
**Renewable energy and energy storage to offset diesel generators at expeditionary contingency bases**

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Abstract

Expeditionary contingency bases (non-permanent, rapidly built, and often remote outposts) for military and non-military applications represent a unique opportunity for renewable energy. Conventional applications rely upon diesel generators to provide electricity. However, the potential exists for renewable energy, improved efficiency, and energy storage to largely offset the diesel consumed by generators. This paper introduces a new methodology for planners to incorporate meteorological data for any location worldwide into a planning tool in order to minimize air pollution and carbon emissions while simultaneously improving the energy security and energy resilience of contingency bases. Benefits of the model apply not just to the military, but also to any organization building an expeditionary base – whether for humanitarian assistance, disaster relief, scientific research, or remote community development. Modeling results demonstrate that contingency bases using energy efficient buildings with batteries, rooftop solar photovoltaics, and vertical axis wind turbines can decrease annual generator diesel consumption by upward of 75% in all major climate zones worldwide, while simultaneously reducing air pollution, carbon emissions, and the risk of combat casualties from resupply missions.

Keywords

Expeditionary/contingency base planning; renewable energy and storage; energy efficiency

1 Introduction

The US Department of Defense (DOD) is the single largest consumer of fuel worldwide (1,2). In 2011 alone, the US military spent a reported $20 billion on air conditioning in Iraq and Afghanistan (3). Much of this cost was merely for transporting energy: “To power an air conditioner at a remote outpost in landlocked Afghanistan, a gallon of fuel has to be shipped into Karachi, Pakistan, then driven 800 miles over 18 days to Afghanistan on roads that are sometimes little more than ‘improved goat trails… and you've got risks associated with moving the fuel almost every mile of the way’” (3). In fact, for every gallon of fuel used in Afghanistan, seven gallons were needed to transport it there (4). Moreover, 18% of all US Army casualties in Iraq and Afghanistan were related to ground resupply operations, and between 2003 and 2007 alone, attacks on logistics convoys resulted in over 3000 wounded or killed in action (5,6). In
addition, the logistics required to assure energy security at military contingency bases (often called forward operating bases, or “FOBs”) is no small measure. In the first months of 2008, over 241,000 troops and over 200,000 contractors were deployed to the US Central Command theater of operations, and, at various times, over 500 FOBs existed in Iraq and Afghanistan (7–9). Approximately one-third of all wartime fuel is used by generators at FOBs, so there exists an opportunity to reduce the inefficiency of current energy consumption (10). As one general implored: “unleash us from the tether of fuel” (11).

After nearly 20 years, the United States still has FOBs in both Iraq and Afghanistan. Based on this commitment, the US Congress has enacted laws regarding the fully-burdened cost of fuel, energy resilience, and energy security (12). Briefly, the fully burdened cost of fuel is the commodity price plus the total cost of all personnel and assets required to move and protect the fuel from point of purchase to point of use; energy resilience is the ability to avoid, mitigate, and/or recover from anticipated and unanticipated energy disruptions; and energy security is having assured access to sufficient energy for mission essential requirements when and where it is needed (12). Given this mandate and limited resources in general, new methods are needed to substantially reduce energy consumption and cost for expeditionary bases.

The authors’ hypothesis is that significant reductions in required diesel resupply at expeditionary bases can be achieved by incorporating renewable energy, energy efficiency, and energy storage. Benefits include improved energy security and resilience, as well as reductions in capital and operations costs, air pollution, carbon emissions, and fuel-related convoy casualties. This paper presents a new optimization model that uses input data for base parameters (e.g., size, level of service, and climate), energy storage, solar photovoltaics (PV), Vertical Axis Wind Turbines (VAWTs), and the US Army’s new energy efficient building
(“hut”) design based on Structural Insulated Panels (SIPs). The model provides planners the
capability to study the impacts of building construction, commercial energy storage systems,
solar PV, wind turbines, Air Source Heat Pumps (ASHPs), and scale at any climate location
worldwide.

Previous studies have investigated only specific aspects of energy use at FOBs or unique
non-military applications. One study found that reducing energy demand, removing the
requirement for a spinning reserve, and allowing generators to operate at 100% of their rated
load produced the best results, while energy storage systems had effectively no impact on
generator run-hours or fuel consumption (13). This study drew conclusions from assessments of
2- or 24-h periods modeled using a theoretical optimization based on efficiency curves for a
common military generator (13). A related theoretical optimization study concluded that using
multiple sizes of generators, adding energy storage systems, and incorporating solar PV arrays
all produced significant fuel savings (14). The US Army has invested in model FOBs where it
can study innovative applications, such as the Future Capabilities Integration Laboratory
(formerly the Base Camp Integration Laboratory) at Fort Devens, Massachusetts. One study
used both theoretical modeling and tests at this FOB laboratory to conclude that the most
impactful technologies were smart microgrids and energy efficient shelters, while noting that the
assumed improved baseline conditions of larger FOBs resulted in lower savings (10). Other
studies from both academic institutions and government suggest that microgrids with energy
storage and scheduling management alone can reduce fuel consumption at FOBs by 20% to 30%
(15,16). In addition, researchers are developing optimization models for non-military
applications, such as the Food-Energy-Water Microgrid Optimization with Renewable Energy
(FEWMORE), which is meant to minimize capital, maintenance, and operations costs for remote
Arctic communities (17). These studies are useful, but they are limited in that they investigate a specific scenario or package of technologies. What remains missing is a tool where military planners can define their own combinations of available technologies to be employed in a desired location, be able to quantify potential benefits versus cost, and determine the best solution for a FOB before it is built.

This study is unique from previous studies in that it develops a new optimization model to take input data for contingency base parameters and quantify benefits from reduced fuel consumption to include reductions in costs, air pollution, carbon emissions, and fuel-related convoy casualties. Energy storage, solar PV, and climate parameters are found in other studies, but this model goes further to also investigate the use of VAWTs, new “SIP huts,” and expands the climates considered to include the polar region. The model provides planners the capability to study the impacts of building construction, commercial energy storage systems, solar PV, wind turbines, ASHPs, and scale. Rather than being limited to a specific time frame, the model uses year-long meteorological data for each of 8,760 h in a year and allows planners to test their own solutions for a potential contingency base located anywhere in the world.

Results demonstrate the imperative of bridging the gap between generalized planning factors and previous research with limited time scales or pre-defined technology packages. This model relies only on common and/or open-source software to facilitate knowledge transfer and use by both planners and researchers alike. Ultimately, the intent is to develop rules of thumb for manuals such that planners can better use energy efficiency, storage, and renewables at expeditionary bases to improve energy resilience while reducing air pollution, carbon emissions, and combat casualties. For a list of nomenclature and an in-depth description of all focus areas,
methods, assumptions, derivations, and calculations, please reference the Supplementary Information.

2 Methods

There are three major parts to this analysis: pre-processing, the optimization process, and post-processing. Pre-processing uses Microsoft Excel to receive model input for key design parameters and data files. The optimization process reads specific data from the pre-processing spreadsheets and uses IBM’s CPLEX solver within an optimization code written using the Julia programming language. The Julia code then passes results from the optimization process to another Excel spreadsheet for compilation, post-processing analysis, and graphing of results.

2.1 Pre-processing

2.1.1 FOB parameters

The pre-processing process begins in Microsoft Excel by allowing user input for FOB parameters. Data for the FOB’s location comes from Typical Meteorological Year version 3 (TMY3) data files. The model includes an example location for each of the five major Köppen Climate Classification Zones (A–E), but it also has a tab where users can input TMY3 data for any other desired location. Critical information from the TMY3 data file includes the location’s latitude and longitude and hourly values for outdoor dry bulb temperature, ground reflectance (albedo), air pressure, and wind speed (18). Data for planning factors, to include size, building square footage requirements, and peak power requirements, come from military publications (19–22). FOB size is based on the unit, population, and land area needed. From small to large, contingency base sizes include platoon, company, battalion, brigade, and support area (see Table 4 in Supplementary Information). Building square footage requirements include
needs for billeting, tactical operations centers, dining facilities, gymnasiums, shops, medical aid stations, laundry facilities, and so on. Peak power estimates depend upon the level of service provided at the FOB, typically referred to as basic, expanded, or enhanced. Data for construction type and energy efficiency of buildings come from studies on experimental buildings and test facilities at the US Military Academy and the US Army Corps of Engineers (23,24). These data permit calculation of the thermal index of construction options using established methods, which involves calculating the R-values for all windows, doors, walls, ceilings/roofs, and floors comprising the building envelope, as well as using blower door test data to calculate infiltration (25,26). The unimproved South West Asia (SWA) Hut serves as the baseline structure and the model calculates the cost of additional lumber and insulation for improved SWA Huts as well as the cost of specialty panels for SIP Huts.

**2.1.2 Electrical load**

Next, the model generates a mock load for the analysis, relying upon previous studies with a 24-h load profile (10). However, rather than repeat the same load every day throughout the year, this model introduces a randomized variable that serves to vary the load from the baseline profile within established boundaries. Furthermore, the model decreases this load profile to make it represent lighting and plug loads, but then also introduces Heating, Ventilation, and Air Conditioning (HVAC) loads that increase the load profile even further, while ensuring the FOB’s location and climate impacts overall load requirements. The model allows for a user-defined building internal temperature set-point, which, when combined with TMY3 data for ambient temperature and construction type thermal index values, facilitates the calculation of space conditioning requirements. Rather than military-grade Environmental Control Units, this model considers the use of more efficient civilian ASHPs. Manufacturers of ASHPs publish
values for the Heating Seasonal Performance Factor (HSPF) and the Seasonal Energy Efficiency Rating (SEER), which relate the average coefficient of performance over the heating and cooling seasons, respectively. This model applies a correction to HSFP and SEER values to reflect the impact of climate on ASHP performance based on the 99% heating and 1% cooling design temperatures for each location (27–29). This calculates a more accurate HVAC load that is currently absent from most planning factors.

2.1.3 Renewable energy resources available

The model next calculates the renewable energy resources, namely solar and wind, available at a specified location. For the solar resource analysis, a user can define an analysis year, from which the model calculates the Julian day and century (2000 standard epoch). The model uses this time data, the location’s latitude and longitude, and astronomical equations (30) to calculate the Sun’s position at every hour of the year. The model uses three different methods to calculate solar position (31–33); example results are compared in the Supplementary Information. The model then calculates the total insolation on a collector, which is the summation of direct, diffuse, and reflected radiation, for both clear- and all-sky insolation scenarios. By comparing the three methods and two scenarios, one can draw conclusions about model complexity versus precision of results. Furthermore, future application of this model to an experimental FOB will allow the analysis of that precision against measured data to assess model accuracy. To determine solar PV electricity production, this model adopts the National Renewable Energy Laboratory’s (NREL’s) PVWatts methodology for calculating transmittance through anti-reflective coatings and the glass of PV panels, as well as a correction for the cell operating temperature (34). The model adopts the same efficiency levels (module and inverter efficiencies and other system losses like soiling, shading, snow, mismatch, wiring, connections,
light-induced degradation, nameplate rating error, age, and availability) and PV characteristics (nominal operating cell temperature, power temperature coefficient) as used in NREL’s PVWatts program for premium PV panels. The model also allows for user input on both rooftop and utility solar installations. For rooftop installations, this study uses buildings oriented with panels facing solar south (if in the Northern Hemisphere, opposite for the Southern Hemisphere) and the panels have tilt angles equal to the roof pitch. Cost estimates use published data for residential installations and include the capital cost of panels, mounts, inverters, wiring, and all balance of plant equipment, less any tax benefits generally included in such reported values.

For the wind resource analysis, a user can select a wind turbine and input manufacturer’s published data for the power curve; rotor swept area; height; rated and maximum power output; cut-in, cut-out, and survival wind speeds; and efficiency (35). This study uses VAWTs, as opposed to Horizontal Axis Wind Turbines (HAWTs), due to their ability to achieve higher wind farm power densities and lower hub heights (36). Using gin poles and winches, it is likely possible to erect VAWTs in the field without lift assets, even with turbines weighing several hundred pounds. Alternatively, the US Army has cranes and trained operators that could help install VAWTs. Users of the model can define a friction coefficient from a pre-defined drop-down list to account for surface ground conditions, although this study uses the “1/7th rule-of-thumb” for open land throughout the analysis for all locations. The model takes hourly air pressure and wind speed data from the TMY3 files to calculate the hourly corrected air density and wind speed at turbine mid-point height. Using an estimated number of turbines (user-defined with consideration of total FOB land area requirements from published planning factors), spacing, an estimated utilization factor, and an aerodynamic loss factor, the model calculates the annual wind farm energy production and capacity factor. Cost estimates use published
manufacturer catalogue prices for turbines, controllers, inverters, towers, and ancillary equipment (37).

2.1.4 Energy storage

For energy storage, the model allows for input on battery characteristics, to include energy capacity, continuous and peak power, and charge/discharge round-trip efficiency. These parameters can reflect either centralized or distributed energy storage solutions. This study uses data for distributed batteries installed in buildings that can be connected to rooftop solar (38); however, in either case, the model treats all batteries as being fully connected on a FOB microgrid. Additionally, this paper takes manufacturer-reported “useable capacity” to mean 100% of the modeled battery’s range of charge/discharge. However, users of the model can just as easily input their own maximum depth of charge/discharge in order to model the use of controls that can help prolong the lifetime of batteries, which may or may not be important to planners based upon the FOB’s purpose.

2.1.5 Diesel generators

The model uses 60 kW diesel generators for platoon- and company-sized FOBs and 840 kW prime power diesel generators for battalion-, brigade-, and support area-sized FOBs (39,40). Users can define a percent overage of diesel generator capacity in order to allow for redundancy, specifically to facilitate repairs, maintenance, and downtime. In addition, users can input minimum and maximum load fractions to define allowable generator loading conditions. The cost of diesel is based on current prices, historical trends, and studies on (and the legal requirement to use) the fully burdened cost of fuel (12,41–44). The fully burdened cost of fuel is highly dependent upon the costs of transport, personnel, sustainment, and air and ground force protection in addition to the cost of the fuel itself. Due to the large sensitivity this has on the cost
analysis, this study adopts a conservative approach and uses a dollar per gallon value that reflects only the fuel commodity, transport, sustainment, and ground force protection components. This value is just $\frac{1}{3}$rd the estimated base case fully burdened cost of fuel in Iraq in FY07 (or $\frac{1}{4}$th that value when adjusted to FY20 dollars) (45). Nevertheless, sensitivity in fuel cost only affects the estimated simple payback results. When considering resilience, the model’s reported percent reduction in the volume of diesel consumed is unaffected by cost, which is further explained in section 2.2.

### 2.2 Optimization process

A text editor (Atom), runs integrated development environment (IDE) software (Juno), which itself uses a statistical programming language (Julia), to execute IBM’s optimization solver software (CPLEX) (46–49). All programs are open-source, with the exception of IBM’s CPLEX, which is offered free of charge to students and academics.

The Julia code pulls data from the pre-processing spreadsheets for use in mixed integer linear programming (MILP) with binary variables for diesel generators (on/off). The optimization program seeks to minimize the total cost of diesel and any curtailment, subject to the following constraints, variable constraints, and expressions (see the Supplementary Information for mathematical representation and code):

#### Constraints:

1. The overall FOB energy balance at every hour is such that the summation of the battery energy used, the total energy produced by all diesel generators that are on, the energy from solar PV, and the energy from wind turbines is equal to the summation of energy demand (load), energy stored in batteries, and energy curtailed.

2. The initial battery energy storage starts at the minimum (i.e., zero).
3. The battery energy balance is such that the energy stored at the beginning of the next hour is equal to the battery energy stored at the beginning of the current hour, plus battery energy stored in that hour, less battery energy used in that hour.

4. The battery energy stored in any hour cannot exceed the summation of the energy produced by the diesel generators, solar PV, and wind turbines in that hour.

5. The battery energy used in any hour cannot exceed the battery energy stored at the beginning of that hour.

6. The diesel generators can run only within a user-specified minimum and maximum load fraction to avoid wet stacking and severe underloading of generators.

Variable constraints:

1. Diesel generator on/off is binary.

2. The energy stored in the batteries at any hour must be greater than or equal to the minimum (zero, a positivity constraint) and less than or equal to the maximum battery capacity.

3. Limitations on battery charging/discharging rates limit the energy stored/used from the batteries in any hour.

4. Energy produced in any hour by the diesel generators and energy curtailed have positivity constraints (the model changes the sign for curtailment to negative later in post-processing for graphing purposes).

Expressions calculate the:

1. Hourly energy produced by all generators turned on.

2. Hourly diesel cost of all generators turned on.

3. Total penalty cost for any curtailment.
4. Total fuel cost and curtailment penalty.

Output from the optimization process includes hourly energy produced by diesel generators, the energy storage level in batteries at the beginning of the hour, the battery energy consumed in that hour, and energy curtailed (if any) as well as the annual volume of diesel consumed and the corresponding annual fuel cost. The optimization program also serves to transfer key data needed from pre-processing spreadsheets to post-processing spreadsheets for further analysis, to include the hourly power load, solar and wind power production, and the additional upfront costs for more energy efficient buildings, battery energy storage, solar PVs, and wind farms used in each scenario.

2.3 Post-processing

Post-processing involves compiling output data from multiple runs of the optimization process in order to graph the results. The results for each climate zone shown in section 3 required compiling a minimum of 51 different iterations, 17 per hut type (unimproved SWA Hut, improved SWA Hut, and SIP Hut). To get a general sense of possible solutions, this study assumes maximum limits for battery, solar, and wind nameplate installations:

1. Up to ten batteries (13.5 kWh storage capacity each) can be installed in each building (hut).

2. Solar PV can only be installed on hut roofs with industry-recommended offsets from roof edges.

3. Wind farms can take up no more than 10% of the prescribed land area for each contingency base size.

These assumed maximum installations are then divided into quarter increments to run simulations using zero, ¼, ½, ¾, and 100% for batteries alone, solar alone, and wind alone.
Additional simulations use all three added to a FOB’s energy portfolio simultaneously in the same incremental amounts. Depending upon the comparison examined, a baseline scenario (generally unimproved SWA Hut construction) defines what improvements can occur. The calculation of simple payback lines uses projected Consumer Price Index (CPI) values for the next two years (50). The model does not change the cost of diesel in future years because data show that, for the ten years between 2009 and 2018, the annual average global price of diesel fluctuated (both positive and negative) between a low of $3.22 gal\(^{-1}\) and a high of $4.35 gal\(^{-1}\) (43). The uncertainty in diesel cost, even without considering the fully burdened cost of fuel, makes forecasting a diesel cost value over the short term have little to no useful meaning.

3 Results

Figure 1 consolidates results for a battalion-sized FOB with an expanded level of service, representing a middle FOB size and median level of service. Figure 1 shows three graphs for each of the five major Köppen Climate Classification Zones (A-E), each corresponding to a selected building construction type. Graphs at left illustrate the baseline condition of unimproved SWA Huts (what the military typically builds in expeditionary environments), in the middle are improved SWA Huts (with additional insulation), and at right are very energy efficient SIP Huts (a new design being tested by the US Army). The left vertical axis reflects annual savings in diesel fuel consumed in millions of dollars. The right vertical axis converts this value to the annual offset of energy demand from diesel generators. The horizontal axis reflects estimates for additional upfront capital cost. In general terms, increasing along the x-axis translates to more money invested upfront, while increasing along the y-axis translates to more money saved. The lines radiating from each baseline condition describe the potential benefits of incorporating battery energy storage alone (green), rooftop solar PV alone (amber),
wind turbines alone (blue), or all three in combination (black). The length of the amber and blue lines indicate design-specified limits on nameplate installations, namely available rooftop area for PV and 10% of estimated FOB land area requirements for wind farms. The green lines have a point at which their slope flattens, representing the point at which additional battery nameplate installations continue to cost more upfront but do not provide additional benefit in reducing diesel consumption (batteries can only store energy, not generate it). In addition, yearly simple payback lines indicate that solutions above (to the left) of each line represent options with a positive return on investment (ROI) within that timeline. Only 1-, 2-, and 3-year simple payback lines are shown, although planners may or may not know an anticipated lifetime for a contingency base.

Full explanations of all assumptions are in the Supplementary Information; however, there are three major considerations that deserve note here. First, although the US Army estimated the fully burdened cost of fuel in Iraq as between $9 and $45 gal\(^{-1}\) depending upon delivery distance and type of protection (ground or air) used (51,52), this study uses a fully burdened cost of fuel of just $8.32 gal\(^{-1}\), which reflects only some of those costs converted to FY17 dollars (44). This model does not include component costs for materiel and personnel; it is assumed the Soldiers are already deployed, their salaries are already paid, and the military vehicles are already purchased and transported to the theater of operations. Although there is an opportunity cost in that the Soldiers and materiel could be used to accomplish other tasks if they were not conducting resupply convoy missions, that is beyond the scope of this study. Second, construction labor costs are not considered. SIP huts are faster to build than SWA Huts (24), but additional labor is required for unpacking and installing batteries, solar PV, and wind turbines. Third, the additional upfront transportation costs of additional building materials, batteries, PV
panels, wind turbine components, and all ancillary equipment are not covered here. All of these factors are highly dependent upon the actual location of a FOB and are best left for further analysis, if desired. The focus here is on the energy resilience of a FOB once it is established. The model uses cost valuation only as a proxy for resilience due to its usefulness in the optimization process and its proportionality to the amount of fuel that must be purchased, transported, delivered, and stored at a FOB in a reliable manner.

