# Modeling sprays by the method of Laplace transforms

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In many applications, a moving fluid carries a suspension of droplets of a second phase which may change in size due to evaporation or condensation. If the number of such particles is very large, it may be practically impossible to explicitly compute all of the fluid and particle degrees of freedom in a numerical simulation of the system. A method for reducing the computational size under these circumstances is presented by representing the particle cloud by means of a distribution function in the particle radius and co-ordinates. This distribution function evolves according to a Fokker Planck equation. It is shown that the Laplace Transform of the distribution function satisfies an integral differential equation that may be conveniently solved numerically in conjunction with the usual LES equations.

## 1. The model

### 1.1. *The equation for the distribution function*

It is assumed that the filter width $\ell$ of the LES is much larger than particle sizes or inter-particle distances, so that a cube of side $\ell$ still contains enough particles that the state of the system may be described by the distribution function, $n_p(r, \mathbf{x}, t)$. Here $n_p(r, \mathbf{x}, t)\, d\mathbf{x} dr$ is the number of particles of radius between '$r$' and '$r + dr$' that are contained in an elementary volume '$d\mathbf{x}$' around location '$\mathbf{x}$' at time $t$. If, further, the particles are inertialess, so that they are advected passively by the flow and they do not undergo coalescence or break up (these effects can be incorporated later), an evolution equation for $n_p$ may be written as

$$\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \mathbf{u}) = -\frac{\partial}{\partial r}(n_p \dot{r}) + \nabla \cdot (D \nabla n_p) \tag{1.1}$$

In (1.1), $\dot{r}$ is generally a function of $r$ as well as other scalar and vector fields (such as density of water vapor or fuel mass fraction, temperature etc.) and $D$ is an "eddy diffusivity" representing diffusive spreading by the unresolved subgrid scales. We will write the law of variation of $r$ as:

$$\frac{dr}{dt} = -G(r)\phi(T, Y) \tag{1.2}$$

where $G(r)$ is a known function and $\phi$ is a known function of temperature $T$, vapor mass fraction $Y$ and possibly additional scalars. For example, with evaporating fuel drops in a gas turbine in the dilute phase we have $G(r) = 1/r$ and $\phi = \Lambda(T - T_0)$ where $T_0$ is the boiling point temperature. Equation (1.1) is a Fokker-Planck equation for particle conservation in the four-dimensional phase space $(x, y, z, r)$. It couples with the fluid and species/temperature transport equations through the source term in the equations for mass/energy that are of the general form $\int_0^\infty (...) n_p \, dr$ where (....) denotes the flux of the appropriate quantity onto a particle of size '$r$'. In principle, one needs to simulate (1.1)

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for $n_p$, together with the equations of compressible flow. However, since solving partial differential equations in four-dimensional space involves a large increase in computational cost, a reduced description is desired. This is considered next.

## 1.2. *Representation by Laplace Transforms*

Let us take the Laplace Transform (LT) of both sides of equation (1.1), where the LT is defined as

$$\hat{n}_p(s, \mathbf{x}, t) = \int_0^\infty n_p(r, \mathbf{x}, t) \exp(-sr)\, dr. \tag{1.3}$$

Here $s$ is restricted to be greater than $s_0$ where $s_0$ is some real number for which the above integral converges. The Laplace inverse is given by

$$n_p(r, \mathbf{x}, t) = \frac{1}{2\pi i} \int_{s-i\infty}^{s+i\infty} \hat{n}_p(s, \mathbf{x}, t) \exp(rs)\, ds. \tag{1.4}$$

Then (1.1) becomes:

$$\frac{\partial \hat{n}_p}{\partial t} + \nabla \cdot (\hat{n}_p \mathbf{u}) = \phi \int_0^\infty \exp(-sr) \frac{\partial}{\partial r}(n_p G)\, dr + \nabla \cdot (D \nabla \hat{n}_p) \tag{1.5}$$

