Optical Transmission through Arbitrarily Located Subwavelength Apertures on Metal Films

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Abstract: We present a novel efficient model to analyze the optical properties of multiple subwavelength apertures arbitrarily located on metal films. The model is verified by computing extraordinary optical transmission through an aperiodic nanoslit array.

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1. Introduction

Following the first experimental discovery of extraordinary optical transmission (EOT) through an array of subwavelength holes in an optically thick metal film [1], a large body of work has investigated EOT and related phenomena in various geometries [2-4]. While the initial studies on EOT focused on infinitely large periodic arrays, it was subsequently demonstrated that arrays with less or even no periodicity also exhibit similar EOT properties [2], as well as unique functionalities such as planar lens focusing [5] and wavelength splitting [6]. Despite these findings, accurate analysis of truly aperiodic 2D arrays has been extremely demanding, and only limited attempts have been reported to date [3,4]. This is mainly due to the coexistence of two different types of waves, i.e., the surface plasmon polariton (SPP) and the quasi-cylindrical wave (quasi-CW, also called Norton wave or creeping wave), which has hindered simple analytical modeling of multiple-aperture interactions [3].

Here, we present a novel efficient and accurate model for computing optical properties of multiple subwavelength apertures that are arbitrarily distributed on a metal film. This model should therefore enable optimized design of a variety of novel components [5,6]. The model treats each aperture as a modal source radiator, which has its own characteristic radiation patterns linked to individual eigenmodes of the aperture. By deriving these radiation patterns numerically, the model inherently includes complicated contributions from both the SPP and the quasi-CW. The model is verified in an example case of an aperiodic 10-slit array.

2. Modal source radiator model

Fig. 1(a) shows the schematic of the structures analyzed in this work, where a rectangular nanoslit is assumed as the example geometry of apertures. We start by expressing the relative dielectric tensor of the entire system as \( \varepsilon(r) = \varepsilon_0 + \Delta \varepsilon(r) \). Here, \( \varepsilon_0 \) describes the background layered structure (i.e., an unpatterned metal layer and top and bottom dielectric layers) without the apertures and \( \Delta \varepsilon \) denotes the perturbation to this system associated with adding apertures; \( \Delta \varepsilon = \varepsilon_1 - \varepsilon_0 \) inside the aperture and 0 elsewhere. Based on the Lippmann–Schwinger equation, the magnetic field at the top interface (which we denote here as interface I) is described using Dirac notation as

\[
[H^I] = [H^0_I] + k^I_z \Delta G^I |E\rangle,
\]

where \( H^I \) is the magnetic field at the top interface in the absence of apertures and the linear operator \( G^I \) describes the Green’s dyad in the background layered structure that relates the radiated magnetic field at the top interface to the electric field \( E \) inside the aperture. A similar equation is obtained at the bottom interface (interface II).

Now, we express the field inside the metal layer as a superposition of the eigenmodes of the apertures, i.e., the guided modes that would propagate vertically in \( \pm z \) directions inside the aperture, so that

\[
|E\rangle = \sum_{a} (A_{a} e^{i \alpha_z} |e_{a}^+\rangle + B_{a} e^{-i \alpha_z} |e_{a}^-\rangle), \quad |H\rangle = \sum_{a} (A_{a} e^{i \alpha_z} |h_{a}^+\rangle + B_{a} e^{-i \alpha_z} |h_{a}^-\rangle).
\]