Figure 1 shows that the energy efficiency of buildings is critically important for bases outside of the tropics, although even the tropics can expect a positive ROI depending upon how long the FOB is in use (see simple payback lines). In addition, incorporating battery energy storage is the next best investment if done alone. In each scenario, there is a point at which additional energy storage is no longer useful in reducing diesel consumption because, without renewables, diesel generators must still produce power for the FOB. The batteries provide a benefit by allowing generators to work at their optimal capacity rather than be forced to follow a load or even dump load. However, once sufficient batteries are installed to reach the potential of this benefit, additional batteries simply result in more upfront cost with no return. Batteries can store and release energy, but they cannot generate energy. Additionally, batteries have a round-trip (charging, discharging, and inverter) efficiency which introduces energy loss.

The results shown allow for a maximum load of 100% rated capacity for each generator, although the model facilitates imposing a limit (e.g., 80%) in order to leave a spinning reserve for peak loads as done by many microgrid management systems (13,16). Rather than underloading a generator, microgrid management software can divert excess energy generation to battery storage for use later on when generators are turned off, and batteries can serve the role of providing peak power within their discharge limits. One can also see in Figure 1 that the
benefits of PV and wind turbine installations are location-specific with wind performing better in some locations and solar in others. Which resource performs best depends upon the FOB’s specific location (not necessarily the climate zone) and is determined using data from each location’s TMY3 data file. For the SIP Hut FOB in a continental climate, an independent solution reflects nameplate installations of 6.9 MWh storage, 94 kWDC solar, and 155 kW wind. Combinations of batteries, solar, and wind need not adhere to the proportional increases shown by the black line stretching from zero to the assumed maximum. This independent solution achieves an ROI within one year and offsets 60% of annual diesel consumption. Planners can use this model to test different scenarios and find a solution to fit any given situation.
Potential for increased energy resilience as measured by reduced reliance on diesel generators for electric power.

Results are for a battalion-sized FOB with an expanded level of service arranged by climate zone and construction type. The baseline business-as-usual scenario uses only diesel generators for power production and unimproved SWA Hut construction. The cost of diesel uses a fully burdened cost of fuel of $8.32 gal⁻¹. Simple payback lines use a CPI of 2.4% at two years and 2.6% at three years with no change in the cost of diesel due to typical +/- price fluctuations. Solutions above (to the left) of simple payback lines represent positive ROI within the defined time period. y-axis values change between climate zones.
From any of the simulated scenarios, one can produce graphs like those shown in Figure 2 to investigate the FOB’s energy portfolio balance over a desired time period. Figure 2 shows a support-area sized FOB with an enhanced level of service in an arid (dry) climate on 21 June, on or about the summer solstice. The time, in hours of the year, is on the horizontal axis and power, in megawatts, is on the vertical axis. Lines denote power whereas shaded areas represent the product of power and time, i.e., energy in megawatt-hours. Figure 2(a) shows the business-as-usual scenario, which would require up to 17x 840 kW generators running for at least 3 h of the day to follow and meet loads. Figure 2(b) shows that this same FOB can reduce to a maximum of just 4x 840 kW generators running for 4 h of the day with zero generators needed for ten of 24 h. The reduction in needed generators is due to three factors: first, a decreased load from more energy efficient SIP Huts, which decreased HVAC requirements; second, the application of both solar and wind power to meet load requirements; and third, battery energy storage facilitating the management of generators switching on or off.

Furthermore, it is interesting to note an occurrence of wind power production exceeding the total wind farm nameplate rating. This occurrence is the result of wind speeds within that favorable range of the wind turbine power curve (see Figure 12 in the Supplementary Information) where output power exceeds the nameplate rating, which can actually be 35% higher for some wind turbines (35).
Figure 2. Potential for increased energy resilience as measured by reduced reliance on diesel generators for electric power.

Results are for a support area-sized FOB (6,000+ people) with an enhanced level of service in an arid (dry) climate on 21 June. (a) uses unimproved SWA Huts and only diesel generators for power, resulting in load following with a requirement of 17x 840kW generators for several hours of the day. (b) uses energy efficient SIP Huts, which decreases energy demand due to lower HVAC loads. Adding 41 MWh battery energy storage, 11.3 MW_{DC} solar PV, and 9.3 MW wind turbines results in a significant reduction of diesel generators required with zero generators required for ten of 24 h.
Figure 3 illustrates a potential pitfall when incorporating renewables without energy storage. In this case, a company-sized (300-person) FOB with a basic level of service is in an arid climate on 21 June with 570 kWDC PV and no battery energy storage. The result is similar to the “duck curve” first shown by the California Independent System Operator in 2013 (53), so called because the line showing net load on the generators looks like the silhouette of a duck with its tail on the left, its back in the middle, and its neck, head, and bill on the right. In situations where renewable power production exceeds load requirements and there is no energy storage available, the result is curtailed energy – a reduction in energy output from what could have been produced. Also problematic is the combination of a setting sun during the hours of typically increased loads, which requires a rapid ramp rate for diesel generator-supplied power in the late afternoon/evening.

![Figure 3. The “duck curve.” Results are for a company-sized FOB with a basic level service in an arid (dry) climate on 21 June. The incorporation of solar PV without energy storage creates a scenario where the FOB cannot use excess production and must curtail it.](image-url)
Figure 4 shows the impact of scale and energy efficiency of buildings for platoon-, company-, battalion-, brigade-, and support area-sized FOBs with an expanded level of service in a continental (cold) climate with zero renewables or energy storage. The size of each marker reflects the comparative size in FOB population. Shown are three scenarios: going from unimproved SWA Huts to improved SWA Huts, going from unimproved SWA Huts to SIP Huts, and going from improved SWA Huts to SIP Huts. For a FOB located in a continental (cold) climate, the extra energy efficiency of SIP Huts results in annual savings over both the unimproved and improved SWA Hut construction options. These graphs illustrate two important concepts. First, savings from energy efficient buildings scale with base size. Second, the relative degree of savings decreases when the energy efficiency of baseline construction improves, which correlates with the Pacific Northwest National Laboratory’s (PNNL’s) study where they assumed the largest bases start off with an improved baseline condition and, consequently, their modeled savings for the largest bases were lower than all the others. In order to maintain a constant point of comparison, this study maintains a baseline of unimproved SWA Huts using only diesel generators for all scenarios.
Figure 4. The impact of scale and energy efficient construction on savings. Results are for platoon-, company-, battalion-, brigade-, and support area-sized FOBs with an expanded level service in a continental (cold) climate, each with reference to a FOB of the same size using the baseline construction type shown. Solutions above (to the left) of the simple payback line represent positive ROI within one year. Although this study uses different buildings, the concept illustrated by the difference between the middle and right graphs agrees with PNNL’s findings in Engels et al. (10) that savings will be less when a higher baseline scenario is assumed.

Table 1 illustrates the capabilities of this model to predict offset energy demand and reduced costs, air pollution, carbon emissions, and casualty prevention for a specified scenario and climate. Shown are results for a battalion-sized FOB with an expanded level of service, SIP Hut construction, and nameplate installations of 6.86 MWh storage, 1.89 MWDC solar PV, and 1.55 MW VAWTs. Such installations require significant additional upfront transportation. Of the styles modeled in this paper, it would take about 508 batteries, 6100 solar panels, and 774 wind turbines, all with associated equipment (foundations, poles, mounting hardware, inverters, wiring, etc.) to satisfy these specifications. The extra building materials for SIP Huts alone would require 121 additional 20 ft shipping containers. Nevertheless, reduced diesel consumption means fewer vehicles and convoy missions later. Using values from Eady et al. (54) for the volume of fuel per truck and considering a FOB in an arid climate, 131 fewer fuel
trucks would be required over the course of a year, and this does not include the numerous additional force protection assets required to support those trucks on multiple convoys. Also, this value is largely dependent upon the fuel trucks used and their volumetric capacity. If the trucks used were a standard military fuel servicing tanker carrying 2500 gallons of fuel, the result would increase to 322 fewer fuel trucks. In any case, additional transport upfront reduces reliance on resupply later.
Table 1. Benefits of implementing energy efficient buildings, energy storage, and renewables at FOBs in terms of offset energy, diesel savings, reduced air pollution and carbon emissions, and avoided casualties.

Results are for a battalion-sized FOB (1,000 people) with an expanded level of service (a median estimated load requirement) using 840 kW diesel generators. Improvements include energy efficient SIP Hut construction (+$1.62 million), and nameplate installations of 6.86 MWh battery storage (+$3.86 million), 1.89 MWDC solar PV (+$7.55 million), and 1.55 MW wind turbines (+$5.60 million), for a total of $18.6 million in additional upfront capital expense. All table values are in terms of annual offset FOB-only diesel requirements for a single FOB with 1,000 people. For reference, in 2QFY08, there were over 441,000 military and civilian contractors deployed in support of contingency operations. In practice, savings will be even greater due to reduced vehicle fuel consumption for logistical transport requirements, which will vary significantly dependent upon the fuel’s point of purchase and the FOB’s location.

<table>
<thead>
<tr>
<th>Savings per 1,000-person FOB, i.e., reductions in...</th>
<th>Units</th>
<th>Tropical</th>
<th>Arid</th>
<th>Temperate</th>
<th>Continental</th>
<th>Polar *</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Energy</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Energy demand</td>
<td>GWh year⁻¹</td>
<td>9.29</td>
<td>10.8</td>
<td>11.7</td>
<td>14.1</td>
<td>37.2</td>
</tr>
<tr>
<td>Diesel required</td>
<td>% year⁻¹</td>
<td>83</td>
<td>80</td>
<td>75</td>
<td>80</td>
<td>88</td>
</tr>
<tr>
<td>Diesel cost, low</td>
<td>FY20 USD year⁻¹</td>
<td>$1.84 million</td>
<td>$2.14 million</td>
<td>$2.32 million</td>
<td>$2.79 million</td>
<td>$7.37 million</td>
</tr>
<tr>
<td>Diesel cost, this paper</td>
<td>FY20 USD year⁻¹</td>
<td>$5.75 million</td>
<td>$6.70 million</td>
<td>$7.26 million</td>
<td>$8.74 million</td>
<td>$23.0 million</td>
</tr>
<tr>
<td>Diesel cost, high</td>
<td>FY20 USD year⁻¹</td>
<td>$22.5 million</td>
<td>$26.2 million</td>
<td>$28.4 million</td>
<td>$34.2 million</td>
<td>$90.1 million</td>
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<tr>
<td><strong>Air pollution</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Non-methane hydrocarbons, NMHC</td>
<td>tonne year⁻¹</td>
<td>1.76</td>
<td>2.05</td>
<td>2.23</td>
<td>2.68</td>
<td>7.07</td>
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<td>Nitrogen oxides, NOₓ</td>
<td>tonne year⁻¹</td>
<td>6.22</td>
<td>7.24</td>
<td>7.86</td>
<td>9.46</td>
<td>24.9</td>
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<tr>
<td>Particulate matter, PM</td>
<td>tonne year⁻¹</td>
<td>0.28</td>
<td>0.32</td>
<td>0.35</td>
<td>0.42</td>
<td>1.12</td>
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<tr>
<td>Carbon monoxide, CO₂</td>
<td>tonne year⁻¹</td>
<td>32.5</td>
<td>37.8</td>
<td>41.1</td>
<td>49.4</td>
<td>130</td>
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<td><strong>Carbon emissions</strong></td>
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<td></td>
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<tr>
<td>Methane, CH₄</td>
<td>tonne year⁻¹</td>
<td>0.28</td>
<td>0.33</td>
<td>0.36</td>
<td>0.43</td>
<td>1.14</td>
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<tr>
<td>Nitrous oxide, N₂O</td>
<td>tonne year⁻¹</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.22</td>
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<tr>
<td>Combined carbon dioxide equivalent, CO₂</td>
<td>tonne year⁻¹</td>
<td>7080</td>
<td>8240</td>
<td>8940</td>
<td>10,760</td>
<td>28,400</td>
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<tr>
<td><strong>Casualties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WIA or KIA</td>
<td># people year⁻¹</td>
<td>0.20</td>
<td>0.23</td>
<td>0.25</td>
<td>0.30</td>
<td>0.80</td>
</tr>
</tbody>
</table>

FOB: forward operating base

*a Uses unimproved SWA Huts as the baseline, like all other climate zones, in order to provide consistency of the compared scenario; however, it is unlikely uninsulated buildings would ever be used in such a climate.

*b Uses the Defense Logistics Agency’s (DLA’s) (42) FY20 United States Dollar (USD) standard purchase price for diesel #2 (fuel only).

*c Uses the FY20 USD for DLA purchase price of diesel #2, plus transport, sustainment, and force protection (ground) costs (42,44).

*d Uses the fully burdened cost of fuel from a FY07 Iraq base case adjusted to FY20 USD and includes costs used in this paper (see *) along with personnel and force protection (air) costs (44,55).

*e Assumes generators meet minimum US Environmental Protection Agency (EPA) Tier IV standards (56).

*f Assumes all particulate matter (PM) ≤ 10μm (PM10) and 97% of PM is smaller than 2.5μm (PM2.5) in accordance with US EPA nonroad compression-ignition engine modeling (57).

*g Uses US EPA values for direct emissions from stationary combustion sources, diesel #2 (58).

*h Uses 100-yr global warming potential values from the Intergovernmental Panel on Climate Change (IPCC) (59).

*i Uses an Army Environmental Policy Institute (AEPI) report and FY07 values for Wounded In Action (WIA) and Killed In Action (KIA) fuel-related convoy statistics in Iraq and Afghanistan (54).
4 Discussion

With regard to previous work, these findings largely confirm those in the PNNL (10) and Naval Postgraduate School (NPS) (13,14) studies, except for the conclusion in the NPS study (13) that energy storage systems have little impact. The findings here suggest that, depending upon the climate zone, incorporating energy storage is the second best improvement after implementing energy efficiency measures.

With regard to energy security and energy resilience, the findings suggest some qualitative benefits. This study assumes that the current solution, using diesel generators, will remain the primary means for electric power production with little to no change in total generator nameplate rating or on-FOB fuel storage. Once FOB commanders gain confidence in the reliability of sustainable energy and as storage costs decline, the military can expand this solution, potentially up to 100% clean, renewable energy. In the meantime, if FOBs consume fuel at a slower rate by incorporating energy efficiency and renewables, then multiple benefits will result, including the following:

1. Energy security will improve due to the additional power generation assets on hand and the ability to maintain fuel storage tanks at or near their full capacity; that is, generators can be shifted from a primary (or only) means of electricity production to backup or peaking roles.
2. The time between mandatory fuel resupply missions will lengthen, which reduces:
   a. The risk of enemy attacks on logistics convoys or transportation infrastructure (bridges, roads) to disrupt resupply.
   b. The amount of fuel consumed to deliver (and protect the delivery of) fuel, which contributes to the fully-burdened cost of fuel.
   c. The risk of injury or loss of life for those conducting dangerous resupply missions.
d. Operations and maintenance costs for logistics, to include freeing up personnel for other missions, and reducing wear and tear on logistics vehicles, vehicle maintenance, and so on.

Furthermore, incorporating additional energy storage and renewables results in power generation and energy storage distributed across the FOB yet managed through a microgrid. Together, the proposed system decreases the likelihood of outages due to generators being down for maintenance, fuel shortages, or enemy attacks destroying critical power nodes using spot generation configurations.

This model is at the macro level with a focus on the microgrid’s total load. The model assumes all loads and phases are properly balanced, and it neglects transmission losses since generators are located near their loads. As shown in Figure 2(b), the model does not penalize generators for having to start-up or shut-down, which would require additional time and fuel and would increase fuel consumption in both the baseline and diversified energy portfolio scenarios. The model uses a constant value for fuel consumption (gal h$^{-1}$) as reported by generator manufacturers, which reflects approximate consumption at optimal or rated load. Since the baseline scenario has generators following the load, this assumption overestimates fuel usage. However, because the model does not require generators to leave a spinning reserve, the model simultaneously underestimates fuel usage in the baseline scenario. In the diversified energy portfolio scenario, batteries can reduce or even potentially eliminate the need for a spinning reserve. Further refinement of the model may include functions for decreasing generator fuel consumption at lower loading and incorporating generator controls, specifically to reduce the frequency of start-up and shut-down and force more storage from excess generation.
The model addresses several gaps in military planning factors. First, peak power is currently estimated based on peak power per person planning factors, which vary according to expected level of service (basic, expanded, or enhanced) (20). However, as shown in Figure 1, building energy efficiency and climate also have large impacts on energy requirements. To get a better estimate of actual load requirements, the model takes planning factors and separates the total load into two parts: lighting + plug loads and HVAC loads (see Section 2 and Supplementary Information). Second, there are no planning factors for incorporating renewables on FOBs. This model allows for planners to download open-source solar and wind data from online databases (18, 60–62) and better design FOB energy portfolios for any desired location.

The optimization model has “perfect foresight” because it solves using a known load demand profile and known environmental conditions (insolation, wind speed, temperature, etc.) with data for every hour of the year. The model reduces computational time required by breaking the optimization of 8760 h a year into 12 discrete optimization problems, one for each month, of about 730 time steps each. One drawback to this method is that the optimization strives to use stored battery energy by the end of each month. However, this also serves to limit perfect foresight to one-month at a time. To maximize resilience, it is conceivable that the best course of action might be to keep batteries near their full state of charge with variation only to avoid curtailment of renewables or to turn off underloaded generators. Nevertheless, in order to share the model and facilitate planners running it on their assigned workstations, managing computational time is imperative.

5 Conclusion

The results suggest three rules-of-thumb for planners when incorporating energy storage and renewables for resilience on expeditionary bases:
1. Efficiency is number one. Additional upfront capital expense for improved building
construction can significantly reduce fuel demand with rapid payback. Planners should
consider the local climate and expected FOB lifetime when determining a strategy.

2. Invest in energy storage next. FOBs can either centralize or distribute batteries.
Compatibility with microgrid controls, deployability, and operations/maintenance
requirements may dictate the appropriate choice. Even if a FOB uses no renewables, energy
storage allows for the reduction (or elimination) of a spinning reserve for peak loads; allows
for generators to work at their optimal capacity; and reduces underloading, wet stacking, and
other maintenance issues. Energy storage is also critical for incorporating renewables, next,
to avoid curtailment.

3. Adding renewables will help eliminate nearly all reliance on diesel generators. Wind and
solar are complementary in nature and their combined installations increase the annual
number of hours of renewable power production.

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**Supplementary material**
Supplementary material for this article is available online.

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Supplementary Information

Renewable energy and energy storage to offset diesel generators at expeditionary contingency bases

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Nomenclature

Acronyms and abbreviations

ACH or ach: Air Changes per Hour
AEPI: Army Environmental Policy Institute
AHRI: Air-conditioning, Heating, and Refrigeration Institute
AMMPS: Advanced Medium Mobile Power Sources
ANSI: American National Standards Institute
AR: Anti-Reflective
ASHP: Air Source Heat Pump
ASHRAE: American Society of Heating, Refrigerating and Air-Conditioning Engineers
ATP: Army Training Publication
BIOF: Bio-Foam
CERL: Construction Engineering Research Laboratory
CO: Carbon monoxide
CO₂: Carbon dioxide
COP: Coefficient Of Performance
CPI: Consumer Price Index
DCAS: Defense Causality Analysis System
DFAC: Dining Facility
DLA: Defense Logistics Agency
DMDC: Defense Manpower Data Center
DST: Daylight Saving Time
ECU: Environmental Control Unit
EPRI: Electric Power Research Institute
EPA: Environmental Protection Agency
EPS: Expanded Polystyrene
ERDC: Engineer Research and Development Center
FOB: Forward Operating Base
FY: Fiscal Year
GMT: Greenwich Mean Time
GWP: Global Warming Potential
H₂SO₄: Sulfuric acid
HAWT: Horizontal Axis Wind Turbines
HQDA: Headquarters, Department of the Army
HSPF: Heating Season Performance Factor
HVAC: Heating, Ventilation, and Air-Conditioning
IBM: International Business Machines
IPCC: Intergovernmental Panel on Climate Change
J2000.0: Julian Calendar 2000 Standard Epoch
KIA: Killed In Action
LBNL: Lawrence Berkeley National Laboratory
LDSS: Load Demand Start Stop
LSD: Low Sulfur Diesel
MHE: Material Handling Equipment
MILP: Mixed Integer Linear Programming
MWR: Morale, Welfare & Recreation
NaN: Not a Number
NMHC: Non-Methane Hydrocarbons
NOAA: National Oceanic and Atmospheric Administration
NOX: Nitrogen Oxides
NREL: National Renewable Energy Laboratory
NSRDB: National Solar Radiation Data Base
OEF: Operation Enduring Freedom
OFS: Operation Freedom’s Sentinel
OLE: Object Linking and Embedding
OIF: Operation Iraqi Freedom
OIR: Operation Inherent Resolve
OND: Operation New Dawn
OSB: Oriented Strand Board
PHBV: Poly-hydroxybutyrate-co-hydroxy valerate
PM: Particulate Matter
PSC: Primary Switching Center
PUR: Polyurethane
PX: Post Exchange
SDC: Secondary Distribution Center
SEA: South East Asia
SEER: Seasonal Energy Efficiency Factor
SIP: Structural Insulated Panel
SO2: Sulfur dioxide
SO3: Sulfur trioxide
SOx: Sulfur Oxides
SPA: Solar Position Algorithm
SPF: Seasonal Performance Factor
SWA: South West Asia
TOC: Tactical Operations Center
TOC: Total Organic Compounds
TRISO: Tri-structural ISOtropic
ULSD: Ultra-Low Sulfur Diesel
USACE: United States Army Corps of Engineers
USCRN: United States Climate Reference Network
USD: United States Dollar
USMA: United States Military Academy
UTC: Coordinated Universal Time
VAWT: Vertical Axis Wind Turbine
WIA: Wounded In Action
WIND: Wind Integration National Dataset
XPS: Extruded polystyrene
Symbols and subscripts

