

Electroabsorption of highly confined systems: Theory of the quantum-confined Franz–Keldysh effect in semiconductor quantum wires and dots

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Semiconductor quantum wells are known to show large electroabsorption (e.g., the quantum-confined Stark effect) that results in low-energy optical modulating and switching devices. We show theoretically that the electroabsorption and associated electrorefraction in lower dimensional structures could be much larger, suggesting very low energy devices. We illustrate the theory with specific calculations for hypothetical GaAs-like quantum wires and dots.

Electroabsorption in quantum wells (QW's), consisting of alternate thin (~ 100 Å) layers of two semiconductors, has been a subject of much recent interest, both for physics and applications.¹⁻⁴ The electroabsorption mechanism for fields perpendicular to the layers, the quantum-confined Stark effect^{1,2,4} (QCSE), is empirically markedly different in character from electroabsorption in bulk semiconductors (e.g., the Franz–Keldysh effect) (see, e.g., Ref. 5), allowing many novel optical devices, including light modulators and low-energy switches. There is also considerable current interest in structures confined in more directions, such as "one-dimensional" quantum "wires" and "zero-dimensional" quantum "dots" (QD's) or "boxes" (see, e.g., Ref. 6 and references therein). One question of interest here is whether this confinement offers any advantages over bulk or QW structures for nonlinear optical applications.⁶ In this letter, we discuss electroabsorption in quantum wires and dots, relating it to that in QW's; the relation between QW and bulk electroabsorption mechanisms is already qualitatively well understood.⁵ To understand the possible advantages of such confinement, we also consider specifically the extreme case of small QD's. We show that the electroabsorptive and electrorefractive effects in such structures, if they could be made, would be greatly enhanced over even the QW effects.

As a simplifying illustrative approximation, we neglect excitonic effects (i.e., the electron-hole Coulomb attraction), as is done for the bulk Franz–Keldysh electroabsorption, and as has been used to illustrate the relation between bulk and QW effects,⁵ resulting in the quantum-confined Franz–Keldysh effect (QCSE)⁵ model. We consider specifically a cuboidal structure, with infinitely high potential barriers at the box surfaces, and for simplicity presume a "two-band" semiconductor with only one isotropic hole mass. The Hamiltonian of the electron-hole pair becomes, in the effective mass envelope function approximation,

$$H = \frac{-\hbar^2}{2m_e} \nabla_e^2 + V_x(x_e) + V_y(y_e) + V_z(z_e) + e\mathbf{F}\cdot\mathbf{r}_e - \frac{\hbar^2}{2m_h} \nabla_h^2 + V_x(x_h) + V_y(y_h) + V_z(z_h) - e\mathbf{F}\cdot\mathbf{r}_h + E_G. \quad (1)$$

Here $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, where \mathbf{i} , \mathbf{j} , and \mathbf{k} are Cartesian coordi-

nate directions; the subscripts e and h refer to electron and hole, respectively; \mathbf{F} is the (uniform) field; E_G is the band-gap energy; e is the modulus of the electronic charge. Here also $V_x(x) = 0$ for $-L_x/2 < x < L_x/2$, and infinity otherwise, where L_x is the box dimension in the x direction, and similarly for $V_y(y)$ and $V_z(z)$ with corresponding box dimensions L_y and L_z . This problem separates *completely*, because of the neglect of the electron-hole Coulomb attraction and the choice of infinite, cuboidal potentials, into Schrödinger equations for each particle and direction:

$$H_{pq} \phi_{pq}^\nu = \left(\frac{-\hbar^2}{2m_p} \frac{\partial^2}{\partial r_{pq}^2} + V_q(r_{pq}) \pm eF_q r_{pq} \right) \phi_{pq}^\nu = E_{pq}^\nu \phi_{pq}^\nu, \quad (2)$$

where $p = e, h$ refers to the particle, $q = x, y, z$ indexes the coordinate direction, ν refers to the ν th eigenstate, and $+$ and $-$ refer to electron and hole, respectively. Now $H = \sum_{pq} H_{pq} + E_G$; the eigenfunctions of the electron-hole pairs are $\Psi^{lmnstu} = \psi_e^{lmn} \psi_h^{stu}$, where $\psi_e^{lmn} = \phi_{ex}^l \phi_{ey}^m \phi_{ez}^n$ and $\psi_h^{stu} = \phi_{hx}^s \phi_{hy}^t \phi_{hz}^u$. The electron-hole pair energy E^{lmnstu} is the sum of all six individual energies E_{pq}^ν plus the band-gap energy E_G .

