MHD turbulence in the presence of a strong magnetic field

By S. C. Kassinos, B. Knaepen AND D. Carati

We consider the case of homogeneous turbulence in a conducting fluid that is exposed to a uniform external magnetic field. When the magnetic Reynolds number is vanishingly small ($R_m \ll 1$), the induced magnetic fluctuations are much weaker than the applied field, and in addition, their characteristic time scale based on their diffusion is much shorter than the eddy turnover time. In this case, it is customary to simplify the governing MHD equations using what is known as the quasi-static (QS) approximation. In practice, the QS approximation is often used even when R_m is moderately high, where its validity is unclear. Here we introduce a new approximation, which we have called the Quasi-Linear (QL) approximation, which is designed to be valid for both small and moderate R_m. The accuracy of both approximations is systematically studied in a series of direct numerical simulations (DNS) of decaying MHD turbulence, in which their predictions are compared with those of the full system of MHD equations. Both approximations are satisfactory for $R_m \lesssim 1$, but the QL approximation is clearly shown to be much more accurate for moderately high values, $1 \lesssim R_m \lesssim 10$.

1. Introduction

1.1. *Motivation and objectives*

The interaction of the turbulence in a conducting fluid with an externally-applied magnetic field at low magnetic Reynolds numbers is important in both Magnetohydrodynamic (MHD) and Magnetogasdynamic (MGD) applications. MHD applications include the use of electromagnetic brakes to control flow unsteadiness during continuous steel casting, and flow-control schemes for submarines. MGD applications involve advanced flow control and propulsion schemes for hypersonic vehicles. In MGD applications, fluid conductivity arises due to thermal ionization of the incoming flow, which in addition can be seeded if desired.

CFD codes used for the prediction of MHD and MGD flows rely on simple turbulence models, like k-ϵ models, with additional *ad hoc* modifications to account for the effects of the magnetic field. This approach neglects the important dynamical role that the structure of the turbulence plays in the interaction between the turbulence and the applied magnetic field. Unfortunately, simple closures using *ad hoc* MHD modifications cannot account for structural effects and, as a result, they tend to be flow-specific, lacking any degree of generality.

Structure-Based Models (SBM) are by construction able to account for the dynamical effects of the energy-containing turbulence structure. Preliminary work in the case of homogeneous unstrained MHD turbulence (Kassinos & Reynolds 1999), has shown that SBM are well suited for use in the prediction of MHD and MGD applications. The task of developing turbulence SBM or other closures for MHD and MGD applications can be simplified by taking advantage of approximations to the governing equations that are valid for the flow regimes that are typically encountered in technological applications.

The objective of this work is to explore two different approximations that can be used for flow regimes of relevance to MHD and MGD applications. For vanishingly small magnetic Reynolds numbers ($R_m \ll 1$), the induced magnetic fluctuations are much weaker than the applied field and their characteristic time scale, based on their diffusion, is much shorter than the eddy turnover time. A classical approximation for decaying MHD turbulence at low R_m is the Quasi-Static (QS) approximation. As recalled in section 3.1, in this approximation, the induced magnetic field fluctuations become a linear function of the velocity filed. In practice, the QS approximation is often used for the prediction of flows even where $1 \lesssim R_m \lesssim 10$, and therefore here we seek to establish the exact range of validity of the QS approximation. In section 4.1, we also introduce a new approximation, which we have called the Quasi-Linear approximation (QL). The QL approximation is designed to be valid even when the Re_m is moderately high. Thus, we seek to establish the range of validity of the QL approximation and also to compare the relative accuracy of the two approximations.

We start by discussing the relevant dimensionless parameters that characterize MHD and MGD flows in greater detail. In section 3.1 we introduce the governing equations and the simplifications associated with the QS approximation. The numerical code and initial conditions used for the numerical experiments are described in section 3.2, while section 3.3 is devoted to a discussion of the results pertaining to the QS approximation. This is followed in section 4.1 by a detailed description of the QL approximation and in section 4.2 by the description of its predictions. A concluding summary is given in section 5.

