# On perfect cloaking

**David A. B. Miller**

*Ginzton Laboratory, Stanford University, 450 Via Palou, Stanford CA 94305-4085*
dabm@ee.stanford.edu

**Abstract:** We show in principle how to cloak a region of space to make its contents classically invisible or transparent to waves. The method uses sensors and active sources near the surface of the region, and could operate over broad bandwidths. A general expression is given for calculating the necessary sources, and explicit, fully causal simulations are shown for scalar waves. Vulnerability to broad-band probing is discussed, and any active scheme should detectable by a quantum probe, regardless of bandwidth.



**OCIS codes:** (260.0260) Physical optics; (350.7420) Waves; (160.4760) Optical properties

## References and links

1. A. Alu and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," Phys. Rev. E **72**, 016623 (2005).
2. U. Leonhardt, "Optical conformal mapping," Science **312**, 1777-1780 (2006).
3. J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling Electromagnetic Fields," Science **312**, 1780-1782 (2006).
4. G. W. Milton, and N.-A. P. Nicorovici, "On the cloaking effects associated with anomalous localized resonance," Proc. Roy. Soc. A **462**, 3027-3059 (2006).
5. E. Wolf and T. Habashy, "Invisible bodies and uniqueness of the inverse scattering problem," J. Modern Opt. **40**, 785-792 (1993).
6. N. A. Nicorovici, R. C. McPhedran, and G. W. Milton, "Optical and dielectric properties of partially resonant composites," Phys. Rev. B **490**, 8479-8482 (1994).
7. A. Alu and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: resonance, tunneling and transparency," IEEE Trans. Antennas Propag. **51**, 2558-2571 (2003).
8. J. E. Ffowcs Williams "Review Lecture: Anti-Sound," Proc. Roy. Soc. London A **395**, 63-88 (1984)
9. P. A. Nelson and S. J. Elliott, *Active Control of Sound* (Academic Press, London, 1992), pp. 290–293.
10. E. Friot and C. Bordier, "Real-time active suppression of scattered acoustic radiation," J. Sound Vib. **278**, 563-580 (2004).
11. E. Friot, R. Guillermin, and M. Winninger, "Active control of scattered acoustic radiation: a real-time implementation for a three-dimensional object," Acta Acust. **92**, 278-288 (2006).
12. Yu. I. Bobrovnitskii, "A new solution to the problem of an acoustically transparent body," Acoust. Phys. **50**, 647-650 (2004).
13. J. A. Stratton and L. J. Chu, "Diffraction Theory of Electromagnetic Waves," Phys. Rev. **56**, 99-107 (1939).
14. J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).
15. M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon, Oxford, 1980).
16. D. A. B. Miller, "Huygens's wave propagation principle corrected," Opt. Lett. **16**, 1370-1372 (1991).
17. C. H. Bennett and G. Brassard, *Proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalore, India, 1984* (IEEE, New York, 1984), 175-179.
18. N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, "Quantum cryptography," Rev. Mod. Phys. **74**, 145-195 (2002).
19. W. K. Wootters and W. H. Zurek, "A single quantum cannot be cloned," Nature **299**, 802 (1982).
20. D. Dieks, "Communication by EPR devices," Phys. Lett. A **92**, 271-272 (1982).

## 1. Introduction

Recent work [1-4], following earlier discussions [5-7], shows that metamaterials could "cloak" regions of space, making them invisible to electromagnetic radiation. For perfect

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invisibility, the wave outside the cloaked region should be everywhere exactly as it would have been if the cloaked region had merely been empty space (or whatever is the "background" medium) instead. As understood in this prior work (e.g., [3]), however, any systems in which the response depends only on the local field can never provide perfect invisibility to broad-band waves. This raises the question of whether such perfect invisibility is ever possible, even in principle. Here we show how perfect cloaking could be achieved at least in a limiting, classical case. Our scheme would measure the waves near the surface of the cloaking volume and excite appropriate surface sources using a novel approach to determine those sources. For such "true" cloaking, each source amplitude necessarily depends on the measurements at all sensing points. We show fully causal simulations for scalar waves, and discuss limitations, including vulnerability to quantum probing.

