Evaluation of subgrid-scale models in terms of time correlations

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In certain applications such as the computation of turbulent sound sources, Large-Eddy Simulation (LES) is required to predict correctly the space-time correlations of the velocity field. A previous study (He, Rubinstein & Wang 2002) has shown that LES with the spectral eddy-viscosity model over-predicts time correlations. In this work, we evaluate the Smagorinsky model, the dynamic Smagorinsky model and the multi-scale LES method in terms of time correlations. The dynamic Smagorinsky model is shown to give better predictions on time correlations than the constant-coefficient Smagorinsky model, which gives significant over-predictions. The results from the multi-scale LES method are in between. The over-predictions are discussed according to the random backscatter and the sweeping hypothesis. Based on those discussions, a history-dependent sub-grid scale model is suggested for time correlations.

1. Motivation and objectives

The goal of this work is to evaluate and develop subgrid-scale (SGS) models for predicting time correlations in turbulent flows. Here by "time correlation" we refer to the two-time, two-point correlation of velocity fields $u_i(\mathbf{x}, t)$ in physical space

$$C(r, \tau) = \langle u_i(\mathbf{x}, t) u_i(\mathbf{x} + \mathbf{r}, t + \tau)\rangle, \tag{1.1}$$

or, equivalently, the two-time correlation of velocity Fourier modes $u_i(\mathbf{k}, t)$ in spectral space

$$C(k, \tau) = \langle u_i(\mathbf{k}, t) u_i(-\mathbf{k}, t + \tau)\rangle. \tag{1.2}$$

where $r = |\mathbf{r}|$ is the magnitude of spatial separation $\mathbf{r}$, $k = |\mathbf{k}|$ is the magnitude of wave number $\mathbf{k}$, and τ is time delay. The turbulence is assumed statistically homogeneous and isotropic, but not necessarily stationary. Hence, the correlation function also depends on t in general.

Time correlation describes temporal statistics of turbulent flows. It has been shown by previous evaluations (Meneveau & Katz 2000) that large-eddy simulation (LES) with an appropriate SGS model is capable of predicting correctly the single-time spatial statistics of turbulent flows, such as turbulent kinetic energy and Reynolds stress. The temporal statistics did not enter such evaluations. However, a major application of LES is the prediction of unsteady flows in which time accuracy, even if in the statistical sense, is important. This emphasis will impose new requirements on SGS modeling. For instance, in the computation of flow-generated sound the Lighthill acoustic analogy (Lighthill 1952) shows that the radiated sound power depends on the space-time properties of turbulence. Therefore, it is necessary to investigate the predictive power of LES in terms of time correlations. A recent study by He *et al.* (2002) has shown that LES with the spectral

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eddy-viscosity model (Chollet & Lesieur 1981) over-predicts the time correlation relative to Direct Numerical Simulation (DNS). The over-prediction, which can be significant, alters the frequency distribution of the sound radiated by turbulence. A follow-up question to ask is then: which of the currently existing SGS models can better predict time correlations, and to what extent?

The dynamic Smagorinsky model (Germano *et al.* 1991) has been applied in many different flows, generally with good results. Therefore, this model is the first choice in our evaluations. The multi-scale LES method of Hughes, Mazzei & Oberai (2001) is also attractive in that the SGS model acts only on the small scales in the resolved field, and thus may have less impact on time-correlations, particularly at large spatial separations. We also use the classical Smagorinsky model (Smagorinsky 1963) to calculate time correlations, in order to verify our previous results and compare it with the dynamic Smagorinsky model and the multi-scale LES method.

The above SGS models are mainly based on energy balance. For example, the eddy-viscosity coefficients in the Smagorinsky models are determined by the energy-balance equation. They are not required to satisfy the governing equations for time correlations, which is the root cause of potential errors. As an extension of the present work, we incorporate the physical mechanism for time correlations into SGS modeling, and suggest a history-dependent Smagorinsky model.

