

# Spatiotemporal control of ultrashort optical pulses by refractive–diffractive–dispersive structured optical elements

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Structured optical elements that control the spatial and temporal characteristics of femtosecond light pulses are analyzed and synthesized. We show that unique spatiotemporal effects can be attained based on the diffraction, refraction, and dispersive effects that appear in the femtosecond regime. We argue that the design requirements for ultrafast optics are beyond the achromatization considerations that are usually applied to incoherent illumination because of the need to consider coherent effects. Despite fundamental limitations in the space–time control of ultrashort pulses, we show the potential of this technique to improve simultaneously the spatial and the temporal resolution of a lens and to generate ultrafast pulse sequences. © 2001 Optical Society of America

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There has been a growing interest in the generation of temporally shaped optical waveforms for applications such as optical communication, nonlinear spectroscopy, and the study of ultrafast processes in matter. Various techniques for temporal shaping have been developed during the past decades. These include linear spectral filtering,<sup>1</sup> linear direct space-to-time conversion,<sup>2</sup> volume holography,<sup>3</sup> nonlinear ultrafast optical processors,<sup>4</sup> and multilayer mirrors.<sup>5</sup> The temporal shape of a focused pulse can also be affected by the spatial distribution of the pulse passing through the focusing lens.<sup>6</sup>

Nevertheless, in some applications such as quantum dynamic control, microscopy, and materials processing the spatial characteristics are critical as well. The classical spectral pulse shaper<sup>1</sup> can be modified to shape waveforms simultaneously both in one spatial dimension and in time.<sup>7</sup> This approach is attractive for generating complex signals, but it requires a full optical system. Another approach<sup>8</sup> utilizes multiply exposed volume holograms to generate ultrafast frames during the reconstruction process.

It would, however, also be interesting to control the spatiotemporal shape of ultrashort pulses with only a single, thin optical element that could be fabricated with existing microfabrication techniques that permit mass production. Accordingly, in this Letter we investigate the possibility of controlling spatial and temporal features of ultrashort pulses simultaneously through the use of a single structured optical element (SOE), i.e., a thin element that is composed of layers of refractive, diffractive, and dispersive structures.

We consider an ultrashort light pulse incident upon a thin element that modulates amplitude and phase; we observe its propagation in the half free space behind it, as shown in Fig. 1. As opposed to the situation found in classical diffractive-optical problems, the broadband and coherent character of the illumination produces additional effects: (1) The difference between phase and group velocity within the material produces time delays among different portions of the wave propagating along different paths within the SOE. (2) Different delays also appear along different paths in free-space propagation because of the

spatial extension of the wave. (3) The group-velocity dispersion leads to pulse stretching (or compression). (4) Refraction and diffraction at the boundaries modulate the wave fronts in a wavelength-dependent way because of dispersion. As a result, the spatial intensity distribution at a given observation plane differs dramatically for different frequency bands, and the temporal characteristics at each location may be completely different from those of the incoming pulse. Note, for example, that a lens designed for a small focal spot in space with incoherent broadband light may well substantially distort a coherent pulse at the focus because of the effects of group velocity.<sup>9,10</sup>

In this Letter we shall assume that the scalar and thin-element approximations are valid. These approximations permit the analysis and synthesis of thin elements as long as the features are much larger than the wavelength of the incident light. Consequently, we consider the field behind the SOE described by the Helmholtz wave equation:  $[\nabla^2 + k^2(\omega)]U(\mathbf{r}, \omega) = 0$ , where  $\mathbf{r} \equiv (x, y, z)$  are the Cartesian coordinates and  $z$  is the direction of propagation,  $\omega$  is the temporal frequency,  $k(\omega)$  is the wave number, and  $U$  is the (scalar) field amplitude. The field across a plane immediately behind the SOE is calculated as

$$U(x, y, 0, \omega) = T(x, y, \omega)U_i(x, y, 0, \omega), \quad (1)$$

where  $T$  is a frequency-dependent transmittance function and  $U_i$  is the incident field. The diffracted field for each frequency component can be calculated from a decomposition of  $U$  in plane waves:

$$U(x, y, z, \omega) = \mathbf{F}^{-1}\{P(\omega, f_\rho)\mathbf{F}[u(x, y, 0, \omega)]\}, \quad (2)$$

