Non-locality of scale interactions in turbulent shear flows

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The scale interactions in turbulent shear flow are evaluated using a channel flow DNS database at $Re_\tau = 590$. The velocity field is filtered using a sharp cut-off filter in the Fourier space, and the relative magnitudes of the advective terms in the small scale equations, derived from Navier-Stokes equations, are computed as a function of wavenumber. We show that the interactions at small scales are non-local in the direction of the mean flow, dominated by the advection of small scales by large scale vortices. This non-locality of interactions suggests that the subgrid-scale model of Dubrulle and Nazarenko (1997) for 2D turbulence is likely to be applicable to turbulent shear flows as well.

1. Introduction

Direct numerical simulation (DNS) based on the Navier-Stokes equations is only possible for relatively low Reynolds numbers. Therefore, filtering is often needed to capture the large-scale structure of the flow, hoping that the effect of small scale fluctuations can be more easily modeled. This is the basis for the classical large eddy simulations (LES) in which the subgrid scale (SGS) stresses are calculated through a statistical prescription. The most frequently used closure in LES is the Smagorinski model in which the SGS stresses are modeled in terms of a turbulent eddy viscosity with a model constant to be calibrated for specific flows.

The presence of the unknown model constant makes this procedure inappropriate for unfamiliar and non-measurable flows such as protoplanetary disks. Hence, it is desirable to develop new models in which arbitrary parameters are eliminated or at least have less impact on the dynamic evolution of the resolved scales. The dynamic SGS model of Germano *et al.* (1991) provides one such example. It employs a *test* filter to calculate the model constant as a function of time and position.

In 1997 Dubrulle and Nazarenko introduced an alternative subgrid scale model for 2D turbulence (hereafter referred to as DN model) based upon a generalization of the Rapid Distortion Theory. This model relies on linearized Navier-Stokes equations forced through the energy cascade mechanism. The assumption underlying this model is that the nonlinear interactions at small scales are mainly non-local and thus have to be treated exactly. This contrasts with the classical Kolmogorov theory of turbulence in which the main interactions are between comparable scales. In the DN model, the less important local interactions are represented by a turbulent viscosity, which plays a secondary role and can be neglected in some special situations (e.g. 2D turbulence).

Laval *et al.* (1999) implemented this model for 2D isotropic turbulence computations. The model gives a much cleaner treatment of the small scales than standard LES

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models and remains cheap in memory and computation time. In 3D homogeneous turbulence, numerical tests have also been performed by Laval *et al.* (2000). They showed that non-local interactions indeed dominate the dynamics, generating coherent structures and intermittency. The local interactions are responsible for energy saturation of the structures and can be modeled by a simple turbulent viscosity.

In many astrophysical and geophysical flows, both 2D and 3D dynamics coexist: when the rotation becomes dynamically important, i.e. for scales larger than the Taylor-Proudman scale, the turbulent eddies have a 2D behavior. On the other hand, these flows are characterized by a strong mean shear flow, which is not the case in the homogeneous turbulence studied by Laval *et al.* Intuitively, we would think that the presence of this mean flow enhances the advection of the small scales and thus renders the non-local approximation even better than in 3D homogeneous turbulence. We present here quantitative support of this idea based on *a priori* tests performed using a DNS database of a turbulent channel flow at $\mathrm{Re}_\tau = 590$ (Moser *et al.* (1999)).

2. The Dubrulle and Nazarenko model

The Navier-Stokes equations for an incompressible flow are:

$$\begin{aligned} \partial_t v_i + v_j \partial_j v_i &= -\frac{1}{\rho}\partial_i p + \nu \partial_j \partial_j v_i \\ \partial_i v_i &= 0 \end{aligned} \tag{2.1}$$

where v_i is the velocity component in the i^{th} direction, p the pressure, ρ the (constant) density, and ν the kinematic viscosity. As in standard LES, we define resolved and subgrid scale quantities using a filter function $G(\mathbf{x}, \mathbf{x}')$:

$$\begin{aligned} v_i(\mathbf{x}, t) &= V_i(\mathbf{x}, t) + v_i'(\mathbf{x}, t) \\ V_i(\mathbf{x}, t) &\equiv \overline{v_i(\mathbf{x}, t)} = \int G(\mathbf{x}, \mathbf{x}') v_i(\mathbf{x}', t) d\mathbf{x}' \end{aligned} \tag{2.2}$$

where V_i and v_i' are the resolved and subgrid scale velocity components in the i^{th} direction. The filter function used in the present report is a sharp cut-off in the Fourier space.

