Relationship between the superprism effect in one-dimensional photonic crystals and spatial dispersion in nonperiodic thin-film stacks

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We derive that, in all-pass (unity amplitude transmittance or reflectance) one-dimensional layered structures, the tangent of the group-propagation angle, the group delay, and the stored energy are approximately proportional. Thus photonic crystal superprisms and spatial dispersion in nonperiodic photonic nanostructures are generally related to wavelength-dependent stored energy in the stack. © 2005 Optical Society of America

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Miniaturization of optical systems for spectroscopy and communications demands compact and cost-effective components. Several groups have noted that a rapid change of the group-propagation angle with wavelength—a superprism effect—observed in one-dimensional (1-D),[1,2] two-dimensional,[1,3,4] and three-dimensional[5] photonic crystals may allow compact high-dispersion devices to be constructed. We have concentrated on 1-D stacks, since they are easy to fabricate. The change in the group-propagation angle (Fig. 1) can be used for wavelength multiplexing or demultiplexing by spatial beam shifting. In Ref. 6 we showed that nonperiodic photonic nanostructures can also be designed for high and controllable spatial dispersion. Stack designs with a wavelength-dependent penetration depth, a change in the stored energy, or both were used. Here we show that the dispersion is related to a change in the stored energy in the stack with wavelength in all all-pass periodic and nonperiodic stacks, demonstrating the similarity between the superprism effect and spatial dispersion effects in nonperiodic photonic nanostructures.

We derive this result in two steps. First we show that spatial dispersion and temporal dispersion are approximately proportional. Next we use Tellegen's theorem relating the group delay to the stored energy in the structure. Thus we conclude that a rapid change in effective group-propagation angle with wavelength necessarily corresponds to a rapid change in stored energy with wavelength for all-pass periodic and nonperiodic 1-D nanostructures.[7]

An all-pass system is one that influences only signal phase, not amplitude. For steady state, the energy propagation direction is given by group-propagation angle θ_{gr}, which is related to group velocities v_{gx} in the x and v_{gz} in the z directions by Eq. (1) for isotropic materials[6,8]

$$\theta_{gr} = \tan^{-1}(v_{gx}/v_{gz}). \quad (1)$$

The group velocities are calculated as $v_{gx} = \partial\omega/\partial\beta|_K$ and $v_{gz} = \partial\omega/\partial K|_\beta$, where ω is the frequency and β and K are the wave vectors in the x and z directions, respectively. For a plane wave incident at a particular angle on parallel, flat surfaces, β is constant and is fixed equal to the vacuum value of $\beta = \omega \sin(\theta_{in})/c$. θ_{in} is the propagation angle in vacuum, and c is the vacuum speed of light. For an infinite stack, $K(\omega, \beta)$ can be calculated using Bloch theory. For finite stacks a transfer matrix method gives $K(\omega, \beta)$, v_{gx}, and v_{gz}, which are in this case effective quantities.[6,9]

Consider first the infinite periodic stack of Fig. 2. It shows a stop band between 783 and 931 nm, where no propagating modes exist. An infinite photonic crystal is an all-pass system for propagating modes. Just outside the stop band, the group-propagation angle changes rapidly with wavelength.

From Eq. (1) we see that a change in group-propagation angle has to correspond to a change in group velocities. Figure 3 plots v_{gx} and v_{gz} for wavelengths just below the stop band. Group velocity v_{gx} along the layers is approximately constant, while that perpendicular to the layers, v_{gz}, changes rapidly with wavelength.[6,9,10] Thus the dispersion is closely linked to the layered structure in the z direction, and we call this type of dispersion, structural. Such dispersion is quite different from material dispersion, where (in an isotropic material) both group velocities change proportionally and dispersion is due solely to Snell's law.

Group delay τ_{group} through an all-pass stack of thickness L is given by[6,11]

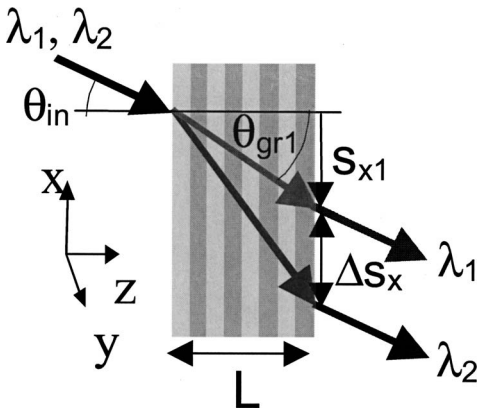

Fig. 1. Superprism effect in a 1-D photonic crystal.

