Numerical investigation of the acoustic behavior of a multi-perforated liner

By J. Eldredge†, M. Shoeybi AND D. Bodony

The acoustic response of a turbulent flow through a multi-perforated liner is computed with incompressible Large-Eddy Simulation (LES). The effect of an array of apertures is accounted for by simulating a single jet with periodic conditions in both directions tangential to the plate. Flows that are parallel to the plate are included in the regions above and below the aperture, which is tilted in the tangential flow direction as in practical film-cooling liners. The mass flow rate through the aperture is forced with a small sinusoidal perturbation superposed on a mean component. The acoustic behavior is determined by measuring the fluctuating pressure difference across the aperture that results from the forcing. In this work, two different forcing frequencies are considered. The transfer function between forcing and response, which represents the acoustic impedance of the liner, is calculated for these frequencies. Good agreement is found when compared with existing theory, when the latter is modified for the thickness and tilting of the aperture.

1. Introduction

Multi-perforated liners are commonly used in combustion systems for providing film cooling of the walls. These liners consist of a regular array of submillimeter apertures, across which a pressure difference forces a mean jet of cool air into the hot combustor. The apertures are often tilted downstream, so that the cool jets coalesce to form a film adjacent to the wall; this film does not mix substantially with the hot combustion products. A numerical study of this role of the multi-perforated liner is described in another report in these proceedings (Mendez *et al.* 2006). It has long been recognized that these liners also serve an acoustic role, and in fact can mitigate the growth of thermoacoustic instabilities by damping incident acoustic waves. Systematic optimization of this damping, for example by tuning the liner geometry, requires an accurate description for the acoustic behavior of the liner. Numerical simulation serves a valuable role for a parametric study of the acoustic characteristics of a multi-perforated liner.

The frequency-dependent behavior of the liner is described by the acoustic impedance. A purely analytical approach for seeking this impedance is complicated by the turbulent flow in the vicinity of the aperture, as well as the large number of flow and geometric parameters that affect the behavior. However, under certain simplifying assumptions about the acoustic mechanisms, a theoretical treatment is possible. In particular, if the liner porosity—the fraction of the total liner area that is open—is small, then the individual apertures can be assumed to act in isolation. This assumption enables investigation of a single aperture in an infinite wall.

The behavior of the aperture is described by its Rayleigh conductivity, K_R, which is

† Mechanical and Aerospace Engineering Department, University of California, Los Angeles

defined as

$$K_R = -\frac{i\omega\bar{\rho}\hat{Q}}{\hat{p}_+ - \hat{p}_-}. \tag{1.1}$$

In this definition, ω is the angular frequency, $\bar{\rho}$ is the mean density in the vicinity of the aperture, $\hat{p}_+$ and $\hat{p}_-$ are the amplitudes of pressure fluctuations above and below the aperture, respectively, and $\hat{Q}$ is the amplitude of volume flow rate fluctuations through the aperture (positive when from the $-$ to $+$ side). Note that we are using the convention that all fluctuating quantities are proportional to $\exp i\omega t$. The Rayleigh conductivity can be be adapted into a homogeneous liner impedance, provided that the acoustic wavelength is large compared to both the characteristic size of the apertures and the distance between them.

Howe (1979) developed a Rayleigh conductivity for a circular aperture in an infinitely thin wall with a high Reynolds jet issuing from it. He hypothesized that pressure fluctuations in the liner vicinity produce fluctuating vorticity shed from the rim of the aperture. The model was derived by assuming that these vorticity fluctuations are convected from the rim in a cylindrical vortex sheet at a mean velocity, U_c. The result of his analysis is the expression

$$K_R = 2R(\gamma + i\delta), \tag{1.2}$$

where R is the radius of the aperture, and γ and δ are functions of the Strouhal number, $\mathrm{St} = \omega R/U_c$, given by