Using this filtering operation, one obtains two coupled sets of equations: one for the resolved scales and the other for the unresolved (subgrid) scales. The resolved scale equations are obtained by a mere filtering of Eq. (2.1):

$$\begin{aligned} &\partial_t V_i + \partial_j \overline{V_i V_j} + \partial_j \overline{V_i v_j'} + \partial_j \overline{v_i' V_j} + \partial_j \overline{v_i' v_j'} \\ &\quad = -\frac{1}{\rho}\partial_i P + \nu \partial_j \partial_j V_i \\ &\partial_i V_i = 0 \end{aligned} \tag{2.3}$$

The subgrid scale equations are obtained by subtracting equations (2.3) from (2.1):

$$\begin{aligned} &\partial_t v_i' + \partial_j V_i V_j + \partial_j V_i v_j' + \partial_j v_i' V_j + \partial_j v_i' v_j' \\ &\quad - \partial_j \overline{V_i V_j} - \partial_j \overline{V_i v_j'} - \partial_j \overline{v_i' V_j} - \partial_j \overline{v_i' v_j'} \\ &\quad = -\frac{1}{\rho}\partial_i p' + \nu \partial_j \partial_j v_i' \\ &\partial_i v_i' = 0 \end{aligned} \tag{2.4}$$

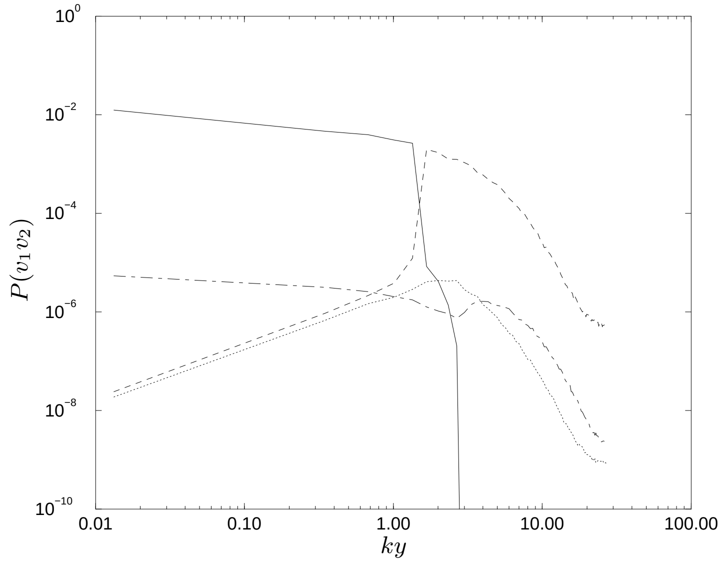

FIGURE 1. Wavenumber spectra of the four Reynolds shear stress components involving the streamwise and the wall-normal velocities, at $y^+ = 100$, for a cut-off wavenumber of $300m^{-1}$ (adapted from Carlier *et al.* (2000)): ——— V_1V_2; - - - - V_1v_2'; ········ $v_1'V_2$; — — — $v_1'v_2'$.

Several terms contribute to the nonlinear interactions: *non-local* terms involving the product of a resolved scale and a subgrid scale component, and a *local* term involving two subgrid scale components. The essence of subgrid scale modeling is to keep only the *non-local* terms and to model the local term by a turbulent viscosity.