\( A[ft^2] \): area
\( A \left[ \frac{w}{m^2} \right] \): apparent extraterrestrial insolation
\( A_{doors}[ft^2] \): area of doors
\( A_{framing}[in^2] \): cross-sectional area consisting of framing
\( A_{PV,FOB}[m^2] \): total FOB PV area
\( A_{PV,Hut}[ft^2] \): available solar PV area on hut roof
\( A_{PV,Hut,SIP}[ft^2] \): available solar PV area on a SIP Hut roof
\( A_{PV,Hut,SWA}[ft^2] \): available solar PV area on a SWA Hut roof
\( A_{PV,U}[m^2] \): total FOB utility solar PV area
\( A_{total}[in^2] \): total cross-sectional area
\( A_{VAWT}[m^2] \): swept area of VAWT
\( A_{walls}[ft^2] \): area of walls
\( A_WF[m^2] \): land area required for wind farm
\( A_{WT} \left[ \frac{m^2}{turbine} \right] \): land area required by an individual turbine
\( ACH50 \left[ \frac{air\ changes}{hr} \right] \): number of air changes per hour at an air pressure of 50 Pascal
\( AST[hr] \): apparent (true) solar time
\( c \left[ Btu \left[ \frac{lb}{F} \right] \right] \): specific heat of air
\( c_{battery} \left[ \frac{S}{battery} \right] \): cost per battery and all balance of plant components
\( c_C[S] \): penalty cost of curtailment
\( c_D[S] \): cost of diesel
\( c_F \left[ \frac{S}{gal} \right] \): cost of diesel fuel per gallon
\( c_{generator} \left[ \frac{S}{generator} \right] \): cost per generator and all balance of plant components
\( c_{generator,FOB}[S] \): total cost of all generators and balance of plant equipment on the FOB
\( c_{PV} \left[ \frac{S}{W_{DC}} \right] \): PV cost on a per watt, direct current basis
\( c_{PV,FOB}[S] \): total PV cost for a FOB
\( c_T[S] \): total cost of diesel and curtailment
\( C[-] \): sky diffuse factor
\( C[deg] \) or \( [rad] \): Sun’s equation of center
\( C_{P,h}[-] \): wind turbine power coefficient in hour, \( h \)
\( CEF \left[ \frac{grams\ GHG}{gal\ diesel} \right] \): carbon emissions factor of a greenhouse gas
\( CF[\%] \): capacity factor
\( CFM50 \left[ \frac{ft^3}{min} \right] \): cubic feet per minute of air movement at 50 Pascal pressure
\( COP[-] \): coefficient of performance
\( CTCCF[-] \): cell temperature coefficient correction factor
\( D[m] \): wind turbine diameter
\( \text{DHI} \left[ \frac{W}{m^2} \right] \): diffuse horizontal irradiance, same as \( I_{DH} \left[ \frac{W}{m^2} \right] \)

\( \text{DNI} \left[ \frac{W}{m^2} \right] \): direct normal irradiance, same as \( I_B \left[ \frac{W}{m^2} \right] \)

e\left[ - \right] \): eccentricity of Earth’s orbit

\( E[\text{min}] \): equation of time

\( E_0 \left[ \frac{W}{m^2} \right] \): extraterrestrial radiant flux

\( E_{\text{battery,FOB,\text{max}}} \left[ \text{kWh} \right] \): maximum battery energy capacity across all batteries, same as \( E_{\text{battery,\text{max}}} \left[ \text{kWh} \right] \)

\( E_{\text{battery,\text{max}}} \left[ \frac{\text{kWh}}{\text{battery}} \right] \): maximum energy that can be stored by an individual battery

\( E^h \left[ \text{kWh} \right] \): energy stored across all batteries in hour, \( h \)

\( E^h_{\text{in}} \left[ \text{kWh} \right] \): additional energy stored across all batteries in hour, \( h \)

\( E^h_{\text{\text{max}}} \left[ \text{kWh} \right] \): maximum battery energy capacity across all batteries, same as \( E_{\text{battery,FOB,\text{max}}} \left[ \text{kWh} \right] \)

\( E^h_{\text{\text{min}}} \left[ \text{kWh} \right] \): minimum battery energy capacity across all batteries

\( E^h_{\text{out}} \left[ \text{kWh} \right] \): energy consumed across all batteries in hour, \( h \)

\( E^c \left[ \text{kWh} \right] \): energy curtailed in hour, \( h \)

\( E^D \left[ \text{kWh} \right] \): energy produced by all diesel generators in hour, \( h \)

\( E^D_{i,h} \left[ \text{kWh} \right] \): energy produced by individual diesel generator, \( i \) in hour, \( h \)

\( E_{\text{\text{max}}}^D \left[ \text{kWh} \right] \): maximum energy produced from diesel generators in hour, \( h \)

\( E_{\text{\text{min}}}^D \left[ \text{kWh} \right] \): minimum energy produced from diesel generators in hour, \( h \)

\( E^L \left[ \text{kWh} \right] \): energy load profile for the FOB in hour, \( h \)

\( E^W \left[ \text{kWh} \right] \): energy produced from wind turbines in hour, \( h \)

\( E_{SC} \left[ \frac{W}{m^2} \right] \): solar constant

\( E_{WF,\text{annual}} \left[ \text{kWh} \right] \): energy produced by a wind farm over one year

\( F_{\text{ach}} \left[ - \right] \): ach factor to convert to normal, ambient pressure

\( F_{\text{\text{framing}}} \left[ \% \right] \): framing factor

\( F_{\text{\text{L&PL}}} \left[ \% \right] \): factor for determining lighting and plug load

\( F_{\text{pp}} \left[ \frac{\text{kW}}{\text{person}} \right] \): per person peak power planning factor

\( FE_{\text{\text{\text{diesel,60kw}}} \left[ \frac{\text{gal}}{\text{hr}} \right] \): fuel efficiency of a 60 kW generator

\( GWP \left[ \frac{\text{grams CO}_2}{\text{grams GHG}} \right] \): global warming potential of a greenhouse gas

\( H \left[ \text{m} \right] \): height

\( H_0 \left[ \text{m} \right] \): reference height

\( HA \left[ \text{deg} \right] \) or \( [ \text{rad} ] \): hour angle

\( HA_{\text{SR}} \left[ \text{deg} \right] \) or \( [ \text{rad} ] \): hour angle at sunrise

\( HA_{\text{SR/SS}} \left[ \text{deg} \right] \) or \( [ \text{rad} ] \): hour angle at sunrise/sunset

\( H\text{SPF} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): heating season performance factor

\( H\text{SPF}_{\text{corrected}} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): climate corrected HSPF

\( H\text{SPF}_{\text{rated}} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): nameplate rating for the HSPF
\[ I_0 \left[ \frac{W}{m^2} \right] \]: extraterrestrial solar insolation just beyond the Earth’s atmosphere
\[ I_B \left[ \frac{W}{m^2} \right] \]: direct beam insolation on the Earth’s surface, same as DNI \[ \left[ \frac{W}{m^2} \right] \]
\[ I_{BC} \left[ \frac{W}{m^2} \right] \]: direct beam insolation on a collector
\[ I_{BH} \left[ \frac{W}{m^2} \right] \]: direct beam insolation on a horizontal surface
\[ I_c \left[ \frac{W}{m^2} \right] \]: total insolation on a collector
\[ I_{CT} \left[ \frac{W}{m^2} \right] \]: total insolation on a collector corrected for transmittance
\[ I_{DC} \left[ \frac{W}{m^2} \right] \]: diffuse insolation on a collector
\[ I_{DH} \left[ \frac{W}{m^2} \right] \]: diffuse insolation on a horizontal surface, same as DHI \[ \left[ \frac{W}{m^2} \right] \]
\[ I_{RC} \left[ \frac{W}{m^2} \right] \]: reflected insolation on a collector

\[ J_C[\text{century}] \]: Julian century
\[ JD[\text{day}] \]: Julian day
\[ k[-] \]: optical depth
\[ L[kW] \]: load
\[ L_0[\text{deg}] \] or \[ L[\text{rad}] \]: Sun’s geometric mean longitude
\[ L_B[kW] \]: baseline load
\[ L_{BL,PL}[kW] \]: baseline lighting and plug load
\[ L_{cool}[kW] \]: cooling load
\[ L_{h,max}[kW] \]: maximum load of any hour, \( h \)
\[ L_{heat}[kW] \]: heating load
\[ L_{HVAC}[kW] \]: HVAC load
\[ L_{L+PL}[kW] \]: lighting and plug load
\[ L_{Peak}[kW] \]: peak load required
\[ L_{Peak,PL}[kW] \]: peak lighting and plug load requirement
\[ L_{PNNL,h}[kW] \]: hourly load from PNNL experiments
\[ L_{PNNL,max}[kW] \]: maximum load from PNNL experiments

\[ LAT[\text{deg}] \]: latitude of site, °N of the Equator, negative in the Southern Hemisphere
\[ LF[\%] \]: loss factor
\[ LON[\text{deg}] \]: longitude of site, °E of Greenwich, negative in the Western Hemisphere
\[ LSM[\text{deg}] \]: longitude of local standard time meridian, °E of Greenwich, negative in the Western Hemisphere

\[ LST[\text{hr}] \]: local standard time
\[ m[-] \]: air mass ratio
\[ M[\text{deg}] \] or \[ M[\text{rad}] \]: Sun’s geometric mean anomaly
\[ M_{load}[\%] \]: load multiplier
\[ M_{Peak,PL,d}[kW] \]: modified daily peak lighting and plug load

\[ MW_{air} \left[ \frac{\text{g}}{\text{mol}} \right] \]: molecular weight of air
\[ n \left[ \frac{\text{# air changes}}{\text{hr}} \right] \]: number of air changes per hour
\[ n[\text{day}] \]: Julian day number (integer, 1… 365 or 366)
\[ n[\text{huts}] \]: number of huts
\( n \) [people]: number of people
\( n \) [turbine]: number of wind turbines
\( n_{air} \): refractive index of air
\( n_{AR} \) [-]: refractive index of anti-reflective coating
\( n_{battery,FOB} \) [batteries]: number of batteries installed on a FOB
\( n_{battery,FOB,\text{max}} \) [batteries]: maximum number of batteries on a FOB
\( n_{bldg} \) [buildings]: number of buildings on the FOB
\( n_{generator,FOB} \) [generators]: number of generators on the FOB
\( n_{\text{random}} \) [-]: random number
\( NOCT[^{\circ}\text{C}] \): nominal operating cell temperature
\( P_{B,\text{out}} \) [kW]: peak power output from all FOB batteries, same as \( P_{\text{battery,FOB,peak}} \) [kW]
\( P_{\text{battery,continuous}} \) [\( \frac{kW}{\text{battery}} \)]: continuous power output from an individual battery
\( P_{\text{battery,FOB,continuous}} \) [kW]: continuous power output from all FOB batteries
\( P_{\text{battery,FOB,peak}} \) [kW]: peak power output from all FOB batteries, same as \( P_{B,\text{out}} \) [kW]
\( P_{\text{battery,peak}} \) [\( \frac{kW}{\text{battery}} \)]: peak power output from an individual battery
\( P_{\text{generator}} \) [\( \frac{kW}{\text{generator}} \)]: nameplate rating of a diesel generator
\( P_h \) [atm]: air pressure in hour, \( h \)
\( P_{PV} \) [kW\text{AC}]: AC power production from solar photovoltaics
\( P_{W,h} \) [W]: power in the wind in hour, \( h \)
\( P_{WF,h} \) [W]: power produced by a wind farm in hour, \( h \)
\( P_{WF,r} \) [kW]: wind farm rated power
\( P_{WT,r} \) [\( \frac{W}{\text{turbine}} \)]: nameplate power rating of an individual wind turbine
\( P_{r,\text{VAWT},h} \) [\( \frac{W}{\text{turbine}} \)]: rated output power of an individual VAWT in hour, \( h \)
\( PALA \) [%]: percent above load allowed
\( PBLA \) [%]: percent below load allowed
\( PD_{WF} \) [W/m\(^2\)]: power density of wind farm
\( PDML_{h} \) [%]: percent of daily maximum hourly load
\( POBM \) [%]: percent overage for backup and maintenance
\( PTC \) [%/\( ^{\circ}\text{C} \)]: power temperature coefficient
\( q \) [Btu/hr]: heat transfer rate
\( Q \) [deg] or [rad]: approximate atmospheric refraction & upper-limb adjustment
\( r_{f} \) [gal/hr]: rate of diesel fuel consumption
\( R[AU] \): Sun’s radius vector
\( R \) [m\(^3\).atm/K.mol]: ideal gas constant
\( R_{2x4} \) [hr.ft\(^2\).\(^{\circ}\text{F} \)/Btu]: thermal resistance through a 2x4 stud
\( R_{\text{cvi,wall}} \) [hr.ft\(^2\).\(^{\circ}\text{F} \)/Btu]: indoor radiative-convective thermal resistance of wall surfaces
\( R_{cvo,avg} \) [hr.ft\(^2\).\(^{\circ}\text{F} \)/Btu]: outdoor average radiative-convective thermal resistance of wall surfaces
\( R_{\text{insul}} \left[ \frac{\text{hr-ft}^2\cdot{^\circ}\text{F}}{\text{Btu}} \right] \): thermal resistance through insulation

\( R_{\text{OSB}} \left[ \frac{\text{hr-ft}^2\cdot{^\circ}\text{F}}{\text{Btu}} \right] \): thermal resistance through OSB

\( R_{\text{T,walls,cavity}} \left[ \frac{\text{hr-ft}^2\cdot{^\circ}\text{F}}{\text{Btu}} \right] \): thermal resistance of the cavity (non-framing portion) of walls

\( R_{\text{T,walls,stud}} \left[ \frac{\text{hr-ft}^2\cdot{^\circ}\text{F}}{\text{Btu}} \right] \): total thermal resistance through the stud framing of wall

\( S[-] \): wind turbine spacing factor

\( SC \left[ \frac{W}{m^2} \right] \): solar constant

\( SD[m] \): wind turbine spacing

\( \text{SEER} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): seasonal energy efficiency ratio

\( \text{SEER}_{\text{corrected}} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): climate corrected seasonal energy efficiency ratio

\( \text{SEER}_{\text{rated}} \left[ \frac{\text{Btu}}{\text{Wh}} \right] \): nameplate rating for the seasonal energy efficiency ratio

\( SF_{\text{Hut}} \left[ \frac{ft^2}{\text{Hut}} \right] \): square footage of a hut

\( SF_{\text{reqd}} \left[ \frac{ft^2}{\text{person}} \right] \): square footage area requirement

\( SF_{\text{reqd}} \left[ \frac{ft^2}{\text{person}} \right] \): square footage area requirement per person

\( SLD[\text{mi}] \): sunlight duration

\( SN[\text{day}] \): solar noon

\( SPF_{\text{cooling}}[-] \): seasonal performance factor for cooling

\( SPF_{\text{cooling,corrected}}[-] \): climate corrected seasonal performance factor for cooling

\( SPF_{\text{heating}}[-] \): seasonal performance factor for heating

\( SPF_{\text{heating,corrected}}[-] \): climate corrected seasonal performance factor for heating

\( T_{\Delta}[^{\circ}\text{F}] \): difference between outdoor dry bulb and indoor set point temperatures

\( T_{\Delta\text{cool}}[^{\circ}\text{F}] : T_{\Delta} \) for cooling

\( T_{\Delta\text{heat}}[^{\circ}\text{F}] : T_{\Delta} \) for heating

\( T_{a}[^{\circ}\text{F}] \): ambient or outdoor dry bulb temperature

\( T_{\text{cell}}[^{\circ}\text{C}] \): temperature of photovoltaic cell

\( T_{db}[^{\circ}\text{C}] \): outdoor dry bulb temperature

\( T_{db,1\%d}[^{\circ}\text{F}] \): ASHRAE 1\% design cooling temperature

\( T_{db,99\%d}[^{\circ}\text{F}] \): ASHRAE 99\% design heating temperature

\( T_{i}[^{\circ}\text{F}] \): internal temperature

\( T_{\text{set}}[^{\circ}\text{F}] \): internal set point temperature

\( T_{SR}[\text{day}] \): sunrise time

\( T_{SR,ACT}[\text{day}] \): apparent sunrise time, clock time

\( T_{SR,AST}[\text{day}] \): apparent sunrise time, solar time

\( T_{SR,G,CT}[\text{day}] \): geometric sunrise time, clock time

\( T_{SR,G,ST}[\text{day}] \): geometric sunrise time, solar time

\( T_{SS}[\text{day}] \): sunset time

\( T_{SS,ACT}[\text{day}] \): apparent sunset time, clock time

\( T_{SS,AST}[\text{day}] \): apparent sunset time, solar time

\( T_{SS,G,CT}[\text{day}] \): geometric sunset time, clock time

\( T_{SS,G,ST}[\text{day}] \): geometric sunset time, solar time
\(TC[min]\): time conversion (between AST and LST)

**Thermal Index** \(\frac{Btu}{\text{day} \cdot \text{ft}^2 \cdot \circ F}\): thermal index per square foot of building floor area

**Total \(UA_e\)** \(\frac{Btu}{\text{hr} \cdot \text{ft}^2 \cdot \circ F}\): equivalent total thermal conductance of building envelop components and infiltration

\(TZ[-]\): time zone

\(U\) \(\frac{Btu}{\text{hr} \cdot \text{ft}^2 \cdot \circ F}\): thermal conductance per unit area

\(U_{avg \text{, walls}}\) \(\frac{Btu}{\text{hr} \cdot \text{ft}^2 \cdot \circ F}\): average thermal conductance of walls

\(UA_{ceiling/roof}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): thermal conductance through the ceiling or roof

\(UA_{doors}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): thermal conductance through doors

\(UA_{floor}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): thermal conductance through the floor

\(UA_{infiltration}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): equivalent thermal conductance due to infiltration

\(UA_{value}\) \(\frac{Btu}{\text{hr} \cdot \circ F \cdot \text{building}}\): thermal conductance per building

\(U_{walls, cavity}\) \(\frac{Btu}{\text{hr} \cdot \text{ft}^2 \cdot \circ F}\): thermal conductance through the cavity (non-framing portion) of walls

\(U_{walls, stud}\) \(\frac{Btu}{\text{hr} \cdot \text{ft}^2 \cdot \circ F}\): thermal conductance through the stud framing of walls

\(UA_{walls, SWA \text{ hut, improved}}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): thermal conductance of the walls in an improved SWA Hut

\(UA_{windows}\) \(\frac{Btu}{\text{hr} \cdot \circ F}\): thermal conductance through windows

\(UF[\%]\): utilization factor

\(v\) \(\frac{m}{s}\): wind speed at hub height

\(v_0\) \(\frac{m}{s}\): wind speed at anemometer or reference height

\(v_h\) \(\frac{m}{s}\): wind speed at hub height in hour, \(h\)

\(v_{cut-in}\) \(\frac{m}{s}\): wind turbine cut-in wind speed

\(v_{cut-out}\) \(\frac{m}{s}\): wind turbine cut-out speed

\(V\) \(\frac{\text{ft}^3}{\text{air change}}\): volume of air exchanged during one air change

\(y_{i, h}\): binary variable for generator on/off in hour, \(h\)

\(Y[-]\): ratio of clear-sky diffuse irradiance on a vertical surface to clear-sky diffuse irradiance on a horizontal surface

\(\alpha[-]\): friction coefficient for ground surface condition

\(\alpha[\text{deg}]\) or \([\text{rad}]\): Sun’s right ascension

\(\beta[\text{deg}]\) or \([\text{rad}]\): Sun’s altitude (elevation) angle

\(\beta_{corrected}[\text{deg}]\) or \([\text{rad}]\): Sun’s elevation (altitude) angle corrected for atmospheric refraction

\(\Gamma[\text{deg}]\) or \([\text{rad}]\): parameter in the equation of time

\(\delta[\text{deg}]\) or \([\text{rad}]\): Sun’s declination

\(\Delta T[^\circ F]\): difference in temperature

\(\varepsilon_0\): mean obliquity of the ecliptic
$\varepsilon_{\text{corrected}}$[$\text{deg}$ or $\text{rad}$]: mean obliquity of the ecliptic corrected for solar parallax
$\eta_B$[-]: round trip efficiency of batteries
$\eta_{\text{Inverter}}$[-]: efficiency of DC/AC inverter
$\eta_{\text{PV panel}}$[-]: efficiency of PV panels
$\eta_{\text{PV system}}$[-]: efficiency of PV system
$\theta$[$\text{deg}$ or $\text{rad}$]: incidence angle
$\theta_1$[$\text{deg}$ or $\text{rad}$]: level one incidence angle
$\theta_2$[$\text{deg}$ or $\text{rad}$]: level two incidence angle
$\theta_3$[$\text{deg}$ or $\text{rad}$]: level three incidence angle
$\lambda$[$\text{deg}$ or $\text{rad}$]: Sun’s true longitude
$\lambda_a$[$\text{deg}$ or $\text{rad}$]: Sun’s apparent longitude
$\nu$[$\text{deg}$ or $\text{rad}$]: Sun’s true anomaly
$\Sigma$[$\text{deg}$ or $\text{rad}$]: collector tilt angle
$\tau_{\text{AR}}$[-]: transmittance through anti-reflective coating
$\tau_{\text{cover}}$[-]: transmittance through cover
$\tau_{\text{glass}}$[-]: transmittance through glass
$\phi_C$[$\text{deg}$ or $\text{rad}$]: collector azimuth angle
$\phi_S$[$\text{deg}$ or $\text{rad}$]: Sun’s azimuth angle
$\phi_Z$[$\text{deg}$ or $\text{rad}$]: Sun’s zenith angle
$\Omega$[$\text{deg}$ or $\text{rad}$]: longitude correction value for nutation and aberration
$\rho$[$\frac{\text{lb}}{\text{ft}^3}$]: air density
$\rho_h$[$\frac{\text{kg}}{\text{m}^3}$]: air density in hour, $h$
$\rho$[-]: albedo or ground reflectance
1. Supplementary methods, derivations, and equations

This section provides detailed calculations and explanations of necessary assumptions such that a reader can verify the results presented within the main manuscript. It is organized to facilitate a reader’s understanding of the model, and it can be used as a handbook in adapting the model for further research. The order of presentation follows that of the Methods section in the main manuscript.