$$\gamma + i\delta = 1 + \frac{(\pi/2)\mathrm{I}_1(\mathrm{St})\mathrm{e}^{-\mathrm{St}} + \mathrm{i}\mathrm{K}_1(\mathrm{St})\sinh(\mathrm{St})}{\mathrm{St}\left[(\pi/2)\mathrm{I}_1(\mathrm{St})\mathrm{e}^{-\mathrm{St}} - \mathrm{i}\mathrm{K}_1(\mathrm{St})\cosh(\mathrm{St})\right]}, \tag{1.3}$$

where I_1 and K_1 are modified Bessel functions. The mechanism for acoustic absorption—conversion into vortical energy—is enhanced by the convection of the vorticity fluctuations by the mean jet, which ensures that the fraction of energy absorbed is independent of the amplitude of the incident sound (in contrast with the non-linear absorption mechanism in the absence of a mean jet, which becomes more effective for stronger incident sound waves).

Though expression (1.2) is nominally designed for an infinitely thin wall in an otherwise stagnant medium, a modified form has been successfully applied to problems with more general conditions. Hughes and Dowling (1990) verified its use with experiments on acoustic waves impinging normally on a perforated screen. Eldredge and Dowling (2003) extended its application to grazing waves, and showed good agreement with experiments on plane waves traveling in a lined duct. In this latter work, the inclusion of a small mean duct flow did not significantly affect on the results.

The effect of thickness in these works was accounted for in the respective models by including the inertia of the aperture fluid with a straightforward reactance term. Jing and Sun (2000) attempted to improve on this simple approach by numerically computing the Rayleigh conductivity for an aperture in a thick wall. They used the same hypothesis as Howe (1979) regarding vorticity fluctuations confined to a steady vortex sheet, and used a panel method to solve the Helmholtz equation in the geometry. However, their presumed configuration of the vortex sheet, a vena contracta anchored at the rim of the aperture entrance, is questionable, and it is unclear if their results led to an improvement in the model.

In addition to liner thickness, the effects of several other geometry and flow parameters are also in question. The aperture tilting that is conventional in a cooling liner most

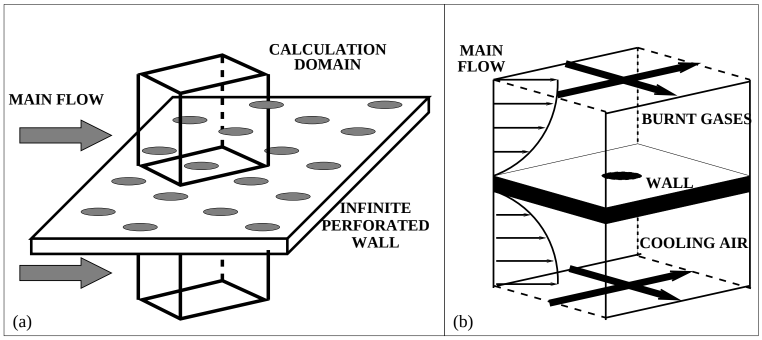

FIGURE 1. Adapted from Mendez *et al.* (2005): Numerical configuration of the multi-perforated liner. (a) Computational domain of single jet relative to the infinite array (b) Bi-periodic directions depicted with arrows, in domain extended equally above and below the aperture.

likely affects the shape of the jet and the associated transport of vorticity. The behavior of an extended array of jets may differ from an uncorrelated combination of isolated jets. Finally, the influence of the tangential flow on either side of the aperture is unclear. High-fidelity numerical simulation can address each of these factors in detail.

The objectives of this study are to

- evaluate the use of incompressible LES as a tool for exploring the acoustic characteristics of a multi-perforated liner with non-idealized flow conditions, and
- compare the LES-computed Rayleigh conductivity with the existing model (1.2).

The target of study for the numerical simulations, described in Section 2, will include many of the geometry and flow features that have been neglected or simplified in previous studies. The results of these simulations will be presented and discussed in Section 3, and compared to Howe's model (with appropriate modification).