The savings in memory achieved with this kind of model are not obvious since all the scales are computed. However the linearity of the subgrid scale equations allows several simplifications. First, they may be decomposed into localized linear modes, which can exploit the intermittency and inhomogeneity of the small scales, thereby decreasing the number of modes needed to reconstruct the subgrid field. Also, the linearity enables the use of semi-Lagrangian schemes of integration, which provides a significant reduction of the computational cost via an increase of the integration time step (see Laval *et al.* (1999) for details of 2D simulations).

3. A priori tests in a channel flow

Carlier *et al.* (2000) have studied the relative importance of local and non-local terms in a high Reynolds number wind tunnel boundary layer at a momentum-thickness based Reynolds number equal to 20600. The results are displayed in Fig. 1, which shows the power spectra of the four instantaneous Reynolds stress components involving v_1 and v_2, defined as $P(v_1v_2) = |\widehat{v_1v_2}|^2$ where the caret denotes Fourier transform in the plane parallel to the wall at $y^+ = 100$. The horizontal axis represents the dimensionless wavenumber $k = (k_1^2 + k_3^2)^{1/2}$ (normalized by $1/y$). One sees that for wavenumbers larger than the cut-off, wavenumber ($= 300m^{-1}$), The non-local term V_1v_2' indeed dominates over the other terms by several orders of magnitudes. However, the hot wire anemometry technique used in the experiment prevents both the measurements of other components of the velocity field and the computation of spatial derivatives, thereby preventing a direct

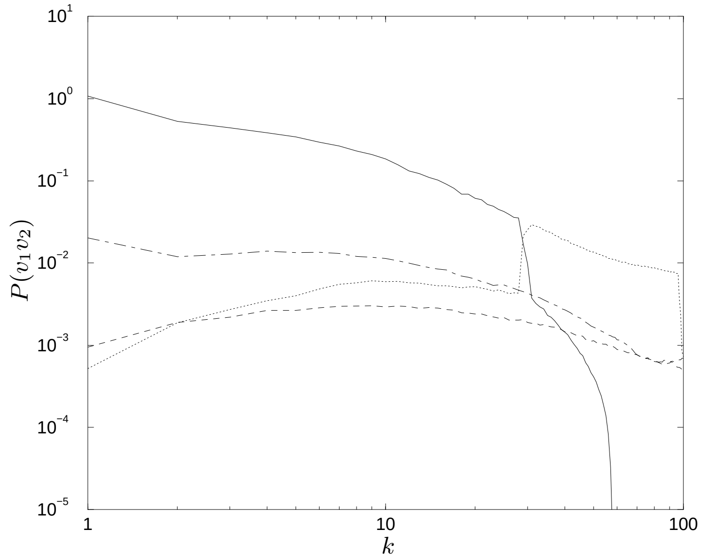

FIGURE 2. Wavenumber spectra of the four Reynolds stress components involving the streamwise and the wall-normal velocities, at $y^+ = 50$, for a cut-off wavenumber of 30: ——— V_1V_2; ········ V_1v_2'; – – – – $v_1'V_2$; —— — $v_1'v_2'$.

check of the non-local hypothesis. It is, therefore, of interest to complement their study via *a priori* tests performed using a DNS database.

The database used in the present study is from the channel flow DNS of Moser *et al.* (1999) at $\mathrm{Re}_\tau = 590$ based on the friction velocity and channel half-width. The DNS was conducted on a grid of $384 \times 257 \times 384$, covering a computational box of size 2π, 2, π in the streamwise (x_1), wall-normal (x_2 or y), and spanwise (x_3) directions, respectively. All the variables used are dimensionless, with velocities normalized by the mean friction velocity and spatial coordinates normalized by the channel half-width. The strong inhomogeneity of the mesh in the wall-normal direction prevents the use of Fourier analysis in that direction. Therefore, filtering was performed only in the planes parallel to the wall. A sharp cut-off filter with cut-off wavenumber $k = 30$ was used to separate large and small scales.