Using integration by parts on the term involving the integral,

$$\frac{\partial \hat{n}_p}{\partial t} + \nabla \cdot (\hat{n}_p \mathbf{u}) = -\phi \lim_{\epsilon \to 0} G(\epsilon) n_p|_{r=\epsilon} + s\phi \int_0^\infty n_p G \exp(-sr)\, dr + \nabla \cdot (D \nabla \hat{n}_p) \tag{1.6}$$

The value of the first term on the right of the equality depends on $G(r)$ and is discussed a little later. The integral term on the right has a simple form for certain kinds of evaporation laws $G(r)$:

**Examples:**

($a$) If the evaporation (or condensation) rate is controlled by the flux of heat (or vapor) to the drop and if the droplet surroundings may be regarded as locally isotropic then $G(r) = 1/r$. In this case

$$\int_0^\infty n_p G \exp(-sr)\, dr = \int_0^\infty \frac{n_p}{r} \exp(-sr)\, dr = \int_s^\infty \hat{n}_p(\xi)\, d\xi \tag{1.7}$$

where we have made use of the following property of the Laplace Transform: if $\hat{f}(s)$ is the Laplace transform of $f(x)$ then the Laplace Transform of $f(x)/x$ is $\int_s^\infty \hat{f}(\xi)\, d\xi$.

($b$) Another useful example is the situation of a linear growth rate, $G(r) = a + br$ where $a$ and $b$ are constants. Here

$$\int_0^\infty n_p G \exp(-sr)\, dr = \int_0^\infty (a + br) n_p \exp(-sr)\, dr = a\hat{n}_p - b\frac{\partial \hat{n}_p}{\partial s} \tag{1.8}$$

where we have made use of the following property of the Laplace Transform: if $\hat{f}(s)$ is the Laplace transform of $f(x)$ then the Laplace Transform of $-xf(x)$ is $\hat{f}'(s)$.

($c$) The function $G(r) = \exp(-\alpha r)$ has the property of a quasi-linear growth rate at small values of $r$ but at large $r$ the growth rate rapidly goes to zero. In this case also the integral has a simple form,

$$\int_0^\infty n_p G \exp(-sr)\, dr = \int_0^\infty n_p \exp(-\alpha r) \exp(-sr)\, dr = \hat{n}_p(s + \alpha) \tag{1.9}$$

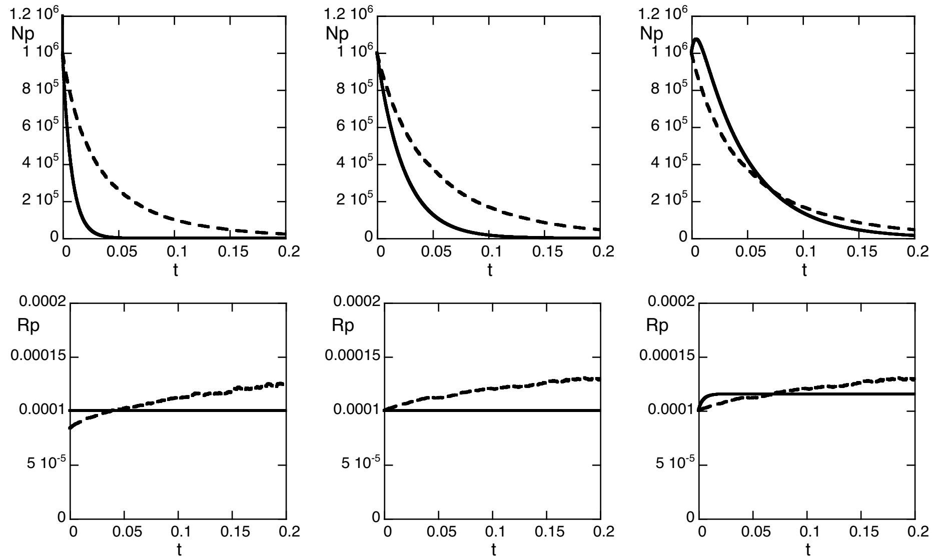


FIGURE 1. *Top Panel:* The number of particles as a function of time (sec) for (left to right) Cases 1,2 and 3 (DNS: dashed line, Model: solid line). *Lower Panel:* same for mean radius (meters)

where use was made of the following property of the Laplace Transform: if $\hat{f}(s)$ is the Laplace transform of $f(x)$ then the Laplace Transform of $\exp(ax)f(x)$ is $\hat{f}(s-a)$.