The solutions of Eq. (2) are Airy functions,^{2,5} and hence this problem is exactly solvable for all field directions. The strength of the optical absorption (as expressed through the imaginary part ϵ_2 of the complex dielectric constant $\epsilon = \epsilon_1 + i\epsilon_2$) seen with an optical field inside the QD is

$$\epsilon_2(\hbar\omega) = C \sum_{l,m,n,s,t,u} |\langle \psi_e^{lmn} | \psi_h^{stu} \rangle|^2 \delta(\hbar\omega - E^{lmnstu}), \quad (3)$$

where⁵ $C = 8\pi^2 |p_{cv}|^2 e^2 / m_0^2 \omega^2 V$. Here p_{cv} is the interband momentum matrix element, m_0 is the free-electron mass, and V is the box volume. Incidentally, it is simple to derive sum rules similar to those for QW's⁴; the sum of all absorption strengths from a given electron level to all the hole levels is conserved with field (and similarly from each hole level to all electron levels), and the overall absorption is also conserved.

The simplest case to consider is that with the field along one coordinate direction (say x); then all transitions with

$m \neq t$ or $n \neq u$ are forbidden. As with QW's, at zero field only the transitions with $l = s$ are allowed, but with increasing field all of the transitions with $l \neq s$ become progressively stronger. For illustration, we have evaluated various Airy function solutions and their overlap integrals, and calculated the absorption $\alpha \propto \epsilon_2$ (assuming constant refractive index and neglecting variations $\propto \omega$ in the proportionality constant), replacing the singular δ functions by Lorentzians. Sets of results are shown in Fig. 1. Here we took

$m_e = 0.0665m_0$ and $m_h = 0.34m_0$, as appropriate for a two-band GaAs-like semiconductor, and used a Lorentzian half-width at half-maximum of 3 meV; this particular broadening is chosen only for illustrative clarity. For 10^5 V/cm applied along a 150 Å direction, we are in a moderately extreme condition for a QW (see, e.g., Ref. 5).

The $150 \times 1400 \times 1600$ Å³ cuboid shows behavior essentially identical to a 150 Å QW. With decreasing size down to $150 \times 175 \times 200$ Å³, the spectra become more and more concentrated into a few lines, as expected for a QD. Because the spectra with field also show forbidden transitions, there are many more lines in these cases, however, and the effect of quantum confinement in concentrating the oscillator strength into a small number of lines is partly lost. As the size of one of the dimensions is increased from 200 up to 3200 Å, the spectra gradually approach those of a quantum wire. [Following an analysis similar to that above for Eq. (3), but retaining a continuum density of states in one dimension (e.g., the z direction), gives the characteristic Van Hove singularity behavior seen in Fig. 1 for $150 \times 175 \times 3200$ Å³.] At the bottom of Fig. 1, we can see the anisotropy of the electroabsorption in a noncubic box, here $100 \times 175 \times 300$ Å³. For the field along the 100 Å direction, we obtain strongly quantum-confined behavior; for the field along the 300 Å direction, the low-energy absorption has degenerated into a weak absorption tail made up from many forbidden and allowed transitions.

Excitonic effects will generally enhance the absorption strength of individual lines in QD's; similarly, they will make the Van Hove singularities more enhanced in quantum wires. When the field is applied along a direction where the box dimension is comparable to or greater than the bulk exciton size, the lines or peaks will broaden because of the effective exciton field ionization.² If, however, a QD is much smaller than the bulk exciton, the e - h Coulomb attraction ($\propto 1/L$ where L is the dot's linear dimension) becomes rigorously negligible compared to the confinement energy ($\propto 1/L^2$), and this attraction is not strong enough to perturb the wave functions significantly. Consequently, the above QCFK picture actually becomes identical to the full excitonic QCSE for this case (except for the "binding energy" shift $\sim e^2/\epsilon L$ that is negligible compared to other terms). We recently concluded⁶ that, at least for the direct gap III-V semiconductors, it was theoretically possible to have a QD smaller than the bulk exciton, yet still large enough to be modeled as a crystalline semiconductor, as required for the present theory. In Fig. 2 we show the resulting spectra at various fields in the vicinity of the lowest transition ($l = m = n = s = t = u = 1$) for a hypothetical $100 \times 45 \times 45$ Å³ GaAs cuboid surrounded by infinite potential barriers. We have used a Lorentzian half-width at half-maximum of 0.5 meV.⁶

Because the Schrödinger equations [Eq. (2)] are identical to those for motion perpendicular to the layers in a QW,² the shifts of the peaks in the present QD case are identical to those in a QW of the same thickness L . However, the absorption that is being moved is much larger in the QD because of the increased confinement in the other directions. In the case of the QW, the entire subband is moved by the electric field,

