Better choices than optical angular momentum multiplexing for communications

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Using optical angular momentum (OAM) beams, Shi et al. (1) demonstrate multiple acoustic channels between arrays of sources and detection points. Even better channels with fewer resources are, however, possible. OAM is not an additional degree of freedom beyond normal spatial multiplexing; using only the OAM "topological charge," l particularly limits the possible spatial channels, with no obvious advantage (2).

Spatial multiplexing in optics is well known (see ref. 3 for a history). For continuous source and receiver functions on circular or rectangular surfaces, prolate spheroidal functions are optimal (3, 4). For arbitrary problems (3), the singular-value decomposition (SVD) of the coupling operator G between the source and receiver spaces gives the optimum orthogonal channels or "communications modes," with a sum rule (and generalized diffraction limit) $S = \mathrm{Tr}(\mathrm{G}^{\dagger}\mathrm{G})$ for the channel coupling strengths. Such truly orthogonal channels exist for any linear wave system (5, 6); they can be established automatically (7) and have fundamental importance in radiation laws (8).

For communications between parallel, concentric surfaces at wavelength λ, the number of available spatial channels within diffraction (and sum rule) limits is, approximately (3, 9), $N_C \sim A_S A_R / L^2 \lambda^2$, where A_S and A_R are the source and receiving surface areas, separated by distance L. This number is not increased by the use of OAM beams; such beams are merely another basis for describing spatial channels.

If only l is used to distinguish beams, omitting the additional orthogonal radial functions, then the OAM basis is not complete, and much of the "space-bandwidth product" (as in N_C) is not utilized. Even imaging an array of sources onto an array of detectors should be better (2).

We can construct an $N_R \times N_S$ coupling matrix G between N_S point sources and N_R point receivers at sets of positions $\mathbf{r}_{Sj}$ and $\mathbf{r}_{Ri}$, respectively, using the scalar Helmholtz wave equation Green's function (3) to give matrix elements $\mathrm{G}_{ij} = \exp(-2\pi i|\mathbf{r}_{Sj} - \mathbf{r}_{Ri}|/\lambda)/4\pi|\mathbf{r}_{Sj} - \mathbf{r}_{Ri}|$. The SVD of G yields orthogonal vectors of source amplitudes that couple to corresponding orthogonal vectors of received amplitudes, with power coupling strengths $\propto |s_m|^2$, where s_m are the corresponding singular values and $S = \mathrm{Tr}(\mathrm{G}^{\dagger}\mathrm{G}) \equiv \sum_m |s_m|^2$.

Calculating for 5×5 planar square arrays of 25 sources and receivers on 3.5 λ centers, and with the arrays separated by 100 λ, yields nine exactly orthogonal channels, all with similar power couplings (i.e., within ×1.5 or 1.5 dB) and using a total of ~86.5% of S. Assigning an effective area of $(3.5\lambda)^2$ to each source and receiver also gives $N_C \simeq 9$. These numbers show good use of space-bandwidth product and/or S. (Six additional orthogonal channels up to approximately −7.7 dB weaker account for a further ~12.8% of S.) Compared with Shi et al. (1), this gives more channels ($9 > 8$) with fewer sources ($25 < 4 \times 16 = 64$) and receiver points ($25 < 61$ to 64 in figure S5 of ref. 1), with smaller cross-sectional areas of source and receiver arrays, and avoiding the up to −8.5 dB cross talk of ref. 1. [The bounding areas for sources and detection in Fig. 1 of ref. 1 imply $N_C \sim \pi \times (12)^2 \times (22.4)^2/(100)^2 \sim 23 \gg 8$, showing relatively poor utilization of space-bandwidth product in ref. 1.]

The study by Shi et al. (1) provides an important demonstration of spatial multiplexing, but we should avoid restricting just to OAM beams; generally, we have both better options and powerful techniques to establish them.

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