Quasi-Ballistic Thermal Transport Across MoS$_2$ Thin Films

Aditya Sood, Feng Xiong, Shunda Chen, Ramez Cheaito, Feifei Lian, Mehdi Asheghi, Yi Cui, Davide Donadio, Kenneth E. Goodson, and Eric Pop

ABSTRACT: Layered two-dimensional (2D) materials have highly anisotropic thermal properties between the in-plane and cross-plane directions. Conventionally, it is thought that cross-plane thermal conductivities ($\kappa_\parallel$) are low, and therefore c-axis phonon mean free paths (MFPs) are small. Here, we measure $\kappa_\parallel$ across MoS$_2$ films of varying thickness (20–240 nm) and uncover evidence of very long c-axis phonon MFPs at room temperature in these layered semiconductors. Experimental data obtained using time-domain thermoreflectance (TDTR) are in good agreement with first-principles density functional theory (DFT). These calculations suggest that ~50% of the heat is carried by phonons with MFP > 200 nm, exceeding kinetic theory estimates by nearly 2 orders of magnitude. Because of quasi-ballistic effects, the $\kappa_\parallel$ of nanometer-thin films of MoS$_2$ scales with their thickness and the volumetric thermal resistance asymptotes to a nonzero value, ~10 m$^2$ K GW$^{-1}$. This contributes as much as 30% to the total thermal resistance of a 20 nm thick film, the rest being limited by thermal interface resistance with the SiO$_2$ substrate and top-side aluminum transducer. These findings are essential for understanding heat flow across nanometer-thin films of MoS$_2$ for optoelectronic and thermoelectric applications.

KEYWORDS: Phonon, mean free path, MoS$_2$, cross-plane, thermal conductivity, time-domain thermoreflectance

Two-dimensional (2D) van der Waals (vdW) layered solids have highly unusual thermal transport properties due to their unique crystal structure. While atoms within a layer are bonded covalently, adjacent layers are coupled via weak vdW interactions. This leads to a strong anisotropy in thermal conductivity, with the in-plane (along the layers) conductivity $\kappa_\parallel$ being significantly larger than the cross-plane (across the layers, or along the c-axis) conductivity $\kappa_\perp$. For example, in bulk graphite, h-BN, and MoS$_2$, anisotropy ratios ($\kappa_\parallel/\kappa_\perp$) as high as ~300, 200, and 50, respectively, have been reported at room temperature. Owing to their high $\kappa_\parallel$, in-plane thermal transport in vdW layered materials has received significant attention, motivated in part by potential applications in heat spreading. In contrast, fundamental aspects of cross-plane thermal transport remain relatively underexplored, despite its relevance to nanoelectronics and energy harvesting applications. For example, self-heating plays a key role in limiting the performance of field effect transistors (FETs) made of 2D materials. While some studies have characterized heat flow at single vdW interfaces, very little is known about the physics of “intrinsic” cross-plane thermal transport across multiple vdW layers in layered thin films. Achieving a better understanding of this is critical to realizing the potential of 2D electronics, as previous work on multilayer MoS$_2$ transistors has shown enhancements in device mobility with increasing channel thickness (up to approximately tens of nanometers). In such devices, charge screening and large interlayer electrical resistance can lead to the localization of current within the top few layers, such that the dissipated heat must flow across multiple vdW interfaces before entering the substrate. It is therefore essential to understand the thickness dependence and fundamental limits of cross-plane thermal transport.
transport in vdW layered solids, particularly in materials like MoS2.

A key quantity that determines thermal transport in the cross-plane direction of a material is the range of phonon mean free paths (MFPs) that carry heat. A simple estimate of the so-called gray MFP \( \langle \lambda \rangle \) can be made using the kinetic theory, \( \kappa_z = (1/3)C_v \Lambda_z \) for MoS2, using a heat capacity \( C_v \sim 2 \text{ MJ} \text{ m}^{-3} \text{ K}^{-1} \), the average sound velocity of cross-plane acoustic modes \( v_z \sim 2400 \text{ ms}^{-1} \), and the cross-plane bulk conductivity \( \kappa_z \sim 2-5 \text{ W m}^{-1} \text{ K}^{-1} \), gives a MFP of around 1.5–4 nm, which corresponds to a thickness of 2–6 layers. A similar calculation for graphite gives a gray MFP estimate of around 3 nm, corresponding to 9 layers. This would imply that size effects (i.e., thickness dependence of \( \kappa_z \)) should be negligible for films thicker than \( \sim 10 \text{ nm} \), i.e. that the cross-plane thermal conductivity should be constant in this thickness regime. However, recent molecular dynamics (MD) simulations \(^{16,17}\) and experimental measurements of \( \kappa_z \) in graphite \(^{16,20}\) have suggested surprisingly long \( c \)-axis MFPs, on the order of approximately hundreds of nanometers.

