities of several pairs can result in enhanced growth rates. Moreover, the configurations studied in these works are investigated with a view to implementing them in actual wing designs.

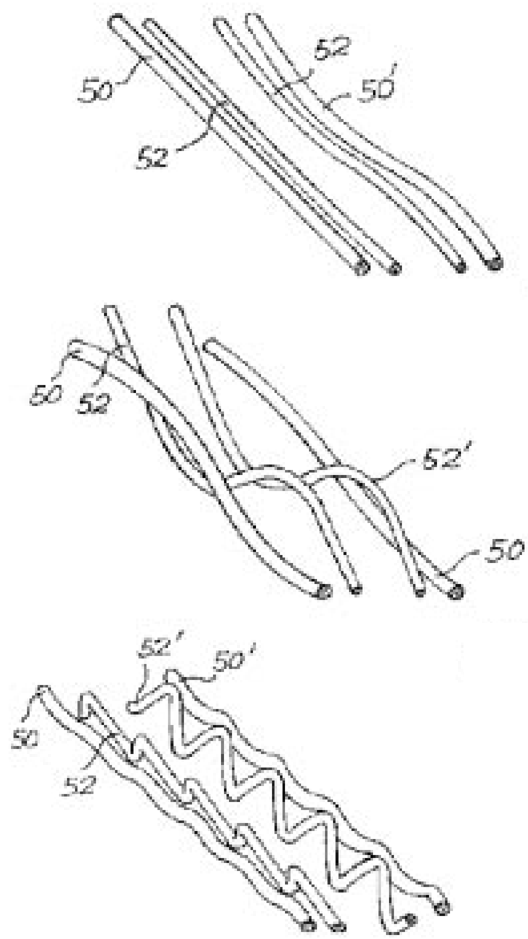

FIGURE 2. Three types of instabilities according to Crouch (1997). From top to bottom: long wave, transient growth, and short wave instabilities.

One of the findings reported in Crouch (1997) and Rennich & Lele (1998) is the extreme sensitivity of the overall dynamics with respect to the initial state of the vortex pairs. In Crouch (1997), the most effective transient growth was achieved when the outboard pair was not initially perturbed, while in Rennich & Lele (1998), early reconnection was obtained for a particular value of the inboard vortices separation.

This motivated our attempt to perform a more systematic parameter search and identify the wake system which would produce the largest instability growth. In other words, our goal was to revisit the above studies from the point of view of optimization.

The tools used in the present work are evolution strategies and viscous vortex methods.

On the one hand, evolution strategies have proven to be a flexible tool for optimization of unsteady flow dynamics when traditional gradient-based methods would be very difficult to implement. One the other hand, vortex methods are well adapted to wake

FIGURE 3. Initial stage (left) and reconnection for a configuration of two vortex pairs of opposite sign, according to Rennich & Lele (1998). Courtesy of S. Lele.

simulation as they require the discretization of only the region of vorticity. Note that the work of Rennich & Lele (1998) is in part based on vortex filament method. Viscous vortex methods offer the advantage of enabling calculations all the way to reconnection (Cottet *et al.* 2000).

An outline of the paper is as follows: Section 2 recalls the basic features of evolution strategies and vortex methods; Section 3 presents our findings, and Section 4 is devoted to a discussion of results and future plans.

2. Approach

2.1. *Evolution strategies*

We want to minimize $f(X), X \in R^N$. Basic one-member evolution strategies (ES) consist in performing successive mutations on the vector X followed by the evaluation of f. The new vector is then selected or rejected depending on whether it improves (in which case the mutation is said to be successful) or does not improve the value of f. The mutation consists of a random walk of the vector X, the size of which depends on the success rate of the mutation. The algorithm can be represented as the following iteration:

$$X_{t+1} = \begin{cases} X_t + \sigma_t Z_t & \text{if } f(X_t + \sigma_t Z_t) \leq f(X_t), \\ X_t & \text{otherwise.} \end{cases}$$

In the above formula, Z_t denotes a random Gaussian vector with zero mean and unit standard deviation. In order to speed the convergence, the radius σ_t is updated according to the success rate of the previous iterations. A high success rate means that one is far away from the minimum and induces an increase in σ_t. In this work we have implemented the so-called 1/5 rule: the variance is increased if the success ratio during the last iterations is greater than 1/5. In order to achieve faster convergence, mutation can be done in an anisotropic fashion on the various components of the parameter vector. This leads to the so-called covariance matrix adaptation technique. We attempted this technique in the last stage of the iterations.