1.A. Experimental design and background

1.A.1. Objective

The authors’ objective in conducting this research is to examine the potential benefits of incorporating renewable energy resources, like solar photovoltaics and wind, along with energy storage to diversify the energy portfolio and improve the energy resilience of military forward operating bases (FOBs) built under expeditionary circumstances to support contingency operations.

1.A.2. Purpose

The purpose is to quantify potential benefits in terms of reducing the cost of fuel and the dependency on generators for electricity generation at FOBs. The model provides a means of investigating different energy portfolio scenarios under different conditions, to include annual climate, renewable resource availability, and building construction techniques.

1.A.3. Approach

The authors consider the following scenarios in this analysis: five specific locations that represent the five major Köppen Climate Zones around the world; the 8,760 hour-by-hour conditions at each location to include temperature, wind speed, and solar radiation; the thermal
conductivity of building construction options; and combinations of diesel generators, batteries, wind turbines, and solar PV for power.

1.A.4. Background

Militaries conduct expeditionary operations, often in austere environments. This necessitates having the ability to rapidly deploy and establish FOBs from which to conduct contingency operations. These FOBs are places where troops can conduct both operational planning and sustainment support operations, like maintaining equipment, eating, sleeping, and receiving medical treatment. To effectively conduct these operations, a constant and reliable access to energy is required, and this is generally accomplished through a network of diesel generators. Unfortunately, this method is inefficient and lacks resilience. Furthermore, energy inefficient buildings on FOBs place undue strain on an already fragile system.

The military of the future is expected to consume even greater amounts of energy in war. Directed-energy weapons (laser cannons), electromagnetic rail guns (capable of firing projectiles at hypersonic speeds using magnetic fields generated by electric currents to slide a projectile between rails inside a barrel), electric vehicles, drones, communications and computer systems, and more will all require vast amounts of energy (63). Some believe the answer lies with mobile micro (1 to 10 MW) nuclear power plants using TRISO (TRi-structural ISOtropic) fuel particles made of uranium, carbon, and oxygen and coated with multiple layers of carbon and ceramic (63–65). Proponents of this option cite the Navy as an example of successfully operating nuclear power plants for military applications (63). However, FOBs for land based operations are often sited adjacent to or within populous areas of a host nation, not in the middle of an ocean in international waters. Regulations for operations, global transportation of materiel and waste, and abidance by international treaties make application of this solution difficult.
Even more important is survivability on the battlefield. To improve energy resilience, diversification and distribution of energy production and storage may be a better course of action rather than consolidation into a potential single point of failure. The current solution that relies upon diesel generators as the only source of power is costly, risky, and lacks resilience. Should enemy actions or natural events, like extreme weather or earthquakes, prohibit resupply convoys, a FOB can be stranded without diesel resupply for days.

In addition to this unstable reliance on power from diesel generators, energy inefficient buildings severely increase the load placed on the system. A study by Noblis revealed that “FOB structures are inefficient, with significant demand placed on generators to power systems to heat or cool tents with no insulation” (66). It is important to remember that “temporary” structures built for contingency operations may end up lasting for many years.

1.B. Orientation to the model

1.B.1. Process

The process begins by pre-processing user input parameters in Microsoft Excel to create data for expected load, solar power production, and wind power production by hour throughout a year. Next, a program using the Julia programming language pulls that data and uses IBM’s CPLEX solver to optimize the best use of pre-defined energy portfolio resources while trying to minimize the operations cost of diesel. The Julia program determines the optimal hourly schedule for diesel generators to be on or off, when battery energy should be stored or used, and the overall annual cost, which directly relates to the amount of diesel fuel consumed. The program passes output data back to Microsoft Excel for graphing using pre-templated charts and diagrams.
1.B.2. Locations studied

The military is expected to be capable of operating under expeditionary conditions worldwide. Therefore, in order to demonstrate the usefulness of this model to a wide range of climate conditions, the authors specifically chose a location from each of the five major Köppen climate classification groups. The five major groups are tropical (A), arid or “dry” (B), temperate (C), continental or “cold” (D), and polar (E). Subgroups, designated by the second and sometimes third letters, are further defined by seasonal temperature and precipitation patterns within the major classification groups.

Table 2 shows the five locations the authors selected for this study. The first and second columns show the station location name and number used by NREL’s National Solar Radiation Data Base (NSRDB).

<table>
<thead>
<tr>
<th>NSRDB station location name</th>
<th>Station #</th>
<th>Köppen climate type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honolulu International Airport, Hawaii</td>
<td>911820</td>
<td>Tropical</td>
</tr>
<tr>
<td>Twentynine Palms, California</td>
<td>690150</td>
<td>Arid (dry)</td>
</tr>
<tr>
<td>Dulles International Airport (Washington DC), Virginia</td>
<td>724030</td>
<td>Temperate</td>
</tr>
<tr>
<td>Stewart Airfield, New York</td>
<td>725038</td>
<td>Cold (continental)</td>
</tr>
<tr>
<td>Barrow Wiley Post – Will Rogers Memorial Airport, Alaska</td>
<td>700260</td>
<td>Polar</td>
</tr>
</tbody>
</table>

1.B.3. Variables investigated

The authors built numerous parameters into this model that can be made variables and changed as desired to investigate their impact upon FOB energy. For this paper, however, the authors established a scope that includes seven variables for investigation: location climate, level of service, unit size, baseline construction type, FOB PV size, number of wind turbines, and number of batteries. Each variable is explained in detail in Section 1.C.
1.B.4. Uncertainty analysis

When conducting optimization modeling, a balance is necessary between accuracy of results and computational time. For any scenario with \( m \) binary decision variables (i.e., \( m \) generators that can either be on or off), there could be up to \( 2^m \) function evaluations necessary to determine the optimal solution by enumeration. Fortunately, methods exist to eliminate some evaluation possibilities within the optimization process. Nevertheless, conducting such an optimization over all 8,760 hours of a year at once can be computationally expensive. By assuming that the FOB can use all stored battery energy by the end of every month, it is possible to break the year-long analysis into a series of 12 month-long analyses of approximately 730 hours each. This also has a side benefit of facilitating partial year analyses if desired. This is an acceptable approximation because the model starts from zero battery energy storage each month, requiring a new build-up of stored energy from either diesel generators or renewables.

Additionally, when calling the CPLEX solver, model users have the option of defining a time limit for processing which, as just described, applies to each month. The default allowance for CPLEX runtime is \( 10^{75} \) seconds. Considering most scientists believe the universe is 13.8 billion years old (4.3x\(10^{17}\) seconds), it is prudent to establish runtime limits if possible without sacrificing required accuracy. A sensitivity analysis of CPLEX processing time included taking a scenario and comparing solutions after imposing time limits of 5, 10, 20, 30, 60, 90, 120, and 3,600 seconds per monthly optimization. Interestingly, for battalion-sized FOBs with an expanded level of service, the optimal solution found was the same whether it took two minutes (ten seconds per month) or 12 hours (3,600 seconds per month) to solve. Only at imposed time limits below ten seconds per monthly optimization did discrepancies result. However, this sensitivity analysis is only applicable to the computer used and the tested scenario: a battalion-
sized FOB with an expanded level of service, which covers the primary results summarized in Figure 1. Changing the FOB size and level of service results in changes to the number of baseline diesel generators, which has significant impacts to computational processing time required. Further checks on sensitivity analysis required simulations and graphed results for hourly load; diesel, solar, and wind power generation; battery storage and use; and curtailment over the course of one year for FOBs of different sizes and levels of service. Due to breaking the overall optimization into twelve independent optimizations, one could visually see aberrations along monthly periods when the allowed computational time was insufficient. As with all modeling, one must carefully inspect results before accepting them.

1.C. Input data, derivations, and calculations

Location climate data are from TMY3 data files publicly available from NREL’s NSRDB (18). These data include 8,760 hourly values for meteorological measurements and solar radiation. It is important to note, however, that the data represents typical, not extreme conditions. Therefore, the data is only useful for understanding scenarios under typical conditions. These data are not acceptable for designing FOBs for operation under worst-case climate scenarios (67). Because of this, it can sometimes be helpful when analyzing results from this model to download and apply data for a specific year at a specific location. This can be done using data available from NREL’s NSRDB and Wind Integration National Dataset (WIND) Toolkit websites (61,62). By simply downloading the desired data and pasting it into the appropriate column of the original TMY3 data file, one can, for instance, see how using solar radiation, albedo, and wind speed data from the previous year makes an appreciable difference (if any) from using the “typical” TMY3 data.
1.C.1. Basecamp data

Contingency bases, as opposed to “enduring” or “permanent” bases, currently have many labels (bases, base camps, intermediate staging bases, forward operating bases, patrol bases, and combat outposts, to name a few) (68). In this paper, the authors use the common term of “forward operating base” or “FOB” to mean any contingency base of any size or intended duration.

1.C.1.A. Level of service

The US Army defines a FOB’s “level of service” in Army Training Publication (ATP) 3-37.10 as “basic” (60 to 180 days), “expanded” (180 days to 2 years), or “enhanced” (2 to 10 years) (20). With increasing duration, additional and/or improved services are offered at a FOB. Consequently, the peak power planning factor also increases, as shown by Table 3.

<table>
<thead>
<tr>
<th>FOB level of service</th>
<th>Peak power planning factor (kW person⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>1.5</td>
</tr>
<tr>
<td>Expanded</td>
<td>2.5</td>
</tr>
<tr>
<td>Enhanced</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Data from DoD (20).

1.C.1.B. Unit size

FOBs are generally designed to meet the needs of a specified unit size. The US Army defines five sizes of FOBs with approximate populations, as shown by Table 4.

<table>
<thead>
<tr>
<th>Unit size</th>
<th>Population (approximate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platoon</td>
<td>50</td>
</tr>
<tr>
<td>Company</td>
<td>300</td>
</tr>
<tr>
<td>Battalion</td>
<td>1,000</td>
</tr>
<tr>
<td>Brigade</td>
<td>3,000</td>
</tr>
<tr>
<td>Support area</td>
<td>6,000+</td>
</tr>
</tbody>
</table>

Data from DoD (20).
1.C.1.C. Construction type

Housing militaries in the field using temporary structures that are inexpensive and rapidly constructed is not a new problem. During the First World War, the British met this need by developing the “Nissen Hut.” During World War II, the US developed the “Quonset Hut.”(69) In the Vietnam War, the US created a new hut design based on standard lumber sizes and plywood which could be more easily shipped and rapidly constructed. These “South East Asia Huts,” (or more commonly known as “SEA Huts”) were adopted and standardized across the service branches of the military. Today, they are known as “South West Asia Huts” or “SWA Huts,” the name change reflecting slight design alterations necessitated by a different environment (i.e., removing screens on SEA Hut walls that facilitated stack-driven ventilation in hot, humid climates but would result in poor indoor air quality if employed in a dusty, desert environment). Although building designs for contingency operations have been improved over many years, they remain today very energy inefficient. Although SWA Huts can be insulated, generally using fiberglass batt insulation, they are often left uninsulated as other construction projects in other areas take precedence and the functioning building is left as-is.

Research spearheaded at the United States Military Academy (USMA) at West Point, New York resulted in a new, highly energy efficient hut design based on Structural Insulated Panels, or “SIPs.” Although the first iteration of the “SIP Hut” used the same dimensions and gable roof style of the SWA Hut, the latest designs incorporate small ballistic glass windows to allow for daylighting and a shed roof to allow for maximum PV installation given proper building siting and orientation. The SIP Hut can be assembled by a crew of four people in a matter of hours, from a level site to the full building envelop complete. Life cycle assessments of the SIP Hut shows significant savings over the SWA Hut, especially after five years of use.
Despite its energy efficiency improvements over the SWA Hut, the SIP Hut has not yet been largely adopted for several reasons. At ~2.5 times the volume of materials per structure, the SIP Hut requires significantly more logistics to transport. The SIPs themselves require a contract from a manufacturer and materials are not as easily accessible. Additionally, the foam used as insulation in the SIPs means a significant amount of non-biodegradable material would be left behind in a host nation after contingency operations are complete.

To overcome the issue of non-biodegradable foam, research is being conducted into biodegradable SIPs, or “bio-SIPs”. One study considered SIPs that used polyhydroxybutyrate-co-hydroxyvalerate (PHBV) instead of oriented strand board (OSB); bio-foam (BIOF) instead of extruded polystyrene (XPS), expanded polystyrene (EPS), or polyurethane (PUR); and Super Sap, a 37% bio-based epoxy, instead of industrial construction adhesive. The PHBV-BIOF showed a measured thermal conductivity of approximately 0.041 W m⁻¹ K⁻¹ (0.284 Btu hr⁻¹ ft⁻² °F⁻¹). Using American terminology, this equates to a conductive resistance of ~3.52 hr ft² °F Btu⁻¹ in⁻¹ (R-3.52 in⁻¹). Since the test panels were 209.52 mm thick (~8.25 in), this equates to a total R-value of ~29.0 hr ft² °F Btu⁻¹ (R-29). Research on a different type of bio-SIP using recycled fiber claims ~R-7 in⁻¹ panel thickness, suggesting a total R-45.5 for a 6.5 in-thick panel. This is an area of developing research that may have beneficial applications to military contingency basing. Although bio-based composites currently have inferior structural properties and resistance to degradation as compared to most traditional building materials, they can take advantage of optimized cellular shapes and geometries to improve their mechanical properties of strength and stiffness. At this point in time, however, bio-SIPs are not yet ready for application in military huts. Consequently, the authors leave the application of bio-SIP huts to future work.
The SIP Hut design has gone through several iterations, and different roof pitches are often used on the SWA Hut. For this paper, the authors simply estimate the conditioned space in both improved and unimproved SWA Huts as the product of the external dimensions for height, width, and length. This assumes that the ceiling is sealed off in both variants with ½ in OSB or plywood, though that is not always the case. For the SIP Hut, the model uses the shape of version four, where the height varies from 7 ft to 8 ft from side to side, or an average of 7.5 ft high.

The authors conduct a thermal analysis of the effects of different construction types using (1). The model takes a defined annual internal temperature set point and then uses the hourly ambient temperature from the TMY3 data files to calculate the change in temperature from the internal conditioned air space to the outdoor ambient temperature, $\Delta T$.

\[
q \left[ \frac{Btu}{hr} \right] = U \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circF} \right] \cdot A[ft^2] \cdot \Delta T[\circF]
\]

The authors further calculate the “UA-value” for each building type by considering the thermal resistivity of building components and the possible paths of heat flow into/out of each building. Figure 5 shows an example of the two heat flow paths possible in an improved SWA Hut wall, driven by the difference between the internal and ambient outdoor temperatures.

![Figure 5. Example of heat flow paths through the walls of an improved SWA Hut](Image)

20
The heat flow differs from that shown in Figure 5 for different building components and different building types. For example, in the improved SWA Hut, the floor differs from the walls because it is not insulated and there is no external OSB; the floor consists of just OSB overlaid on 2x4 floor joists. The ceiling differs from the walls in that there is no OSB on top of the 2x4 ceiling joists. In the unimproved SWA Hut, there is no insulation and no internal OSB on the walls. In the SIP Hut, the walls, floor, and roof are simply the panels themselves. In all cases, the combined convective-radiative heat transfer coefficients, $R_{cv}$, in units of hr ft$^2$ °F Btu$^{-1}$, are estimated for ordinary building surfaces as 0.61 for the ceiling, 0.68 for the walls, and 0.92 for the floor for internal, $i$ values (25). Outside, $o$ values in the same units are generally taken as 0.17 for the winter assuming 15 mph winds and 0.25 in the summer assuming 7.5 mph winds (25). This model simply takes the average of these outside values and applies the result to every hour of the year. $R_{cvo}$ values do not apply to the ceilings of the SWA Huts because this is an enclosed area not directly exposed to wind. This value does apply, however, to the roof of the SIP Hut, which is exposed to wind.

To determine the weighting factors between the two general paths illustrated in Figure 5, i.e., through the framing and through the cavity, the authors use (2), which examines a 1 ft x 1 ft cross-section of wall, ceiling, or floor with framing spaced at 16 in on-center. Because there is additional framing from headers and cripples over doors, jack studs, the double top plate, and the bottom sole plate in walls, hangars between joists in the floor, and blocking between ceiling joists, the model increases the framing factor estimate to 15% in walls and 12% in the floor and ceiling.
The thermal conductance, $U$-value, is the inverse of the thermal resistivity, $R$-value. The $R$-value can be added in series (the $U$-value cannot be added in series), and its inverse taken to find the overall $U$-value for a heat transfer path. Therefore, one can calculate the $UA$-value for the walls using (3) through (9). These equations can be adjusted to match the conditions described above for the different buildings’ walls, doors, windows, ceilings, roofs, and floors as appropriate.

\[
 UA_{walls,SW,Atu,improved} \left[ \frac{Btu}{hr \cdot \circ F} \right] = U_{avg,walls} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right] \cdot A_{walls}[ft^2]
\]

where:

\[
 A_{walls}[ft^2] = (2 \cdot (\text{width}[ft] \cdot \text{height}[ft])) - A_{doors}[ft^2] + (2 \cdot (\text{length}[ft] \cdot \text{height}[ft]))
\]

\[
 U_{avg,walls} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right] = \left( F_{framing}[\%] \cdot U_{walls,stud} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right] \right) + \left( 1 - F_{framing}[\%] \right) \cdot U_{walls,cavity} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right]
\]

\[
 U_{walls,stud} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right] = \frac{1}{R_{T,walls,stud} \left[ \frac{Btu}{hr \cdot ft^2 \cdot \circ F} \right]}
\]
\[ R_{T,wall,stud} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] \]

\[ = R_{cvi,wall} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{OSB} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{2x4} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{OSB} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{cvo,avg} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] \]

(7)

\[ U_{walls,cavity} \left[ \frac{Btu}{hr \cdot ft^2 \cdot ^\circ F} \right] = \frac{1}{R_{T,wall,cavity} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right]} \]

(8)

\[ R_{T,wall,cavity} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] \]

\[ = R_{cvi,wall} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{OSB} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{insul} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{OSB} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] + R_{cvo,avg} \left[ \frac{hr \cdot ft^2 \cdot ^\circ F}{Btu} \right] \]

(9)

Infiltration (air leakage) can account for significant loss of conditioned air in buildings, and different construction techniques lend to different rates of infiltration. The author conducted experiments on full-scale models of a SIP Hut and SWA Hut (sometimes referred to as a barracks Hut or “B Hut”) in 2013. Later, in 2015, the US Army Corps of Engineers, Engineer Research and Development Center, Construction Engineering Research Laboratory (USACE, ERDC, CERL) conducted experiments on a different set of SIP and SWA Huts in Champaign, Illinois. USACE’s experiments resulted in infiltration values within the extremes calculated by the author at USMA (Table 5). This makes sense because USMA’s SIP Hut used expanding foam insulation between all panels (floor, walls, and roof) whereas USACE’s SIP Hut was only partially sealed with roofing caulk, tape, and foam. On the other hand, USMA’s SWA Hut was
uninsulated, meaning it did not have a ceiling installed with fiberglass insulation lofted above nor wall insulation with an inside layer of OSB. This represents how many SWA Huts were built and used during deployments of the early 21st century. The USACE SWA Hut was improved with insulation in the walls and ceiling.