These studies motivate the following key questions: (1) Are long \( c \)-axis phonon MFPs a general feature of other vdW-layered systems, like the transition metal dichalcogenides (TMDs) such as MoS2? (2) Can experimental observations of long cross-plane phonon MFPs in vdW materials be explained by first-principles calculations? Density functional theory (DFT) has recently proven to be very effective in understanding fundamental aspects of thermal transport in covalently bonded systems like Si,\(^{21}\) but similar studies are lacking for vdW-layered solids, especially quantitative comparisons with cross-plane thermal measurements. (3) What is the impact of cross-plane ballistic transport and related size effects on the thermal resistance of thin-film TMD devices? For monolayers, it is understood that interfaces dominate cross-plane transport.\(^{8-11}\) However, the transition from interface-dominated to bulk-like transport across multilayer TMDs remains unclear to date.

In response, here we probe the spectrum of heat-carrying \( c \)-axis phonon MFPs in MoS2, a vdW layered semiconductor. Through time-domain thermoreflectance (TDTR) \(^{22}\) measurements of the thickness-dependent cross-plane thermal conductivity in single-crystalline films, we show that the \( c \)-axis phonon MFPs are at least \( \sim 10 \)s of nm long, significantly larger than kinetic theory estimates. Using first-principles DFT calculations we uncover that nearly 80% of the heat at room temperature is carried by phonons with MFPs in the range 10 to 500 nm. Furthermore, we show that by suitably defining a characteristic thermal length scale, our thickness-dependent \( \kappa_z \) data (with film thickness \( t \) ranging from 20 to 240 nm) are consistent with TDTR data on bulk MoS2 crystals reported previously\(^{15,17,23}\) (with thermal penetration depth \( \delta_f \) ranging from 200 nm to 1 \( \mu \)m). Taken together, we find good agreement between the combined data set and DFT predictions over a broad spectrum of thermal length scales, from 20 nm to 1 \( \mu \)m. Finally, using our measured values of the metal/MoS2 and MoS2/substrate interface resistances, we estimate the impact of cross-plane quasi-ballistic phonon transport on the total thermal resistance of multilayer MoS2 devices. These calculations reveal that contrary to what is typically assumed, the total thermal resistance of few nanometer thick films is not entirely interface-dominated; the lower limit is set by the ballistic resistance across the thickness of MoS2, which is estimated to be \( \sim 10 \text{ m}^2 \text{ K GW}^{-1} \).

**Experimental Procedure.** Single crystalline MoS2 films were exfoliated onto SiO2 (90 nm) on p-doped Si substrates using micromechanical exfoliation. Exfoliation yielded several MoS2 films of different thicknesses on a single \( \sim 1 \text{ cm}^2 \) chip. Suitable films were identified using optical microscopy, and their thicknesses measured using atomic force microscopy (AFM). An \( \sim 80 \text{ nm} \) thick Al transducer was patterned and deposited onto the samples using electron-beam (e-beam) lithography and e-beam evaporation, respectively, for TDTR measurements (see sample schematic in Figure 1a and Methods). We also patterned Al onto bare regions of the SiO2/Si substrate adjacent to the MoS2 during the same