## 2. Cloaking and local sources

It is, of course, easy to make an object invisible by building a box around it. The box prevents the wave from getting to the object – the field is excluded from the volume inside the box – and so the wave cannot sense the object. Generally, though, the box itself would still be visible, and so we would always know there was something there. For perfect invisibility, the box should be invisible also. The recent meta-material work [2,3] gives a partial solution to making the box invisible, valid for narrow bandwidths. As an alternative to a box made from some material, we could instead position active sources on the surface of a volume to exclude the wave from the volume by cancellation. (By active sources we mean ones whose value is set as a result of measurements of the wave.) Such an approach is well known in acoustics (see Refs. [8] and [9] for reviews, and Refs. [10-12] for representative recent work), where such cancellation is sometimes called "antinoise" [8]. Typically, however, such sources also change the wave outside the volume, changes we could regard as "scattering" [10,11], and so the presence of this active cloaking is also usually detectable. It is important to emphasize that mere exclusion of a wave from a volume, even by active sources, does not guarantee invisibility of the volume itself.

These problems of cloaking or invisibility are particularly difficult when we require causality; in such perfect, causal cloaking, waves must appear to pass through a volume as if it were empty or transparent space, even with no advance knowledge of those probing waves. If our method relied on such advance knowledge, an adversary could simply change the probing wave without telling us and then discover our attempted cloaking. In this causal case, no general solution is apparently known even in acoustics for making a volume invisible, though recent work has used some advance knowledge together with control loops to minimize scattering [10,11].

A key point to understand is that there is no material with which we can coat our volume to provide perfect invisibility – there can be no perfect "invisible paint". Recent work [2, 3] shows that a shell of meta-material could achieve cloaking for monochromatic electromagnetic waves. Meta- and conventional materials all have essentially local response, however; the sources excited at any point in the material by waves depend only on the wave at or very near to the same point. Any approach based on such local response is easily discovered in principle using transmission of a pulsed probe, even for materials with broad-band response. A simple causal argument proves this, as illustrated explicitly in Fig. 1(a).

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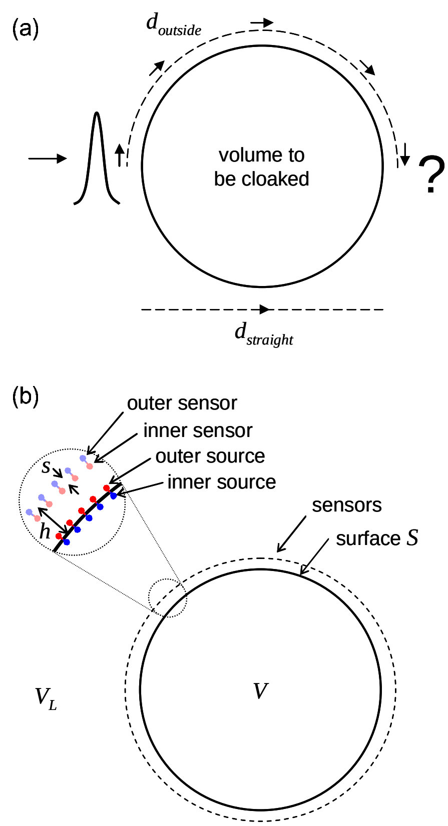


Fig. 1. (a). Illustration of a pulse incident on a volume to be cloaked solely on the basis of local response near the surface. (b) Arrangement of sources and sensors for cloaking of scalar waves.