Table 5. Comparison of Minneapolis blower door test results on different variants of SIP and SWA Huts in New York (2013) and Illinois (2015)

<table>
<thead>
<tr>
<th>Measurement</th>
<th>USMA Experiments (2013)</th>
<th>USACE ERDC CERL Experiments (23)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SIP Hut v2</td>
<td>SWA Hut</td>
</tr>
<tr>
<td>CFM50 (cfm @ 50 Pa)</td>
<td>109</td>
<td>4,580</td>
</tr>
<tr>
<td>ACH50 (air changes per hour @ 50 Pa)</td>
<td>1.70</td>
<td>67.1</td>
</tr>
<tr>
<td>Infiltration, n (# air changes per hour)</td>
<td>0.09</td>
<td>3.35</td>
</tr>
</tbody>
</table>

The heat flow due to infiltration is given by (10) where the density of air, $\rho$, is 0.075 lb ft$^{-3}$, the specific heat of air, $c$, is 0.24 Btu lb$^{-1}$ °F$^{-1}$, and the internal conditioned air space volume, $V$, is calculated as described above for each building. Note that (10) is of the same format as (1), and the product of $\rho cnV$ Btu hr$^{-1}$ °F$^{-1}$ is equal to the “UA-value” of infiltration (11).

$$q \left[ \frac{Btu}{hr} \right] = \rho \left[ \frac{lb}{ft^3} \right] \cdot c \left[ \frac{Btu}{lb \cdot ^\circ F} \right] \cdot n \left[ \frac{# \text{ air changes}}{hr} \right] \cdot V \left[ \frac{ft^3}{air \ change} \right] \cdot \Delta T [^\circ F]$$  \hspace{1cm} (10)

$$UA_{infiltration} \left[ \frac{Btu}{hr \cdot ^\circ F} \right] = \rho \left[ \frac{lb}{ft^3} \right] \cdot c \left[ \frac{Btu}{lb \cdot ^\circ F} \right] \cdot n \left[ \frac{# \text{ air changes}}{hr} \right] \cdot V \left[ \frac{ft^3}{air \ change} \right]$$  \hspace{1cm} (11)
Whether one uses the CFM50 one-point test to directly measure the air flow rate at 50 Pa pressure differential, as was done at USMA, or one uses the multi-point test to develop a log-log plot of air flow ($ft^3/min$) vs pressure difference (Pa) with multiple measurements of air flow at different pressure differentials, as was done at USACE, (12) and (13) are used to calculate the air changes per hour (ach) due to infiltration. For the ach factor in (13), the model uses the “Princeton” estimate of 20 (73). A more refined estimate based on location, building height, and wind shielding created by the Lawrence Berkeley National Laboratory (LBNL) could also be used, with values ranging from 14.0 to 29.4 for single story structures (74).

\[ ACH50 \left[ \frac{\text{air changes}}{\text{hr}} \right] = \frac{\text{CFM50} \left[ \frac{ft^3}{\text{min}} \right] \cdot \frac{60 \text{ min}}{1 \text{ hr}}}{V \left[ \frac{ft^3}{\text{air change}} \right]} \]  

(12)

\[ n \left[ \frac{\text{air changes}}{\text{hr}} \right] = \frac{ACH50[\text{air changes}]}{F_{ach}[-]} \]  

(13)

When using multi-point test data, one must first obtain the CFM50 (i.e., the air flow at 50 Pa pressure difference) by plotting the data, creating a trendline equation, and then using (14) (75).

\[ \text{CFM50} \left[ \frac{ft^3}{\text{min}} \right] = \text{air leakage coefficient} \left[ \frac{ft^3/\text{min}}{Pa} \right] \cdot (50[Pa])^{\text{pressure exponent}[-]} \]  

(14)

The authors use the ach under natural conditions for the unimproved SWA Hut using data from USMA’s experiments, for the improved SWA Hut using USACE’s experiments, and for the SIP Hut using the average of USMA’s and USACE’s experiments. This matches measured
data with the exact building types, except in the case of the SIP Hut where values are averaged across the different versions that have been built and tested to this point.

One can now modify (1) to form (15) and calculate an estimated rate of heat transfer through a building’s envelope driven by the difference in temperature from the internal temperature to the outdoor ambient temperature (17). The total UA-value “equivalent” is the summation of all UA-values calculated above for walls, doors, windows, ceiling or roof, floor, and infiltration (16). The internal temperature is equal to the thermostat “set” temperature, which is a constant 70°F year-round. This model neglects the effects of internal temperature gains (from occupants and equipment), the effects of the timing of thermal transience through building components, and shading due to solar position. Such refinements are not expected to have large impacts given the purpose of this model, but they can be added in future work.

\[
q \left[ \text{Btu/hr} \right] = \text{Total } UA_e \left[ \text{Btu/hr} \cdot \text{°F} \right] \cdot \Delta T[\text{°F}]
\]

\[
\text{Total } UA_e \left[ \text{Btu/hr} \cdot \text{°F} \right] = (UA_{\text{walls}} + UA_{\text{doors}} + UA_{\text{windows}} + UA_{\text{ceiling/roof}} + UA_{\text{floor}} + UA_{\text{infiltration}}) \left[ \text{Btu/hr} \cdot \text{°F} \right]
\]

\[
\Delta T[\text{°F}] = (T_i[\text{°F}] - T_a[\text{°F}])
\]

Whereas the UA-value is a good means of comparing the energy efficiency of buildings of the same size (applicable in this study), the thermal index is a convenient way to compare the energy efficiency of buildings of different sizes. The thermal index is calculated using (18).
Table 6 shows a summary of the UA-values and thermal indices for the buildings considered in this study.

\[
\text{Thermal Index} = \frac{Btu}{day \cdot ft^2 \cdot ^\circ F} = \frac{24 \frac{hr}{day} \cdot Total \text{ UA}}{A_{floor} \frac{Btu}{hr \cdot ^\circ F}}
\]

Table 6. UA-values and thermal indices by building type

<table>
<thead>
<tr>
<th></th>
<th>Unimproved SWA Hut</th>
<th>Improved SWA Hut</th>
<th>SIP Hut</th>
</tr>
</thead>
<tbody>
<tr>
<td>UA-value [\frac{Btu}{hr \cdot ^\circ F}]</td>
<td>1,381</td>
<td>534</td>
<td>77</td>
</tr>
<tr>
<td>Thermal Index [\frac{Btu}{day \cdot ft^2 \cdot ^\circ F}]</td>
<td>65</td>
<td>25</td>
<td>4</td>
</tr>
</tbody>
</table>

1.C.1.C.1. Building Cost

A 2015 study on SWA Huts and SIP Huts by USACE reported that the material cost of a SIP Hut was $14,500, whereas the material cost of an unimproved SWA Hut was $6,500, a difference of $8,000 (23). This report did “not include electrical, lighting, or HVAC systems” but reasoned that “the actual HVAC systems used in contingency bases vary from military issued Environmental Control Units (ECUs) to relatively inexpensive commercial, locally procured split systems” (23). The USACE report did acknowledge, however, that the SIP Hut’s labor cost is, on average, less than one-third that of a SWA Hut due to its speed of construction (~$1,700 for a SIP Hut vs ~$5,300 for a SWA Hut, a difference of ~$3,600) (23).

A 2014 thesis on the life cycle assessment of SIP Huts vs SWA Huts reported that the material cost of a SIP Hut was $17,400, whereas the material cost of an unimproved SWA Hut was $6,300, a difference of $11,100. This difference is due to the specialized SIPs that must be custom made by a SIP manufacturer. This report also did not include interior components, but it did report that a SWA Hut’s ECU would cost ~$19,000 whereas a SIP Hut’s ECU would cost.
~$9,500, a difference of $9,500 per hut. The improved energy efficiency of the SIP Hut allows for much smaller ECUs to be installed, which lowers two major costs: an upfront capital cost for the purchase, transport, and installation of the ECUs, and an ongoing operations cost in terms of electricity required to run the ECUs. This thesis reported that the SIP Hut’s construction/labor cost was ~$1,700 vs ~$2,900 for the SWA Hut, a difference of ~$1,200. Finally, the thesis notes that the materials for five SWA Huts can be packed and shipped within a 40 ft shipping container, also known as a Container Express, “CONEX” or International Standard Organization (ISO) container, whereas the materials for just two SIP Huts can fit in the same container. Therefore, a SIP Hut likely costs about 2.5 times that of a SWA Hut to transport, though that will depend on many factors to include the modes of shipment used.

This model neglects electrical and lighting systems as they will be mostly similar between the building styles. Also not included is the cost of construction; it is assumed military personnel will primarily provide the labor in all cases. Without knowing the actual location of a FOB and the modes of transport required, the authors avoid offering any initial cost benefit to SWA Huts for transport or any initial cost benefit to SIP Huts for being able to use significantly less expensive ECUs or less labor. A rough estimate is that, if the increased cost of transportation for a SIP Hut does not exceed 2.5 times the difference in required ECU cost (~$23,750 per hut), assumptions will favor the SWA Hut in terms of initial cost. This paper adopts the latest published values for cost: $6,500 per unimproved SWA Hut and $14,500 per SIP Hut. Further, the authors estimate the additional material cost for insulation in the walls and ceiling and OSB on interior walls at ~$900 per hut, making an improved SWA Hut cost ~$7,400.
1.C.1.D. Building area requirements

The model uses planning factors for building area requirements published in the *Base Camp Student Guide* by USMA’s Department of Civil & Mechanical Engineering (19). These values are, in turn, consolidated from various US Air Force, US Army, and US Central Command published standards. In terms of square feet per person, the model uses values of 80 ft² for billeting, 3 ft² for Tactical Operations Centers (TOCs), 14 ft² for Dining Facilities (DFACs), 3.3 ft² for gymnasiums and other Morale, Welfare & Recreation (MWR) facilities, 1.2 ft² for Post eXchanges (PXs), and 2.4 ft² for medical, dental, and other aid stations. It is assumed that other building types, like those for laundry, maintenance, and other uses, are non-conditioned spaces. However, the model allows for users to change these parameters as desired.

Using the selected unit size (see Table 4) and (19), the model calculates the total building area required for a FOB. All huts are the standard 16 ft by 32 ft size (512 ft²), and the number of huts required is calculated using (20). Results from (20) are rounded up to the nearest integer, and then the total building square footage required is recalculated using (21).

\[
S_{\text{reqd}}[ft^2] = S_{\text{reqd}} \left[ \frac{ft^2}{\text{person}} \right] \cdot n[\text{people}]
\]

\[
n[huts] = \frac{S_{\text{reqd}}[ft^2]}{S_{\text{Hut}} \left[ \frac{ft^2}{\text{hut}} \right]}
\]

\[
S_{\text{Buildings}}[ft^2] = S_{\text{Hut}} \left[ \frac{ft^2}{\text{hut}} \right] \cdot n[huts]
\]
1.C.2. Power load requirements

U.S. Army planning factors for power production use 1.5 kW person\(^{-1}\) for basic, 2.5 kW person\(^{-1}\) for expanded, and 3.5kW person\(^{-1}\) for enhanced levels of service. Using these factors, the total power production capabilities needed for a FOB is estimated using (22). Unfortunately these planning factors do not account for climate; they assume the same power would be required for a FOB in a tropical climate as would one in a polar climate and require further adjustment.

\[
L_{\text{peak}}[kW] = F_{pp} \left[ \frac{kW}{\text{person}} \right] \cdot n[\text{people}]
\]  

(22)

1.C.2.A. Lighting and plug loads

To address the lack of climate effects in determining the peak power requirement using military planning factors, the authors split a FOB’s power load into two components: one for HVAC loads and one for everything else (mostly lighting and plug loads). Because the peak load calculated from planning factors and (22) is supposed to be for all loads, including general HVAC requirements, the authors sought a way to refine the estimate. In U.S. residential and commercial buildings, HVAC loads account for ~38% of energy use; the rest (~62%) goes toward lighting, water heating, refrigeration, electronics, cooking, cleaning, and other plug load requirements (25). To be conservative and include a factor of safety in this estimate, the authors chose to add an additional 10% and a lighting and plug load factor of 72%, which is applied using (23). The authors believe this assumption is acceptable because the purpose is to compare portfolio options, and the effect will be applied across all FOB options, existing and proposed. Also, should future energy requirements be assessed as greater than current planning factors,
then this model will already account for at least some portion of that. The model allows users to change this factor as desired.

\[
L_{\text{Peak,L&PL}}[kW] = F_{L&PL}[\%] \cdot L_{\text{Peak}}[kW]
\]

Because these loads are not constant, it is important to create a mock load profile for the analysis. To do so, the authors adopted a load profile used by PNNL in studies on energy for a 150-person FOB, which in turn had been adopted from a previous report by TIAX (10). For each hour of a day, the load was recorded for the PNNL FOB, \(L_{PNNL,h}\). Each hourly load measurement was then divided by the maximum hourly load experienced during the 24-hour period, \(L_{PNNL,max}\) to determine an hourly percent of daily maximum load, \(PDML_h[\%]\) (24). A modified peak daily lighting and plug load, \(M_{\text{Peak,L&PL,d}}\), is calculated using (25), which acknowledges that the maximum load experienced during a typical day is only a percentage of the peak lighting and plug load, i.e., a buffer exists in planning factors, and the maximum expected load is not experienced every day. This is accomplished by dividing the peak lighting and plug load from (23) by \(1 + \) a buffer percentage, which is called the “percent above load allowed, \(PALA\).” Finally, the product of the hourly percent daily maximum load and the modified peak daily lighting and plug load (26) yields a baseline hourly lighting and plug load, \(L_{B,L&PL,h}\). This hourly value is based on PNNL’s 150-person FOB yet scaled to whatever user input selections have been made for the FOB unit size and level of service in the model.

\[
PDML_h[\%] = \frac{L_{\text{PNNL,h}}[kW]}{L_{PNNL,max}[kW]}
\]
\[ M_{\text{Peak,L\&PL,d}}[kW] = \frac{L_{\text{Peak,L\&PL}}[kW]}{(1 + PALA[\%])} \]  

(25)

\[ L_{B,L\&PL,h}[kW] = PDM_{h}[\%] \cdot M_{\text{Peak,L\&PL,d}}[kW] \]  

(26)

This, however, is just a daily baseline load profile. One could apply the same load every day, but that would not be realistic. Operations will cause the actual load profile to shift up and down from the baseline. Consequently, the user can assign a percent below load allowed, \( PBLA \) and a percent above load allowed, \( PALA \). The model then uses a random number generator in Microsoft Excel to randomize the actual load profile shape within \( +PALA[\%]/-PBLA[\%] \) of the scaled baseline.

To get a random decimal in Microsoft Excel (27), one can use the RAND() function. Because Excel’s RAND() function updates frequently, the authors have done it once and then used paste-special with the values to keep the same randomized scaling factors for each hour from trial to trial. (Note that, from here on, all calculations are generally done for each hour, so nomenclature will simply drop the “h” subscript.)

\[ 0.0 \leq n_{\text{random}}[-] \leq 1.0 \]  

(27)

In order to get a random number in Excel that is between two specified values, A and B, one can use the formula: \( A + (B-A) \times \text{RAND()} \). Therefore, to get a modified, randomized hourly lighting and plug load within the specified percent below/above load allowed of the baseline lighting and plug load, use (28). In this study, the authors modeled 30% allowed both above and below the mock load.
\[
M_{Load}[\%] = n_{random}[-] \cdot ((1 + PALA[\%]) - (1 - PBLA[\%])) + (1 - PBLA[\%])
\]

(28)

\[
L_{L+PL}[kW] = M_{Load}[\%] \cdot L_{B,L&PL}[kW]
\]

(29)

The result of all these modifications is a lighting and plug load profile for all 8,760 hours of the year that is based off from an example FOB load profile yet scaled to the size and level of service FOB under investigation. Further, the maximum lighting and plug load experienced is the peak lighting and plug load from (23), and the minimum lighting and plug load is the defined \( PBLA[\%] \) of the minimum scaled baseline lighting and plug load.

1.C.2.B. HVAC loads

In addition to the lighting and plug loads, the FOB will have HVAC loads that are dependent upon the environmental conditions at that location. In this study, the authors model the use of ASHPs for heating and cooling needs, though, often, the military uses ECUs. ECUs are ruggedized, heavy duty ASHPs. One example ECU has the capability of maintaining, within a tent, a maximum indoor temperature of 26°C (79°F) while in cooling mode and a minimum indoor temperature of 10°C (50°F) while in heating mode with ambient temperatures ranging anywhere from -19 to 49°C (-2 to 120°F), though the manufacturer does not expound upon the insulation applied (if any) to the tent (76). As discussed, with more energy efficient buildings like the SIP Hut, the military can use smaller, more efficient civilian ASHPs. Table 7 provides a side-by-side comparison of an example ECU and an example ASHP.
Table 7. Example ECU and ASHP nameplate/rating performance characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Example Military Grade ECU (76)</th>
<th>Example Civilian ASHP (27)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooling capacity (BTU hr(^{-1}))</td>
<td>60,000</td>
<td>22,000</td>
</tr>
<tr>
<td>SEER (Btu Wh(^{-1}))</td>
<td>15</td>
<td>20.5</td>
</tr>
<tr>
<td>Heating capacity at 47°F (Btu hr(^{-1}))</td>
<td>55,000</td>
<td>25,000</td>
</tr>
<tr>
<td>Heating capacity at 17°F (Btu hr(^{-1}))</td>
<td>28,600</td>
<td>18,200</td>
</tr>
<tr>
<td>Operating temperature range</td>
<td>-19°C to 49°C</td>
<td>-20°C to 46°C</td>
</tr>
<tr>
<td></td>
<td>-2°F to 120°F</td>
<td>-4°F to 115°F</td>
</tr>
<tr>
<td>HSPF (Btu Wh(^{-1}))</td>
<td>8.2</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Heat pumps work both as a heater and as an air conditioner. Consequently, they are sold with two separate ratings for efficiency, the HSPF and the SEER. Both are based on the concept of a Coefficient Of Performance (COP), which is a unitless ratio of heat energy output divided by electrical energy input (30). For “cooling output,” the heat energy is being moved to and expelled outdoors when the heat pump is in air conditioning mode.

\[
COP[-] = \frac{\text{Heat output}}{\text{Electrical energy input}}
\]

(30)

The HSPF and SEER are given using (31), which uses different units of energy in the numerator and denominator convenient to North American standard units of heat energy (Btu) and electric energy (Wh).

\[
HSPF \text{ or } SEER \left[\frac{BTU}{Wh}\right] = \frac{\text{Total seasonal heat energy transferred (output)[Btu]}}{\text{Total seasonal electric energy required (input)[Wh]}}
\]

(31)

Dividing the HSPF by the conversion factor between Btu and Wh yields a seasonal heating average COP, given by (32), which is known in Europe as the “Seasonal Performance
Factor” (SPF) (77). The same goes for the seasonal cooling average COP to define a “cooling SPF” in (33).

\[
SPF_{heating}[\cdot] = \frac{HSPF \left[ \frac{BTU}{Wh} \right]}{3.41214 \left[ \frac{BTU}{Wh} \right]}
\]

(32)

\[
SPF_{cooling}[\cdot] = \frac{SEER \left[ \frac{BTU}{Wh} \right]}{3.41214 \left[ \frac{BTU}{Wh} \right]}
\]

(33)

The HSPF and SEER ratings are established using standard laboratory tests defined by the American National Standards Institute (ANSI), the Air-conditioning, Heating, and Refrigeration Institute (AHRI), and the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) in ANSI/AHRI Standard 210/240 (78). A heat pump’s performance at any instant depends upon the environmental conditions. There are many reductions in efficiency that occur, to include defrosting requirements, temperature differentials, fan operation, duty load cycling, and using electric resistance supplemental heat at temperatures below the balance point (79). Consequently, the HSPF is meant to measure average performance over the entire heating season. When used for cooling, the performance of a heat pump is generally greater than when it is used for heating because of the smaller temperature difference between inside and outside and the fact that heat is being extracted from the warm interior rather than from very cold outdoor air (28).

It has been noted that the nameplate HSPF and SEER values were “never intended… to be used to estimate energy use across climates,” but they have been, which “can lead to erroneous conclusions on their relative merit” (28). The problem arises because published
“rated” or “nameplate” HSPFs are for AHRI Climate Zone IV only, which has 2,000 to 2,500 annual heating load hours and an outdoor design temp of 5°F (28,78,80). Of the locations in this study, just Stewart Airfield, New York and Dulles Airport, Virginia are within climate zone IV. Twentynine Palms, California is within Climate Zone III, and AK and HI are in zones not even defined within ANSI/AHRI Standard 210/240 (78).

Another problem with the manufacturer’s published HSPF is that established testing procedures and calculations incorporate a correction factor “C” equal to 0.77 in order to reduce the calculated building load required on the heat pump (80). This factor was introduced to correct test results to better match measured building loads using a 65°F set point. In reality, however, this difference is caused by internal and solar heat gains, which effectively lowers the balance point temperature (28). The Electric Power Research Institute (EPRI) conducted tests and found that C could actually vary from 0.4 to 1.2 in residences (28).