---

**Figure 1.** (a) Cross-sectional schematic of samples under study, showing the three unknown parameters: \( G_1 \) (TBC between Al and MoS2), \( G_2 \) (TBC between MoS2 and SiO2), and \( \kappa_z \) (cross-plane thermal conductivity of MoS2). The pump (green, 532 nm) and probe (red, 1064 nm) lasers used for TDTR are shown schematically. (b,c) Top view optical micrographs and probe reflectivity maps (taken at a fixed delay time of 0 ps) for 30 and 175 nm thick MoS2 samples, respectively. Probe reflectivity maps are used to locate a uniform region away from the sample edges, where TDTR measurements are taken (white circles). (d,e) TDTR data (symbols) and best fits (lines): normalized in-phase signal \( V_{in} \) at \( f_{mod} = 10 \text{ MHz} \) and \( \sim V_{in}/V_{out} \) ratio at \( f_{mod} = 4 \text{ MHz} \) for 30 and 175 nm thick films, respectively.
measurements on a patterned 4-probe device and the Al was estimated using in-plane electrical conductivity performed at room temperature. The thermal conductivity of the p-type Si substrate, and volumetric specific heat of Al, SiO₂, MoS₂ and Si were taken from literature. The dependence was observed in the thermal conductivity of SiO₂ next to each set of samples, and prior TDTR measurements of bulk MoS₂ by Liu et al. (red triangle), Muratore et al. (blue diamond), and Jiang et al. (magenta squares). Solid and dashed lines are predictions of first-principles DFT calculations with suppression functions based on the BTE (eq 2) and Matthiessen’s rule (eq 3), respectively.

Figure 2. (a) Measured intrinsic cross-plane thermal conductivity κₓ versus film thickness t. (b) Experimental data plotted as a function of the characteristic thermal length scale, which is the smaller of the thickness (t) and thermal penetration depth (dₜ). For our measurements, dₜ ≈ 160 nm in the three thickest films at fₘod = 4 MHz. Also shown are prior TDTR measurements of bulk MoS₂ by Liu et al. (red triangle), Muratore et al. (blue diamond), and Jiang et al. (magenta squares). Solid and dashed lines are predictions of first-principles DFT calculations with suppression functions based on the BTE (eq 2) and Matthiessen’s rule (eq 3), respectively.

The sample stack consists of Al/MoS₂/SiO₂/Si (see Figure 1a). The thicknesses of Al and MoS₂ were measured using ellipsometry to be 90 ± 1 nm. All measurements were performed at room temperature. The thermal conductivity of Al was estimated using in-plane electrical conductivity measurements on a patterned 4-probe device and the Wiedemann–Franz law, κₓ ≈ 170 W m⁻¹ K⁻¹. The thermal conductivity of the p-type Si substrate, and volumetric specific heat of Al, SiO₂, MoS₂ and Si were taken from literature. To reduce the uncertainties associated with slight variations in the laser spot size between measurements on different samples, reference data were taken on the Al/SiO₂/Si regions next to each flake. Adjustments were made in the spot size (<5% variation across samples) to keep the fitted SiO₂ thermal conductivity fixed at 1.4 W m⁻¹ K⁻¹. No fₘod dependence was observed in the thermal conductivity of SiO₂ and thermal boundary conductance (TBC) of the Al/SiO₂ interface (≈ 130 MW m⁻² K⁻¹) for modulation frequencies between 4 and 10 MHz.

In the MoS₂ sample stack, there are four unknown parameters for each sample thickness t. They are the intrinsic cross-plane and in-plane thermal conductivities of the MoS₂ layer, κₓ, κᵧ, and the TBCs at the Al/MoS₂ and MoS₂/SiO₂ interfaces, G₁ and G₂, respectively. The in-plane thermal conductivity is held fixed at κᵣ = 90 W m⁻¹ K⁻¹, based on prior measurements of bulk MoS₂ crystals by Liu et al. Here, the authors had measured a spot-size dependent κᵣ likely due to the partial exclusion of ballistic phonons with in-plane MFPs larger than the spot diameter. Our estimate for κᵣ is obtained by linearly interpolating their data to an rms spot diameter of 3 μm. To simplify our analysis we assume that κᵣ is independent of t at least within the range of thicknesses (20 nm < t < 240 nm) measured here. This is consistent with previous arguments by Minnich and Gu et al., an assumption further discussed below.