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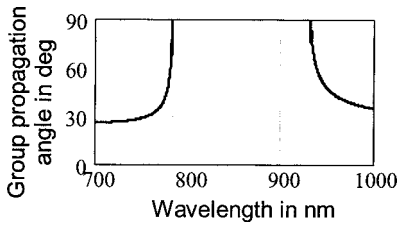

Fig. 2. Group-propagation angle as a function of wavelength for an infinite 1-D photonic crystal consisting of alternating 167 nm SiO_2 ($n=1.46$ at 830 nm) and 110 nm Ta_2O_5($n=2.06$) layers at $\theta_{in}=45°$ and of p polarization calculated by Bloch theory.

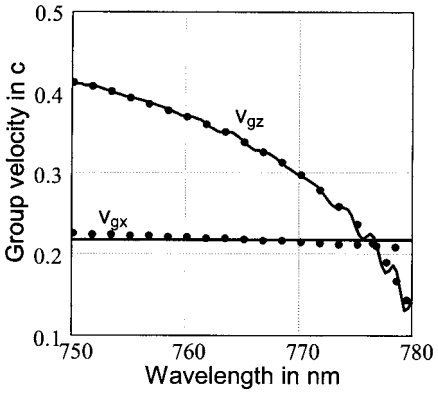

Fig. 3. Group velocities v_{gx} and v_{gz} in units of the speed of light, c, as a function of wavelength. Dotted curves were calculated by Bloch theory for the infinite stack in Fig. 2. The solid v_{gx} curve represents a constant-value approximation. The solid v_{gz} curve was calculated from the stored energy in the periodic region of the stack in Fig. 4.

$$\tau_{group}=\frac{L}{v_{gz}}=\frac{L\tan(\theta_{gr})}{v_{gx}}=\frac{s_x}{v_{gx}}, \tag{2}$$

where s_x is the spatial shift along the x direction. Equation (2) also shows how group delay is related to group-propagation angle θ_{gr} and shift s_x through Eq. (1). Thus, since v_{gx} is approximately constant for structural dispersion, spatial shift and group delay are approximately proportional. The stack shown in Fig. 1 will exhibit both spatial dispersion and temporal dispersion.[6]

The relationship between total stored energy in the stack W_{tot} and the group delay was previously derived for linear, time-invariant all-pass electrical networks[12] and microwave circuits[13] from Tellegen's theorem as well as for electromagnetic waves with sufficiently long pulse durations and negligible self-interference effects[9,14,15]:

$$\tau_{group}=\frac{W_e+W_m}{P}=\frac{W_{tot}}{P}. \tag{3}$$

In Eq. (3), W_e (W_m) is the electrical (magnetic) energy stored in the system and P is the incident power. Equation (3) states that group delay and stored energy are proportional for all linear and time-invariant all-pass systems, including both periodic and nonperiodic linear all-pass stacks. Combining Eqs. (2) and (3), we conclude that both spatial and temporal dispersion correspond to a change in the stored energy with wavelength. Since an infinite photonic crystal is an all-pass structure, the superprism effect is therefore an energy storage effect; i.e., a larger change in the stored energy with wavelength gives a larger change in the propagation direction with wavelength and vice versa.

To illustrate the wavelength dependence of stored energy, we calculate the energy distribution in a 120 period 1-D photonic crystal. To guarantee reflection-free coupling (and hence all-pass behavior) into and out of this finite stack, we add impedance-matching regions on both sides. Figure 4(a) graphs the layer sequence for the stack. Impedance matching is achieved by two 40 period regions with the same Bragg wavelength as the center region but a slowly increasing amount of high-index material per period.[6,7]

Figure 4(b) plots the transmittance for this stack, showing that it is indeed close to unity outside the stop-band region and justifying the treatment as an all-pass system. Figure 5 plots the E-field amplitude squared as a function of the position in the stack. We see a standing-wave-like pattern with increasing E-field amplitudes as well as longer spatial periods for wavelengths closer to the stop-band edge. Note that this pattern is due not to reflections at the end surfaces but to structural dispersion. We can verify this by estimating the group velocity v_{gz} from the stored energy in the finite periodic region and comparing the result to v_{gz} calculated using Bloch theory.