The direct path, $d_{straight}$, through the volume in Fig. 1(a) is necessarily shorter than any path, $d_{outside}$, round the surface. If we exclude any wave propagation through the volume, the soonest any wave can get to the far side, and, with purely local response, the soonest any sources on the far side can respond, is through the longer path, $d_{outside}$. Hence it is not possible to reproduce the correct form of the pulse just on the far side of the volume. No matter what response we choose for the material or locally responding sources on the far side of the volume, the "transmitted" pulse must be delayed at least in part, leading at least to some distortion. There would always be some kind of "shadow" region in which a "transmitted" pulse is at least changed. Those changes allow us to detect any attempted cloaking based on locally responding materials or sources. This argument is changed in detail but not in substance if the cloaking material or sources have finite thickness. No locally responding material or sources can perfectly cloak the volume. Equivalently, any cloaking by any locally responding material will always give rise to some scattering.

A solution in principle to perfect invisibility is known in acoustics [9], but it requires advance knowledge of what the incoming wave would be on the surface if the volume and its cloaking material or sources were simply replaced by empty space. This is not therefore a causal solution; we will describe this result later, and we can call it "predetermined cloaking". The practical and fundamental problem is that any locally responding sources (whether active sources or responses of materials) around a finite body always change (or partially scatter) the wave, as we can see in the "shadowing" in Fig. 1(a), and so such an approach can never be perfect if we insist it is to be causal.

Hence, if perfect causal cloaking is to be possible, it must require non-local sources, that is, sources whose strength depends on the wave amplitude at other points. (Non-local impedances have recently also been considered for acoustic waves [12], though only for a monochromatic case.) The core question is whether we can deduce, in a self-consistent and causal manner, what those sources should be. Below, we provide a general solution to this problem, showing how such sources can be chosen, and we demonstrate the resulting cloaking numerically for the case of scalar waves.

### 3. Excluding scalar waves from a volume

Consider first how to exclude a scalar wave from a volume $V$ based only on surface sources deduced from local values of the field at the surface $S$. We presume that, in the region outside the volume $V$, the wave $\phi$ is propagating subject to the usual homogeneous scalar wave equation $\nabla^2\phi-(1/c^2)(\partial^2\phi/\partial t^2)=0$ where $c$ is the wave velocity. To exclude the wave from the volume, we can now add sources to the surface of the volume (formally giving an inhomogeneous equation). For scalar waves of local amplitude $\phi$, for example, point sources of strength $-\partial\phi/\partial n$ per unit area and dipole sources oriented perpendicular to the surface and of strength $\phi$ per unit area on $S$ will exclude the field from the volume [8,9,13-16]. Here $n$ is in the direction normal to the surface and outward from $V$. Formally, the initial conditions on this problem are the (mathematically) known initial incident field. The surface sources will formally be retarded and will obey the radiation condition. Such sources function as "perfect absorbers" for scalar waves; on a large planar surface, for example, such sources would give exactly no reflection of a normally incident plane wave.

Note also that, if such a set of sources $\mathrm{p}$ will exactly "stop" a wave on the outside of a surface $S$ to give no wave inside $S$, then the sources $-\mathrm{p}$ would exactly recreate that same wave starting on the inside of the surface $S$; this is the same as saying that the sources $\mathrm{p}$ generate a wave that exactly cancels the original wave inside the surface $S$. Note that such sources work for stopping and creating waves; sources constructed by the above prescription make no explicit distinction between these two processes.

Hence, we could imagine a scheme as in Fig. 1(b). Pairs of sensors some small distance $h$ outside $V$, oriented perpendicular to the surface $S$, are aligned with corresponding pairs of sources straddling $S$, with, for simplicity, the same separations $s$ in each pair. The difference between the measured waves at these two sensors gives the wave gradient $\partial\phi/\partial n$, and their average gives the amplitude $\phi$. We emulate dipole sources per unit area with appropriate opposite values on the two sources in a pair, and can also choose to emulate a point source by assigning half of its value to each of the elements in the source pair. Then we get a particularly simple formula for calculating the resulting inner and outer source amplitudes $p_{in}$ and $p_{out}$ respectively based on the inner and outer measured wave amplitudes $f_{in}$ and $f_{out}$ respectively from the corresponding sensor pair, namely

$$p_{out}=\frac{f_{in}\delta a}{s}\,;\;\; p_{in}=-\frac{f_{out}\delta a}{s} \tag{1}$$

where $\delta a$ is the effective element of area on $S$ "occupied" by a given source pair.