Our analysis was performed in two steps: first, we computed the four components of v_1v_2 and their power spectra $P(v_1v_2)$ $(= |\widehat{v_1v_2}|^2)$ as a function of wavenumber $k = (k_1^2 + k_3^2)^{1/2}$, analogous to those plotted in Fig. 1, thereby allowing an estimate of the influence of the Reynolds number. The results are presented in Fig. 2. Then, we computed the advective terms in the three subgrid scale equations (2.4), complete with the spatial derivatives and summation over indices. The derivatives were calculated using spectral methods (Fourier in the streamwise and spanwise directions, and Chebychev in the wall-normal direction) consistent with the original DNS. The results, again expressed in terms of their wavenumber spectra $P(v_j\partial_j v_i) = |\widehat{v_j\partial_j v_i}|^2$, are shown in Figs. 3, 4 and 5. Notice that although the examples shown in Figs. 2–5 are for $y^+ = 50$, the same calculations have been repeated for $y^+ = 20$ and $y^+ = 100$ as well, and the results are found to be very similar to those depicted in the figures.

From a comparison of Figs. 1 and 2, one sees that the main influence of the higher Reynolds number (Fig. 1) is to increase the dominance of the non-local term with respect to the others. This is a consequence of the increased scale separation, which has been

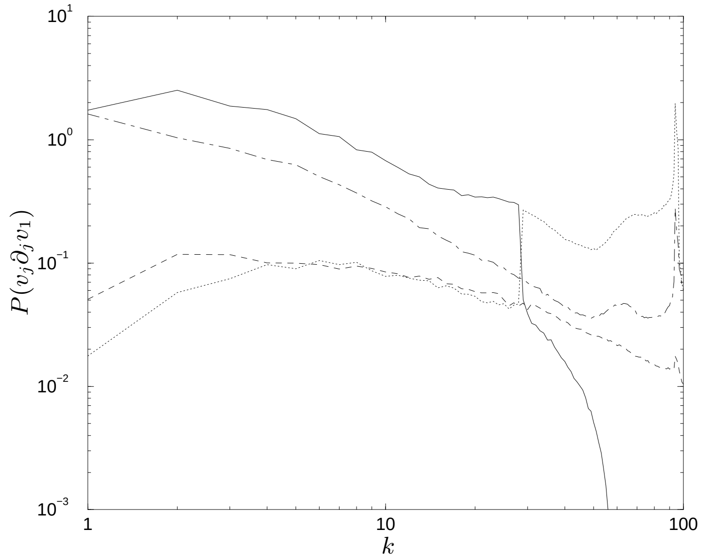

FIGURE 3. Wavenumber spectra of the four components of the divergence of the Reynolds tensor projected in the streamwise direction, at $y^+ = 50$, for a cut-off wavenumber of 30: ——— $V_j\partial_j V_1$; ········ $V_j\partial_j v_1'$; – – – – $v_j'\partial_j V_1$; —– — $v_j'\partial_j v_1'$.

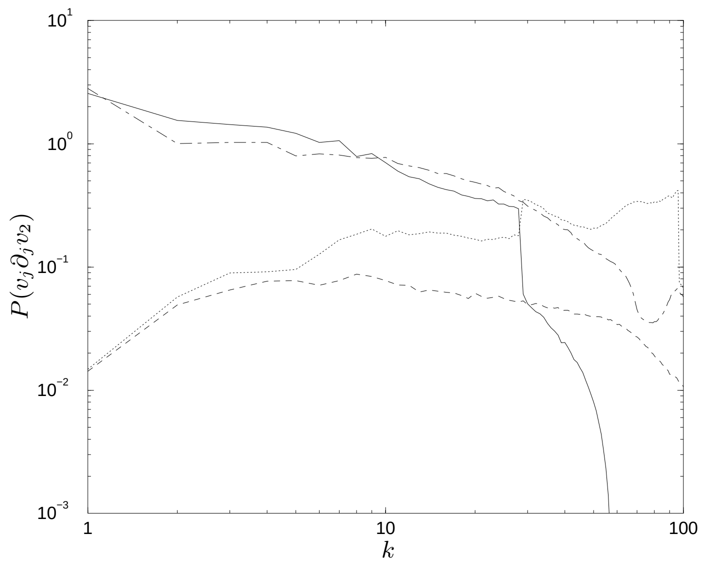

FIGURE 4. Wavenumber spectra of the four components of the divergence of the Reynolds tensor projected in the wall-normal direction, at $y^+ = 50$, for a cut-off wavenumber of 30: ——— $V_j\partial_j V_2$; ········ $V_j\partial_j v_2'$; – – – – $v_j'\partial_j V_2$; —– — $v_j'\partial_j v_2'$.