To better understand the problem of using manufacturer’s reported HSPF and SEER values in spreadsheet calculations for 8,760 hours of a year, consider Figure 6. The manufacturer’s reported COPs under different lab temperature conditions are shown by the singular points in triangles. The dashed lines represent the COP that is effectively used in calculations whenever a model uses just the nameplate/rated HSPF and SEER. Clearly, this can lead to significant modeling errors depending upon actual climate and temperature conditions. One method of improving spreadsheet calculations is to incorporate user-defined approximations for the COP in heating and cooling modes at any temperature, as shown by the solid lines and trend line equations. This facilitates spreadsheet calculations where the inside temperature (thermostat set point) and the outside temperature (dry bulb) are known. However, these straight lines are simply approximations and may give a false sense of accuracy.
In one study, a group of researchers devised another method to address the changes in climate effects on ASHP performance (28). They selected 15 locations across the US that represented all six climate zones covered in the AHRI standards. They ran a model using a single-story, 2,000 ft² building model, and empirically-derived percent decrease factors for the nameplate HSPF and SEER values. For ASHPs with a reported HSPF > 8.5 Btu Wh⁻¹ and a reported SEER > 13.5 Btu Wh⁻¹, they devised the trend lines shown by (34) and (35), which are based on the rated HSPF and SEER values and the installed ASHP location’s 99% design heating temperature and 1% design cooling temperature from ASHRAE (28,29). The R-Square values for the trend lines are 0.9648 and 0.9654, respectively (28).
Using these equations, the calculated climate corrected HSPF and SEER values are shown and plotted in Figure 7. In this case, the red and blue dashed lines extend only for the range of outdoor dry bulb temperatures recorded in the TMY3 data file for each location, whereas the black dashed lines reflect the SPFs that result from the nameplate HSPF and SEER. When one considers that, in residences, heating is rarely done in Hawaii (internal and solar gains tend to make it unnecessary) and that cooling is rarely done in Alaska (the highest temperatures are still within comfort levels), it can now be clearly seen that using the manufacturer’s singular reported HSPF and SEER values across different locations with different climates would produce inaccurate results.
Table 8. HSPF and SEER corrections based on location climate for an ASHP with nameplate ratings of HSPF = 11.2 Btu Wh\(^{-1}\), SPF\(_{\text{heating}}\) = 3.3, SEER = 20.5 Btu Wh\(^{-1}\), SPF\(_{\text{cooling}}\) = 6.0

<table>
<thead>
<tr>
<th>Location state Köppen climate</th>
<th>Hawai\i Am</th>
<th>California Bwk</th>
<th>Virginia Cfa</th>
<th>New York Dfa</th>
<th>Alaska ET</th>
</tr>
</thead>
<tbody>
<tr>
<td>99% Heating T(_{db})</td>
<td>(°F)</td>
<td>(°C)</td>
<td>(°F)</td>
<td>(°C)</td>
<td></td>
</tr>
<tr>
<td>64.5</td>
<td>18.1</td>
<td>0.0</td>
<td>-8.2</td>
<td>-12.7</td>
<td>-34.3</td>
</tr>
<tr>
<td>1% Cooling T(_{db})</td>
<td>(°F)</td>
<td>(°C)</td>
<td>(°F)</td>
<td>(°C)</td>
<td></td>
</tr>
<tr>
<td>88.5</td>
<td>31.4</td>
<td>41.3</td>
<td>32.7</td>
<td>30.6</td>
<td>12.1</td>
</tr>
<tr>
<td>TMY3 T(_{db,\text{max}})</td>
<td>(°F)</td>
<td>(°C)</td>
<td>(°F)</td>
<td>(°C)</td>
<td></td>
</tr>
<tr>
<td>91.9</td>
<td>33.3</td>
<td>43.9</td>
<td>37.2</td>
<td>36.0</td>
<td>16.1</td>
</tr>
<tr>
<td>TMY3 T(_{db,\text{min}})</td>
<td>(°F)</td>
<td>(°C)</td>
<td>(°F)</td>
<td>(°C)</td>
<td></td>
</tr>
<tr>
<td>55.9</td>
<td>13.3</td>
<td>-2.2</td>
<td>-16.1</td>
<td>-20.0</td>
<td>-41.7</td>
</tr>
<tr>
<td>HSPF(_{\text{corrected}})</td>
<td>Btu Wh(^{-1})</td>
<td></td>
<td>(°F)</td>
<td>(°C)</td>
<td></td>
</tr>
<tr>
<td>18.3</td>
<td>5.4</td>
<td>3.2</td>
<td>2.5</td>
<td>2.2</td>
<td>1.4</td>
</tr>
<tr>
<td>SPF(_{\text{heating,corrected}})</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>SEER(_{\text{corrected}})</td>
<td>Btu Wh(^{-1})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17.9</td>
<td>5.2</td>
<td>4.6</td>
<td>5.2</td>
<td>5.3</td>
<td>6.4</td>
</tr>
<tr>
<td>SPF(_{\text{cooling,corrected}})</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Figure 7. ASHP SPF lines for heating and cooling

Figure 8 shows Figure 7 overlaid on Figure 6, which helps to illustrate the two possible methods of improved modeling. One could either use the solid, straight-line estimated COP values for heating and cooling at any hourly outdoor dry bulb temperature, or one could use the location-corrected HSPF and SEER values shown by the dashed lines. Moving forward, the
authors adopt the location-corrected HSPF and SEER values method proposed by Fairey et al. (28).

![Figure 8. ASHP COP and SPF lines for heating and cooling](image)

**Figure 8. ASHP COP and SPF lines for heating and cooling**

The model can now proceed with the analysis by first taking the outdoor dry bulb temperature values from the appropriate TMY3 data file and converting to degrees Fahrenheit. Next, (36) is used to find the temperature difference between the outdoor ambient temperature and the inside thermostat set point. Negative results mean heating is required; positive means cooling.

\[
T_{\Delta[^\circ F]} = T_{db[^\circ F]} - T_{set[^\circ F]}
\]

(36)

To facilitate spreadsheet operations, columns for both heating and cooling requirements in each hour are used. The heating requirement is given by (37) using an if-then statement and an absolute value operation to make all cooling hours equal to zero heating load and to convert
the negative results from (36) to positive values. (38) is used to calculate the heating load, making use of the equivalent UA-value for buildings, the number of huts, and the climate corrected HSPF values previously calculated.

\[
T_{\Delta \text{heat}}[^\circ F] = \begin{cases} 
0[^\circ F] & \text{if } T_{\Delta}[^\circ F] \geq 0 \\
|T_{\Delta}[^\circ F]| & \text{if } T_{\Delta}[^\circ F] < 0 
\end{cases}
\]  

(37)

\[
L_{\text{heat}}[kW] = \left( \frac{T_{\Delta \text{heat}}[^\circ F] \cdot UAe \left[ \frac{\text{Btu}}{\text{hr} \cdot ^\circ F \cdot \text{building}} \right] \cdot n_{\text{bldg[buildings]}}}{HSPF_{\text{corrected}} \left[ \frac{\text{Btu}}{\text{Wh}} \right]} \right) \cdot \left[ \frac{1kW}{1,000W} \right] \cdot [1\text{hr}]
\]

The same operation is used for cooling loads using (39) and (40).

\[
T_{\Delta \text{cool}}[^\circ F] = \begin{cases} 
0[^\circ F] & \text{if } T_{\Delta}[^\circ F] \leq 0 \\
T_{\Delta}[^\circ F] & \text{if } T_{\Delta}[^\circ F] > 0 
\end{cases}
\]  

(39)

\[
L_{\text{cool}}[kW] = \left( \frac{T_{\Delta \text{cool}}[^\circ F] \cdot UAe \left[ \frac{\text{Btu}}{\text{hr} \cdot ^\circ F \cdot \text{building}} \right] \cdot n_{\text{bldg[buildings]}}}{SEER_{\text{corrected}} \left[ \frac{\text{Btu}}{\text{Wh}} \right]} \right) \cdot \left[ \frac{1kW}{1,000W} \right] \cdot [1\text{hr}]
\]

(40)

The HVAC load power requirement is given by (41), where one of the two terms is always going to equal zero since only heating or cooling can occur in any given hour.

\[
L_{\text{HVAC}}[kW] = L_{\text{heat}}[kW] + L_{\text{cool}}[kW]
\]

(41)

Finally, the total power load for the FOB at every hour is calculated as the summation of the plug and lighting load and HVAC load (42).
1. C. Batteries

For modeling purposes, the authors use lithium ion batteries with the same specifications as Tesla Powerwalls for energy storage. Although centralized energy storage options, such as Tesla Powerpacks, could also be used, the use of Powerwalls represents a decentralized, and therefore likely more resilient, application.

The model allows for users to input a number of batteries used on the FOB. Since up to ten Powerwalls can be connected together in a building, the model suggests a maximum number of batteries defined using (43) (38).

\[
\text{n}_{\text{battery, FOB, max}}[\text{batteries}] = \text{n}[\text{huts}] \cdot 10 \left( \frac{\text{batteries}}{\text{hut}} \right)
\]

The most recent published costs for one Tesla Powerwall are $6,500 for the battery itself and an additional $1,100 for supporting hardware (38). As with all other costs aspects in this model, the authors neglect transport costs and installation construction/labor costs. The capital cost of batteries is estimated using (44).

\[
\text{c}_{\text{battery, FOB}}[\$] = \text{n}_{\text{battery, FOB}}[\text{batteries}] \cdot \text{c}_{\text{battery}} \left( \frac{\$}{\text{battery}} \right)
\]

Each battery has a useable capacity of 13.5 kWh with 5 kW continuous (7 kW peak) power capability. Tesla reports a round-trip efficiency of 90%, which uses a 3.3 kW charge/discharge and AC to/from the battery (81). For modeling, the model uses this reported value, which best represents charging from a diesel generator or wind turbine producing AC to
the microgrid. This efficiency will be a conservative estimate for battery charging from solar power production, which is already in DC and need not be inverted before battery charging.

Although batteries are installed in individual huts, this model considers the storage and discharge of battery energy as if there were one large battery on the FOB, i.e., battery charging/discharging is not limited to within the boundaries of individual huts. Therefore, the maximum limit of battery energy storage possible at any one time on the FOB is given by (45), the total continuous FOB battery power output possible by (46), and the total peak FOB battery power output possible by (47). The minimum battery energy storage is zero.

\[
E_{battery, FOB, max} [kWh] = E_{battery, max} \left[ \frac{kWh}{battery} \right] \cdot n_{battery, FOB} \text{[batteries]}
\]

(45)

\[
P_{battery, FOB, continuous} [kW] = P_{battery, continuous} \left[ \frac{kW}{battery} \right] \cdot n_{battery, FOB} \text{[batteries]}
\]

(46)

\[
P_{battery, FOB, peak} [kW] = P_{battery, peak} \left[ \frac{kW}{battery} \right] \cdot n_{battery, FOB} \text{[batteries]}
\]

(47)

1.C.4. Diesel generators

FOBs are generally powered by diesel generators that are either sourced as equipment organic to the unit, by specialized prime power units, or by civilian contract (82). When the generators themselves are reliable, when they are properly loaded and maintained, and when there is a consistent and secure supply of fuel available, this solution has proved acceptable.

Nevertheless, many problems exist that threaten energy security and energy resilience when diesel generators are the only solution. To ensure that power generation capabilities
Exceed power demand, base planners design for and install excess generators. This can be problematic. A study by Noblis, an independent company hired to collect data on actual FOB requirements in Iraq and Afghanistan in 2010, found that “the supply of power generated far exceeds demand at most FOBs. At one base, the 5 MW of demand was met by 19 MW of capacity with 196 generators running at 30% capacity and consuming 15,431 gallons of fuel per day” (66). Beyond being inefficient, running generators in such a fashion causes wet stacking and is detrimental to their overall maintenance and operational life (83). Wet stacking occurs when a diesel engine is operated on light loads and does not attain its correct operating temperature (84). When the diesel engine runs below its designed operating temperate for extended periods (typically by working at less than 30% of the nameplate power rating), unburned fuel is exhausted and noticed as wetness in the exhaust system (85). Furthermore, loading is often done incorrectly for 3-phase AC power, with one phase being overloaded and the others underloaded simply due to users not knowing any better.

Diesel generators are rated as standby, prime, or continuous. Standby generators are sized for a maximum of 70% to 80% of the average load factor for 200 to 500 hours per year (86,87). Prime power generators are meant to run continuously for an unlimited number of hours per year with an average load factor of no more than 70% the prime rating with 10% overload allowed for a maximum of one hour in 12 and no more than 25 hours per year (87). Continuous generators also have unlimited hours of usage but require load management to ensure a non-varying load factor of 70% to 100% the nameplate power rating. The typical demand is 100% for 100% of operating hours, with non-operating hours required for maintenance (87). In this paper, the authors consider two types of generators commonly used at military FOBs: a 60 kW Advanced Medium Mobile Power Sources (AMMPS) generator and an
The 840 kW generator. The AMMPS is a tactical power system organic to many units and is used during the initial stages of establishing a FOB, during the final stages of closing a FOB, and throughout the duration of a smaller FOB’s existence. Once a large FOB is established, larger generators are incorporated, which frees up smaller generators for use in other mission requirements (82). The 840 kW generator is a prime power, medium voltage system used for large FOBs that requires transformers to convert voltage to user levels (88).

Power generation at FOBs is usually accomplished by “spot generation” where a set of loads is connected to a single generator sized to meet the peak demand of those specific loads. Unfortunately, this tends to result in underloading and wet stacking as previously discussed (13). In the past, generators were seldom networked into a comprehensive power grid, but today the US Army is investigating the use of a Load Demand Start Stop (LDSS) microgrid that can aggregate loads between multiple generators, which leads to more optimal loading and the ability to shut down unneeded generators (13). Although it may seem logical and more efficient under normal conditions to generate all primary power at a centrally located power plant, using a more dispersed layout for military bases reduces risk by decreasing the likelihood of total loss of generating capability due to an attack (21).

The model first selects the appropriate generator type based upon unit size and level of service, according to Table 9. All platoon- and company-sized FOBs use the 60 kW AMMPS generators, while battalion-, brigade-, and support area-sized FOBs use the 840 kW generators.
Table 9. Generator type and published planning factors based on FOB unit size and level of service

<table>
<thead>
<tr>
<th>Unit</th>
<th>Size (# people)</th>
<th>Level of service planning factors (kW person(^{-1})) &amp; resulting FOB requirements (kW)</th>
<th>Generator type (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Platoon</td>
<td>50</td>
<td>Basic, 1.5 75 125 175</td>
<td>60</td>
</tr>
<tr>
<td>Company</td>
<td>300</td>
<td>Expanded, 2.5 450 750 1,050</td>
<td>60</td>
</tr>
<tr>
<td>Battalion</td>
<td>1,000</td>
<td>Enhanced, 3.5 1,500 2,500 3,500</td>
<td>840</td>
</tr>
<tr>
<td>Brigade</td>
<td>3,000</td>
<td></td>
<td>840</td>
</tr>
<tr>
<td>Support area</td>
<td>6,000</td>
<td></td>
<td>840</td>
</tr>
</tbody>
</table>

Next, the user can input a desired power generation percent overage for backup and maintenance, \(POBM\)\(\%\). This is used to determine the total number of generators that a FOB should have, according to published planning factors, using (48).

To ensure enough generators for the model in the most extreme conditions, the authors elected to use an overage value of 50\%. Although this may seem excessive, it is prudent for several reasons. First, as previously discussed, the published planning factors do not account for building construction or climate type, so the more extreme environments studied in this paper will require resources beyond which the planning factors suggest. Second, referring back to the Noblis study of recent FOBs in Iraq and Afghanistan, having 19 MW of generation capacity to meet a 5 MW load is equivalent to an overage of 280\%, far more than this model uses. This assumption allows for generators on a FOB to be shut down for maintenance, repair, or replacement without compromising meeting mission load requirements. With regard to operating load, the model allows for user input as to the minimum and maximum load fraction. As discussed, generators can exceed their rated load capacity by \(~10\%\) for limited periods of time, and they can operate at lower than 50\% of their rated load capacity but will end up wet stacking over time. For this model, the authors establish a minimum load fraction of 30\% and a maximum load fraction of 100\%. 

46
\[ n_{\text{generator, FOB}}[\text{generators}] = (1 + POBM[\%]) \cdot \left( \frac{L_{h,\text{max}}[kW]}{P_{\text{generator}}[kW]} \right) \]

Although these 60 kW and 840 kW generators are already within specific units’ organic equipment inventories, they often do not have enough to meet all of a FOB’s needs, especially as the FOB progresses to higher levels of service. It is helpful, therefore, to be able to estimate the capital cost of generators, transformers, and other distribution equipment, which can later be compared to the capital cost of batteries and renewable power generation assets. The cost of one 60 kW AMMPS is \(~$25,000\) and the cost of one 840 kW generator is \(~$450,000\). In addition to this, 840 kW generators require a Primary Switching Center (PSC) that costs about \(~$57,000\) to transfer power to the microgrid as well as multiple Secondary Distribution Centers (SDCs) that costs about \$23,000 each to transform voltage to the user-level. Multiple generators can connect to one PSC, and multiple SDCs can be connected to one PSC. For this paper, the authors estimate an 840 kW generator cost (with required downstream equipment) using (49).

\[ c_{\text{generator}}[\frac{\$}{\text{generator}}] = \$450,000 + \left( \frac{1}{3} \right) \cdot \$57,000 + (5) \cdot \$23,000 \]

The total capital cost of diesel power generation is estimated using (50). (On a side note, a quick survey of long-term rental costs for diesel generators in the U.S. reveals a cost of about \$0.50 kW^{-1}\text{day}^{-1} \text{ (89). That equates to over }$3.8 \text{ mil yr}^{-1} \text{ for a Support area-sized FOB at an enhanced level of service, and this does not include the cost of backup generators, transformers, power distribution, fuel, operations and maintenance, transportation for delivery and removal, etc.)} Despite calculating these costs, they are not used in cost comparisons. Again, it is assumed that these resources will be installed on the FOB in all scenarios, whether they are used or not.
Manufacturers often report a singular value for the fuel consumption rate of generators, like 60 gal hr\(^{-1}\) for the 840 kW generator (40,90). However, this value represents fuel economy at the rated load. Fuel economy is actually non-linear, and the approximate fuel efficiency of a 60 kW AMMPS operating between 70% and 100% of its rated capacity is \(\sim 13.1 \text{ kWh gal}^{-1}\) (10). At 100% load, the 60 kW produces \(\sim 13.43 \text{ kWh gal}^{-1}\). Using (51), the average fuel consumption for a 60 kW AMMPS at full load is estimated as \(\sim 4.5 \text{ gal hr}^{-1}\). Therefore, fourteen 60 kW generators running at 100% capacity to produce 840 kW would require \(\sim 62.5 \text{ gal hr}^{-1}\), which is within \(\sim 4\%\) of the fuel efficiency of an 840 kW generator operating at its rated load.

\[
FE_{\text{diesel,60kW}} \left[\frac{\text{gal}}{\text{hr}}\right] = \frac{60 \left[\frac{\text{kW}}{\text{hr}}\right]}{13.43 \left[\frac{\text{kWh}}{\text{gal}}\right]} = 4.47 \left[\frac{\text{gal}}{\text{hr}}\right]
\]

1.C.4.A. Fuel costs and consumption

The cost of diesel fluctuates over time and also varies from region to region. The IEA’s 2019 World Energy Prices Statistics Report states that, for the ten years between 2009 and 2018, the annual average global price for diesel ranged from a minimum of 0.85 USD L\(^{-1}\) in 2016 to a maximum of 1.15 USD L\(^{-1}\) in 2009 (43). The world average was about 1.1 USD L\(^{-1}\) (~4.16 USD gal\(^{-1}\)) in 2018. In March 2020, one organization tracking the energy prices in 163 countries around the world showed that there were 115 countries below this value and 48 above, with a maximum price of 6.81 USD gal\(^{-1}\) (41).

However, research has shown that just the purchase price of the fuel commodity itself is an inappropriate metric for military applications (45,66,91). United States Code (Title 10,
Subtitle A, Part IV, Chapter 173, Subchapter I, Section 2911) states, “the Secretary of Defense shall require that the life-cycle cost analysis for new capabilities include the fully burdened cost of fuel during analysis of alternatives and acquisition program design trades” (12). It further defines “the term ‘fully burdened cost of fuel’ [as] the commodity price for fuel plus the total cost of all personnel and assets required to move and, when necessary, protect the fuel from the point at which the fuel is received from the commercial supplier to the point of use” (12).

Several models and instructions exist for how to calculate and implement the fully burdened cost of fuel (44,92,93). Starting at the Defense Logistics Agency (DLA) – Energy website, the FY20 standard purchase price of diesel was $2.66 gal⁻¹ and the moving purchase price was $2.91 gal⁻¹ (42). Note that the standard purchase price is established prior to the start of the FY for budgetary purposes and cannot be compared to purchase price, whereas the moving purchase price is the average purchase cost in DLA’s inventory, which changes daily. One frequently cited model on the fully burdened cost of fuel in Iraq considered a base case for FY07 where the fuel commodity itself was 2.14 USD gal⁻¹ (44). On top of this, average costs for materiel ($0.40 gal⁻¹), transport ($1.06 gal⁻¹), personnel ($2.28 gal⁻¹), sustainment ($0.13 gal⁻¹), force protection - ground ($3.03 gal⁻¹) and force protection - air ($16.25 gal⁻¹) were added to come to a total fully burdened cost of fuel of $25.29 gal⁻¹. If these projections are extended to FY20 using each value’s relation to the fuel commodity price, the fully burdened cost of fuel is $34.51 gal⁻¹. Instead, simply converting the FY07 values to FY20 values using a Consumer Price Index (CPI) inflation calculator, the fully burdened cost of fuel is $32.55 gal⁻¹. In this paper, the authors take a very conservative approach and use a fully burdened cost of fuel value of $8.32 gal⁻¹, which takes the current moving purchase price of $2.92 gal⁻¹ and adds $1.36 gal⁻¹ for transport costs (to include fuel for convoy trucks), $0.17 gal⁻¹ for sustainment (to include maintenance of convoy
vehicles), and $3.88\,\text{gal}^{-1}$ for ground force protection (like gun truck and weapons platform operations). As with all other components of this model, the authors neglect the cost of materiel that is already within the military’s inventory, the labor costs paid to personnel, and further assume that force protection via air is not required.

The model uses a binary decision variable to determine, at every hour, if each one of the generators is on or off. If on, the model calculates the fuel consumed and takes the summation for the cost of diesel consumed by each generator that is on. The model does not currently consider costs for starting, shutting down, labor, or maintenance, but it does consider reductions in fuel consumption as a direct measure of improved energy security and resilience. Reductions in fuel consumption from generators means reductions in (and reduced reliance on) required logistics convoys, reductions in risk of attack, and reductions in loss of life. If FOBs incorporate renewables and energy storage but are still built with the same number of generators and diesel storage as per business as usual designs, any diesel power generation offset by renewables will mean more on site capability. The unused diesel that remains readily available directly translates to energy security (having access to energy when you need it) and resilience (ability to bounce back if any disruptions do occur).

1.C.5. Solar power production

1.C.5.A. Solar altitude and azimuth

There are various methods for performing calculations to determine the Sun’s position in relation to a solar collector and the amount of incident solar radiation (insolation) that a solar collector can receive. This paper investigates the methods used by three sources: the National Oceanic and Atmospheric Administration’s (NOAA’s) Earth System Research Laboratory, ASHRAE in their 2013 Handbook – Fundamentals (SI edition), and Professor Emeritus Gilbert
Masters in his book *Renewable and Efficient Electric Power Systems* (31–33). Through investigating these three methods, the authors highlight some differences in approach between them as well as comment on yet other methods, specifically NREL’s Solar Position Algorithm (SPA), which is largely based on the book *Astronomical Algorithms* by Jean Meeus (30,94).