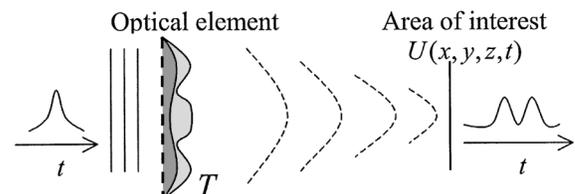


Fig. 1. Graphic representation of the spatiotemporal pulse-shaping problem.

where  $P(\omega, f_\rho) = \exp\{iz[k^2 - (2\pi f_\rho)^2]^{1/2}\}$ ,  $f_\rho$  is the radial spatial frequency, and  $\mathbf{F}$  is a two-dimensional Fourier transform. The resultant frequency-dependent spatial distribution is transformed into a spatiotemporal field distribution by means of a (temporal) inverse Fourier transformation:

$$u(x, y, z, t) = \mathbf{F}_t^{-1}U(x, y, z, \omega). \quad (3)$$

The transmittance function  $T$  can be evaluated from the geometry and the material properties of the structured layers of the SOE. The overall transmittance is

$$T(x, y, \omega) = T_A(x, y, \omega)T_{P_1}(x, y, \omega) \dots T_{P_n}(x, y, \omega). \quad (4)$$

$T_A(x, y, \omega)$  is an amplitude modulation that includes the action of the aperture and the absorption that corresponds to all the layers. The phase modulation  $T_{P_i}(x, y, \omega)$  includes the spatially and wavelength-dependent phase delay that is responsible for dispersion in the material:

$$T_{P_i}(x, y, \omega) = \exp\left(i\{[n_i(\omega) - 1]\Delta_i(x, y) + \Delta_i^M\} \frac{\omega}{c}\right), \quad (5)$$

where  $\Delta$  is the function that describes the width of the layer and  $\Delta_i^M$  is the width span. Each layer has a uniform index of refraction  $n_i(\omega)$ . We neglect the effect of multiple reflections at the boundaries of the layers. This effect could be dramatic if multiple parallel layers were implemented, but here we consider only a few nonparallel layers with relatively low index contrast.

To calculate the diffracted field we need to know the dispersion characteristics of the material. For weakly dispersive materials we can use a Taylor expansion of  $k_i(\omega) = n_i(\omega)\omega/c$  about the central frequency ( $\omega_0 = k_0c$ ). The zero-order term gives rise to a time delay relative to the passage of light through air. The first-order term is associated with the group velocity and is responsible for the pulse delay. This delay changes with the lateral location. The second-order term is related to the group-velocity dispersion (GVD) and leads to pulse deformation at each lateral location. In addition, all the terms of order greater than zero lead to (spatial) dispersion at the boundaries of the refractive and diffractive surfaces. Note that the procedures presented here are not restricted to a second-order model: When necessary, higher-order terms, a Sellmeier model, or look-up tables for  $k(\omega)$  can be considered. For the thin SOE designs presented below, second- and fourth-order models were compared and showed a very low discrepancy (below 0.3% in the peak values). However, such might not be the case for thicker optical elements or shorter pulses. Finally, we need to specify the spatiotemporal shape of the incident pulse. In the examples below, we considered an incident pulse of spatial plane waves and a Gaussian temporal dependence  $u_i(x, y, 0, t) \propto \exp[-(t/\tau)^2]$ , where  $\sqrt{2 \ln 2} \tau$  is the FWHM of the pulse intensity profile.