From this point on we assume $G(r) = 1/r$ as this case is most commonly encountered in sprays and also because for this form the LT method is particularly simple. If $G(r) = 1/r$, then in order that the limit term in (1.6) be finite, we must have $n_p(0, \mathbf{x}, t) = 0$. Expanding $n_p$ in a Taylor series in $r$ we find:

$$\lim_{\epsilon \to 0} G(\epsilon) n_p|_{r=\epsilon} = \left. \frac{\partial n_p}{\partial r} \right|_{r=0} \tag{1.10}$$

so that for $G(r) = 1/r$ (1.6) becomes

$$\frac{\partial \hat{n}_p}{\partial t} + \nabla \cdot (\hat{n}_p \mathbf{u}) = -\phi \left. \frac{\partial n_p}{\partial r} \right|_{r=0} + s\phi \int_s^\infty \hat{n}_p(\xi, \mathbf{x}, t)\, d\xi + \nabla \cdot (D \nabla \hat{n}_p) \tag{1.11}$$

### 1.3. *Solution Method*

Here we present a method for solving equation (1.11) in the context of an LES calculation. It is important to note that equation (1.11) is *exactly* equivalent to the original equation (1.1). No approximation such as truncation to a finite set of moments in the Moments Method has been invoked until this point. Unlike the Moments Method, here the only approximation arises out of the necessity to solve (1.11) with finite resources. This can be done in a variety of ways, with the usual trade offs between cost and efficiency. We present one such method here modeled on the idea of expanding in a set of suitable basis functions.

Let us write the distribution function as

$$n_p = \sum_{n=1}^{\infty} A_n \frac{r^n}{n!} \exp(-ar) \tag{1.12}$$

where the $A_1, \cdots$ are constants with respect to $r$ but generally functions of position and time. Clearly any well behaved distribution function that vanishes at $r = 0$ and decays rapidly as $r \to \infty$ may be written in this way. The only approximation that we will introduce is to truncate the series (1.12) at some finite number of terms, $n = N$, that is, we assume that

$$A_n = 0 \qquad \text{if } n > N \tag{1.13}$$

The positive number "$a$" is generally arbitrary but we will choose it in a reasonable manner e.g. $a = 1/r_0$ where $r_0$ is some characteristic scale such as the mean particle size at the initial time, $t = 0$. Since the LT of $r^n \exp(-ar)/n!$ is $1/(s+a)^{n+1}$, on transforming (1.12) we get

$$\hat{n}_p = \sum_{n=1}^{\infty} \frac{A_n}{(s+a)^{n+1}} \tag{1.14}$$

On substitution in (1.11) we get

$$\sum_{n=1}^{\infty} \frac{\mathcal{L}[A_n]}{(s+a)^{n+1}} = -\phi A_1 + \phi \sum_{n=1}^{\infty} \frac{sA_n}{n(s+a)^n} \tag{1.15}$$

where for brevity we have introduced the differential operator $\mathcal{L}$ defined by

$$\mathcal{L}[\psi] = \partial_t \psi + \mathbf{u} \cdot \nabla \psi - \nabla \cdot (D \nabla \psi) \tag{1.16}$$