To estimate v_{gz}, we integrate the stored energy per unit area for plane waves in the periodic region and use Eqs. (2) and (3).[10] The estimated v_{gz} is plotted in Fig. 3 and agrees well with the group velocity in the infinite periodic stack. The slight ripple close to the stop band is due to a combination of less than unity transmission and the finite nature of the stack. The group-propagation angle can easily be calculated by approximating v_{gx} with a constant value.

The proportionality among spatial dispersion, temporal dispersion, and stored energy is valid for a much broader range of structures. Equations (1)–(3) are valid for all linear, time-invariant all-pass stacks in steady state. In Refs. 6 and 10 we discussed various nonperiodic structures including a structure incorporating resonators, a chirped structure, and a

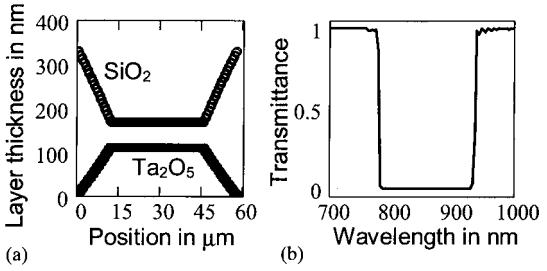

Fig. 4. (a) Layer thickness as a function of position in the stack for a stack with 120 periods of alternating 167 nm SiO_2 and 110 nm Ta_2O_5 layers in the center and 40 period impedance-matching regions on either side. The bulk material on either side is assumed to be SiO_2. (b) Transmittance calculated using transfer matrices.

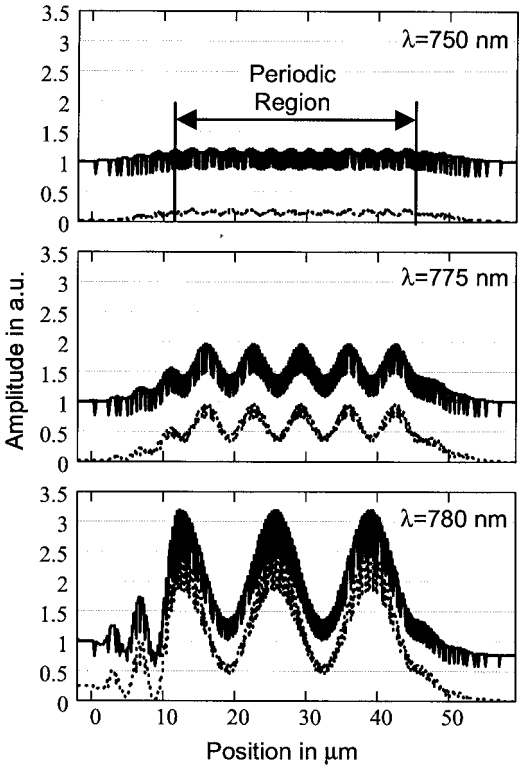

Fig. 5. *E*-field amplitude squared in the x direction of the forward (solid curves) and backward- (dotted curves) propagating waves as a function of the position in the stack from Fig. 4 for three wavelengths.

numerically optimized structure. Figures 6, 8, and 9 of Ref. 6 clearly show the relationship between spatial shift and stored energy. We also verified that v_{gx} is approximately constant with wavelength in this broader class of structure.[10] Since only all-pass stacks (in either transmission or reflection) offer loss-free operation, these are the stacks of greatest practical interest.

In conclusion, we have shown that spatial and temporal dispersion correspond to a change in the stored energy with wavelength. This physical insight allows us to consider how we might modify periodic structures to obtain superior dispersion characteristics. The key is to design stacks that exhibit a wavelength-dependent amount of stored energy. In Refs. 6 and 10 we discussed various design algorithms for obtaining stacks with dispersion. Because of their much larger design freedom, we achieved far superior dispersion characteristics in nonperiodic multilayer stacks. Here we have shown that, even though different design algorithms yield different types of layer sequence, the dispersion is related to a change in the stored energy with wavelength in all cases.

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