2.2. *Viscous vortex methods*

Vortex methods operate on the vorticity formulation of the incompressible Navier-Stokes equations (Cottet & Koumoutsakos 2000). The method we are using is a time-splitting algorithm with alternating advection and diffusion. Advection is achieved by tracking particles along flow trajectories. Particles carry circulation, which is updated to account for stretching. Diffusion is dealt with by vorticity redistribution among nearby particles. To enable fast velocity evaluations, circulations are interpolated on a fixed grid (vortex-in-cell scheme). The Poisson equation is then solved by a Fourier type method, and velocities are obtained by finite-differences on the grid and then interpolated back on particles. Finally, to maintain a smooth particle distribution, which is essential for accuracy, particles are frequently re-meshed on a regular lattice. Systematic comparisons with spectral methods have been done to validate this method as a tool for DNS (Cottet *et al.* 2000).

3. Results

Our study focused on the case of two pairs of co-rotating vortices studied by Crouch (1997). The parameters which the evolution strategy optimized were:

- the initial perturbation amplitude of the tip (ϵ_1) and outboard (ϵ_2) vortices
- the angles of the perturbation planes α_1 and α_2
- the wavelength of the perturbations, λ
- the separation between the two vortices, δ
- the circulation ratio between the outboard and tip vortices, Γ

Quantities were non-dimensionalized by the distance b_0 between the tip vortices and the total circulation. To work with parameters in the same order of magnitude as Crouch (1997), the total perturbation amplitude was constrained to be below 10% of b_0:

$$\sqrt{\epsilon_1^2 + \epsilon_2^2} \leq 0.1.$$

The following additional constraints were imposed to remain within achievable design configurations:

$$0.25 \leq \delta \leq 0.4; \quad 0.5 \leq \lambda \leq 10; \quad 0. \leq \Gamma \leq 0.5.$$

Note that the constraints on λ allow for a wide range of wavelengths, varying from short wavelengths of the order of a few core sizes to long wavelengths of the type found in the Crow instability.

Our goal was to optimize the instability on the tip vortex. To measure its deformation, we computed the average angle, inside the core of the tip vortex, of the vorticity vector relative to the axis of the unperturbed vortex. More precisely, the objective function was given by the formula

$$f = \int dz \int_{A(z)} \frac{\omega_x^2 + \omega_y^2}{\omega_z^2} dA(z),$$

where

$$A(z) = \{(x, y), |\omega(x, y, z)| \geq 1/2|\omega|_{max}\}.$$

Figure 5 shows the convergence history of the evolution algorithm. After iteration 150 the ES algorithm was run with the covariance matrix adaptation technique, which only slightly improved the value of the objective function. It is not clear that at this stage a global optimum has been reached.

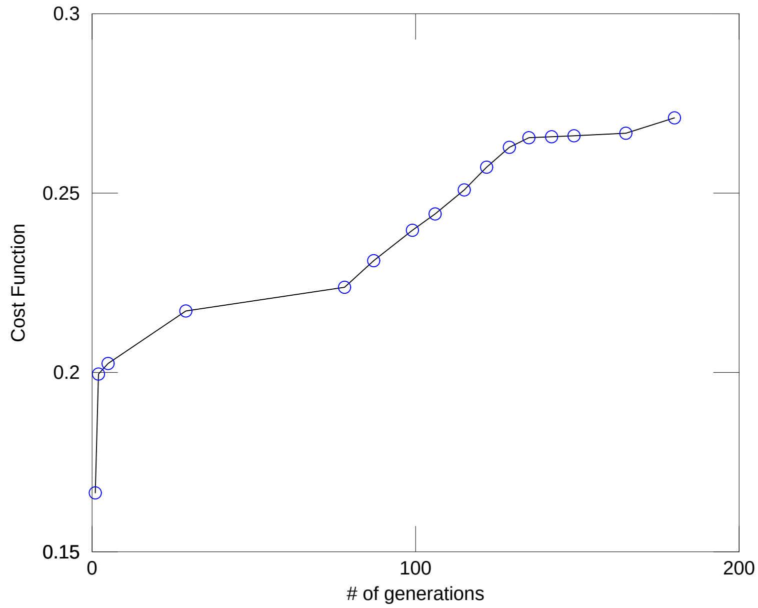

FIGURE 4. Convergence history for the evolution strategy.