2. Numerical configuration

The target of this study is a conventional industrial configuration of a multi-perforated plate for cooling applications. The infinite array of jets is modeled computationally as a single jet in a bi-periodic computational domain. This configuration, depicted in Fig. 1, has been the target of several recent numerical investigations for effusion cooling applications (Mendez *et al.* 2005; Mendez and Nicoud 2005). The previous studies have presented experimental validation of the computational model; a parallel numerical study reported in these proceedings (Mendez *et al.* 2006) focuses on further validation of this model, including a comparison of the compressible AVBP (AVBP 2006) and the incompressible CDP LES (Mahesh *et al.* 2004) codes. The present work uses the latter code. The apertures have circular bore of diameter of 5 mm and are tilted by 60 degrees from vertical in the plate of 10 mm thickness. As seen in Fig. 1, the apertures are arranged in a staggered configuration, so the computational domain presents a diamond-shaped projection on the wall, with long diameter 58.4 mm in the streamwise direction and short diameter 33.7 mm in the transverse direction (thus giving an area of 984 mm^2). The intersections of the tilted circular aperture with the inside and outside of the plate form elliptical holes with major axis (in the tilt direction) of 1 cm and minor axis (in the transverse direction) of 5 mm. Thus, the porosity of the liner is 0.04.

The tangential flows above and below the aperture have Reynolds numbers of 17,750 and 8900, respectively (with Reynolds number defined here based on the streamwise

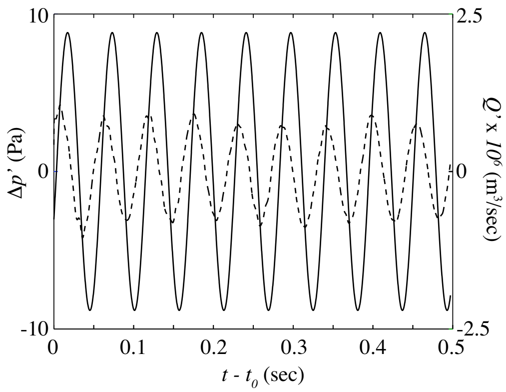

FIGURE 2. Fluctuations in aperture volume flow rate (—) and pressure drop (– – –) for forcing at 180 Hz. Time is measured relative to the instant of introduction of fluctuations.

velocity at the top of the domain, which is approximately 4.29 m/s, and the height of the domain above the aperture, 60 mm). To produce the jet flow, a uniform inflow of 0.112 m/s is introduced at the bottom of the domain. Because of the bi-periodic conditions, the entire mass of this inflow must exit the top of the domain, via an outflow boundary condition. Note that, in cold flow conditions, the bias flow Mach number associated with the resulting jet is 0.008, which justifies the assumption of incompressible flow. The boundary layer that develops from the coalescence of the jet with the tangential flow can be observed in Figs. 7 and 8 of Mendez *et al.* (2006).

The acoustic characteristics of the aperture in these flow conditions are assessed by superposing a small sinusoidal fluctuation on the inflow velocity,

$$v_{\text{in}} = 0.112(1 + A\cos 2\pi f t), \tag{2.1}$$

where $A = 0.02$ and f is the forcing frequency. The simulation is run for a sufficiently long enough time to achieve statistically stationary turbulent flow conditions prior to introducing this inflow perturbation. The Rayleigh conductivity is calculated by measuring the pressure fluctuations that result from this inflow forcing. Specifically, this calculation is based on the difference in the spatially averaged pressure at the top and bottom planes of the domain. A fast Fourier transform of the time series of this pressure difference is used in Eq. (1.1).