This assumption leaves three unknown parameters for each sample: κₓ, G₁, and G₂. To extract a unique value for κₓ we use a combination of V in and ratio (= -V in/V out) signals at two different modulation frequencies, 4 and 10 MHz. This tandem fitting approach is similar to that used by Meyer et al. and is supported by our sensitivity analysis (see Supporting Information Section 1). For films with t < 150 nm, we first estimate G₁ by fixing κₓ and G₂ and fitting the in-phase signal V in (normalized at +100 ps) at the higher fₘod of 10 MHz. Next, fixing G₁ at this value, the voltage ratio data at the lower fₘod of 4 MHz are fit for κₓ and G₂. This process is repeated until the values of κₓ, G₁, and G₂ each change by less than 1% between successive iterations. We verify that the final fit results are not sensitive to the choice of initial values. For films with t > 150 nm, measurement sensitivity to the bottom interface TBC, G₂, is relatively low. For these, we follow the same procedure as above, except that G₂ is held fixed at 21 ± 5 MW m⁻² K⁻¹ based on the thin film results, further discussed below. Our methodology is generally similar to that used by Zhang et al. and Jang et al. for thickness-dependent κᵣ measurements of graphite and black phosphorus, respectively. Error bars are calculated by propagating uncertainties in the assumed thermophysical parameters, mainly the Al thickness (±1 nm) and rms laser spot size (±2%), and for the thick films also G₂ (±5 MW m⁻² K⁻¹).

We note that a recent experimental study reported thickness-dependent in-plane thermal conductivity of MoS₂.
films in the range 2.4 to 37.8 nm. To check whether this thickness-dependence might affect our extraction of \( \kappa \), we also analyzed our data using \( \kappa \) estimated from these results. For the 20 and 34 nm thick films, the resulting change in \( \kappa \) is only \( \sim \) 2\% and \( \sim \) 12\%, respectively. These uncertainties are within the experimental error bars; this further confirms that our assumption of constant \( \kappa \) for all films does not affect the extracted trend of \( \kappa \) versus \( t \).

**Results and Discussion.** Representative TDTR data and model best fits for 30 and 175 nm thick samples are shown in Figure 1d,e, respectively. Figure S2 shows the extracted top and bottom interface TBCs, \( G_1 \) and \( G_2 \) versus \( t \). The MoS\(_2\)/SiO\(_2\) TBCs fall within a narrow range of 16 to 26 MW m\(^{-2}\) K\(^{-1}\); in reasonable agreement with Raman thermometry measurements of monolayer MoS\(_2\) on SiO\(_2\) (14 \pm 4 MW m\(^{-2}\) K\(^{-1}\)) by Yalon et al.\(^{3,4}\) The Al/MoS\(_2\) TBCs are in general higher than MoS\(_2\)/SiO\(_2\) TBCs and also show a larger spread from 30 to 80 MW m\(^{-2}\) K\(^{-1}\) with no systematic trend as a function of \( t \). This larger variability in \( G_1 \) could be a result of varying degrees of surface cleanliness after the e-beam patterning process that is used to define the Al transducer.

Figure 2a plots the extracted cross-plane thermal conductivity \( \kappa \) as a function of layer thickness \( t \); \( \kappa \) for the thickest film \( (t = 240 \text{ nm}) \) is 2.0 \( \pm \) 0.3 W m\(^{-1}\) K\(^{-1}\). This decreases with decreasing film thickness down to 0.9 \( \pm \) 0.2 W m\(^{-1}\) K\(^{-1}\) for \( t = 20 \text{ nm} \), more than a two-fold reduction. Such a dependence of \( \kappa \) on \( t \) is indicative of quasi-ballistic c-axis phonon transport and suggests that the dominant heat-carrying vibrational modes have MFPs of at least tens of nanometers. We note that \( \kappa \) appears to saturate for the three thickest films. As discussed further below, we posit that this occurs due to the finite thermal penetration depth of the TDTR measurement.

Our measured \( \kappa \) values for the thickest films are close to two prior measurements of bulk MoS\(_2\) by Liu et al.\(^{3} \) and Muratore et al.\(^{23} \) who obtained \( \kappa \) of \( \sim \) 2 W m\(^{-1}\) K\(^{-1}\), and \( \sim \) 2.5 W m\(^{-1}\) K\(^{-1}\), respectively, using a TDTR modulation frequency of 9.8 MHz.\(^{23} \) However, these two results are significantly lower than recent measurements by Jiang et al.\(^{17} \) who obtained a bulk \( \kappa \) \( \sim \) 4.8 W m\(^{-1}\) K\(^{-1}\). In addition, our first-principles DFT calculations (described later) obtain a bulk \( \kappa \) \( \sim \) 5 W m\(^{-1}\) K\(^{-1}\), which is in good agreement with the experimental result of Jiang et al.\(^{17} \) and a recent DFT calculation by Lindroth et al.\(^{16} \) that predicted \( \kappa \) \( \sim \) 5.1 W m\(^{-1}\) K\(^{-1}\).