2. Dimensionless parameters

The effects of a uniform magnetic field applied to unstrained homogeneous turbulence in an electrically conductive fluid are characterized by two dimensionless parameters, the first being the magnetic Reynolds number

$$R_m = \frac{vL}{\eta} = (\frac{v}{L})(\frac{L^2}{\eta}). \tag{2.1}$$

Here v is the r.m.s. fluctuating velocity

$$v = \sqrt{R_{ii}/3}, \qquad R_{ij} = \overline{u_i u_j}\,, \tag{2.2}$$

where u_i is the fluctuating velocity, and L is the integral length scale. η is the magnetic diffusivity

$$\eta = 1/(\sigma\mu^*) \tag{2.3}$$

where σ is the electric conductivity of the fluid, and μ^* is the fluid magnetic permeability (here we use μ^* for the magnetic permeability and reserve μ for the dynamic viscosity). Thus the magnetic Reynolds number represents the ratio of the characteristic time scale for diffusion of the magnetic field to the time scale of the turbulence. In the case of vanishingly small R_m, the distortion of the magnetic field lines by the fluid turbulence is sufficiently small that the induced magnetic fluctuations $\mathbf{b}$ around the mean (imposed) magnetic field $\mathbf{B}$ are also small.

One can also define a magnetic Prandtl number representing the ratio of R_m to the hydrodynamic Reynolds number Re_L

$$P_m \equiv \frac{\nu}{\eta} = \frac{R_m}{Re_L}, \qquad Re_L = \frac{vL}{\nu}. \tag{2.4}$$

The second relevant dimensionless parameter is the magnetic-interaction number (or Stuart number),

$$N \equiv \frac{\sigma B^2 L}{\rho v} = \frac{\tau}{\tau_m} \tag{2.5}$$

where B is the magnitude of the magnetic field and ρ is the fluid density. N represents the ratio of the large-eddy turnover time τ to the Joule time τ_m, i.e. the characteristic time scale for dissipation of turbulent kinetic energy by the action of the Lorentz force. N parametrizes the ability of an imposed magnetic field to drive the turbulence to a two-dimensional three-component state. Under the continuous action of the Lorentz force, energy becomes increasingly concentrated in modes independent of the coordinate direction aligned with $\mathbf{B}$. As a two-dimensional state is approached, Joule dissipation decreases because fewer and fewer modes with gradients in the direction of $\mathbf{B}$ are left available. In addition, the tendency towards two-dimensionality and anisotropy is continuously opposed by non-linear angular energy transfer from modes perpendicular to $\mathbf{B}$ to other modes, which tends to restore isotropy. If N is larger than some critical value N_c, the Lorentz force is able to drive the turbulence to a state of complete two-dimensionality. For smaller N, the Joule dissipation is balanced by non-linear transfer before a complete two-dimensionality is reached. For very small N ($N \leq 1$), the anisotropy induced by the Joule dissipation is negligible. Here we consider N in the range $1 - 50$.

In MGD applications relying on thermal ionization without artificial flow seeding, the magnetic Reynolds number ranges from $R_m \sim 10^{-3}$ to $R_m \sim 5$ depending on the vehicle speed. The magnetic-interaction or Stuart number is typically of order unity.

3. The Quasi-Static approximation

3.1. *Equations and assumptions*

If the external magnetic field B_i^{ext} is explicitly separated from the fluctuations b_i, the MHD equations can be written as

$$\partial_t u_i = -\partial_i(p/\rho) - u_j\partial_j u_i + \frac{1}{(\mu^*\rho)}(B_j^{ext} + b_j)\partial_j(B_i^{ext} + b_i) + \nu\Delta u_i, \tag{3.1}$$

$$\partial_t(B_i^{ext} + b_i) = -u_j\partial_j(B_i^{ext} + b_i) + (B_j^{ext} + b_j)\partial_j u_i + \eta\Delta(B_i^{ext} + b_i), \tag{3.2}$$

where p is the sum of the kinematic and magnetic pressures, ν is the kinematic viscosity, ρ is the fluid density, μ^* is the magnetic permeability and η is the magnetic resistivity.

Since here we only consider homogeneous and stationary external magnetic fields, (3.1) and (3.2) reduce to

$$\partial_t u_i = -\partial_i(p/\rho) - u_j\partial_j u_i + \frac{1}{(\mu^*\rho)}b_j\partial_j b_i + \frac{1}{(\mu^*\rho)}B_j^{ext}\partial_j b_i + \nu\Delta u_i, \tag{3.3}$$

$$\partial_t b_i = -u_j\partial_j b_i + b_j\partial_j u_i + B_j^{ext}\partial_j u_i + \eta\Delta b_i. \tag{3.4}$$