The first term on the right of the equality in (1.15) follows by directly differentiating (1.12) with respect to $r$ and then setting $r = 0$. The summation on the right of (1.15) may be rewritten as follows by re-writing the $s$ in the numerator as $s + a - a$:

$$\begin{aligned}
\sum_{n=1}^{\infty} \frac{sA_n}{n(s+a)^n} &= \sum_{n=1}^{\infty} \frac{A_n}{n(s+a)^{n-1}} - a \sum_{n=1}^{\infty} \frac{A_n}{n(s+a)^n} \\
&= A_1 + \frac{A_2}{2(s+a)} + \sum_{n=3}^{\infty} \frac{A_n}{n(s+a)^{n-1}} - \frac{aA_1}{(s+a)} - \sum_{n=2}^{\infty} \frac{aA_n}{n(s+a)^n} \\
&= A_1 + \frac{A_2}{2(s+a)} - \frac{aA_1}{(s+a)} + \sum_{n=1}^{\infty} \left[ \frac{A_{n+2}}{(n+2)} - \frac{aA_{n+1}}{(n+1)} \right] \frac{1}{(s+a)^{n+1}}
\end{aligned} \tag{1.17}$$

On substitution in (1.15) we get

$$\frac{\phi}{(s+a)} \left[ \frac{A_2}{2} - aA_1 \right] + \sum_{n=1}^{\infty} \frac{\left\{ \phi \frac{A_{n+2}}{(n+2)} - \phi \frac{aA_{n+1}}{(n+1)} - \mathcal{L}[A_n] \right\}}{(s+a)^{n+1}} = 0 \tag{1.18}$$

In order that this equation be satisfied for all $s$ we must have,

$$A_{n+2} = \frac{n+2}{n+1} aA_{n+1} + \frac{n+2}{\phi} \mathcal{L}[A_n] \tag{1.19}$$

where $n = 0, 1, 2, \ldots$ and $A_0 \equiv 0$. We now present the theory for several different truncation orders $N$, that is, we retain exactly $N$ coefficients $A_1, A_2, \ldots, A_N$ and set the rest to zero, further, we ignore all equations in (1.19) except for the first $N$.

**CASE** 1 (Two terms)

If we retain the first two terms in the above series of equations and the first two expansion coefficients ($A_1$ and $A_2$), we get

$$0 = A_3 = \frac{3}{2} a A_2 + \frac{3}{\phi} \mathcal{L}[A_1] \tag{1.20}$$

$$A_2 = 2aA_1. \tag{1.21}$$

On eliminating $A_2$ from the first equation this set becomes:

$$\mathcal{L}[A_1] = -a^2 \phi A_1 \tag{1.22}$$

$$A_2 = 2aA_1 \tag{1.23}$$

Since the particle density (irrespective of size) may be written as

$$N_p = \int_0^\infty n_p \, dr = \hat{n}_p|_{s=0} = \frac{3}{a^2} A_1 \tag{1.24}$$

the solution in this approximation may also be expressed as

$$n_p = \frac{a^2 N_p}{3} (r + ar^2) \exp(-ar) \tag{1.25}$$

where $N_p$ satisfies

$$\partial_t N_p + \mathbf{u} \cdot \nabla N_p - \nabla \cdot (D \nabla N_p) = -a^2 \phi N_p \tag{1.26}$$

Thus, at this level of approximation the shape of the size distribution does not change just the number of particles get reduced due to evaporative losses while being advected by the filtered field and undergoing turbulent diffusion induced by the fluctuating velocity.

**CASE** 2 (Three terms)

If we retain the first three terms in the above series of equations and keep the first three expansion coefficients $A_1$, $A_2$ and $A_3$, we get

$$0 = A_4 = \frac{4}{3} a A_3 + \frac{4}{\phi} \mathcal{L}[A_2] \tag{1.27}$$

$$A_3 = \frac{3}{2} a A_2 + \frac{3}{\phi} \mathcal{L}[A_1] \tag{1.28}$$

$$A_2 = 2aA_1 \tag{1.29}$$

which yields after some re-arrangement

$$\mathcal{L}[A_1] + \frac{a^2 \phi}{3} A_1 = 0 \tag{1.30}$$

$$A_2 = 2aA_1 \tag{1.31}$$

$$A_3 = 2a^2 A_1 \tag{1.32}$$

Once again we get a single scalar transport equation to solve. The distribution function is

$$n_p = A_1 \left( r + ar^2 + \frac{a^2}{3} r^3 \right) \exp(-ar) \tag{1.33}$$

where $A_1$ is related to $N_p$ by

$$N_p = \frac{5}{a^2} A_1 \tag{1.34}$$

Once again, at this level of description, the shape of the distribution function does not change.