If the source and sensor pairs are sufficiently dense, such an active "quasi-cloaking" scheme of local measurements and resulting driven sources on $S$ does indeed exclude all waves from the volume $V$, as can be seen in the simulation in Fig. 2(c). But it does not make the volume invisible; the transmitted wave on the far (right) side of $V$ is significantly changed [compare the original wave with no cloaking in Fig. 2(b)]. We see the kind of "shadowing" perturbation in the field that we would have expected from the argument in Fig. 1(a). Though we could not see inside $V$, we could deduce at least that there is something on the surface $S$.

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Though the sources eliminate the wave inside $V$, they do actually influence the wave outside $V$, even though these are the sources we would expect to use for a "perfect absorber".

### 4. Method for calculating sources for true, causal cloaking

To see how to achieve "true" cloaking, we first write the relation between waves and sources in a more general linear algebra form valid for any kind of linear wave. We represent the wave (at all times and places) as a vector of values $\mathsf{f}$, and the values of the various surface sources (at all times and for all the different sources near the surface $S$) as a vector $\mathsf{p}$. The sources that eliminate the waves from inside $V$, as in the quasi-cloaking scheme above, can then formally be deduced from the local measured wave values through a linear operator $\mathsf{C}$ that is local in both space and time, i.e.,

$$\mathsf{p} = \mathsf{C}\mathsf{f} \tag{2}$$

The above relations, Eq. (1), are an example for scalar waves of the relation Eq. (2). Equation (2) as it stands gives the sources for quasi-cloaking. Explicitly, we could construct a vector $\mathsf{f}$ that contained, in order, the values of the wave at the inner and outer sensors for each sensing pair for each time of interest. $\mathsf{C}$ could then be represented by a diagonal matrix with elements alternating between + and - $\delta a/s$ (where $\delta a$ is possibly different for different source pairs). The result $\mathsf{p} = \mathsf{C}\mathsf{f}$ would contain, in order, the values of the sources at the outer and inner source points for each source pair for each time of interest.

We can also usefully define the (retarded) free-space Green's function operator $\mathsf{G}$, which gives the wave everywhere in response to some source. For example, for scalar point sources, $\mathsf{G} \equiv \delta(|\mathbf{r}-\mathbf{r}_s| - c(t-t_s))/4\pi|\mathbf{r}-\mathbf{r}_s|$ where $\mathbf{r}_s$, $t_s$, $\mathbf{r}$, and $t$ are respectively the source position and time, and the position and time of interest for the resulting wave. $c$ is the wave velocity.

Now imagine that we added on the surface $S$ a set of sources that is chosen to be always exactly equal and opposite to the sources $\mathsf{p}$, i.e., sources $\mathsf{p}_v = -\mathsf{p}$ . Since there are now no net sources on or near the surface, the wave $\mathsf{f}$ is everywhere exactly as it was before, i.e., it is simply the original incident wave $\mathsf{f}_{inc}$, i.e., now $\mathsf{f} = \mathsf{f}_{inc}$. The sources now calculated on the basis of the local wave amplitudes, $\mathsf{p} = \mathsf{C}\mathsf{f}\ (= \mathsf{C}\mathsf{f}_{inc})$, would therefore be the sources that, on their own, would exactly stop the *incident* wave, cloaking the volume, and leaving the wave otherwise unchanged; these sources, $\mathsf{p} = \mathsf{C}\mathsf{f}_{inc}$ are the ones that would give "predetermined" cloaking [9]. If we could deduce such source values on their own (i.e., without the cancelling sources $\mathsf{p}_v$,) as a result of causal measurements, we would therefore have solved the problem of true cloaking. The question is whether we can deduce them in a causal fashion.