For flows at low magnetic Reynolds number, (3.4) can be simplified considerably (see e.g. Roberts 1967). Indeed, by definition, the limit $R_m \ll 1$ describes flows for which non-linear terms resulting from magnetic fluctuations are negligible when compared to the dissipative term in (3.4). This is easily seen by adopting the traditional scalings,

$$\|u_j\partial_j b_i\| = \frac{vb}{L}, \quad \|b_j\partial_j u_i\| = \frac{vb}{L}, \quad \|\eta\Delta b_i\| = \frac{\eta b}{L^2}, \tag{3.5}$$

where $b = \sqrt{\frac{2}{3} b_i b_i}$, and noting that

$$R_m = \frac{vL}{\eta} = \frac{\|u_j \partial_j b_i\|}{\|\eta \Delta b_i\|} = \frac{\|b_j \partial_j u_i\|}{\|\eta \Delta b_i\|}. \tag{3.6}$$

In place of (3.4) we thus have, in the limit $R_m \ll 1$,

$$\partial_t b_i = B_j^{ext} \partial_j u_i + \eta \Delta b_i. \tag{3.7}$$

The so-called *quasi-static* (QS) *approximation* (Roberts 1967) is obtained by further assuming that $\partial_t b_i \approx 0$ in (3.7). To understand how this comes about, let us consider the time scales of the two terms on the right-hand side of (3.7). Since B^{ext} is independent of time, the time scale of $B_j^{ext} \partial_j u_i$ is $\mathcal{T} = L/v$, while the time scale of the diffusion term can be identified with the damping time $\mathcal{T}^* = L^2/\eta$. The ratio of these two time scales is then

$$\frac{\mathcal{T}^*}{\mathcal{T}} = R_m, \tag{3.8}$$

indicating that at low magnetic Reynolds number, diffusion time is much smaller than large-eddy turnover time. This justifies the assumption $\partial_t b_i \approx 0$ since the magnetic fluctuations then adapt instantaneously to the slowly varying velocity field and reach their asymptotic values for which $\partial_t b_i \approx 0$ (see section 4.1 for more details). In the QS approximation, we thus have

$$\eta \Delta b_i = -B_j^{ext} \partial_j u_i. \tag{3.9}$$

Using a Fourier representation for u_i and b_i, this equation is readily solved and yields

$$b_i(\mathbf{k}, t) = i \frac{(B_j^{ext} k_j)}{\eta k^2} u_i(\mathbf{k}, t), \tag{3.10}$$

where we have defined

$$u_i(\mathbf{k}, t) = \sum u_i(\mathbf{x}, t) e^{-i\mathbf{k}\cdot\mathbf{x}}, \quad b_i(\mathbf{k}, t) = \sum b_i(\mathbf{x}, t) e^{-i\mathbf{k}\cdot\mathbf{x}}. \tag{3.11}$$

Since b_i is now expressed completely in terms of u_i, the evolution equation for the velocity field can be explicitly closed. In Fourier representation one gets,

$$\partial_t u_i(\mathbf{k}, t) = -\partial_i p'(\mathbf{k}, t) - [u_j \partial_j u_i](\mathbf{k}, t) - \sigma \frac{(\mathbf{B}^{ext} \cdot \mathbf{k})^2}{\rho k^2} u_i(\mathbf{k}, t) - \nu k^2 u_i(\mathbf{k}, t), \tag{3.12}$$

where $p' = p/\rho$ (consistently with the small magnetic fluctuations assumption, the second-order term $b_j \partial_j b_i$ does not appear in (3.12)).

To summarize, two simplifications are needed in order to reach (3.12). The first consists in neglecting the non-linear terms $u_j \partial_j b_i$ and $b_j \partial_j b_i$ in (3.4). The second is obtained by discarding the time derivative of b_i in (3.7). These two simplifications are consequences of the assumption $R_m \ll 1$ and one should thus expect them to break down when the magnetic Reynolds number is increased. In the next sections, we test the QS approximation by comparing its predictions to those obtained using the full MHD equations (3.3) and (3.4).

3.2. *Numerical code and initial condition*

To test the range of validity of the QS approximation, we have used two different pseudo-spectral codes. The first one simulates the full MHD equations (3.3) and (3.4), while the second one simulates (3.12). All the runs presented here have a resolution of 128^3 cube Fourier modes in a $(2\pi)^3$ computational domain.

The initial condition for the velocity field is common to both codes. The field is initialized in Fourier space and the mode amplitudes are set to match the spectra (see Rogallo 1981 for details),

$$E(k) = 16 \left(\frac{2}{\pi}\right)^{.5} \frac{v_0^2}{{k_p}^5} k^4 \exp(-2k^2/{k_p}^2), \tag{3.13}$$

where we have arbitrarily set $v_0 = 1$ and $k_p = 3$. In order to let the higher-order statistics develop, the flow is then evolved (without any external magnetic field) until the skewness reaches its peak value. At that time, hereafter referred to as t_0, the external magnetic field is switched on.