	α_1	α_2	ϵ_1	ϵ_2	δ	Γ	λ
Optimal parameters	0.47	0.73	0.098	0.008	0.26	0.31	0.72
Parameters in Crouch (1997)	$\pi/4$	$\pi/4$	0.1	0.	0.3	0.4	0.7

TABLE 1. Comparison of the parameters found by the evolution strategy and those studied in Crouch (1997).

The parameter values finally obtained by the ES are listed on Table 6 together with the parameters reported in Crouch (1997) as leading to efficient transient growth. Some striking similarities can be noticed between these two sets of parameters. In particular, the ES has selected perturbations that are mostly located on the tip vortex, confirming the observation in Crouch (1997) of efficient transient growth when the outboard flap vortex was unperturbed. The wavelengths of the perturbations are also very close to the ones given in Crouch (1997).

Finally Fig. 5 shows the evolution of the objective function for various parameter vectors: the two sets of parameters shown in Table 1, parameters similar to the ones found by the ES but with perturbations of same magnitude for the two pairs and a third set of parameters obtained by optimizing on 4 pairs instead of 2 pairs. Theses simulations confirm that, in the early stages of the dynamics, the evolution strategies have picked up the most efficient parameters for two pairs. One can also notice that adding more degrees of freedom to the optimization can pay off and lead to increased efficiency. However, an inspection of the vorticity angle at later times show that the differences between the configurations involving pairs of co-rotating pairs tend to disappear. A similar observation was made in Rennich (1997) by considering a different measure of the perturbation (namely the maximum displacement).

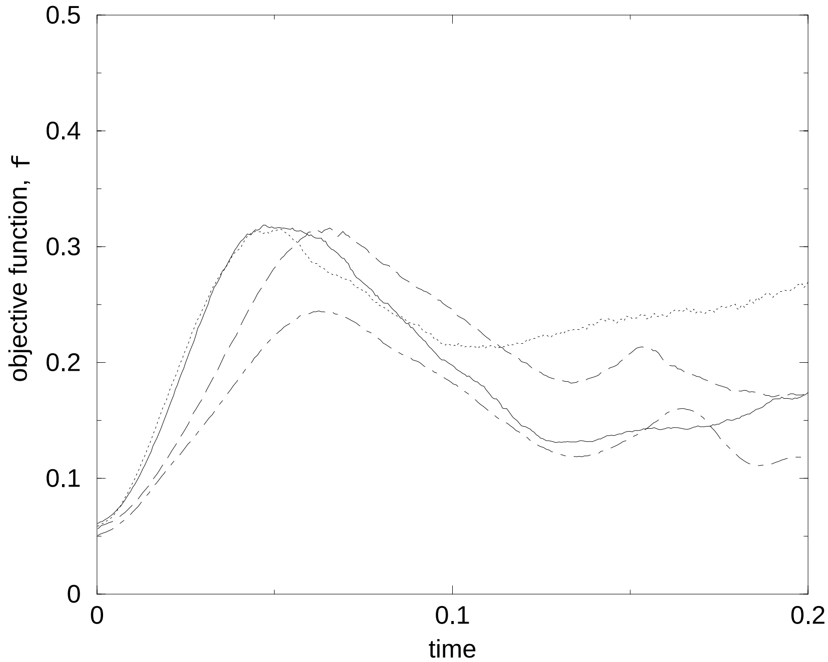

FIGURE 5. Evolution of the objective function. ——— : optimal parameters for 2 pairs; – – – – : parameters of Crouch (1997); ········ : optimal parameters for 4 pairs; —·— : case of 2 pairs with equal initial perturbations.

4. Conclusion

Our goal was to investigate whether optimization techniques could be helpful in determining parameters enhancing vortex break-up in trailing vortices. These preliminary results show that evolution strategies are a valuable tool to explore realistic configurations in a systematic way. Their flexibility makes it easy to modify the number of parameters as desired without having to reformulate the optimization problem. Note, however, that considering configurations involving more than two pairs cannot be done without keeping in mind that these configurations have to be in accordance with current design constraints.

To keep the computational cost at a reasonable level, our study has focused on the preliminary stages of the dynamics. However, because of the relative rotations of the pairs, it is not clear that the trends observed initially persist for long times. In other words, parameters leading to the greatest growth rates may not be those which lead to fastest reconnection.

It thus appears necessary to elucidate the relevant time-scale on which optimization should be performed.

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