The wave $\mathsf{f}$ that leads to these sources is the sum of the wave generated by the sources $\mathsf{p}$, the (exactly opposite) waves generated by the sources $\mathsf{p}_v$, and the original wave. Hence, what we can do, instead of actually having the sources $\mathsf{p}_v$, is merely to *calculate* what the wave would be from the sources $\mathsf{p}_v$, and mathematically add it to the signals from the sensors when setting the sources $\mathsf{p}$. The wave from such (now virtual) sources would be $\mathsf{f}_v = \mathsf{G}\mathsf{p}_v = -\mathsf{G}\mathsf{p} = -\mathsf{G}\mathsf{C}\mathsf{f}$. Adding that virtual wave to the real wave at the sensors, we therefore should set our sources for true cloaking to be, with $\mathsf{I}$ as the identity operator,

$$\mathsf{p}_{true} = \mathsf{C}(\mathsf{f} - \mathsf{G}\mathsf{C}\mathsf{f}) = \mathsf{C}(\mathsf{I} - \mathsf{G}\mathsf{C})\mathsf{f} \tag{3}$$

Mechanistically, we have therefore added a *calculated* amount $-\mathsf{G}\mathsf{C}\mathsf{f}$ to the measured $\mathsf{f}$ when determining the values of the sources to put on the surface. Note this "true" cloaking calculation is non-local; operating with $\mathsf{G}$ on $\mathsf{p}_v$ adds the effects from all these virtual sources at all the different points on the surface at the relevant prior times (we formally are integrating or summing over $\mathbf{r}_s$ and $t_s$).

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Equivalently, in the language of scattering, because we can know the sources on the surface at all prior times simply by noting them down as our causal interaction evolves, we can calculate the scattered wave from all prior sources. That scattered wave is $\mathsf{Gp} = \mathsf{GCf}$, and hence we can correct any subsequent sources by subtracting an amount $\mathsf{CGCf}$ so as to cancel the scattered wave, as in Eq. (3). To the extent that our calculation points in time are close, we can make this cancellation in principle as close to perfect as we wish. Cancelling the scattered wave leaves the wave outside the volume as the original wave. In essence, in a fully self-consistent manner, we have calculated rather than physically propagated the wave through the volume (though this is a slight understatement – we have also compensated for all the scattered waves from the surface sources).