**CASE** 3 (Four Terms)

In this case we have

$$0 = A_5 = \frac{5}{4}aA_4 + \frac{5}{\phi}\mathcal{L}[A_3] \tag{1.35}$$

$$A_4 = \frac{4}{3}aA_3 + \frac{4}{\phi}\mathcal{L}[A_2] \tag{1.36}$$

$$A_3 = \frac{3}{2}aA_2 + \frac{3}{\phi}\mathcal{L}[A_1] \tag{1.37}$$

$$A_2 = 2aA_1. \tag{1.38}$$

After some algebra, these equations may be re-written as

$$A_2 = 2aA_1 \tag{1.39}$$

$$A_4 = 4aA_3 - 8a^3A_1 \tag{1.40}$$

with the following pair of coupled equations from $A_1$ and $A_3$:

$$\mathcal{L}[A_1] + a^2\phi A_1 = \frac{\phi}{3}A_3 \tag{1.41}$$

$$\mathcal{L}[A_3] + a^2\phi A_3 = 2\phi a^4 A_1 \tag{1.42}$$

It is useful to write the particle number density $N_p$ and the mean particle radius $R_p$ in terms of the $A_n$. This is conveniently done using the result

$$-xf(x) \longrightarrow \hat{f}'(s) \tag{1.43}$$

where the arrow indicates the L.T. Thus,

$$-\int_0^\infty rn_p \exp(-sr)\, dr = \frac{\partial \hat{n}_p}{\partial s} \tag{1.44}$$

so that

$$N_p = \hat{n}_p\big|_{s=0} \tag{1.45}$$

$$R_p = -\left.\frac{1}{\hat{n}_p}\frac{\partial \hat{n}_p}{\partial s}\right|_{s=0} \tag{1.46}$$

Therefore,

$$N_p = \frac{A_1}{a^2} + \frac{A_2}{a^3} + \frac{A_3}{a^4} + \frac{A_4}{a^5} \tag{1.47}$$

$$= \frac{5}{a^4}\left(A_3 - a^2A_1\right) \tag{1.48}$$

$$N_pR_p = -\left.\frac{\partial \hat{n}_p}{\partial s}\right|_{s=0} = 2\frac{A_1}{a^3} + 3\frac{A_2}{a^4} + 4\frac{A_3}{a^5} + 5\frac{A_4}{a^6} \tag{1.49}$$

$$= \frac{4}{a^5}\left(6A_3 - 8a^2A_1\right). \tag{1.50}$$

Here, the distribution function is determined by the two parameters $A_1$ and $A_3$ the evolution of which is determined by (1.41) and (1.42):

$$n_p = \left[A_1\left\{r + ar^2 - \frac{a^3}{3}r^4\right\} + A_3\frac{r^3}{6}(1+ar)\right]\exp(-ar) \quad (1.51)$$

Initial values of $A_1$ and $A_3$ may be chosen based on the values of $R_p$ and $N_p$ or may be chosen based on some other consideration, such as matching the peak of the distribution to the observed peak at $t = 0$. For convenience (1.48) and (1.50) may be inverted to express $A_1$ and $A_3$ directly in terms of the physically measurable quantities $N_p$ and $R_p$:

$$A_1 = \frac{3a^2}{5}N_p - \frac{a^3}{8}N_pR_p \quad (1.52)$$

$$A_3 = \frac{4a^4}{5}N_p - \frac{a^5}{8}N_pR_p \quad (1.53)$$

In order to ensure that the distribution function has no negative regions, $A_1$ and $A_3$ should be chosen at $t = 0$ so that they are both non-negative as well as such that $A_3 > 2a^2A_1$ at all points in space (which guarantees the positivity of $A_4$). Clearly, from (1.48) and (1.50) this also ensures the positivity of $N_p$ and $R_p$. If $A_1$ and $A_3$ are determined from (1.52) and (1.53) using known $N_p$ and $R_p$ values, then,

$$A_3 - 2a^2A_1 = a^4N_p\left(\frac{aR_p}{8} - \frac{2}{5}\right). \quad (1.54)$$

Therefore, in order to guarantee the positivity of the distribution function, $a$ must be chosen such that

$$a > \frac{16}{5}\frac{1}{R_p} = \frac{3.2}{R_p} \quad (1.55)$$

at the initial time (for example, the choice $a = 4/R_p$ may be reasonable).