For the full MHD case, an initial condition for b_i has to be chosen at $t = t_0$. Here we have made the choice $b_i(t_0) = 0$. In other words, our simulations describe the response of an initially non-magnetized turbulent conductive fluid to the application of a strong magnetic field. The corresponding completely-linearized problem has been described in detail in Moffatt (1967). For the QS approximation case, an initial condition for b_i is of course not required since the equation for the velocity field is completely closed. In this case, the initial condition is in fact implicitly given by (3.10) at $t = t_0$. One could then argue that the two codes do not simulate the same flow since they do not have the same initial condition for the magnetic field. However, the independence of the QS approximation of the initial magnetic field is precisely one aspect that is interesting to test. If the flow behaves according to the QS approximation, the magnetic field (using full MHD) should very rapidly converge to the value given by (3.10).

The only free parameters that remain to be specified are the kinematic viscosity ν, magnetic diffusivity η and external 'magnetic field' strength $B^{ext}/\sqrt{\mu^* \rho}$ (by convention we chose the magnetic field to be orientated in the z direction). The kinematic viscosity is $\nu = 0.003$ for all the runs and the rest of the parameters are specified in table 1, along with the corresponding values of the interaction number and magnetic Reynolds number at $t = t_0$. These last two quantities are calculated using (2.1) and (2.5), taking into account the fact that at $t = t_0$ (i. e. at the end of the initial decay during which $B^{ext} = 0$), we have $v(t_0) = 0.984$ and $L(t_0) = 0.787$.

All the runs can be grouped according to the initial value of the Stuart number. For the first five runs we have $N(t_0) = 1$; for the next five $N(t_0) = 10$ and finally for runs 11-15 we have $N(t_0) = 50$.

3.3. *Results*

In this section we present some results obtained by performing the simulations detailed in section 3.2.

3.3.1. *Kinetic energy decay*

In figure 1 we present the time evolution of the normalized kinetic energy,

$$E_K = \frac{1}{E_K(0)} \int d\mathbf{x} \frac{1}{2} u_i(\mathbf{x}) u_i(\mathbf{x}). \tag{3.14}$$

From (3.12), it is clear that the behavior of u_i in the QS approximation does not depend on the magnetic Reynolds number but only on the Stuart number (all other parameters being constant). The reason for this is of course that the QS approximation corresponds to

#	η	$\frac{B^{ext}}{\sqrt{\mu^* \rho}}$	$N(t_0)$	$R_m(t_0)$
1	7.75	3.11	1	0.1
2	.387	.696	1	2.0
3	.258	.568	1	3.0
4	.155	.440	1	5.0
5	.0775	.311	1	10.0
6	7.75	9.84	10	0.1
7	.387	2.20	10	2.0
8	.258	1.80	10	3.0
9	.155	1.39	10	5.0
10	.0775	.984	10	10.0
11	7.75	22.0	50	0.1
12	.387	4.92	50	2.0
13	.258	4.02	50	3.0
14	.155	3.11	50	5.0
15	.0775	2.20	50	10.0

Table 1. Summary of the parameters for the different runs performed

the implicit limit $R_m \to 0$. As expected, in each set of runs at fixed $N(t_0)$ the agreement between full MHD and the QS approximation gets better as the Reynolds decreases. At $R_m = 0.1$ the agreement is nearly perfect. At intermediate values, $R_m = 2$ and above, there is a quite severe discrepancy: the rate at which the QS approximation dissipates kinetic energy is too high.

It is also interesting to note that the difference between full MHD and the quasi-static approximation at higher magnetic Reynolds numbers is not very sensitive to the value of the Stuart number.