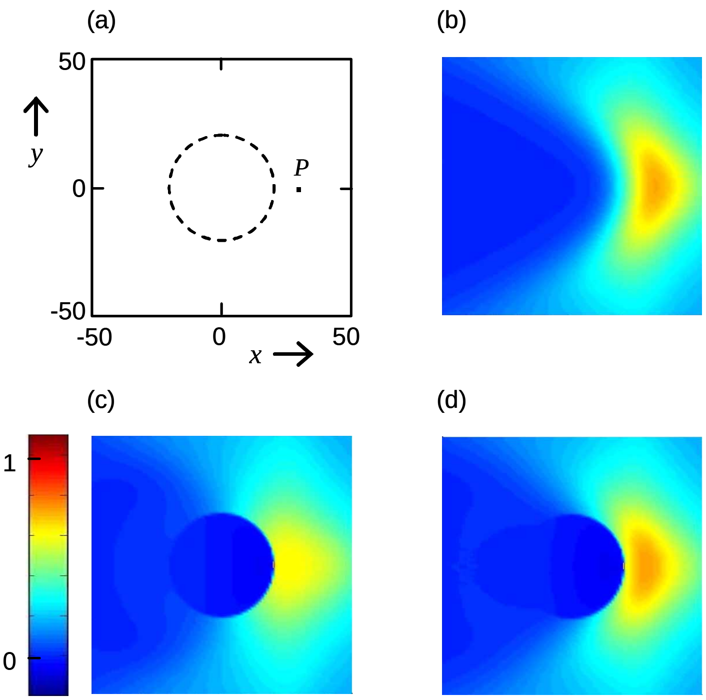


Fig. 2. Simulation results, shown for an equatorial (*x*-*y*) plane of the spherical volume to be cloaked, at a time $t$ = +25 in the still figures, for the propagation of a pulse from left to right through the volume. The vertical color bar shows the false color scale. (a) Coordinates, position of cloaked volume (dashed circle), and point $P$ for Fig. 3 results. (b) Movie of original unperturbed pulse (876 KB), focused approximately at the center of the volume, and arriving there approximately at time $t$ = 0. (c) Movie of approximate "quasi-cloaking" (930 KB), using surface sources with values derived only from local measurements. Significant distortion of the transmitted pulse is seen. (d) Movie of approximate "true cloaking" (894 KB), with surface source values calculated from the measured wave at current and past times at all the measurement points near the sphere surface; the propagated pulse is then substantially unchanged.

## 5. Causal simulations for scalar waves

In Fig. 2, we show a simulation of the cloaking. A short pulse wave, approximately Gaussian in time with an amplitude full width at half maximum (FWHM) of ~ 16.7 units, is traveling from the left and is focused towards the center. The wave velocity is one unit of distance per unit time, and there is one unit of time between each step of the simulation. Here, we construct a spherical cloaking volume of radius 20 units, distributing 3,264 source pairs approximately evenly in angle, and, at a radius $h = 2$ units further out, at the same angles, corresponding pairs of sensing points, with pair separation along the radii of $s = 0.1$ units for sense pairs and source pairs. The pictures show the resulting wave values on the equatorial plane of the sphere. (Note that the method described here is not, however, restricted to spherical surfaces.) We need finite separation $h$ between sources and sensors because we are approximating distributed sources per unit area with lumped source pairs, and the sensors consequently have to be separated from the sources by an amount comparable to or larger than the lateral separation of the sources for such an approximation to work.

To give some sense of dimensions for this example simulation, suppose we choose distance units of 1 cm in the simulation. Then we have a 40 cm diameter spherical cloaked volume with sensors 2 cm outside that sphere, and with an area of $\delta a \sim 1.54\ \mathrm{cm}^2$ "occupied" by each source pair on the sphere surface. If we consider ordinary acoustic waves in air, with a sound velocity of ~ 340 m/s, then the time units are (1 cm)/(340 m/s) = 29.4 µs, the time taken to propagate sound across the sphere would be 1.18 ms, and the pulse amplitude FWHM would be ~ 0.5 ms in this simulation, corresponding to frequency bandwidths in the kHz range.

We proceed step by step in time in a fully causal simulation using measured wave values to calculate the corresponding source values. For the "quasi-cloaking" case, the value chosen for each source at the current time is based only on known current wave measurements at the local sensors. For the "true cloaking" case, we use the local wave measurements together with the scattered wave correction as in Eq. (3) (i.e., $-\mathbf{GCf} = -\mathbf{Gp}$) calculated from the prior source amplitudes that we have recorded from previous steps in the simulation. In calculating the wave values from given sources, we linearly interpolate between the two time delays that bracket the actual (non-integer) time delay associated with the distance of interest between source, measurement, or graphing points. We also allow 2 units of time for the information from the sensors to propagate the 2 units of distance to the corresponding sources.

We chose to perform this simulation by calculating, at each time step and each sensor, the waves from each source, at an appropriately retarded time, using the Green's function $\mathbf{G}$. Note that we did not use a finite-difference time-domain approach, avoiding a calculation mesh and any possible problems with real source boundary conditions. The incident pulse was also directly generated by an explicit shell of sources to the far left rather than by defining a pulsed wave itself.

Because of the finite distance $h$, the cloaking calculations are necessarily approximate; we measure the wave at a finite distance from $S$, rather than exactly on $S$. Though no information is required to flow faster than the wave velocity in this simulation, we do presume instantaneous calculation in this example. Figure 3 shows the resulting pulse as a function of time at a point $P$ (Fig. 2) that is 29.5 units to the right of the center of the volume.