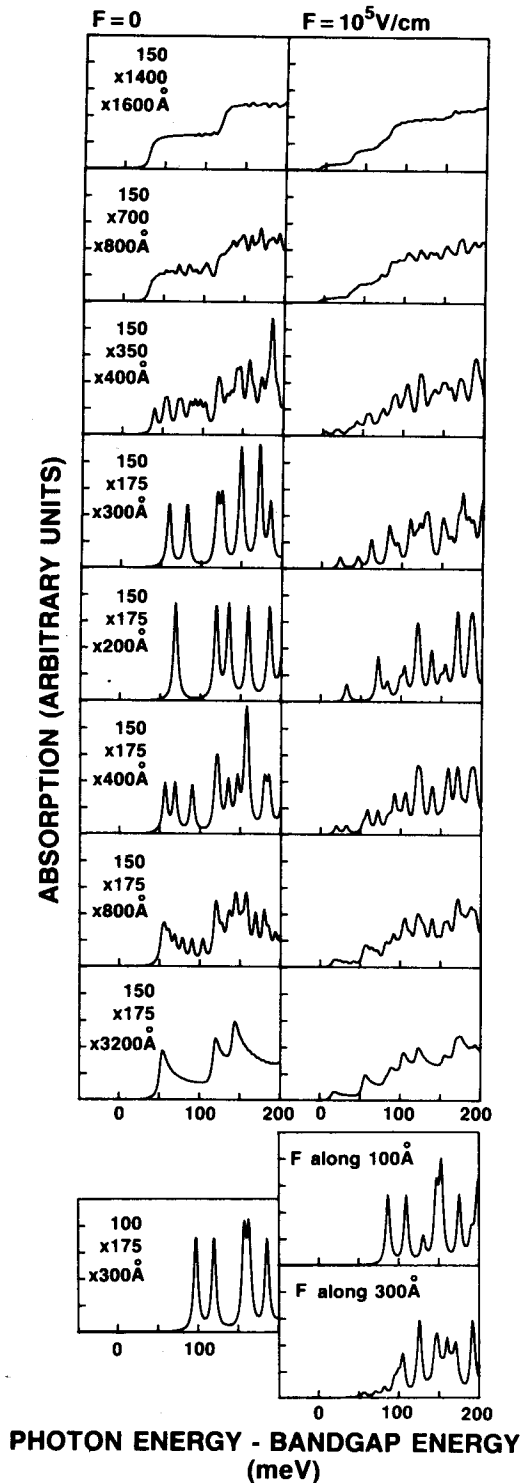


FIG. 1. Absorption with and without field F calculated for a GaAs-like semiconductor box in the quantum-confined Franz-Keldysh approximation. Field F is along the 150 Å direction except as noted.

but only the bottom edge is used for modulation. The situation in the wires and dots is that the states that formerly constituted the rest of the subband are now concentrated more strongly. The energy required to make this change in absorption or refraction is the stored electrostatic energy. Because the changes in absorption and refraction per unit field are larger, the quantum dot or wire device would require less energy for operation than the corresponding QW device. This energy scaling contrasts with the absorption saturation case⁶ where the change in absorption per unit energy is essentially the same in the QD as in the QW, although larger total changes of absorption and refraction can be made in the QD; in the present electroabsorption case, such large changes are also possible, but there is also a reduction in energy per unit change of absorption and refraction.

As to absolute numbers, the present calculation of a hypothetical (but not unphysical) structure is primarily meant to show trends. It is clear, however, that large benefits are possible; the absorption changes in Fig. 2 are more than an order of magnitude larger than the changes in absorption of a quantum well of the same thickness (100 Å) at the same fields. Another advantage of QD's is that the larger refractive index changes possible in this case could allow a refractive modulator capable of inducing half a wavelength of path length change in a single pass of light propagating perpendicular to the layers, a feature that opens up many options in refractive devices. This is not currently feasible in QW structures with total thickness of $\sim 1 \mu\text{m}$ without large absorption losses.⁷

All of the analysis in this letter considers only the optical properties in terms of uniform optical fields *inside* the QD. Of course, such microstructures can also show strong local field effects (see, e.g., Ref. 6) that can be particularly dramatic in the vicinity of the resonance, but their precise consequences depend very much on the density and shape of the microstructures and the medium in which they are embedded, and we will not consider them further here.