Arguably the single most important function of a passive element in femtosecond optics is that of a lens. It is well known that singlet lenses produce

pulse spreading in time relative to the input field and beam spreading in space relative to the diffraction limit achievable with monochromatic light.<sup>9,10</sup> Pulse spreading is caused by the difference in propagation time between the pulse front and the phase front and by the GVD in the material. The pulse width is also wider at later times because of the apodization effect caused by the propagation-time difference.<sup>9,10</sup> These spatiotemporal effects are shown in Fig. 2(a) for a spherical lens (BK7 glass,  $f/15$ , 30-cm focal length) and a 15-fs pulse centered at 620 nm.

A partial solution to this problem is to use achromatic doublets. However, these elements are thick and introduce a large GVD. Moreover, standard achromats possess residual chromatic aberrations that lead to additional spreading. We optimized an achromatic doublet (BK7-SF11) for the previously described short pulse. The result [Fig. 2(b)] showed a 37% increase in peak intensity.

Diffractive lenses, by contrast, have small GVD effects because they have much less material. However, they possess a strong chromatic dispersion that leads to pulse expansion and spreading. We investigated the possibility of combining phase diffractive effects with the refractive and dispersive effects that appear in the singlet lens. We designed a SOE consisting of a refractive-diffractive pair to improve the focusing function in two ways, namely, to compensate for the chromatic first-order dispersion and to reduce the material thickness, which led to a significantly reduced GVD. The focal lengths of the diffractive and the refractive structures were optimized numerically to yield the maximal spatiotemporal focusing, as shown in Figs. 2(b) and 2(c). The SOE was designed based on the material properties of BK7 glass. One of its surfaces is refractive and corresponds to a spherical lens of focal length 31.37 cm, whereas the second surface is diffractive and corresponds to a Fresnel lens of focal length 6.85 m (see Fig. 3). The diameter is 2 cm. Compared with those of a singlet lens, the peak intensity is 67% larger, the

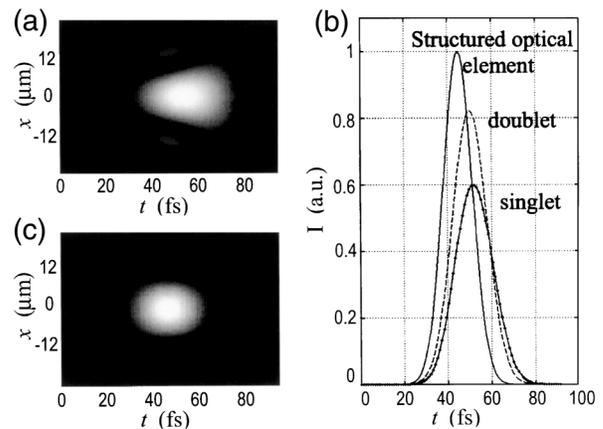


Fig. 2. Response in the focal plane for a 15-fs incident pulse (focal length at 30 cm,  $f/15$ ): (a) Spatiotemporal response of a singlet spherical lens. (b) Temporal response on axis of the singlet ( $\Delta t = 20.5$  fs), the doublet ( $\Delta t = 18.5$  fs), and the SOE ( $\Delta t = 15.6$  fs). The time origin is arbitrary. (c) Spatiotemporal response of the optimized SOE. All the pulses's durations ( $\Delta t$ ) are FWHM.

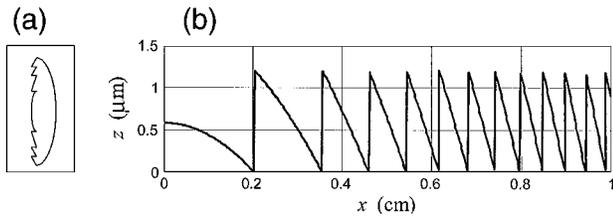


Fig. 3. (a) Schematic representation of the optimized SOE. (b) The actual diffractive surface relief.