The objective of performing solar position calculations is to determine the solar altitude and azimuth angles for each hour of the year, which describe the position of the Sun from the perspective of an observer (or solar collector) on the Earth’s surface. Consequently, the process of calculating the Sun’s position from this reference point requires transitioning from ecliptic coordinates to equatorial coordinates and then to the solar altitude and azimuth angles. To assist in understanding the calculations that follow, Table 10 provides a brief description of critical terms, listed in the general order in which they are used.
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Julian Day</td>
<td>The number of days (including fraction) between the start of the Julian Calendar on January 1, 4713 BC to the day of interest.</td>
</tr>
<tr>
<td>OLE Automation Date</td>
<td>The numerical day (including fraction of days) from the start of January 1, 1900; used in Microsoft applications.</td>
</tr>
<tr>
<td>Julian Century</td>
<td>The number of centuries (including fraction) from a starting reference point, generally the 2000 Standard Epoch (J2000.0) starting on January 1, 2000.</td>
</tr>
<tr>
<td>“Apparent”</td>
<td>In terms of reference of an observer (or solar collector) on Earth where the Sun and stars appear to move across the sky in a 24-hour period.</td>
</tr>
<tr>
<td>Plane of the Ecliptic</td>
<td>An imaginary plane that contains the Earth’s orbit around the Sun (or, in “apparent” terms, the Sun’s rotation around the Earth).</td>
</tr>
<tr>
<td>Perihelion</td>
<td>The point of the Earth’s orbit around the Sun where the distance between the two bodies is shortest.</td>
</tr>
<tr>
<td>Aphelion</td>
<td>The point of the Earth’s orbit around the Sun where the distance between the two bodies is greatest.</td>
</tr>
<tr>
<td>Sun’s Geometric Mean Longitude</td>
<td>The degree of the Earth’s orbital progression around the Sun in the plane of the ecliptic measured as if the orbit was a perfect circle and 0° starts at the point of orbit when the vernal equinox occurs.</td>
</tr>
<tr>
<td>Sun’s Geometric Mean Anomaly</td>
<td>Same as the Sun’s Geometric Mean Longitude except 0° starts at the perihelion.</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>A dimensionless parameter between 0 and 1 that describes the amount of “flatness” in shape, i.e., the deviation from a perfect circle, of an ellipse.</td>
</tr>
<tr>
<td>“Mean”</td>
<td>Assumes a perfectly circular orbit, i.e., the Earth’s orbit has a constant angular velocity.</td>
</tr>
<tr>
<td>“True”</td>
<td>Uses the actual elliptical orbit, i.e., accounts for changes in angular velocity in the Earth’s orbit.</td>
</tr>
<tr>
<td>Equation of Center</td>
<td>A measure, in degrees, of difference between “mean” and “true” positions.</td>
</tr>
<tr>
<td>Sun’s True Longitude</td>
<td>Same as the Sun’s Geometric Mean Longitude except now accounts for the elliptical shape of the Earth’s orbit and the resulting change in angular velocity by adding the Equation of Center correction.</td>
</tr>
<tr>
<td>Sun’s True Anomaly</td>
<td>Same as the Sun’s Geometric Mean Anomaly except now accounts for the elliptical shape of the Earth’s orbit and the resulting change in angular velocity by adding the Equation of Center correction.</td>
</tr>
<tr>
<td>Sun’s Radius Vector</td>
<td>The distance between the center of the Sun and the center of the Earth.</td>
</tr>
<tr>
<td>Nutation</td>
<td>The variance of the orientation of the Earth’s axis of rotation over time, principally caused by the gravitational forces of the Sun and Moon.</td>
</tr>
<tr>
<td>Aberration</td>
<td>The variance in the apparent position of the Sun to an observer on Earth due to the change in the angular velocity of the Earth’s orbit during its revolution around the Sun.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Sun’s Apparent Longitude</td>
<td>Same as the Sun’s True Longitude except now in terms of apparent position relative to an observer (or solar collector) on Earth to include corrections for nutation and aberration.</td>
</tr>
<tr>
<td>Celestial Equator</td>
<td>A plane of reference that cuts through the Earth’s equator and upon which lies a circle of any size that is equidistant from the celestial poles.</td>
</tr>
<tr>
<td>Mean Obliquity of the Ecliptic</td>
<td>The angle formed between the plane of the ecliptic and the plane through the celestial equator.</td>
</tr>
<tr>
<td>Solar Parallax</td>
<td>The difference in the Sun’s position as seen from the center of the Earth (upon which primary calculations are performed) and from the surface of the Earth.</td>
</tr>
<tr>
<td>Sun’s Right Ascension</td>
<td>The coordinate position of the Sun corresponding to ecliptic longitude.</td>
</tr>
<tr>
<td>Sun’s Declination</td>
<td>The coordinate position of the Sun corresponding to ecliptic latitude.</td>
</tr>
<tr>
<td>Sidereal Time</td>
<td>Astronomical timekeeping system where one sidereal day is approximately 23 hours, 56 minutes, 4.0905 seconds.</td>
</tr>
<tr>
<td>Solar Hour Angle</td>
<td>An expression of time, in angular measurement of degrees, from a reference point of Solar Noon (where the Solar Hour Angle equals 0°); related to the Sun’s Right Ascension by Local Sidereal Time.</td>
</tr>
<tr>
<td>Solar Time</td>
<td>The hour angle of the apparent Sun at a given instant in time, which moves along the plane of the ecliptic at a varying rate due to the eccentricity of the Earth’s orbit.</td>
</tr>
<tr>
<td>Mean Time</td>
<td>The hour angle of a fictitious sun that moves along the plane of the celestial equator at a uniform rate by assuming that the Earth’s orbit is perfectly circular.</td>
</tr>
<tr>
<td>Equation of Time</td>
<td>The difference between Solar Time and Mean Time.</td>
</tr>
<tr>
<td>Zenith</td>
<td>An imaginary point on the celestial sphere directly above an observer’s (or solar collector’s) location on the surface of the Earth.</td>
</tr>
<tr>
<td>Solar Zenith Angle</td>
<td>The angle formed by drawing a line from the center of the Sun to an observer’s position on the surface of the Earth to the Zenith; a complementary (adds to 90°) angle with the Solar Elevation Angle.</td>
</tr>
<tr>
<td>Solar Elevation (Altitude) Angle</td>
<td>The altitude (up/down) of the Sun from the perspective of an observer (or solar collector) on the surface of the Earth; a complementary (adds to 90°) angle with the Solar Zenith Angle.</td>
</tr>
<tr>
<td>Solar Azimuth Angle</td>
<td>The direction (left/right) of the Sun from the perspective of an observer (or solar collector) on the surface of the Earth.</td>
</tr>
</tbody>
</table>
Each of the three methods takes a different approach. NOAA’s approach is illustrated by Figure 9. Critical path values are shown in bold, black text with solid flowchart arrows, whereas calculated values not essential to finding the solar altitude and azimuth angles are shown in gray text with dashed flowchart arrows.
Figure 9. NOAA approach to finding the solar altitude and azimuth angles
The Sun’s position is dependent upon time, which can be expressed in terms of the Julian Century. Microsoft Excel uses an Object Linking & Embedding (OLE) Automation format for time and date values, assigning a numeric value for an entered date equal to the number of days on the Gregorian Calendar since the last moment of (midnight on) December 31, 1899. Astronomical calculations, however, use the Julian Calendar, which starts from midday on January 1st, 4713 BC (30). In order to convert to the Julian Date from the OLE Automation date, one must add 2,415,018.5 (95). To get the appropriate fraction of the day due to the local hour (given as hours past local midnight), one must take the local hour (which in OLE Automation format returns a value equal to the fraction of one day) and adjust for the offset given by the time zone, where the value for the time zone hour is negative in the Western Hemisphere and its fraction of a day is given by dividing by 24 hours. The Julian Day is calculated using (52).

\[
JD[day] = Date[day, \text{OLE Automation format}] + 2,415,018.5[day] + Hour[day, \text{OLE Automation format}] - \left(\frac{TZ[hr]}{24[hr/day]}\right)
\]

To calculate the Julian Century for the 2000 standard epoch (J2000.0 epoch) using (53), consider that January 1, 2000 began at Julian Day 2,451,545 and a Julian Year has 365.25 days.

\[
JC[century] = \frac{JD[day] - 2,451,545[day]}{36,525\left[\frac{day}{century}\right]}
\]

An “epoch” is an instant in time chosen as an origin or reference point, so the numeric values in the NOAA equations that follow are based on using the Julian Century value calculated in (53), which is a decimal fraction of time where 0.0 equals the start of the Gregorian Year 2000.
and 1.0 equals one century beyond. Following the flow shown in Figure 9, one can now calculate the Sun’s geometric mean longitude and geometric mean anomaly, the mean obliquity of the ecliptic, the eccentricity of Earth’s orbit, and a longitude correction for nutation and aberration.

The geometric mean longitude and geometric mean anomaly measure how far around an orbit a body has progressed beyond a given reference point. The Earth orbits the Sun in an ellipse as opposed to a circle. The degree of the ellipse (i.e., how flattened the ellipse is from a perfect circle) is described by its eccentricity. The “plane of the ecliptic” is an imaginary plane that contains the Earth’s orbit around the Sun. If the Earth’s orbit around the Sun was in a perfect circle, then the Sun would appear to traverse the ecliptic at a constant speed. However, because the Earth’s orbit is an ellipse with eccentricity, the angular velocity of orbit is fastest when at perihelion (shortest distance to the Sun) and slowest when at aphelion (furthest distance from the Sun).

The Sun’s geometric mean longitude is given by (54) and defines the position of the Sun if the Earth’s orbit was a perfect circle using as a reference point when the vernal equinox occurs (i.e., in March when the Northern and Southern Hemispheres receive equal amounts of daylight and the Sun’s rays are parallel along the celestial equator). At the start of the J2000.0 epoch, the geometric mean longitude of the Sun was 280°27′59.26″ (or 280.46646 decimal degrees). Note that numeric results from (54) can be quite large. In order to correct results such that they are within 0° to 360° of a single revolution, one can use the MOD() function in Excel, e.g., “=MOD(equation,360)” which returns the remainder of the result of the equation after it has been divided by 360.
\[ L_0[\text{deg}] = 280.46646[\text{deg}] + JC[\text{century}] \cdot 36,000.76983 \left( \frac{\text{deg}}{\text{century}} \right) \\
\quad + 0.0003032 \left( \frac{\text{deg}}{\text{century}^2} \right) \cdot (JC[\text{century}])^2 \]

The geometric mean anomaly, given by (55), is the position of the Sun if the Earth’s orbit was a perfect circle and the reference point is measured from 0° at the perihelion. Here again, results can be quite large, and one can use the MOD() function to keep results between 0° and 360°. Note that “anomaly” refers to the nonuniform (or “anomalous”) apparent motion of the Sun along the plane of the ecliptic (96).

\[ M[\text{deg}] = 357.52911[\text{deg}] + 35,999.05029 \left( \frac{\text{deg}}{\text{century}} \right) \cdot JC[\text{century}] \\
\quad - 0.0001537 \left( \frac{\text{deg}}{\text{century}^2} \right) \cdot (JC[\text{century}])^2 \]

The change in eccentricity of the Earth’s orbit over time is very small and is given by (56).

\[ e[-] = 0.016708634[-] - JC[\text{century}] \cdot 0.000042037 \left( \frac{-}{\text{century}} \right) \\
\quad + 0.0000001267 \left( \frac{-}{\text{century}^2} \right) \cdot (JC[\text{century}])^2 \]

The Sun’s true anomaly is the angle formed by drawing a line from the center of the Sun to the center of the Earth and another line from the center of the Sun along the perihelion to a point on the Earth’s orbit around the Sun. The mean anomaly is the angle formed by drawing a line from the center of the Sun along the perihelion to a point on the Earth’s orbit around the Sun and another line from the center of the Sun to the point at which the Earth would be at should its orbit have a constant angular velocity. Put another way, the mean anomaly assumes a constant
speed whereas the true anomaly uses the actual speed that varies around the elliptical orbit. The
difference in position between the two is called the “equation of center” and is given by (57).

\[
C[\text{deg}] = \sin(M[\text{rad}]) \\
\quad \times \left( 1.914602 - 0.004817 \frac{\text{century}}{\text{century}} \cdot JC[\text{century}] + 0.000014 \frac{\text{century}^2}{\text{century}^2} \cdot (JC[\text{century}])^2 \right) \\
\quad + \sin(2 \cdot M[\text{rad}]) \cdot \left( 0.019993 - 0.000101 \frac{\text{century}}{\text{century}} \cdot JC[\text{century}] \right) \\
\quad + \sin(3 \cdot M[\text{rad}]) \cdot 0.000289
\]

Note that Excel executes calculations in radians mode, so although the result is in
degrees, each of the angular inputs (in this case the geometric mean anomaly) must be first
converted from degrees to radians using \( \frac{\pi[\text{rad}]}{180[\text{deg}]} \). Although this can be done inside equations in
Excel using the RADIANS() function, it is often clearer to make another column alongside all
results in degrees to also show results in radians, though this does increase file size. For clarity,
this paper presents all equations in terms of the values and units that must be used for proper
spreadsheet calculations.

Having calculated the equation of center, one can now calculate the Sun’s true longitude
and the Sun’s true anomaly using (58) and (59), respectively.

\[
\lambda[\text{deg}] = L_o[\text{deg}] + C[\text{deg}]
\]

\[
\nu[\text{deg}] = M[\text{deg}] + C[\text{deg}]
\]

Next, calculate the Sun’s radius vector, which is the distance between the center of the
Sun and the center of the Earth. It is measured in astronomical units (AU), a unit of length equal
to 149,597,870,700m, which is roughly equal to the distance from the Earth to the Sun. Because
this distance changes throughout the Earth’s orbit, results from (60) will vary from slightly above to slightly below 1.0 throughout the year.

\[ R[AU] = \frac{1.000001018[AU] \cdot (1 - (e[-])^2)}{1 + e[-] \cdot \cos(\nu[rad])} \]  

One could simply use the result for the Sun’s true longitude in further calculations, but it can be slightly improved by correcting for nutation and aberration. Nutation is the variance of the orientation of the Earth’s axis of rotation over time, which is principally caused by the gravitational forces of the Sun and Moon. Aberration is the variance in the apparent position of the Sun to an observer on Earth due to the change in the angular velocity of the Earth’s orbit during a revolution (“apparent” refers to the observer’s perspective that the Sun and stars appear to move across the sky). The correction is given by (61), and the Sun’s apparent longitude is given by (62).

\[ \Omega[deg] = 125.04[deg] - \left(1,934.136 \left[\frac{deg}{century}\right] \cdot JC[century]\right) \]  
\[ \lambda_a[deg] = \lambda[deg] - 0.00569 - (0.00478 \cdot \sin(\Omega[rad])) \]

The obliquity of the ecliptic is the angle between the plane of the ecliptic and a plane through the celestial equator. Since this method uses the J2000.0 standard epoch, the baseline reference is 23°26′21.448″ (30). This can be seen in (63) for the mean obliquity of the ecliptic and in (64) where that equation is rewritten for spreadsheet operations.

\[ \varepsilon_0 = 23°26′21.448 - 46″.8150 \cdot JC[century] - 0″.00059 \cdot (JC[century])^2 + 0″.001813 \cdot (JC[century])^3 \]
\[\varepsilon_0[\text{deg}] = 23[\text{deg}] + \left( \frac{26[\text{min}] + \left( 21.448[\text{sec}] - \left( 46.812\frac{\text{sec}}{\text{century}} \cdot J[\text{century}] + 0.00059 \frac{\text{sec}}{\text{century}} \cdot (J[\text{century}])^2 - 0.001812 \frac{\text{sec}}{\text{century}} \cdot (J[\text{century}])^3 \right) \right)}{60\frac{\text{sec}}{\text{min}} \cdot \frac{\text{min}}{\text{deg}}} \right)\]

Because calculations thus far have considered a perspective from the center of the Earth and an observer (or solar collector) is actually on the surface of the Earth (a distance of the Earth’s radius), one must correct for this displacement in the point of reference perspective. The displacement in apparent position viewed along two different lines of sight is called “parallax,” and the obliquity of the ecliptic is corrected using (65).

\[\varepsilon_{\text{corrected}}[\text{deg}] = \varepsilon_0[\text{deg}] + 0.00256 \cdot \cos(\Omega[\text{rad}])\]

One can now convert from the ecliptic coordinates to equatorial coordinates using (66) in order to find the Sun’s right ascension and (67) to find the Sun’s declination. The Sun’s right ascension is the coordinate position of the Sun corresponding to longitude, and the Sun’s declination is the coordinate position of the Sun corresponding to latitude. Note that, in Excel, (66) uses the \text{ATAN2}(a,b) function, which equals \(\tan^{-1}\left(\frac{b}{a}\right)\) and allows for \(a\) to equal 0. The solar declination, which is the angle formed between the plane of the celestial equator and a line drawn from the center of the Earth to the center of the Sun, varies between approximately \(\pm 23.45^\circ\) throughout the year (the tilt of the Earth’s axis).

\[\alpha[\text{rad}] = \tan^{-1}\left(\frac{\cos(\varepsilon_{\text{corrected}}[\text{rad}]) \cdot \sin(\lambda_a[\text{rad}])}{\cos(\lambda_a[\text{rad}])}\right)\]

61
\[
\delta [rad] = \sin^{-1}(\sin(\varepsilon_{corrected}[rad]) \cdot \sin(\lambda_a[rad]))
\]

It is important to now reconcile the difference between apparent solar time and mean solar time, which is done using the equation of time. Meeus provides an equation for the equation of time in Chapter 28 of his book *Astronomical Algorithms* where, presumably to simplify the complexity of the equation, he introduces a variable \( y \) such that

\[
y = \tan^2 \left( \frac{\varepsilon_{corrected}}{2} \right)
\]

(30). NOAA calculations follow this equation and have published calculators that display values for “\( \text{var} \, y \)” (31). For simplicity of terminology and spreadsheet columns, the model does not show “\( \text{var} \, y \)” but instead performs this calculation directly within an expanded form of the equation of time as shown by (68).

\[
E[\text{min}] = 4 \left[ \frac{\text{min}}{\text{deg}} \right] \cdot \left( \frac{180}{\pi} \right) \left[ \frac{\text{deg}}{\text{rad}} \right]^2 \times \left( \tan \left( \frac{\varepsilon_{corrected}[\text{rad}]}{2} \right) \right)^2 \times \sin(2 \cdot L_o[\text{rad}]) - 2 \cdot e[-] \\
\times \sin(M[\text{rad}]) + 4 \cdot e[-] \times \left( \tan \left( \frac{\varepsilon_{corrected}[\text{rad}]}{2} \right) \right)^2 \times \sin(M[\text{rad}]) \times \cos(2 \cdot L_o[\text{rad}]) - 0.5 \times \left( \tan \left( \frac{\varepsilon_{corrected}[\text{rad}]}{2} \right) \right)^2 \times \sin(4 \cdot L_o[\text{rad}]) - 1.25 \cdot (e[-])^2 \times \sin(2 \cdot M[\text{rad}])
\]

(68)

The solar hour angle is an expression of time, in terms of angular measurement (degrees). It uses a reference point of solar noon where the solar hour angle is equal to \( 0^\circ \). Negative hour angles represent times before solar noon and positive hour angles represent times after solar noon. The solar hour angle is related to the Sun’s right ascension by local sidereal time. Sidereal time is an astronomical timekeeping system where one sidereal day is approximately 23 hours, 56 minutes, 4.0905 seconds. For the special cases of sunrise and sunset, the solar zenith angle is set to \( \pm 90.833^\circ \) (\( \pm 1.5853 \) radians), which is greater than \( 90^\circ \) because of corrections for atmospheric refraction and the size of the solar disk (97). (Note that the zenith is an imaginary
point on the celestial sphere directly above an observer’s location on the surface of the Earth and the solar zenith angle is formed by drawing a line from the center of the Sun to an observer’s position on the surface of the Earth to the zenith.) One can calculate the solar hour angle at sunrise using (69). The solar hour angle at sunset is simply the negative of the solar hour angle at sunrise with solar noon being directly between the two at 0°. Note that, in this special case of calculating the hour angle at sunrise and sunset, NOAA’s equation results in a positive number that corresponds with sunrise and a negative number that corresponds with sunset (opposite the general sign convention for hour angles), which follows through in (71) to (73). Also, if the location is at a large enough latitude, then there may be times during the year when the Sun never rises (around the winter solstice) nor sets (around the summer solstice).

\[
H_{a_{sr}}[deg] = \left(\frac{180}{\pi}\right)[deg][rad]
\cdot \left(\cos^{-1}\left(\frac{\cos(1.5853[rad])}{\cos(LAT[rad]) \cdot \cos(\delta[rad])} - \tan(LAT[rad]) \cdot \tan(\delta[rad])\right)\right)
\]

One can calculate the time at which solar noon (the point at which the Sun is at its highest point during the day) occurs using (70). Note that this equation is built for spreadsheet operations where units of a fraction of a day will return time in terms of hh:mm:ss. There are 1,440 minutes in one day, and the halfway point (solar noon) occurs at 720 minutes. Corrections for the local time zone and the equation of time are included.

\[
SN[day] = \frac{720[min] - 4[deg \cdot LON[deg]] - E[min] + TZ[hr] \cdot 60[min]}{1,440[min]} - 1[day]
\]
One can also calculate the time of sunrise and time of sunset using (71) and (72), respectively.

\[
T_{SR}[\text{day}] = \frac{SN[\text{day}] \cdot 1,440 \left[ \frac{\text{min}}{\text{day}} \right] - H A_{SR}[\text{deg}] \cdot 4 \left[ \frac{\text{min}}{\text{deg}} \right]}{1,440 \left[ \frac{\text{min}}{\text{day}} \right]}
\]  
(71)

\[
T_{SS}[\text{day}] = \frac{SN[\text{day}] \cdot 1,440 \left[ \frac{\text{min}}{\text{day}} \right] + H A_{SR}[\text{deg}] \cdot 4 \left[ \frac{\text{min}}{\text{deg}} \right]}{1,440 \left[ \frac{\text{min}}{\text{day}} \right]}
\]  
(72)