In conclusion, we have discussed the general form of quantum wire and dot electroabsorption, and illustrated how it relates to QW behavior. We have also given specific results for the extreme case of small QD's. The calculations are idealized in that we neglect surface effects and defects that may well dominate in current practical attempts to make such structures, and we are not aware of any method of fabricating the uniform arrays of identical QD's in the materials systems that we have treated here. The resulting electroabsorptive and electrorefractive effects are, however, shown to benefit from the quantum confinement to an even greater degree than that recently proposed for nonlinear absorption. The calculation is an existence proof that much better electro-optical effects are possible than are currently available. We also believe that the principles involved here that give rise to these enhanced electro-optic effects are not peculiar to our specific example, and other highly confined structures may acquire similar benefits. Such enhanced ef-

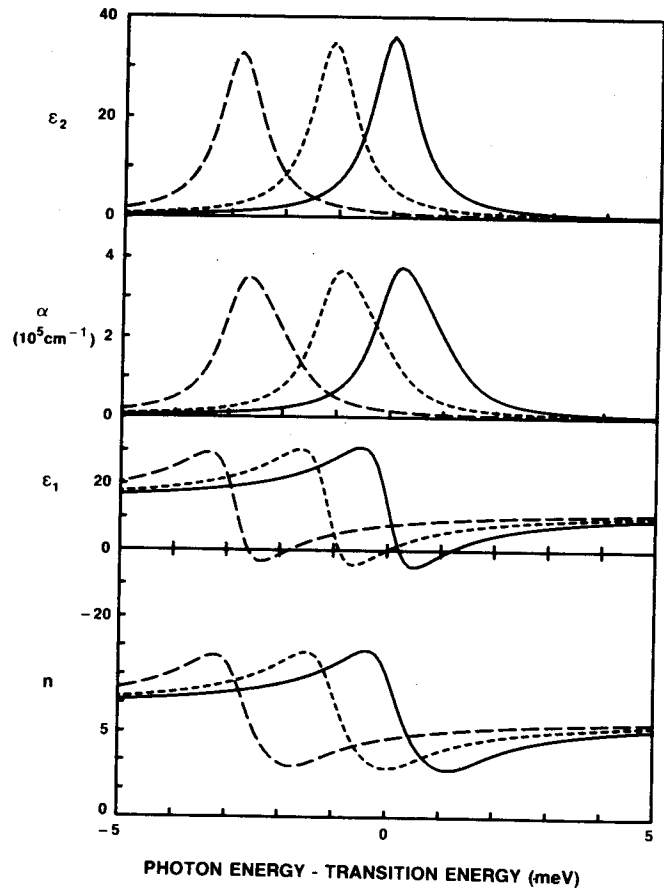


FIG. 2. Theoretical spectra, in the vicinity of the lowest transition, of the imaginary part of the dielectric constant, ϵ_2 , the absorption coefficient α , the real part of the dielectric constant, ϵ_1 , and the refractive index n , for a $100 \times 45 \times 45 \text{ \AA}^3$ GaAs-like quantum dot with field along the 100 Å direction: solid line, 0 V/cm; short dashed line, 3×10^4 V/cm; long dashed line, 5×10^4 V/cm. A transition energy of ~ 1.9 eV, a line full width of 1 meV, and a background dielectric constant of 13 are assumed.

fects would permit electroabsorptive and electrorefractive modulators and optical switches with even lower energy requirements than existing quantum well devices.

¹For a review, see D. A. B. Miller, D. S. Chemla, and S. Schmitt-Rink, in *Optical Nonlinearities and Instabilities in Semiconductors*, edited by H. Haug (Academic, Orlando, 1988), pp. 325-359.

²D. A. B. Miller, D. S. Chemla, T. C. Damen, A. C. Gossard, W. Wiegmann, T. H. Wood, and C. A. Burrus, *Phys. Rev. B* **32**, 1043 (1985).

³D. A. B. Miller, D. S. Chemla, T. C. Damen, T. H. Wood, C. A. Burrus, A. C. Gossard, and W. Wiegmann, *IEEE J. Quantum Electron.* **QE-21**, 1462 (1985).

⁴D. A. B. Miller, J. S. Weiner, and D. S. Chemla, *IEEE J. Quantum Electron.* **QE-22**, 1816 (1986).

⁵D. A. B. Miller, D. S. Chemla, and S. Schmitt-Rink, *Phys. Rev. B* **33**, 6976 (1986).

⁶S. Schmitt-Rink, D. A. B. Miller, and D. S. Chemla, *Phys. Rev. B* **35**, 8113 (1987).

⁷J. S. Weiner, D. A. B. Miller, and D. S. Chemla, *Appl. Phys. Lett.* **50**, 842 (1987).