predicted and discussed in Dubrulle & Nazarenko (1997). When the other components of the velocity fields and the derivatives are taken into account, one may note that the non-local hypothesis remains valid for the subgrid scale equations (2.4) in the streamwise (Fig. 3) and spanwise (Fig. 5) directions, while it becomes clearly violated for the equation in the direction normal to the wall (Fig. 4). This is consistent with the belief that the

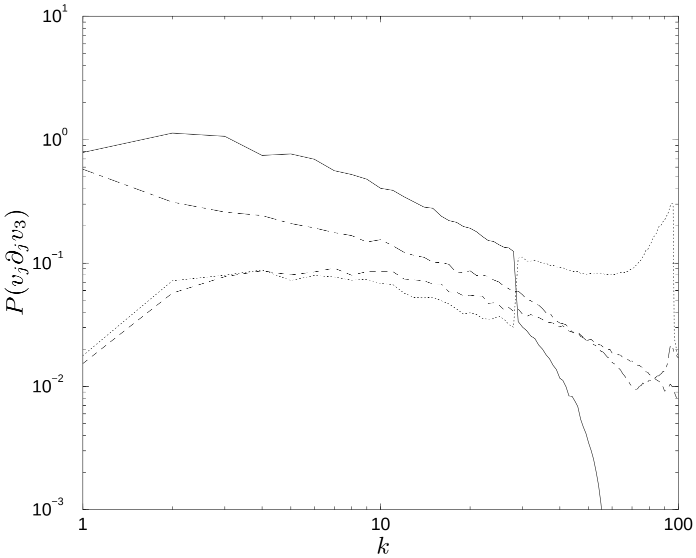

FIGURE 5. Wavenumber spectra of the four components of the divergence of the Reynolds tensor projected in the spanwise direction, at $y^+ = 50$, for a cut-off wavenumber of 30: ——— $V_j \partial_j V_3$; ········ $V_j \partial_j v'_3$; – – – – $v'_j \partial_j V_3$; —–— $v'_j \partial_j v'_3$.

presence of a mean flow (here in the streamwise direction) favors the development of non-local interactions even at moderate Reynolds numbers. In the absence of a mean flow (like in the direction normal to the wall, or in 3D homogeneous turbulence), the local terms need to be taken into account. On the whole, these results suggest that, in the streamwise and spanwise directions, the local interaction terms could be safely neglected (as in 2D turbulence), while in the direction normal to the wall, they should be modeled by a turbulent viscosity as in 3D homogeneous turbulence.

4. Conclusions

We have shown that, based on *a priori* tests in a channel flow, the model developed by Dubrulle & Nazarenko (1997) should be applicable in turbulent shear flows. As expected, nonlinear interactions in the presence of a mean flow are mainly dominated by the advection of small scales by the large scale motions, including the mean stream. The remaining interactions only involve a small fraction of the flow energy, and their dynamics can be modeled by, for example, a turbulent viscosity.

This is of great interest for protoplanetary disks. In fact accretion disks are believed to have many similarities with Couette-Taylor flows (see e.g. Richard & Zahn (1999)). The presence of rotation may induce some differences. However, the rotation tends to force the large scale motions to have a 2D behavior. Given the success of the Laval *et al.* (1999) model for 2D turbulence, it is hoped that the model will be also valid in 3D rotating shear flows. Obviously, the present *a priori* test results are very encouraging, but further investigations are required to evaluate the influence of rotation more precisely. Dynamical tests are also needed to validate this procedure for 3D turbulence in complex geometries.

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