How can one be sure that if initial conditions are chosen properly to ensure a nonnegative distribution function, it will not evolve into one that no longer satisfies this property? We offer the following argument by way of proof: if we denote $\mathcal{A} = A_3 - 2a^2A_1$, then, using (1.41) and (1.42) one can derive the following equation for $\mathcal{A}$:

$$\mathcal{L}[\mathcal{A}] + \frac{5}{3}a^2\phi\mathcal{A} = \frac{2}{3}a^4\phi A_1. \quad (1.56)$$

Therefore, as long as $A_1$ remains positive, $\mathcal{A}$ will not become negative.

**CASE** 4 (Six Terms)

In this case we have

$$0 = \frac{7}{6}aA_6 + \frac{7}{\phi}\mathcal{L}[A_5] \tag{1.57}$$

$$A_6 = \frac{6}{5}aA_5 + \frac{6}{\phi}\mathcal{L}[A_4] \tag{1.58}$$

$$A_5 = \frac{5}{4}aA_4 + \frac{5}{\phi}\mathcal{L}[A_3] \tag{1.59}$$

$$A_4 = \frac{4}{3}aA_3 + \frac{4}{\phi}\mathcal{L}[A_2] \tag{1.60}$$

$$A_3 = \frac{3}{2}aA_2 + \frac{3}{\phi}\mathcal{L}[A_1] \tag{1.61}$$

$$A_2 = 2aA_1. \tag{1.62}$$

After some algebra, these equations may be re-written as

$$A_2 = 2aA_1 \tag{1.63}$$

$$A_4 = 4aA_3 - 8a^3A_1 \tag{1.64}$$

$$A_6 = 96a^5A_1 - 40a^3A_3 + 6aA_5. \tag{1.65}$$

with the following pair of coupled equations from $A_1$, $A_3$ and $A_5$:

$$\mathcal{L}[A_1] + a^2\phi A_1 = \frac{\phi}{3}A_3 \tag{1.66}$$

$$\mathcal{L}[A_3] + a^2\phi A_3 = 2\phi a^4 A_1 + \frac{\phi}{5}A_5 \tag{1.67}$$

$$\mathcal{L}[A_5] + a^2\phi A_5 = \frac{20}{3}\phi a^4 A_3 - 16\phi a^6 A_1 \tag{1.68}$$

These three coefficients can be related to the three physical parameters $N_p$, $R_p$ and $\langle r^2\rangle$ by using:

$$N_p = \hat{n}_p|_{s=0} = \frac{A_1}{a^2} + \frac{A_2}{a^3} + \frac{A_3}{a^4} + \frac{A_4}{a^5} + \frac{A_5}{a^6} + \frac{A_6}{a^8} \tag{1.69}$$

$$-N_pR_p = \left.\frac{\partial \hat{n}_p}{\partial s}\right|_{s=0} = 2\frac{A_1}{a^3} + 3\frac{A_2}{a^4} + 4\frac{A_3}{a^5} + 5\frac{A_4}{a^6} + 6\frac{A_5}{a^7} + 7\frac{A_6}{a^8} \tag{1.70}$$

$$\begin{aligned} N_p\langle r^2\rangle &= \left.\frac{\partial^2 \hat{n}_p}{\partial s^2}\right|_{s=0} \\ &= 2.3\frac{A_1}{a^4} + 3.4\frac{A_2}{a^5} + 4.5\frac{A_3}{a^6} + 5.6\frac{A_4}{a^7} + 6.7\frac{A_5}{a^8} + 7.8\frac{A_6}{a^9} \end{aligned} \tag{1.71}$$