3.3.2. *Magnetic energy decay*

The difference between full MHD and the QS approximation can result only from the fact that the quasi-static approximation predicts the magnetic fluctuations incorrectly. In order to assess this, we can define two normalized magnetic energies for each full-MHD

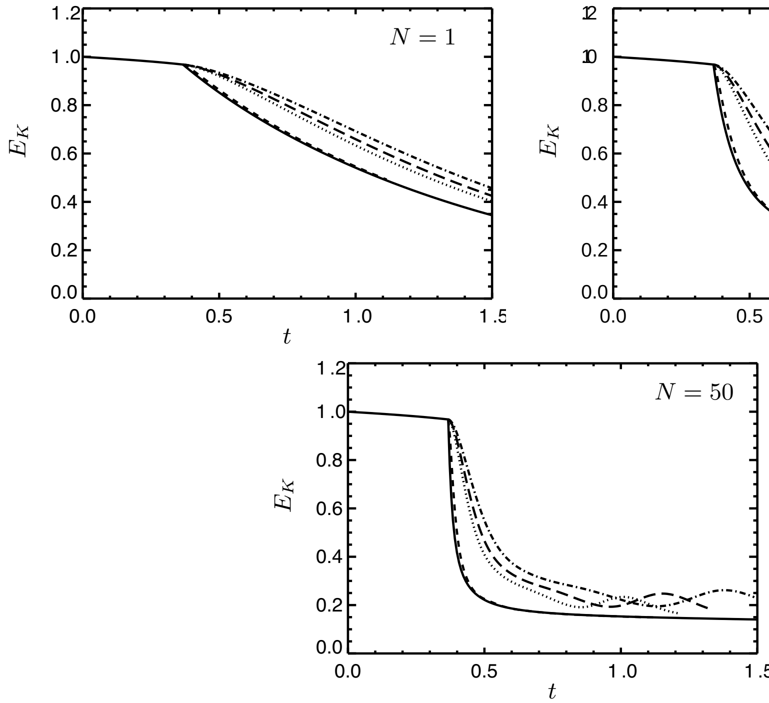

FIGURE 1. Evolution with time of the kinetic energy at different Stuart number and magnetic Reynolds number. ——— quasi-static approximation; - - - - - - $R_m = 0.1$; ········· $R_m = 2$; – – – – $R_m = 3$; —·— $R_m = 5$.

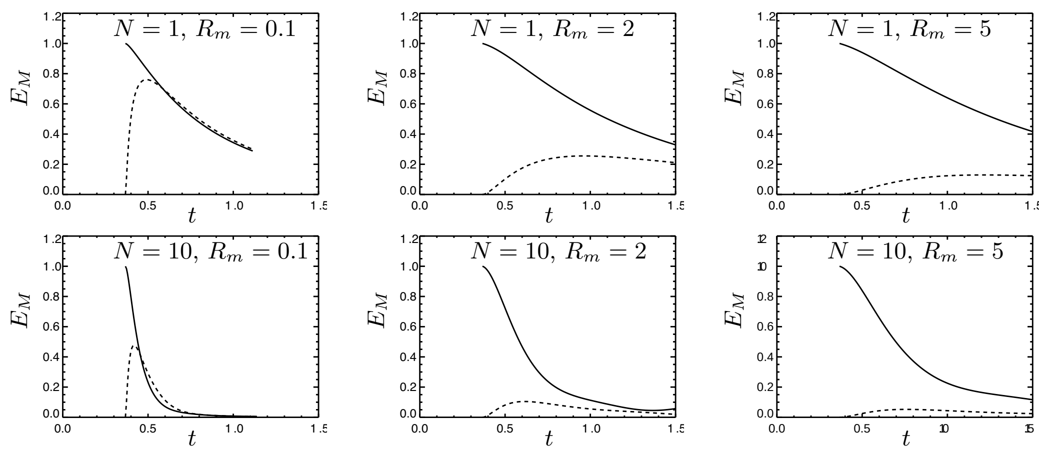

FIGURE 2. Evolution with time of the magnetic energy. ——— 'quasi-static' approximation computed from (3.15); - - - - - - full MHD computed from (3.16).

run: one obtained from the velocity field through (3.10),

$$E_{M1} = \frac{1}{E_{M1}(t_0)} \int d\mathbf{k} \frac{1}{2} \frac{(B_j^{ext} k_j)^2}{\eta^2 k^4} |u_i(\mathbf{k}, t)|^2, \tag{3.15}$$

and one computed directly from the magnetic fluctuations.

$$E_{M2} = \frac{1}{E_{M1}(t_0)} \int d\mathbf{k} \frac{1}{2} |b_i(\mathbf{k}, t)|^2. \tag{3.16}$$

Ideally, we should have $E_{M1} = E_{M2}$.