3. Numerical results

This section presents the results of the numerical simulations of forcing of the inflow velocity. Two separate simulations were conducted, with forcing frequencies of $f = 180$ Hz and $f = 360$ Hz. These frequencies correspond to Strouhal numbers of approximately 0.5 and 1, respectively, when based on the mean jet velocity, $2 \times 0.112/0.04 = 5.60$ m/s (the factor of 2 accounts for the jet tilting by 60°). For Case 1 at 180 Hz, the fluctuating volume flow rate and resulting fluctuations in pressure difference are depicted in Fig. 2. The plot clearly shows the phase lead of the pressure drop oscillations relative to the volume flux. The results for 360 Hz forcing shown in Fig. 3 exhibit an increase in the pressure fluctuations (or, from an alternative perspective, the volume flux for given pressure drop is decreased). Note that grid convergence studies were not performed in the context of

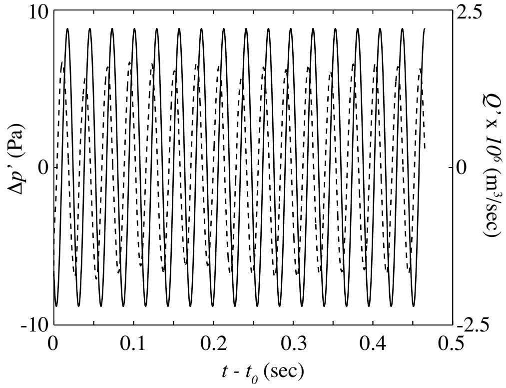

FIGURE 3. Fluctuations in aperture volume flow rate (—) and pressure drop (– – –) for forcing at 360 Hz.

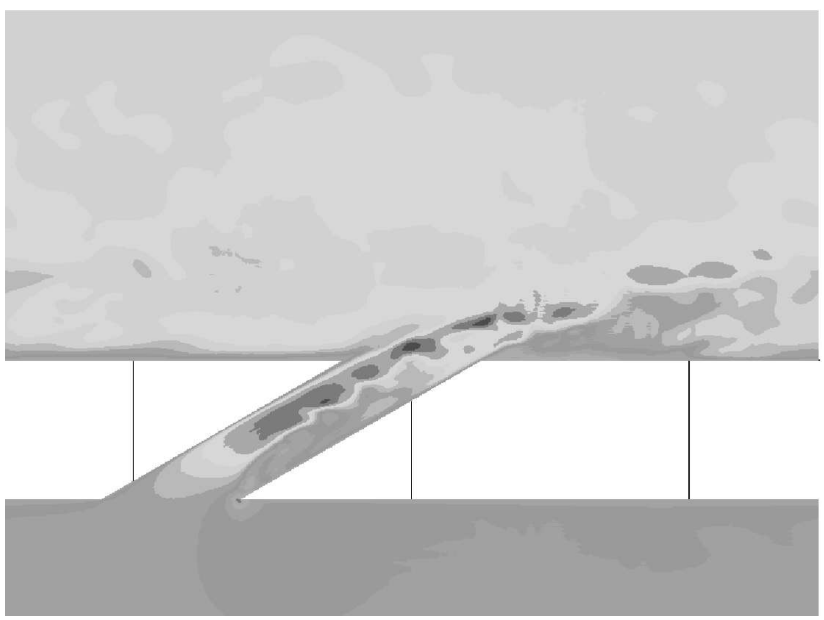

FIGURE 4. Snapshot of streamwise component of velocity from LES with 180 Hz forcing.

the acoustic forcing simulations; however, grid convergence tests were carried out on the unforced simulations that are utilized as initial conditions for the present investigation.

Figure 4 depicts the instantaneous streamwise velocity field with 180 Hz forcing. The jet in the aperture exhibits a varicose structure, which is consistent with the vortex sheet structure hypothesized by Howe (1979). A sequence of several of these snapshots reveals the pulsations experienced by the jet. The flow separates from the downstream lip of the aperture inlet and is primarily confined to the upper portion of the aperture. The jet loses coherence as it exits the aperture and mixes with the turbulent flow tangential to the liner.