To understand possible reasons behind the apparent discrepancy among the different bulk \( \kappa \) measurements and first-principles calculations, we consider the characteristic thermal length scale (i.e., length scale over which the temperature gradient occurs) in the experiments. For TDTR measurements made at a frequency \( f_{\text{mod}} \), this is determined by the thermal penetration depth \( d_p \), which is approximately \( \left( \kappa / \pi C_f \right)^{1/2} \). To calculate \( d_p \) accurately, we solve the full 3D heat diffusion equation in the two-layer Al/MoS\(_2\) stack (see Supporting Information Section 3).

For the case of Liu et al.\(^3 \) and Muratore et al.\(^{23} \) this gives \( d_p \sim 180 \text{ nm} \) (for \( \kappa \sim 2 \text{ W m}^{-1} \text{ K}^{-1} \)), and \( d_p \sim 200 \text{ nm} \) (for \( \kappa \sim 2.5 \text{ W m}^{-1} \text{ K}^{-1} \)), respectively, at \( f_{\text{mod}} = 9.8 \text{ MHz} \). Jiang et al.\(^{17} \) performed TDTR measurements of \( f_{\text{mod}} \)-dependent \( \kappa \) in bulk MoS\(_2\) and observed a reduction in the apparent \( \kappa \) from 4.5 to 3.3 W m\(^{-1}\) K\(^{-1}\) while increasing \( f_{\text{mod}} \) from 1 to 10 MHz. These results were interpreted based on a two-channel model that considers nonequilibrium effects between low and high-frequency phonons that have different thermal conductivities and heat capacities. The interpretation of \( f_{\text{mod}} \)-dependent \( \kappa \) in modulated opto-thermal measurements has been the topic of
much recent discussion.\textsuperscript{17,25,35--40} While the treatment of near-interfacial phonon nonequilibrium deserves further attention, a first order approximation is that the contributions to heat transport of long MFP phonons with $\Lambda_\zeta > d_p$ are suppressed at high $f_{mod}$ thereby lowering the measured apparent $\kappa_z$. This simplification is reasonable for low thermal conductivity solids with relatively broad MFP spectra, as was discussed recently in the context of black phosphorus by Sun et al.,\textsuperscript{41} and applied quantitatively to low thermal conductivity semiconductor alloys by Koh et al.\textsuperscript{35}

In this simplified picture, we replot our thin-film $\kappa_z$ data, along with the bulk data of Liu et al.,\textsuperscript{3} Muratore et al.\textsuperscript{23} and Jiang et al.,\textsuperscript{1} against the thermal characteristic length scale (the smaller of $t$ and $d_p$) as shown in Figure 2b. For our thin-film samples, $d_p$ is calculated using a numerical solution of the 3D heat diffusion equation in the four-layer stack ($\text{Al/MoS}_2$/SiO$_2$/Si). For most of our films, $d_p$ is larger than $t$. However, for the three thickest films $d_p \approx 160$ nm, which is smaller than $t$ ($\approx 175$, $200$, $240$ nm). Details of these calculations are provided in the Supporting Information Section 3. Following this procedure, a combined data set is obtained, where $\kappa_z$ increases from $\sim 0.9$ to $\sim 5$ W m$^{-1}$ K$^{-1}$ for thermal length scales ranging from 20 nm to 1 $\mu$m. This analysis suggests that one possible reason for the discrepancy between different bulk measurements\textsuperscript{3,17,23} could be the dependence of $\kappa_z$ on modulation frequency and the finite thermal penetration depth. However, given that the source of MoS$_2$ crystals is typically geological, one cannot entirely rule out differences in sample quality between the various studies as contributing to the observed $\kappa_z$ variations.

Also shown in Figure 2b is a prediction of $\kappa_z$ from first-principles calculations (described below), where the effect of finite thickness is incorporated with a suppression function calculated by the Boltzmann transport equation\textsuperscript{20} (BTE) and Matthiessen’s rule. These predictions show reasonably good agreement with the combined data set over the full range of characteristic thermal length scales from 20 nm to 1 $\mu$m. The data are thus consistent with theoretical predictions of very long c-axis phonon MFPs, and a broad spectral distribution of vibrational modes.