The comparison between (3.15) and (3.16) for different runs is displayed in figure 2. We observe that at magnetic Reynolds number $R_m = 0.1$, (3.15) and (3.16) predict similar values soon after the field is switched on. This indicates that, in a very short time, the magnetic fluctuations forget their initial state and 'align' with the predictions of the QS approximation (3.10). This is entirely in the spirit of the assumption that at low magnetic Reynolds number the time derivative in (3.7) can be neglected (or rather that it is significant only during a very short transient time). At higher values of the magnetic Reynolds number the transient time becomes longer and the 'true' energy content of the fluctuations never reaches values comparable to those predicted by the QS expression. The same conclusions hold for the runs at Stuart number $N = 50$ (not displayed).

As one expects, this discussion indicates that neglecting the time derivative of b in the induction equation is problematic when the magnetic Reynolds number is increased. In the next section we study this question in more detail.

4. The Quasi-Linear approximation

4.1. *Governing equations*

As was recalled in section 3.1, the final quasi-static induction equation is obtained by dropping the time derivative of the magnetic field in (3.7). The discussion of the previous section suggests that keeping this time derivative might be crucial when the magnetic Reynolds number is increased.

We thus introduce an intermediate approximation which is obtained by considering the following simplified MHD equations:

$$\partial_t u_i = -\partial_i (p/\rho) - u_j \partial_j u_i + \frac{1}{(\mu^* \rho)} B_j^{ext} \partial_j b_i + \nu \Delta u_i, \tag{4.1}$$

$$\partial_t b_i = B_j^{ext} \partial_j u_i + \eta \Delta b_i. \tag{4.2}$$

We call this approximation the *quasi-linear* (QL) *approximation* since we discard only the non-linear terms involving the magnetic field and keep the non-linear convective term in the velocity equation.

If $\partial_t b_i$ is neglected in (4.2) one immediately recovers the quasi-static approximation. In fact, (4.2) is nothing else than a 'heat' equation for the magnetic field with a source term given by $B_j^{ext} \partial_j u_i$. In Fourier space the solution of this equation is easily obtained and reads,

$$b_i(\mathbf{k}, t) = b_i(\mathbf{k}, 0) e^{-\eta k^2 t} + i \int_0^t d\tau k_j B_j^{ext} u_i(\mathbf{k}, \tau) e^{-\eta k^2 (t-\tau)}. \tag{4.3}$$

From the first term on the right-hand side of (4.3), we see that the initial condition for b_i gets damped more rapidly with increasing magnetic diffusivity (if v and L are constant this is equivalent to a decrease in the magnetic Reynolds number). Note that with the initial condition we have chosen for the magnetic field, i.e. $b_i(0) = 0$, this first term vanishes. At high magnetic diffusivity the integral in (4.3) converges to (3.10) and the QS approximation holds. More explicitly, we have the situation depicted in figure

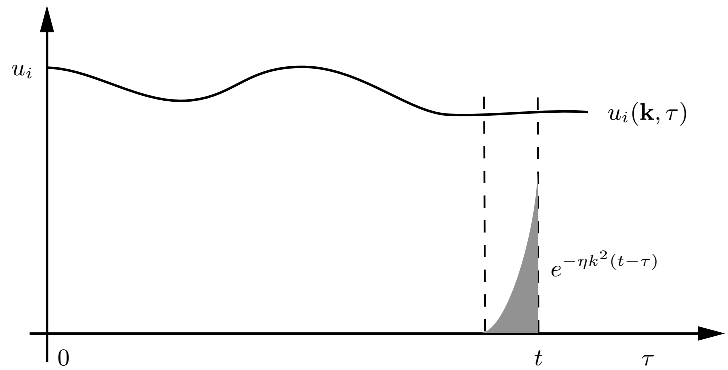

FIGURE 3. Evaluation of $b_i(\mathbf{k}, t)$ in the quasi-linear approximation.

3. The interval between the dashed lines represents the support in which $\exp^{-\eta k^2(t-\tau)}$ is significant and thus where there is some contribution to the integral of (4.3). As η increases, this interval gets smaller, and $u_i(\mathbf{k}, t)$ may be assumed constant in that short period of time. The integration is then immediate and one gets (3.10). Thus, the time history of $u_i(\mathbf{k}, t)$ plays a role only when η is small, in which case the exponential has a wider support.

4.2. *Results*

In order to compare the QL approximation with full MHD, we have performed the same numerical simulations as described in section 3, but this time using (4.1) and (4.2) instead of the QS approximation. The only points we mention here are first, that (4.2) needs an initial condition, so to be consistent with the full MHD case we have set $b_i(0) = 0$; and secondly, we have increased the magnetic Reynolds number as far as $R_m = 10$ for the results presented in this section.