The Rayleigh conductivity computed from these simulations is plotted in Fig. 5 and compared with a thickness-modified model (1.2) of Howe (1979). Note that this modified

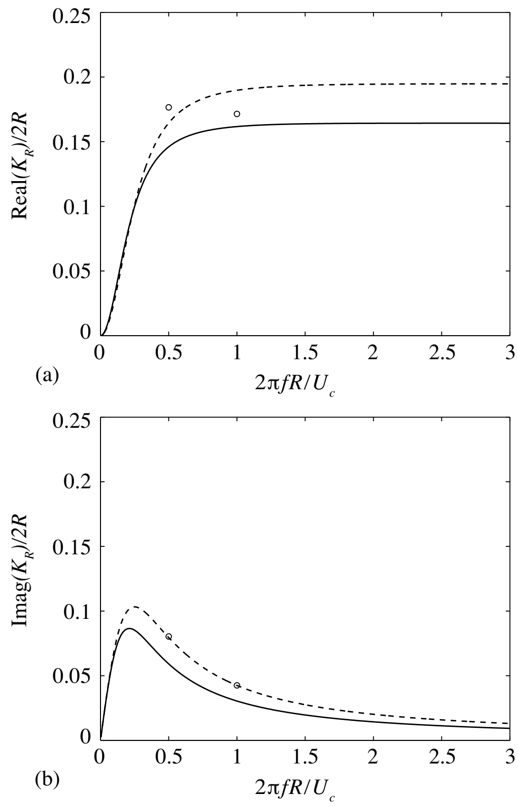

FIGURE 5. (a) Real and (b) imaginary components of Rayleigh conductivity of aperture. Modified Howe conductivity with $t/R = 8$ (—) ; Modified Howe conductivity with $t/R = 6.5$ (– – –); LES results (○).

model can be written as (see, e.g., Jing and Sun 2000)

$$K_{R,\mathrm{mod}} = 2R\left(\frac{1}{\gamma + i\delta} + \frac{2}{\pi}\frac{t}{R}\right)^{-1}. \tag{3.1}$$

Using a thickness of 20 mm, which corresponds to the length of the tilted aperture, leads to $t/R = 8$. The simulation results are reasonably close to the model for this choice. However, the experiments of Jing and Sun (2000) show that the effective t/R decreases as the bias flow increases. This decrease can be attributed to the configuration of the separated jet in the aperture, as well as to the fact that the radius of the aperture is effectively larger than 2.5 mm because of the elliptical intersection of the aperture with the top and bottom of the plate. To illustrate this point, Fig. 5 depicts another plot of Eq. (3.1) using a lower value of $t/R = 6.5$, resulting in much better agreement with the LES results in the imaginary part of the Rayleigh conductivity.

There also exists some ambiguity about the choice of the mean convection velocity, U_c, when calculating the Strouhal number appropriate for the flow. In this work, the mean jet velocity (averaged across the area of the aperture) was chosen for simplicity. However, Fig. 4 shows that this jet is clearly not uniformly distributed across the aperture. But the convection of vortical disturbances by this jet—which is what U_c represents—is likely at some fraction of the jet's mean velocity. These ambiguities should be clarified with further numerical simulations under a variety of geometries and flow conditions. Additionally, we are currently evaluating a larger range of forcing frequencies in the present conditions.

4. Conclusions

In this work we have used incompressible LES to investigate the acoustic characteristics of a multi-perforated liner in the presence of a turbulent flow. The volume flux through the aperture was perturbed sinusoidally and the resulting pressure fluctuations across the aperture were measured. The Rayleigh conductivity of the aperture was computed and compared with a previous theoretical model of Howe (1979), and it was found that the thickness-modified model of Howe (1979) provides a reasonable estimate of the LES-calculated impedance. However, the results suggest that this model needs to be modified to include bias flow and aperture angle considerations. In future work, more simulations will be performed over a broader range of operating conditions and forcing frequencies to further reveal the acoustic behavior of the aperture.

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