2. Numerical setup

We will use a calculation of decaying homogeneous isotropic turbulence to evaluate the Smagorinsky model, the dynamic Smagorinsky model and the multi-scale LES method. The previous evaluation of He *et al.* (2002) was carried out for forced homogeneous isotropic turbulence at a moderate Taylor-scale Reynolds number (Re_λ) of 108 in a cubic box of side 2π, using DNS on a 128^3 grid and LES on a 64^3 grid. That evaluation may have been affected by the forcing scheme, the moderate Reynolds number and the relatively small grid ratio between DNS and LES. These deficiencies are rectified or reduced in the present study.

The new setting is a decaying turbulence of initial $Re_\lambda = 127.4$ in the same cube as before. It is simulated by DNS with grid size 256^3 and LES with grid size 64^3. A standard pseudo-spectral method is used, in which spatial differentiation is by the Fourier spectral method, time advancement is by a second-order Adams-Bashforth method with the same time steps for both DNS and LES, and molecular viscous effects are accounted for by an exponential integrating factor. All nonlinear terms are de-aliased with the two-thirds rule, except those involving SGS eddy viscosity.

The following SGS models are evaluated:

(1) The Smagorinsky model: the Smagorinsky constant is $C_s = 0.22$ and the filter width is set equal to the inverse of the largest effective wave number $k_c = 21$.

(2) The dynamic Smagorinsky model: the Smagorinsky coefficients are determined by the Germano identity. The grid filter width is k_c^{-1} and the test filter width is taken as $2k_c^{-1}$.

(3) The multi-scale LES method: we decompose the filtered Navier-Stokes equations into the large-scale equations for the lower one-half of the Fourier modes and the small-scale equations for the remaining half of the Fourier modes. The Smagorinsky model is applied only to the small-scale equations.

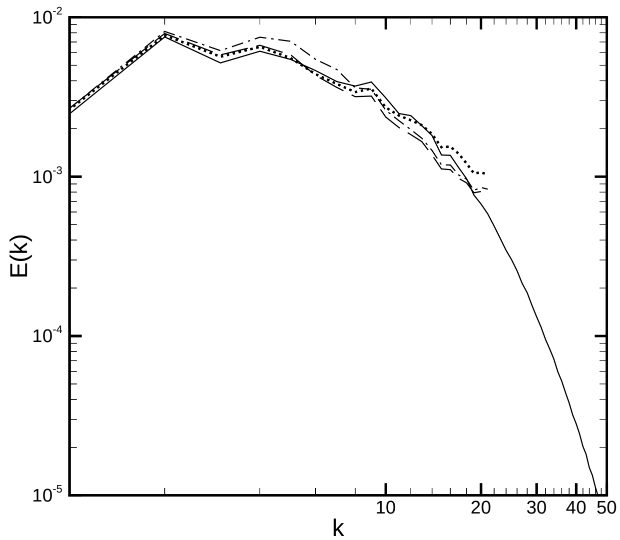

FIGURE 1. Energy spectra at $t = 2.4$. ——— DNS field; - - - - LES with dynamic Smagorinsky model; —·— Multi-scale LES; ········ LES with Smagorinsky model.

The initial condition for DNS is an isotropic Gaussian field with energy spectrum

$$E(k,0) \propto (k/k_0)^4 \exp[-2(k/k_0)^2], \tag{2.1}$$

where $k_0 = 4.68$ is the wavenumber corresponding to the peak of the energy spectrum. The shape of the energy spectrum excludes the effects of the box size. The initial condition for LES is obtained by filtering the initial DNS velocity fields with filter wavenumber $k_c = 64/3 \approx 21$. Therefore, the initial LES and filtered DNS velocity fields are exactly the same. At early stages, the LES and DNS velocity fields are highly correlated due to the same initial conditions. Therefore, the time correlations of the LES velocity field are nearly the same as those of the DNS field. As time progresses, the LES fields become decorrelated from the DNS fields. The difference in time correlations between the LES and DNS velocity fields are then observed.