**First-Principles DFT Calculations.** To gain insight into fundamental aspects of phonon transport processes in MoS$_2$, we perform first-principles DFT calculations in the local density approximation of the exchange and correlation functional. We compute the frequency- and MFP-resolved $\kappa_z$ of MoS$_2$ by solving the phonon BTE with an iterative self-consistent algorithm.\textsuperscript{42} Further details are provided in Methods and Chen et al.\textsuperscript{22}

Calculated phonon dispersion curves for 2H-MoS$_2$ are shown in Figure 3a, which are in good agreement with experimental data.\textsuperscript{43} Figure 3b,c plots the calculated phonon relaxation times and MFPs as a function of phonon frequency. The MFP accumulation function $\kappa_{\text{accum}}$ is calculated as a cumulative integral of the contributions to the total thermal conductivity of phonons with MFPs smaller than a certain value and is plotted in Figure 3d. From these calculations, we infer that more than 50% of the heat at room temperature is carried by phonons with MFPs exceeding 200 nm and nearly 80% is carried by MFPs in the range 10 to 500 nm. In comparison, in silicon,\textsuperscript{44} 80% of the heat at room temperature is carried by phonons with MFPs between 40 nm and 10 $\mu$m.

On the basis of the MFP accumulation function, we calculate the cross-plane thermal conductivity of a film of thickness $t$ as follows

$$\kappa_z(t) = \int_0^{\infty} S(Kn_{\omega}) \kappa_{\text{partial}}(\Lambda_{\omega,0}) d\Lambda_{\omega,0}$$

$$= \int_0^{\infty} \frac{1}{t} N(Kn_{\omega}) \kappa_{\text{accum}}(\Lambda_{\omega,0}) d\Lambda_{\omega,0}$$

(1)

where $\kappa_{\text{partial}}(\Lambda_{\omega,0})$ is the MFP partial contribution function, $Kn_{\omega} = \Lambda_{\omega,0}/t$ is the Knudsen number, $S(Kn_{\omega})$ is the heat flux suppression function, $\kappa_{\text{accum}}(\Lambda_{\omega,0}) = \int_0^{\Lambda_{\omega,0}} \kappa_{\text{partial}}(\Lambda_{\omega,0}) d\Lambda_{\omega,0}$ is the MFP accumulation function, and $N(Kn_{\omega}) = -dS(Kn_{\omega})/dKn_{\omega}$. Two cases are considered for the suppression function $S(Kn_{\omega})$: one is based on a solution to the BTE for cross-plane heat flow in an anisotropic film inspired by the Fuchs–Sondheimer model\textsuperscript{20} (eq 2), and the other is based on Matthiessen’s rule (eq 3).

$$S_{\text{BTE}}(Kn_{\omega}) = 1 - Kn_{\omega} \left(1 - \exp\left(-\frac{1}{Kn_{\omega}}\right)\right)$$

(2)
These are plotted as solid and dashed lines, respectively, in Figure 2b and show good agreement with the experimental $\kappa_z$ data over a large range of characteristic thermal length scales. These results have important implications for the design of thermoelectric devices based on vdW materials, as they suggest that cross-plane heat conduction can be suppressed significantly by the incorporation of defects along the c-axis, such as intercalants and rotationally mismatched layers. The large phonon MFPs predicted and experimentally confirmed here offer a route to high-efficiency thermoelectrics based on nanostructuring of layered 2D materials along the c-axis.

Implications for 2D Device Thermal Characteristics. To understand the impact of cross-plane ballistic phonon transport on thermal characteristics of thin-film MoS$_2$ electronic and optoelectronic devices, in Figure 4a we plot the volumetric cross-plane thermal resistance $R_{\text{MoS}_2} = t/\kappa_z$ the combined interface resistance (Al/MoS$_2$ and MoS$_2$/SiO$_2$) $R_{\text{int}} = 1/G_t + 1/G_{t'}$ and the total thermal resistance $R_{\text{tot}} = R_{\text{MoS}_2} + R_{\text{int}}$ as a function of thickness $t$. This simplification assumes that the total resistance can be decomposed into the separate interfacial and volumetric contributions even though a large fraction of phonons undergoes quasi-ballistic transport across the thickness of the MoS$_2$ film. This assumption, which is also inherent to our data analysis methodology, is consistent with the approach commonly followed in literature when dealing with subcontinuum heat conduction across thin films.