The terms $A_2$, $A_4$ and $A_6$ can be eliminated using (1.65), thus,

$$\frac{1}{7}N_p = \frac{13A_1}{a^2} - \frac{5A_3}{a^4} + \frac{A_5}{a^6} \tag{1.72}$$

$$-\frac{a}{16}N_pR_p = 40\frac{A_1}{a^2} - 16\frac{A_3}{a^4} + 3\frac{A_5}{a^7} \tag{1.73}$$

$$\frac{a^2}{42}\langle r^2\rangle N_p = 123\frac{A_1}{a^3} - 50\frac{A_3}{a^4} + \frac{9}{a^6}A_5 \tag{1.74}$$

where $\langle r^2 \rangle$ is the mean square radius.

### 1.4. *Source terms for vapor equation*

If the evaporating fluid drops are converted entirely into vapor, we have the following transport equation for the vapor mass fraction, $Y$:

$$\rho \left( \frac{\partial Y}{\partial t} + \mathbf{u} \cdot \nabla Y \right) - \nabla \cdot [D_v \nabla(\rho Y)] = g \tag{1.75}$$

where $g$ is the mass of liquid evaporated per unit volume per unit time at location $\mathbf{x}$ at time $t$ and $\rho$ is the gas density and $D_v$ the diffusivity. Droplets in the size range $(r, r+dr)$ contribute the amount $-4\pi\rho_L r^2 \dot{r} n_p dr$ to $g$, where $\rho_L$ is the liquid density. On using (1.2) with $G = 1/r$ and integrating in $r$, we have

$$g = -\int_0^\infty 4\pi\rho_L r^2 \dot{r} n_p \, dr = 4\pi\rho_L \phi N_p R_p \tag{1.76}$$

## 2. Numerical experiments

Some preliminary "Numerical Experiments" were conducted to assess the accuracy of the model. The objective of these tests was to determine how many of the modes $A_1, A_2, A_3, \cdots$ were needed to reproduce with reasonable accuracy the behavior of the full system which was followed using a DNS simulation. The full system in this case was a set of $N = 10^6$ droplets distributed at random within a cubical box that was supposed to contain a gas at a constant temperature. The physical parameters were the same as that of Apte & Ghosal (2004). In the particular set of data being reported here, there was no fluid flow. The droplets simply evaporated "in place" starting from an initial distribution $n_p \sim r \exp(-ar)$. The constant $a$ was chosen as $a = 3.2/R_p$ where $R_p$ was the mean radius. The time evolution of the number of droplets $N_p$ and that of the mean drop radius $R_p$ is shown in Figure 1. It is seen that the models with up to four coefficients $A_1 \cdots A_4$ (Case 3 which contains four coefficients though only two evolution equations) appears to be the minimal system needed for a reasonable prediction of $R_p$ and $N_p$. Figure 2 shows the same data in terms of the PDF.

## 3. Conclusions

The general dynamic equation describing evaporating droplets [equation (1.1)] is recast into an alternate form in terms of the Laplace Transform of the distribution function. In case of several different micro-scale evaporation laws $G(r)$, the evolution equation takes a simple form in the Laplace Transform Space [equation 1.6]. In particular if $G(r) \sim 1/r$, equation (1.11) results. The equation (1.11) is then solved by expanding in a basis of modes and truncating the infinite series of modal equations that result. Each modal equation is an advection-diffusion-reaction type of PDE in three dimensional space. Therefore, the practicality of the method hinges on whether or not a truncated system with just a handful of modes can give reasonably accurate results for practical systems. We have not answered this question, for only repeated applications to diverse systems can begin to provide a picture of the efficacy of this method compared to more standard approaches. However, the fact that the basis involves the Gamma distribution which is in some sense "natural" to droplet size distributions, provides reason to hope that the evolving distribution is representable by just a few modes. The numerical experiment conducted here,