3. Main results

In figure 1, energy spectra are presented at $t = 2.4$. Generally speaking, the results from the Smagorinsky model, the dynamic Smagorinsky model and the multi-scale LES method are in agreement with the DNS result. However, the multi-scale LES method over-predicts a little more than the Smagorinsky and dynamic Smagorinsky models between $k = 3$ and $k = 10$. There is also a slight under-prediction for $k \geq 10$ which is shared by the Smagorinsky model. Beyond $k = 20$, the resolution limit is exceeded, and the LES results are meaningless.

Figure 2 shows the normalized time correlations, $c(k,\tau) = C(k,\tau)/C(k,0)$, of the DNS and LES fields for wavenumbers $k = 7$, 11, 15, and 17, spanning a range of scales from the integral scale to the upper end of the resolved scale. The starting time is $t = 1.4$. The figure confirms our previous observations: LES with eddy-viscosity-type

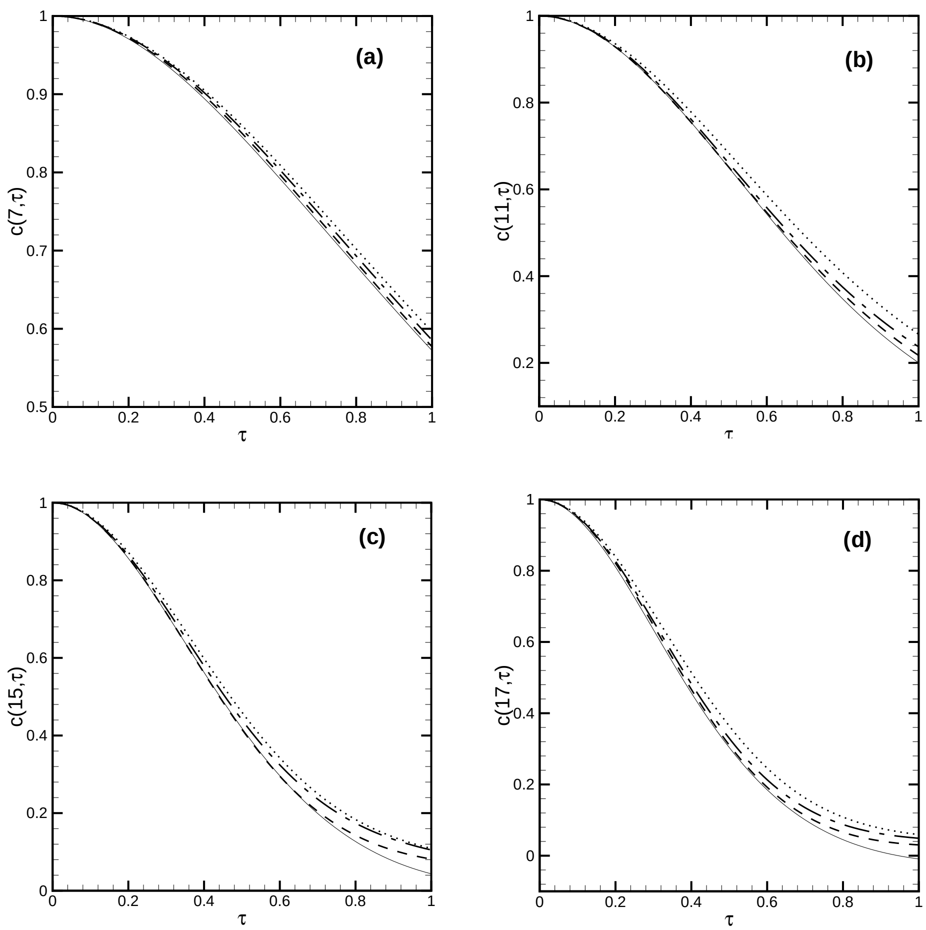

FIGURE 2. Time correlation $c(k, \tau)$ vs time lag τ (start time $t = 1.4$) for (a) $k = 7$, (b) $k = 11$, (c) $k = 15$, (d) $k = 17$. ——— DNS field; - - - - LES with dynamic Smagorinsky model; — - — Multi-scale LES; ········ LES with Smagorinsky model.