We find that $R_{\text{MoS}_2}$ decreases with decreasing thickness but does not go to zero in the limit of zero $t$. This is a direct consequence of quasi-ballistic phonon transport and the diffusive scattering of long MFP phonons at the Al/MoS$_2$ and MoS$_2$/SiO$_2$ interfaces. In Figure 4a, we also plot the calculated MoS$_2$ volumetric resistance as a function of thickness, based on DFT predictions of the phonon MFPs and the BTE suppression function (eq 2). Because of ballistic transport across the film thickness, $R_{\text{MoS}_2}$ saturates at a finite value of $\sim 10$ m$^2$ K$^{-1}$ in the limit of 2−3 monolayers. In the absence of quasi-ballistic effects, i.e., in the diffusive regime, $R_{\text{MoS}_2}$ would have been significantly lower and become vanishingly small in the monolayer limit.

An important consequence of quasi-ballistic effects is that the total thermal resistance is not dominated entirely by the interfaces, even for thin films. In Figure 4b we plot the fractional contribution of the volumetric MoS$_2$ resistance to the total device resistance ($= R_{\text{MoS}_2}/R_{\text{tot}}$) versus $t$. In our experiments, for the thinnest film ($t = 20$ nm), $R_{\text{MoS}_2} \approx 22$ m$^2$ K$^{-1}$ and $R_{\text{tot}} \approx 76$ m$^2$ K$^{-1}$, i.e., nearly 30% of the total thermal resistance is due to the volumetric component. In the diffusive limit with constant $\kappa_z = 5.1$ W m$^{-1}$ K$^{-1}$, this value is only 7% (i.e., the interfaces contribute 93%) for a 20 nm thick film. From a metrology perspective, an important consequence of this effect is our ability to experimentally measure the intrinsic component $\kappa_z$ separately from the interface resistances, down to films as thin as 20 nm. If thermal transport were to remain diffusive, the volumetric resistance component would have been too small compared to the interface resistances, and we would not have been able to extract it uniquely using TDTR.

To estimate the contribution of quasi-ballistic transport to heat flow across thin devices ($t < 20$ nm), we plot the fractional MoS$_2$ volumetric component for different interface resistance values ($R_{\text{int}} = 10, 25, 50$ m$^2$ K$^{-1}$), as shown in Figure 4c. As before, size effects are considered by calculating the thickness-dependent $\kappa_z$ using the BTE suppression function described in eq 2 above. In the extreme scaling limit of 1−2 nm thick films (2−3 monolayers), we estimate this fractional contribution to be as large as $\sim 15\%$, $\sim 25\%$, and $\sim 50\%$ for $R_{\text{int}} = 50, 25, 10$ m$^2$ K$^{-1}$, respectively. This suggests that even if interface quality (and TBCs) of the metal/MoS$_2$ and MoS$_2$/substrate interfaces were to be improved substantially ($R_{\text{int}} \to 0$), cross-plane heat transport would likely still be limited by the ballistic resistance ($= \lim t/\kappa_z$) of the MoS$_2$ film, which is $\sim 10$ m$^2$ K$^{-1}$. Note that this analysis assumes a 3D phonon dispersion for thin films, which may face its limits when the thickness becomes comparable to the phonon wavelengths (see Supporting Information Section 4). Nevertheless, this raises interesting questions about the nature of heat conduction across few-layer thick vdW layered materials, where it is often assumed that interfaces dominate the total cross-plane thermal resistance. Given that the existence of long MFP $c$-axis thermal phonons has been experimentally demonstrated in graphite and predicted theoretically in other TMDs such as WS$_2$ and WSe$_2$, the above argument may be applicable to a large class of ultrathin vdW layered devices (see Supporting Information Section 5).

In conclusion, we reported thickness-dependent cross-plane thermal conductivity measurements of crystalline films of layered MoS$_2$. The cross-plane thermal conductivity shows a strong dependence on thickness in the range of 20−240 nm, revealing quantitative information about the distribution of phonon MFPs along the $c$-axis. Combining our results with previous measurements of $\kappa_z$ in bulk MoS$_2$ and other TMDs such as WS$_2$ and WSe$_2$, the above argument may be applicable to a large range of MFPs, illustrating the predictive power of first-principles phonon calculations for vdW layered materials.