4.2.1. *Kinetic energy decay*

In figure 4 we present the time history of the kinetic energy (as defined by (3.14)) obtained from both full MHD and the QL approximation. As is obvious from the graphs, the quasi-linear theory predicts this diagnostic extremely well, even at magnetic Reynolds number up to $R_m = 10$. The improvement over the quasi-static approximation (also shown in the graphs) is evident.

4.2.2. *Magnetic energy decay*

Figure 5 represents the time evolution of the energy of the magnetic fluctuations (defined by (3.16) without the normalization factor) for various values of the Stuart number and magnetic Reynolds number. Each graph contains one curve for the full MHD case and another for the QL approximation case. The agreement is excellent at magnetic Reynolds number $R_m = 2$ and degrades only slightly for higher values. It is interesting to note that, as the Stuart number is increased, the agreement between full MHD and the QL approximation gets better. As in the case of classical homogeneous sheared flows, we thus see that the further away from equilibrium the flow is, the better it is described by a linear theory. The role of increased shear is played here by the increasing external magnetic field.

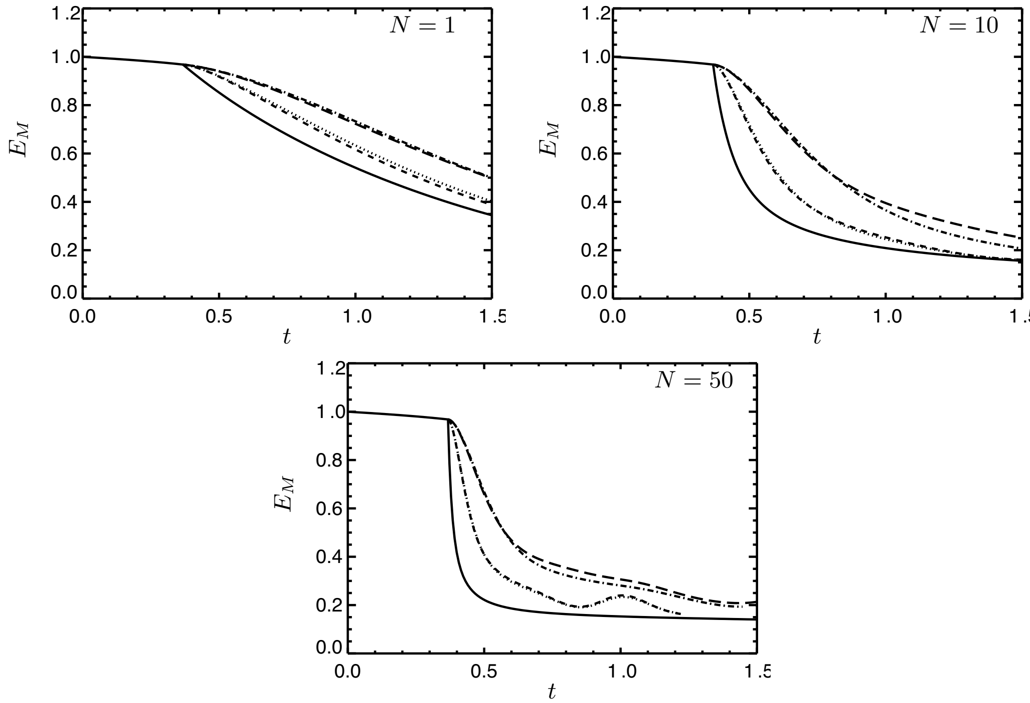

FIGURE 4. Evolution with time of the kinetic energy. ——— quasi-static approx.; ········ $R_m = 2$ full MHD; - - - - - - $R_m = 2$ QL approx.; —·— $R_m = 10$ full MHD; — — — — $R_m = 10$ QL approx.