Here the index \( a \) runs over all the apertures and eigenmodes, \( q_a \) and \( e_{a}^\pm \) \( (h_{a}^\pm) \) are the propagation constants and electric (magnetic) field profiles of these modes, and \( A_{a}, B_{a} \) are the complex amplitudes of the downward (upward) propagating modes. By inserting Eq. (2) in Eq. (1), imposing the field continuity conditions at the two interfaces (I and II), and using the modal orthogonality relation, \( \langle e_{a}|h_{a}^\pm \rangle = \delta_{a \delta} \), we arrive at a linear system of equations for the unknowns \( A_{a}, B_{a} \). All the coefficients in the equations can be derived explicitly for arbitrary locations of apertures, provided that \( q_a, e_{a}^\pm \) and \( G^{\text{new}} |e_{a}^\pm\rangle \) are known. Among these parameters, \( q_a \) and \( e_{a}^\pm \) are derived by solving the eigenmode and \( H^I \) is obtained by simulating the background structure. The only nontrivial terms are \( G^{\text{new}} |e_{a}^\pm\rangle \), which physically represent the radiation patterns at the interfaces I and II generated by the individual eigenmodes.
Instead of solving $G^{m\alpha}$ itself, we propose here that its operated form $G^{m\alpha}[e_{\alpha}]$ can be obtained straightforwardly in most cases of our interest, by numerically simulating three different single-aperture cases as shown in Fig. 1(b): (i) a case with downwardly infinitely thick metal to extract $G^{m\alpha}[e_{\alpha}]$, (ii) a case with upwardly infinitely thick metal to extract $G^{m\alpha}[e_{\alpha}]$, and (c) a case with a finite-thickness metal to extract $G^{m\alpha}[e_{\alpha}]$ and $G^{m\alpha}[e_{\alpha}]$. Once these parameters are obtained for a given aperture geometry, we can store them in a database and use them to solve $\{A_{\alpha}, B_{\alpha}\}$ for arbitrary locations of the apertures. Further details about the model will be given elsewhere [7].

3. Numerical application

To verify the validity of the model, we calculated optical transmission through an aperiodic array of 10 rectangular slits (60 nm $\times$ 300 nm) on a 200-nm-thick silver film as shown in Fig. 2(a) under a normal-incidence $x$-polarized ($E \parallel x$) plane-wave illumination. The dielectric constant of silver was modeled by a Lorentz fit to the experimental values, where the metallic loss was also included. We assumed air ($\varepsilon_1 = 1$) and silica ($\varepsilon_2 = 2.117$) as the top and bottom dielectric layers. In this particular example, we found that the fundamental (least evanescent) mode has dominant contribution to transmission, so that the higher-order modes were neglected. The computation took less than 2 sec per wavelength on a Xeon Quad Core E5506 processor, out of which roughly 1 sec was used for loading $e_{\alpha}$ and $G^{m\alpha}[e_{\alpha}]$. This is a dramatic speed up compared with 3D-FDTD simulation of the entire structure, which took more than 90 min on an NVIDIA C-1060 GPU. Figure 2(b) shows the modal amplitudes $|A_{\alpha}|$ and $|B_{\alpha}|$ inside each slit at a 600-nm wavelength, derived using our model (blue dots and red circles) and 3D-FDTD (crosses). Good agreement is obtained with a relative error of less than 4%. Using the derived values of $\{A_{\alpha}, B_{\alpha}\}$, the transmission spectrum is calculated as shown in Fig. 2(c). The transmission is evaluated at the middle of the silver film and normalized to the total area of 10 slits, following the usual convention [5]. Again, we obtain excellent agreement over a broad spectral range (400-1200 nm) with a normalized error of less than 6%. For comparison, we also plot the normalized-to-area transmission spectrum of a single slit (gray solid). The difference in this case is as large as 34%, indicating the significance of modeling the multi-slit interactions accurately.

4. Conclusions

We have demonstrated a novel efficient method to compute optical properties of a metal film, perforated with arbitrarily located multiple subwavelength apertures. By using the radiation patterns from a single aperture, the interactions between multiple apertures at arbitrary locations were calculated accurately. The model was applied successfully to analyze EOT through an aperiodic 10-slit array with an error of less than 6%.

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Fig. 1: (a) Schematic picture of a metal film perforated with arbitrarily located subwavelength apertures (slits). (b) Series of simulation to extract all four radiation patterns $G^{m\alpha}[e_{\alpha}]$.

Fig. 2. (a) Top view of an aperiodic 10-slit array on a silver film. (b) Modal amplitude inside each slit at 600-nm wavelength, obtained using the model (red), 3D-FDTD (blue), and single-slit approximation (gray).