Importantly, our results show for the first time that diffuse scattering of long MFP phonons imposes a lower limit on the cross-plane thermal resistance of vdW layered thin films. This can have significant implications for the thermal management of multilayer 2D electronics and optoelectronics (e.g., photovoltaics) where thermal transport across the device thickness imposes the primary bottleneck for heat dissipation. Finally, the quantitative knowledge of thermal phonon MFPs obtained here will enable the design of new applications that require engineering of the phonon spectrum. For example, the substantial contribution of long MFP phonons to $\kappa_z$ suggests that the introduction of disorder and defects along the $c$-axis can drastically suppress cross-plane thermal transport without significantly affecting electronic transport. This could have exciting implications for cross-plane thermoelectrics made of layered 2D materials, potentially enabling next-generation energy harvesting and electronics cooling technologies.

Methods. Sample Preparation. Flakes of MoS$_2$ were mechanically exfoliated from bulk crystals (SPI Supplies) onto 90 nm SiO$_2$ on p-type Si substrates (0.001 to 0.005 Ω cm) using a thermal release tape (Nitto-Denko Revalpa). Samples were cleaned with an acetone/2-propanol soak and subsequently annealed in Ar/H$_2$ at 400 °C for 40 min. This was followed by spin coating a double layer of electron-beam (e-beam) resist PMMA 495 K A2/950 K A4 (Microchem).
The metal transducer (nominally 80 nm Al) was patterned by e-beam lithography (Raith 150, 10 kV) and deposited through e-beam evaporation. Lift-off was performed in acetone at 50 °C.

**Ab Initio Calculations.** First-principles phonon calculations of 2H-MoS₂ were carried out in the local density approximation (LDA) of the exchange and correlation functional using the Quantum-Espresso package. Norm-conserving pseudopotentials were used to approximate core electrons. Kohn–Sham wave functions were expanded on a plane wave basis set (cutoff = 100 Ry). Integration of the electronic properties over the first Brillouin zone was performed using 10 × 10 × 4 Monkhorst–Pack meshes of k-points. Structural and cell relaxations were performed by a Broyden–Fletcher–Goldfarb–Shanno quasi-Newton algorithm with a strict convergence criterion of 10⁻⁸ Rydberg/Bohr for maximum residual force component.

Phonon dispersion relations were computed by density functional perturbation theory (DFPT) with 10 × 10 × 4 q-point mesh (see Figure 3a). The computed dispersion curves agree well with neutron diffraction data for bulk MoS₂. For the calculation of lattice thermal conductivity, anharmonic third order interatomic force constants (IFCs) are also necessary besides the harmonic second order IFCs. Third order anharmonic force constants were computed by finite differences for a supercell, which is a 5 × 5 × 1 replica of the unit cell and contains 150 atoms, with an interaction cutoff of 7 Å, including interactions up to the tenth shell of neighbors. Translational invariance of the anharmonic force constants was enforced using the Lagrangian approach. With the second and third order IFCs, the thermal conductivity of MoS₂ was computed by solving the phonon BTE with an iterative self-consistent algorithm, using the ShengBTE code, considering phonon–phonon and isotopic scattering. Convergence was checked with q-point grids up to 45 × 45 × 11. Further details are provided in Chen et al.

**PRESENT ADDRESSES**

(A.S.) Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA.

(F.L.) NG Next Basic Research Laboratory, Northrop Grumman Corporation, Redondo Beach, CA 90278, USA.

**AUTHOR INFORMATION**

Corresponding Authors

*E-mail: adityasood@alumni.stanford.edu.

*E-mail: epop@stanford.edu.

**ORCID**

Aditya Sood: 0000-0002-4319-666X

Shunda Chen: 0000-0002-3506-7507

Yi Cui: 0000-0002-6103-6352

Davide Donadio: 0000-0002-2150-4182

Eric Pop: 0000-0003-0436-8534

**REFERENCES**


(9) Yalon, E.; Aslan, O. B.; Smith, K. K. H.; McClellan, C. J.; Suryavanshi, S. V.; Xiong, F.; Sood, A.; Neumann, C. M.; Xu, X.; Goodson, K. E.; Heinz, T. F.; Pop, E. Temperature-Dependent Thermal Boundary Conductance of Monolayer MoS₂ by Raman