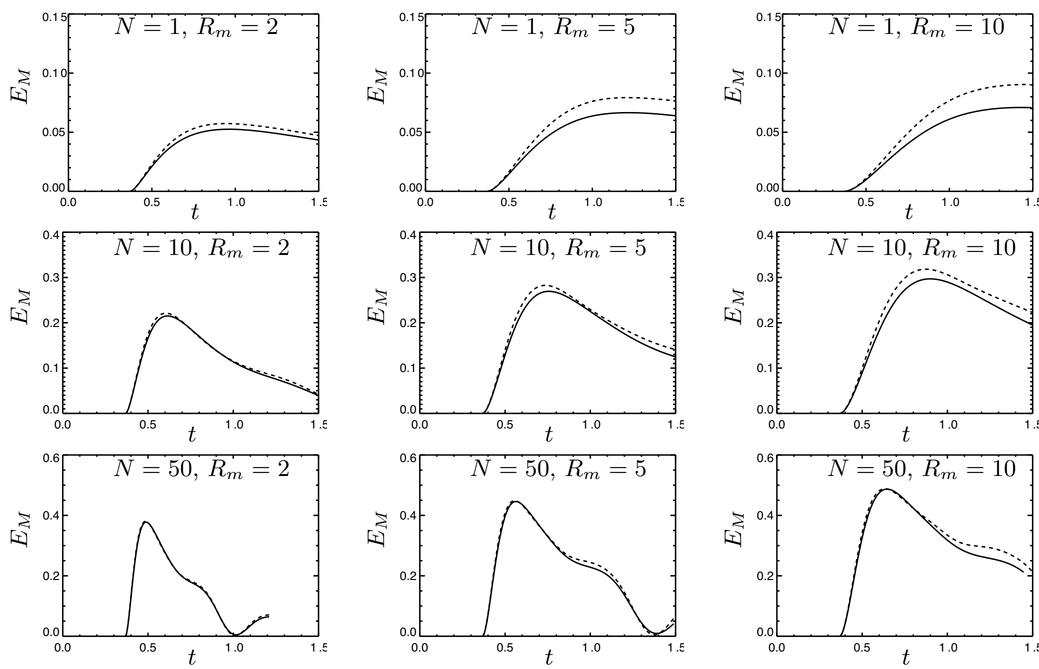

FIGURE 5. Evolution with time of the magnetic energy. ——— full MHD; - - - - - - QL approx.

5. Conclusions and future plans

By studying the case of decaying homogeneous MHD turbulence, we have established that the Quasi-Static (QS) approximation is valid for $R_m \lesssim 1$, but progressively deteriorates as R_m is increased beyond 1. The magnetic Stuart number does not seem to have a strong effect on the accuracy of the QS approximation. That is, at a given R_m, the accuracy of the QS approximation is roughly the same for $N = 1$ as it is for $N = 50$. The QL approximation, as we expected when we proposed it during the Summer Program, performs like the QS approximation for $R_m \lesssim 1$, but has the advantage that it retains excellent agreement with full MHD for $0 \lesssim R_m \lesssim 10$ at least. It should be noted that $R_m = 10$ is the highest value of the magnetic Reynolds number that we tested during the Summer Program.

In terms of computational costs, the QS approximation is clearly the cheapest of the three methods used during our study. It has fewer non-linear terms to evaluate, and the time step needed to advance the flow is governed by the time scale of the velocity field which, for most industrial cases involving liquid metal, is significantly longer than the time scale of the underlying magnetic field.

However, as we have demonstrated, the QS approximation becomes inadequate for conductive flows with moderate magnetic Reynolds numbers such as are, for instance, encountered in MGD applications involving hypersonic vehicles. We have studied another approximation, the *QL approximation*, for use at higher R_m. As with the QS approximation, this approximation assumes small magnetic fluctuations but it tries to resolve the time dependence of these fluctuations explicitly. Our numerical simulations indicate that the QL approximation should be adopted in place of the quasi-static approximation for flows with a moderate value of the magnetic Reynolds number, since in those cases it compares much better with full MHD. In terms of computational cost the QL approximation does not depart enormously from full MHD, but nevertheless allows a reasonable gain since fewer non-linear terms need to be evaluated. The appeal of the QL approximation lies more in the prospect of simpler turbulence models for conductive flows at moderate magnetic Reynolds number. Indeed, the structure of equations (4.1) and (4.2) is simpler than that of the full MHD equations. We thus have a strong hope that devising turbulence models in the framework of the QL approximation should be an easier task. This question will be examined in the coming months and will undoubtedly benefit significantly from the work we have produced during this 2002 Summer Program.

REFERENCES

KASSINOS, S. C. & REYNOLDS, W. C. 1999 Structure-based modeling for homogeneous MHD turbulence. *Annual Research Briefs*, Center for Turbulence Research, NASA Ames/Stanford Univ., 301–315.

MOFFATT, H.K. 1967 On the suppression of turbulence by a uniform magnetic field. *J. Fluid Mech.* **28**, 571–592.

ROBERTS, P. H. 1967 *An Introduction to Magnetohydrodynamics.* Elsevier, New York.

ROGALLO, R.S. 1981 Numerical experiments in homogeneous turbulence. *NASA TM* 81315.