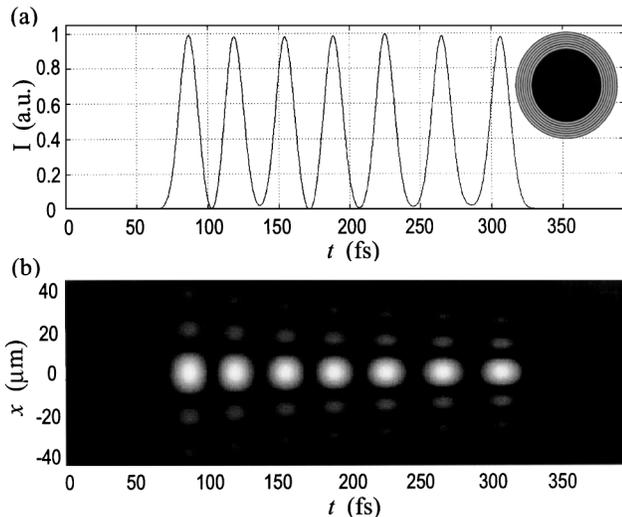


Fig. 4. Generation of ultrafast pulse trains. Inset, schematic representation of the SOE. (a) Axial temporal intensity at 30 cm. (b) Spatiotemporal diagram at the same location (the time origin is arbitrary).

pulse duration (FWHM) is 20% shorter, and the spot size is  $\sim 10\%$  smaller. Figure 2(b) shows the focal temporal behavior of the singlet, doublet, and SOE lenses. It should be noted that, for lower  $f$ -numbers, the pulse spreading caused by singlet lenses as well as the benefits of using a SOE are more dramatic.

We also explored the possibility of time shaping the pulse intensity by using only a binary computer-generated mask to implement the amplitude transmittance function  $T_A$ . The mask consists of a set of ring apertures whose width and location are optimized to produce the desired temporal response. The observation point is at a given distance from the mask. The pulses generated by different rings arrive at the observation point after different time delays. As we are dealing with extremely short pulses, the time delay between the pulses generated by different rings can be significantly longer than the pulse duration itself. Note that there is a one-to-one correspondence between the spatial rings and the temporal pulses generated on axis; thus the SOE can be considered a direct space-to-time converter. Figure 4 depicts the response of such a system when it is optimized to produce a train of seven pulses 30 cm from the mask. In this specific case we used a binary search algorithm. The seven rings' radii transitions (in millimeters) were as follows: 5.430, 5.448; 5.935, 5.952; 6.455, 6.47; 6.914, 6.929; 7.373, 7.387; 7.832, 7.867;

and 8.291, 8.324. The duration of each pulse is basically the same as that of the input pulse. Each central lobe in Fig. 4(b) is diffraction limited relative to the size of each of the rings.

As these designs suggest, different types of SOE can be considered and optimized according to fundamental and technological constraints. The fundamental constraints originate in the wave equation and in the fact that a single thin structure cannot address all the degrees of freedom available in an ultrashort waveform.<sup>11</sup> The detailed structure can be calculated by use of techniques similar to those used in multidimensional diffractive optics design.<sup>12</sup> Note that the design of optical elements for ultrashort pulse illumination is more intricate than for broadband incoherent illumination. The reason is that in ultrashort pulse optics we have to consider the phase relation among the different spectral components to achieve a desired temporal response, a requirement that is not present for incoherent optics.

In conclusion, in the femtosecond regime the diffraction and refraction effects are entangled with dispersive phenomena. SOEs are intended to compensate for and utilize these effects to produce useful optical functions. The analysis and examples that we have shown here are a first attempt to generate a new class of ultrafast optical elements. We have shown explicitly that it is possible to make SOEs with performance for short pulses that exceeds that of conventional lens designs, and we have also illustrated how to use single SOEs for complex pulse shaping, such as in generation of pulse trains.

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