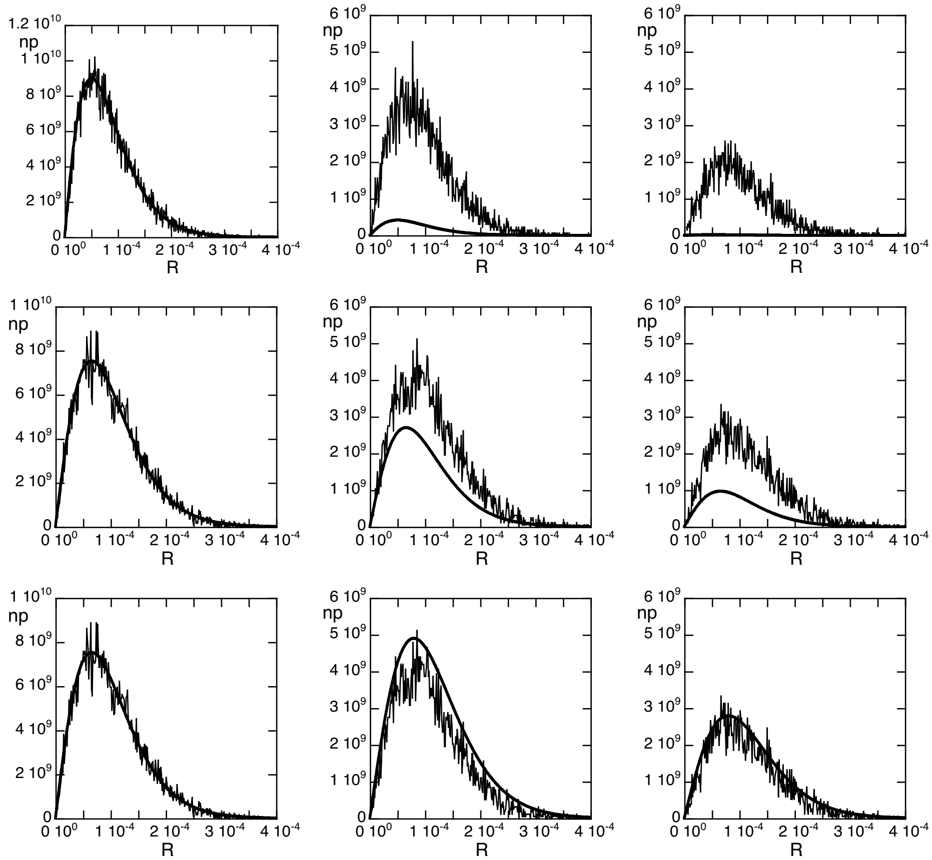

FIGURE 2. The PDFs at three successive times ($t = 0, 0.025, 0.050$ sec) starting with the initial condition. The solid line is the model referred to in the text as Case 1 (top panel), Case 2 (middle panel) and Case 3 (bottom panel).

though lacking most of the complexity of real turbulent flows, points to a result consistent with this expectation. First, we observe that the quality of the prediction improves with the number of modes. Secondly, a system involving four modes (two transport equations) appears to be the minimal set needed for reasonably accurate results. The approach outlined here has the advantage over traditional "presumed pdf" type methods (Lognormal, Gamma, ....) in that no specific form for the PDF need be assumed. Indeed, the modal expansion can be written with multiple scales to represent multimodal distributions. On the other hand, unlike moment methods, no ad hoc moment truncation is invoked. In fact, the basic equation (1.11) is exact, though approximations (such as a finite basis) needs to be invoked for its solution. This provides a "cleaner" theoretical foundation, though we do not know yet if it leads down the road to more efficient numerical schemes. Last, but not least, the approach goes over almost unchanged to the situation where the velocity refers to a filtered velocity of an LES field. In view of these differences with the usual method of spray modeling, it appears that the approach outlined here is worthy of further investigation.

## REFERENCES

Apte, S. & Ghosal, S. 2004 A presumed pdf approach to modeling evaporating or condensing droplets in complex flows. CTR *Annual Research Briefs*, 209–221.