models over-predicts time correlations. The over-predictions can be understood by the following physical arguments: the contribution of unresolved scales to resolved scales can be described as energy dissipation and random backscatter. The eddy-viscosity-type SGS models are designed to model the drain of energy from resolved scales to unresolved scales, but fail to account for the random backscatter from unresolved scales to large scales. This leads to a more coherent field at resolved scales. Therefore, the LES field evolves in a more correlated fashion, in the sense that their time correlations decay more slowly. The extent of over-estimation on time correlations varies with the SGS model, as seen in figure 2. This can be understood by the sweeping effects (Kraichnan 1964): the Eulerian time correlations are dominated by the sweeping velocity. The eddy-viscosity-type models reduce the sweeping velocity by excessive dissipation, which causes the time correlations of the LES fields to decay more slowly than those of the DNS fields. It is observed from our numerical calculations that, among the SGS models evaluated, the Smagorinsky model produces the largest dissipation and thus the smallest sweeping velocity. Therefore, it predicts the slowest correlation decay. The dynamic Smagorinsky model introduces the most appropriate amount of dissipation and hence gives the best predictions on time correlations. The dissipation produced in the multi-scale LES is smaller than the Smagorinsky model but larger than the dynamic Smagorinsky model. Therefore, its predictions for time correlation lie between those of the other two models.

Figure 2 also indicates that the time correlations of the LES fields deviate more from the DNS fields at small scales than at large scales. This is in agreement with the well-known divergence property of Eulerian time correlations discovered by Kraichnan (1964). It implies that the cutoff effects are stronger in the near range than those in the far range.

It is further noted from figure 2 that, although the dynamic Smagorinsky model and the multi-scale LES method give better overall predictions than the Smagorinsky model, all curves near $\tau = 0$ are in good agreement, implying that the Taylor microscales are well predicted by all models. The maximum time delay used in the calculations is $\tau = 1.0$, which is not long enough to catch the zero-crossing points of time correlations. Nonetheless, the relatively large differences of the LES curves relative to the DNS value at $\tau = 1.0$ are still observed, indicating that the decorrelation scales are poorly predicted.

4. Conclusions and future work

We have evaluated the Smagorinsky model, the dynamic Smagorinsky model and the multi-scale LES method in terms of time correlations. Comparatively speaking, the dynamic Smagorinsky model predictions are in better agreement with DNS fields, with some over-prediction of the decorrelation length scales. The Smagorinsky model obviously over-predicts time correlations. The results of the multi-scale LES method, using the Smagorinsky model on the small scale equations, lie in between. However, we believe that the results from the multi-scale LES method could be much improved if an appropriate SGS model is applied to the small scale equations.

Evaluations have also been made on forced turbulence, and similar results are obtained but will be presented in a forthcoming paper (He, Wang & Lele 2002). This is because the sweeping hypothesis is true for both forced and decaying turbulence. However, the sweeping hypothesis may not be valid for general unsteady flows, such as 'kicked' turbulence, where turbulence is forced periodically. In this case, the eddy viscosity coefficients should be history-dependent. Therefore, we suggest a history-dependent Smagorinsky model for time correlations

$$\tau_{ij}(\mathbf{x},t) = \int_{-\infty}^{t} [-2\nu_r(t')\tilde{S}_{ij}(\mathbf{x},t')]w(t')dt' \tag{4.1}$$

where τ_{ij} is the sub-grid scale stress, and $w(t')$ is a weighting function. $\nu_r(t')$ is a history-dependent-eddy-viscosity coefficient, which can be determined by either one time or two time Germano identity. This work is in progress.

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