Here we see that the pulse is significantly distorted on the right after passing "through" the cloaked volume for the "quasi-cloaking" (local calculation) case. In time, it acquires a long tail extending ~ 40 units beyond the original pulse, which is consistent with the wave having to propagate a longer distance round the volume $V$ to stimulate the sources on the right hand side. The approximate "true-cloaking" (global calculation) case shows some small delay and a small negative tail. This delay and tail result from the finite actual delay in the wave propagating up to an additional 2 units out to the sensing points and a further 2 units for that information to propagate back to the sources. If in this simulation we set up the values of the sources on the surface based on prior knowledge of what the unperturbed wave should be on the surface ("predetermined cloaking"), then simulations with the same number of sources show essentially perfect cloaking of the volume, with a transmitted pulse indistinguishable on the scales of Figs. 2 and 3 from the original wave; the minor discrepancies for the

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approximate “true cloaking” in Figs. 2 and 3 do not result from the finite number of sources used, but rather from the finite separation between sensors and sources.

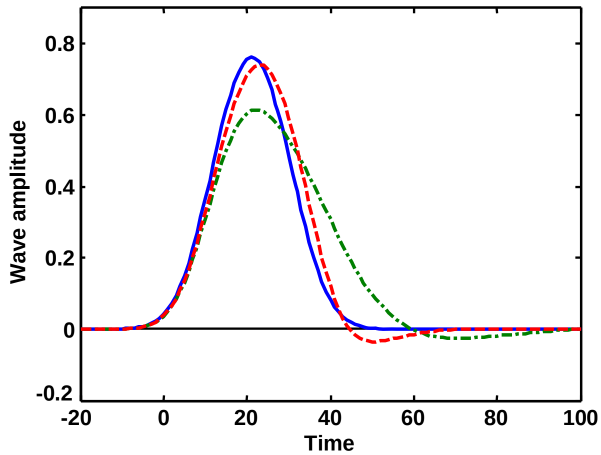


Fig. 3. Simulated wave amplitude as a function of time $t$ at a point $P$ on the $x$ axis 29.5 units to the right of the center, as shown in Fig. 2. Solid line – unperturbed wave. Dashed line – approximate “true cloaking”. Dot-dashed line – approximate “quasi-cloaking”.

Note that, in both the quasi- and true cloaking cases, the shielding of the field from inside $V$ is quite effective, and there is little if any backward scattered wave. Hence, from a practical point of view, both methods could provide good cloaking against reflective probing (as in typical sonar or radar). Both methods also do exclude the wave from the volume.

## 6. Discussion

### *6.1. Sources for cloaking of electromagnetic waves*

With the scalar waves we have considered so far, we have been able to use a simple, exact result in which specific surface sources can terminate or generate any wave. The corresponding mathematical result is known also for electromagnetic waves [13]. In that case, at least for everything other than static electric and magnetic fields, the appropriate surface sources involve surface current densities based on measured values of the two components of the magnetic field in the local plane of the surface, and surface magnetic current densities based on the corresponding two measured in-plane electric field components. Magnetic current densities are physical (even though magnetic monopoles apparently are not), and can be emulated by electrical current loops (or solenoids) with the loop axes in the local surface plane and with appropriate time-varying currents. With the in-plane **E** and **H** components thus determined, Maxwell’s equations then determine all the changing components of the fields perpendicular to the surface, so no other sources are then required for time-varying fields. (Static electric fields can also be handled with the addition of electric charge on the surface, though static magnetic fields are more difficult because of the physical absence of magnetic monopole sources.) Hence, at least in principle, we can construct a similar operator $\mathsf{C}$ with real physical sources on the surface for time-varying electromagnetic fields, and the process of cloaking could otherwise proceed analogously to that above for scalar waves. Note that we now have twice as many sensors and sources (two electric and two magnetic sensors and sources at each point on the surface) as in the scalar case; we would expect this since now we have to be able to handle two distinct polarizations of fields. If we were to set up a simulation analogous to the scalar simulation above, but for electromagnetic waves in free space, and

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take a similar 1 cm distance unit, and hence a 40 cm diameter volume, the time unit would be ~ 33 ps, and the pulse width would be ~ 0.55 ns, corresponding to GHz bandwidths.

All of the sources discussed so far are based on classical generation and detection of wave amplitudes. If we consider waves in the optical domain, it is not clear whether in practice we could take such an approach, though nanoantennas are becoming a possibility. Any quantum mechanical sensing and sourcing would have to be phase coherent with the incident waves.

### *6.2. Propagation and calculation time*

As noted, our approximate "true" cloaking simulation has assumed no additional calculation delay, and also implicitly presumes that the information can propagate across the inside of $V$ at least as fast as the wave velocity outside. For slowly propagating waves, such as acoustic waves, we could send the information at higher speed inside $V$ along wires or optical fibers around the inside of the surface $S$, leaving a clear, usable cloaked volume, and still have time for calculations. For electromagnetic waves in a vacuum, however, it could be difficult to make the calculation delay small compared to the time taken for light to propagate across the volume; also, to avoid excessively perforating the volume with wires, beams, or waveguides, we would certainly have additional propagation delay for information flowing through the volume. Such cloaking would be detectable in transmission with pulses of length comparable to the sum of the additional delays involved. Whether such an approach is then usable depends on the bandwidth or length of the probing signals.

### *6.3 Quantum detection of cloaking*

In general, such cloaking schemes could also be detected by quantum mechanical transmission probes. Consider an approach similar to the BB84 protocol for quantum key distribution [18,19]. There, the presence of eavesdropping by "Eve" in an optical mode between "Alice" and "Bob" can be detected as a result of quantum mechanical collapse onto eigenstates. Eve's measurement of the incident photon to extract information necessarily gives such collapse, and, because of the no-cloning theorem [20,21], she cannot guarantee to restore the original quantum mechanical superposition if she wants to retransmit the photon on to Bob; by random choice of basis representations by Alice and Bob, such collapse is easily detected as an increased error rate.

Since our active cloaking scheme involves measurement of the incident field, it too cannot avoid such collapse, and so any attempt at such cloaking of a volume by "Clovis" can be similarly detected. Even if Clovis is merely implementing a cloaking scheme that has to compensate for loss (as might be the case in meta-material implementations), he cannot do so without similarly creating errors because the no-cloning theorem [20, 21] prevents him from replacing the lost photons with ones in the same general quantum mechanical state.

Note, though, that in cloaking, in contrast to eavesdropping, Clovis has no need to know in a classical sense what the incident field is; as long as the volume is cloaked, Clovis need not classically record the state of the incident field. This speculatively leaves open the possibility that the calculations perhaps could be performed by some quantum computer, avoiding the necessity of quantum mechanical collapse, though the now quantum-mechanical sensing mechanism would need to be loss-less; any actual loss is easily detected classically, and any amplification to overcome loss is easily detected quantum mechanically. Note also that if Clovis is to be quantum-mechanically invisible to the outside for electromagnetic radiation, thus becoming a kind of "dark matter", then, for immunity to quantum probing, the outside must be invisible to him – he must not measure the incoming radiation.

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## 7. Conclusions

In conclusion, we have discussed how non-local response is required for true cloaking of a volume. In "quasi-cloaking", where the sources depend only on local values of the wave, the cloaking can be detected through the distorted transmission of a pulse of length comparable to or shorter than the volume (or by another signal of similar bandwidth). Because materials all have local response, no "invisible paint" is possible for true, causal cloaking. Even quasi-cloaking does, however, apparently give good invisibility to reflective probing.

We have shown how to achieve true cloaking based on wave measurement, giving a general formula, applicable in principle for any kind of linear wave, for how the necessary surface sources should be calculated. We have verified the concept with fully causal simulations for scalar waves. Cloaking for slowly propagating (e.g., acoustic) classical waves has no problems in principle, though cloaking for broad-band electromagnetic waves in a vacuum is challenging because there is little or no additional time available for any calculations or extra propagation. We have discussed also how any schemes that rely on measurement or that have to make up for losses should be detectable with a quantum transmission probe.

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