Because the Earth rotates 1 degree every 4 minutes and the Sun is above the horizon from the hour angle at sunrise to the hour angle at sunset (numerically equivalent to two times the positive value for the hour angle at sunrise), the daily sunlight duration can be calculated using (73). Note that, because the spreadsheet does calculations for every hour of the year and the daily sunlight duration changes throughout the year, the values for each hour across a single day may change by approximately 30 seconds over 24 hours.

\[
SLD[\text{min}] = 4 \left[ \frac{\text{min}}{\text{deg}} \right] \cdot 2 \cdot H A_{SR}[\text{deg}]
\]  
(73)

TMY3 data files are recorded in terms of local standard time (67). Therefore, to convert between the time recorded in the TMY3 data file to true solar time, one must use (74). Recall that Excel will report a value for the local standard time in terms of a fraction of a day. Since results of this equation can be large, one can again use the MOD() function in Excel to keep values between 0 and 1,440 minutes. The apparent solar time can be reported in terms of
decimal hours by dividing the result of (74) by 60 min hr$^{-1}$ or in terms of hh:mm:ss in Excel by dividing the result of (74) by 1,440 min day$^{-1}$ and changing the cell’s format.

\[ AST[\text{min}] = LST[\text{day}] \cdot 1,440 \frac{\text{min}}{\text{day}} + E[\text{min}] + 4 \frac{\text{min}}{\text{deg}} \cdot LON[\text{deg}] - 60 \frac{\text{min}}{\text{hr}} \cdot TZ[\text{hr}] \]  

(74)

Knowing the solar time, one can quickly calculate the hour angle at any given local standard time using the if-then statements of (75). Note that the if-then statement is dependent upon whether it is morning or afternoon and keeps results within $\pm 180^\circ$ of solar noon ($0^\circ$), which covers all $360^\circ$. Also note that (75) will result in hour angles that are negative in the morning and positive in the afternoon, which is contradictory to (69) and (71) through (73), which use the special case of the hour angle at sunrise as a positive value.

\[ HA[\text{deg}] = \begin{cases} \frac{AST[\text{min}]}{4} + 180[\text{deg}], & \text{if } \frac{AST[\text{min}]}{4} < 0 \\ \frac{AST[\text{min}]}{4} - 180[\text{deg}], & \text{if } \frac{AST[\text{min}]}{4} \geq 0 \end{cases} \]

(75)

The solar zenith angle (previously discussed) can now be calculated using (76). Since the solar zenith angle and the solar elevation angle are complementary angles (add to $90^\circ$), the solar elevation angle (also known as the solar altitude angle) can be calculated using (77).

\[ \phi_Z[\text{rad}] = \cos^{-1}(\sin(LAT[\text{rad}]) \cdot \sin(\delta[\text{rad}]) + \cos(LAT[\text{rad}]) \cdot \cos(\delta[\text{rad}]) \cdot \cos(HA[\text{rad}]))) \]  

(76)

\[ \beta[\text{deg}] = 90[\text{deg}] - \phi_Z[\text{deg}] \]  

(77)
Atmospheric refraction is the deviation of light from a straight line as it passes through the atmosphere due to variation in air density across altitude. This bends the Sun’s rays, making it appear to rise approximately 2.4 minutes sooner and set approximately 2.4 minutes later than calculations would suggest (33). For sunrise and sunset calculations, NOAA assumes an atmospheric refraction of 0.833°; for any solar elevation angle (0 to 90°), NOAA approximates atmospheric refraction using (78) (31).

\[
Q[\text{deg}] = \begin{cases} 
\frac{1}{150} \text{[min]} - 0.000355 \text{[min] per rad}, & \text{if } \beta[\text{deg}] > 85 \\
0, & \text{if } 5 < \beta[\text{deg}] \leq 85 \\
(1.735 \text{[min]} - 518.2 \text{[min]}) \cdot \beta[\text{deg}] + 103.4 \text{[min]} \cdot \beta[\text{deg}]^3 - 12.79 \text{[min]} \cdot \beta[\text{deg}]^5 + 0.711 \text{[min]} \cdot \beta[\text{deg}]^7, & \text{if } 0.575 < \beta[\text{deg}] \leq 5 \\
-28.772 \text{[min]} \cdot \frac{\beta[\text{deg}]}{\text{[rad]}}, & \text{if } \beta[\text{deg}] \leq 0.575 \\
\end{cases}
\]

A more correct calculation for the solar elevation angle that accounts for atmospheric refraction can now be performed using (79). This tells us the altitude (up/down) of the Sun from the perspective of an observer (or solar collector) on the surface of the Earth.

\[
\beta_{\text{corrected}}[\text{deg}] = \beta[\text{deg}] + Q[\text{deg}]
\]

Finally, the solar azimuth angle is calculated using (80), which tells the direction (left/right) of the Sun from the perspective of an observer (or solar collector) on the surface of the Earth. NOAA and NREL have, in recent years, changed their method of measuring azimuth angles from south and eastward to north and eastward, i.e., they now report the solar azimuth as degrees clockwise from north (31,94). If the hour angle is positive (afternoon), then one must add 180° to start from north. If the hour angle is negative (morning), one must subtract the result from 540° (360° + 180° = 540°). Using Excel’s MOD() function, this results in solar azimuth angles between 0° and 360° taken clockwise (eastward) from north. To adjust to 0° at solar
south with positive values for before solar noon and negative values for after solar noon, take the
negative of the result from (80) and subtract 180° (81).

\[
\phi_S[rad] = \begin{cases} 
\cos^{-1}\left(\frac{\sin(LAT[rad]) \cdot \cos(\phi_Z[rad]) - \sin(\delta[rad])}{\cos(LAT[rad]) \cdot \sin(\phi_Z[rad])}\right) + 180[deg], & \text{if } HA[deg] > 0 \\
540[deg] - \cos^{-1}\left(\frac{\sin(LAT[rad]) \cdot \cos(\phi_Z[rad]) - \sin(\delta[rad])}{\cos(LAT[rad]) \cdot \sin(\phi_Z[rad])}\right), & \text{if } HA[deg] \leq 0
\end{cases}
\]

(80)

\[
\phi_S[deg] = - (\phi_S[deg] - 180[deg])
\]

(81)

Next, consider the method proposed by Masters, which, as can be seen in Figure 10, is
significantly less complicated than the NOAA method described above (33). Masters’ method
begins by finding the Julian Day, which, as opposed to the Julian Date, is simply a number from
1 to 365 (or 366 in a leap year), that corresponds to the day number in any given Gregorian Year
(see Table 11). Knowing the Julian Day, the Sun’s Declination can be calculated using (82).

Note that the vernal equinox occurs on approximately March 21st (n = 81) and the autumnal
equinox on approximately September 21st (n = 264). Values of 81 and 263.5 result in the Sun’s
declination equal to zero on the equinoxes. The Sun’s declination is positive in the summer and
negative in the winter. Also note that (82) includes a conversion from degrees to radians within
the sine function, which is not needed for a calculator in degrees mode but is required for
Microsoft Excel spreadsheet calculations.

\[
\delta[deg] = 23.45[deg] \cdot \sin\left(360\left[\frac{deg}{yr}\right] \cdot (n[day] - 81[day]) \cdot \left[\frac{\pi[rad]}{180[deg]}\right]\right)
\]

(82)
Figure 10. Masters’ approach to finding the solar altitude and azimuth angles
The equation of time can be approximated using the Julian Day in (83).

\[
E[\text{min}] = 9.87[\text{min}]
\]

\[
\cdot \sin \left( 2 \cdot \frac{360}{364} \cdot \left[ \frac{\text{deg}}{\text{yr}} \right] \cdot \left( n[\text{day}] - 81[\text{day}] \right) \cdot \left[ \frac{\pi[\text{rad}]}{180[\text{deg}]} \right] \right) - 7.53[\text{min}]
\]

\[
\cdot \cos \left( \frac{360}{364} \cdot \left[ \frac{\text{deg}}{\text{yr}} \right] \cdot \left( n[\text{day}] - 81[\text{day}] \right) \cdot \left[ \frac{\pi[\text{rad}]}{180[\text{deg}]} \right] \right) - 1.5[\text{min}]
\]

\[
\cdot \sin \left( \frac{360}{364} \cdot \left[ \frac{\text{deg}}{\text{yr}} \right] \cdot \left( n[\text{day}] - 81[\text{day}] \right) \cdot \left[ \frac{\pi[\text{rad}]}{180[\text{deg}]} \right] \right)
\]

The difference between clock time (local standard time) and solar time (true solar time) is given by (84).
\[ TC[\text{min}] = 4 \left[ \frac{\text{min}}{\text{deg}} \right] \cdot (LSM[\text{deg}] - LON[\text{deg}]) + E[\text{min}] \]

where:

\[ LSM[\text{deg}] = TZ[\text{hr}] \cdot 15 \left[ \frac{\text{deg}}{\text{hr}} \right] \]

It is critical to note that Masters' use of (84) has positive values for the local standard meridian and longitude. For example, San Francisco is located at 122° west of Greenwich in the Pacific time zone where the local standard meridian is 120° west of Greenwich. The TMY3 data file will report a negative for San Francisco’s longitude, -122° where the Universal Time Zone/Greenwich Mean Time (UTC/GMT) offset (the time zone) is -8 hours. Since \(-120° - (-122°) \neq 120° - 122°\), care must be taken to ensure the correct sign is used. The equation used in the spreadsheet is only good for the Western Hemisphere.

Solar time can be calculated from the TMY file’s reported local standard time (clock time) using the result from (84) in (86). Note that (86) will return a result in decimal days, which Excel can report in hh:mm:ss format, and (87) converts that value to decimal minutes, which can be compared to the result from (74).

\[ AST[\text{day}] = LST[\text{day}] + \frac{TC[\text{min}]}{1,440 \left[ \frac{\text{min}}{\text{day}} \right]} \]

\[ AST[\text{min}] = LST[\text{day}] \cdot 1,440 \left[ \frac{\text{min}}{\text{day}} \right] \]
The hour angle at sunrise and sunset is estimated using (88). Because Masters uses the Julian Day to calculate the Sun’s declination, the Sun’s declination is constant throughout each Julian Day (as opposed to changing from hour to hour in the NOAA method), and this consistency continues to maintain a constant hour angle at sunrise for each Julian Day. As with this special case of NOAA calculations, the hour angle at sunrise is a positive value and sunset is a negative value. Note that Masters’ (88) differs from NOAA’s (69) by the omission of the term:

\[
\frac{\cos(1.5853[\text{rad}])}{\cos(LAT[\text{rad}]) \cdot \cos(\delta[\text{rad}])}.
\]

(88)

\[
HA_{SR/SS}[\text{deg}] = \left(\frac{180}{\pi}\right) \left(\frac{\text{deg}}{\text{rad}}\right) \cdot \left(\cos^{-1}(\tan(LAT[\text{rad}]) \cdot \tan(\delta[\text{rad}])))\right)
\]

The term “geometric” refers to geometric relationships that are based on angles measured to the center of the Sun. However, as previously discussed, the size of the solar disk and atmospheric refraction of the Sun’s rays means the Sun will appear to rise sooner and set later than geometric relationships dictate. Masters uses (89) to estimate an adjustment for atmospheric refraction and the “upper-limb” of the rising or setting Sun.

(89)

\[
Q[\text{min}] = \frac{3.467[\text{min}]}{\cos(LAT[\text{rad}]) \cdot \cos(\delta[\text{rad}]) \cdot \sin(HA_{SR/SS}[\text{rad}])}
\]

The geometric sunrise can be found using (90) in terms of solar time and (91) in terms of clock time. Note that, in (90), the sunrise is found by subtracting from solar noon.
The apparent sunrise can be found using (92) in terms of solar time and (93) in terms of clock time. The apparent sunrise is earlier than the geometric sunrise, so one must subtract the adjustment for atmospheric refraction and upper-limb.

\[
T_{SR,G,CT}[day] = T_{SR,G,ST}[day] - \left( \frac{TC[min]}{1,440[min/day]} \right)
\]

The geometric sunset can be found using (94) in terms of solar time and (95) in terms of clock time. In (94), the sunset is found by adding to solar noon (which could also be accomplished by using a negative value for the hour angle at sunrise/sunset).
\[ T_{SS,G,ST}[day] = 0.5[day] + \left( \frac{HA_{SR/SS}[deg]}{15 \frac{deg}{hr}} \right) \left( \frac{15 \frac{deg}{hr}}{24 \frac{hr}{day}} \right) \]

\[ T_{SS,G,CT}[day] = T_{SS,G,ST}[day] - \left( \frac{TC[min]}{1,440 \frac{min}{day}} \right) \]

The apparent sunset can be found using (96) in terms of solar time and (97) in terms of clock time. The apparent sunset is later than the geometric sunset, so this time one must add the adjustment for atmospheric refraction and upper-limb.

\[ T_{SS,A,ST}[day] = T_{SS,G,ST}[day] + \left( \frac{Q[min]}{1,440 \frac{min}{day}} \right) \]

\[ T_{SS,A,CT}[day] = T_{SS,A,ST}[day] - \left( \frac{TC[min]}{1,440 \frac{min}{day}} \right) \]

Masters’ results for geometric sunrise/sunset using clock time are several minutes different from NOAA’s results for sunrise/sunset using local standard time. However, if one were to replace the portion of the hour angle at sunrise/sunset equation that Masters omits, then these values match very closely across the two methods. Nevertheless, if one were to compare Masters’ apparent sunrise/sunset using clock time to NOAA’s sunrise/sunset using local standard
time, the results would still differ. This is because Masters has already corrected for atmospheric refraction and upper-limb by this point, but the NOAA calculations do not correct for this until later.

Masters presents (98) to find the hour angle with the caveat that hour angles prior to solar noon are positive and hour angles after solar noon are negative (which is opposite in sign convention from NOAA’s (75)). To perform Masters’ calculation in Excel, use (99).

\[
HA[deg] = 15 \left[ \frac{deg}{hr} \right] \cdot (\text{hours before solar noon})
\]

\[
HA[deg] = 15 \left[ \frac{deg}{hr} \right] \cdot \left( 12[hr] - (ST[day] \cdot 24 \left[ \frac{hr}{day} \right]) \right)
\]

There is no change from (76) with regard to how Masters calculates the Sun’s altitude angle, except he omits first solving for the zenith angle. Since \(\cos(90° - \theta) = \sin(\theta)\), Masters simply solves for the solar altitude angle using (100).

\[
\beta[deg] = \sin^{-1}(\cos(LAT[rad]) \cdot \cos(\delta[rad]) \cdot \cos(HA[rad]) + \sin(LAT[rad]) \cdot \sin(\delta[rad]))
\]

For the Sun’s azimuth angle, Masters presents (101). He notes that there is a complication using this equation because there may be times when, in the early morning and late evening, it is possible for the Sun’s azimuth to be more than 90° from solar south and, since the inverse of sine is ambiguous, i.e., \(\sin(x) = \sin(180 - x)\), one must use the test given by (102).

\[
\phi_s[rad] = \sin^{-1}\left( \frac{\cos(\delta[rad]) \cdot \sin(HA[rad])}{\cos(\beta[rad])} \right)
\]
condition #1: \[ \cos(HA) \geq \frac{\tan(\delta)}{\tan(LAT)} \rightarrow |\phi_S| \leq 90^\circ \]

condition #2: \[ \cos(HA) < \frac{\tan(\delta)}{\tan(LAT)} \rightarrow |\phi_S| > 90^\circ \]

The test shown in (102) is fine for checking hand calculations, but the absolute value on the solar azimuth in this check provides another complication when trying to automate an Excel worksheet to perform these checks and correct the results. Bottom line, if condition #1 is true, then the solar azimuth is simply given by (101). If condition #2 is true, then another check must be done: if the result from (101) is positive, then the solar azimuth is given by subtracting the result of (101) from 180\(^\circ\); if the result from (101) is negative, then take -180\(^\circ\) and subtract the negative result (see (103)).

\[
\phi_S[\text{deg}] = \begin{cases} 
180[\text{deg}] & - \frac{180}{\pi} \frac{[\text{deg}]}{[\text{rad}]} \sin^{-1} \left( \frac{\cos(\delta[\text{rad}] \cdot \sin(HA[\text{rad}])}{\cos(\beta[\text{rad}])} \right), \text{if } \cos(HA) \geq \frac{\tan(\delta)}{\tan(LAT)} \geq 0 \\
-180[\text{deg}] & - \frac{180}{\pi} \frac{[\text{deg}]}{[\text{rad}]} \sin^{-1} \left( \frac{\cos(\delta[\text{rad}] \cdot \sin(HA[\text{rad}])}{\cos(\beta[\text{rad}])} \right), \text{if } \cos(HA) < \frac{\tan(\delta)}{\tan(LAT)} \wedge \sin^{-1} \left( \frac{\cos(\delta[\text{rad}] \cdot \sin(HA[\text{rad}])}{\cos(\beta[\text{rad}])} \right) < 0 
\end{cases}
\]

ASHRAE presents a slightly more simplified method of finding the solar altitude and azimuth, shown by Figure 11.
ASHRAE begins by calculating the equation of time (again, the difference between a solar time sundial and mean time from a clock running at a uniform rate) using (104) and (105) (32).

\[
E[\text{min}] = 2.2918 \\
\quad \cdot (0.0075 + 0.1868 \cdot \cos(\Gamma[\text{rad}]) - 3.2077 \\
\quad \cdot \sin(\Gamma[\text{rad}]) - 1.4615 \cdot \cos(2 \cdot \Gamma[\text{rad}]) - 4.089 \cdot \sin(2 \cdot \Gamma[\text{rad}]))
\]

where:
\[ \Gamma[\text{deg}] = 360 \frac{\text{deg}}{\text{day}} \cdot \left( \frac{n[\text{day}] - 1[\text{day}]}{365[\text{day}]} \right) \]

Next, ASHRAE calculates true (apparent) solar time using (106), which takes the local standard time, adds the equation of time correction, and then also adds another correction that adjusts for the actual longitude with respect to the location’s local standard meridian (i.e., the longitude at the start of the location’s clock time zone). The equation for the local standard meridian is the same as previously presented in (85).

\[ AST[\text{hr}] = LST[\text{hr}] + \left( \frac{E[\text{min}]}{60[\text{min} hr]} \right) + \left( \frac{LON[\text{deg}] - LSM[\text{deg}]}{15[\text{deg} hr]} \right) \]

This version of the equation better illustrates the sign challenges encountered when using Masters’ equation for time conversion (84). In ASHRAE’s version of the equation, the longitude and local standard meridian terms are transposed, and both are input as degrees east of Greenwich (i.e., negative in the Western Hemisphere). Multiplying both sides of (106) by 60 min hr\(^{-1}\) results in (107). Here, one can see the similarity to Masters’ (84), except for the longitude and local standard meridian terms are transposed, and Masters considers the time conversion in terms of minutes, not hours. Also evident is that the time conversion is the same as in NOAA’s (74), where NOAA simply incorporates the equation for calculating the local standard meridian (85).

\[ 60 \cdot AST[\text{min}] = 60 \cdot LST[\text{min}] + \left( E[\text{min}] + 4 \frac{\text{min}}{\text{deg}} (LON[\text{deg}] - LSM[\text{deg}]) \right) \]
Rather than using (82) to find the solar declination like NOAA and Masters do, ASHRAE uses (108), which gives the same result for the sine function since $81 + 284 = 365$.

$$\delta[\text{deg}] = 23.45[\text{deg}] \cdot \sin\left(360 \left[\frac{\text{deg}}{\text{yr}}\right] \cdot \frac{(n[\text{day}] + 284[\text{day}])}{365 \left[\frac{\text{day}}{\text{yr}}\right]} \cdot \left[\frac{\pi[\text{rad}]}{180[\text{deg}]}\right]\right)$$  \hspace{1cm} (108)

ASHRAE calculates the hour angle using (109), which results in negative hour angle values before solar noon and positive hour angles after solar noon (the same sign convention as NOAA but opposite Masters).

$$HA[\text{deg}] = 15 \left[\frac{\text{deg}}{\text{hr}}\right] \cdot (AST[\text{hr}] - 12[\text{hr}])$$  \hspace{1cm} (109)

ASHRAE calculates the solar altitude angle using the same method as Masters in (100) (repeated here in (110) for clarity). Negative values correspond to nighttime, $0^\circ$ is when the Sun is on the horizon, and $90^\circ$ is when the Sun is directly overhead (32).

$$\beta[\text{deg}] = \sin^{-1}(\cos(LAT[\text{rad}]) \cdot \cos(\delta[\text{rad}]) \cdot \cos(HA[\text{rad}]) + \sin(LAT[\text{rad}]) \cdot \sin(\delta[\text{rad}]))$$  \hspace{1cm} (110)

ASHRAE calculates the solar azimuth angle by its sine and cosine functions (111). As opposed to NOAA, which counts solar azimuth degrees as clockwise (east) of north ($0^\circ$), ASHRAE, like Masters, considers a solar azimuth of $0^\circ$ at solar south. However, opposite Masters, ASHRAE counts positive solar azimuth values for afternoon hours and negative for morning hours. ASHRAE’s Handbook of Fundamentals illustrates an example for calculating the solar azimuth angle using (111), but unfortunately it selects a case where both versions of the equation yield the same result. This will not be the case for every hour of the year, and
ASHRAE does not provide instruction on how to ensure the correct quadrant, sign, and angle are selected for each hour.

\[
\phi_s [\text{deg}] = \sin^{-1} \left( \frac{\sin(HA) \cdot \cos(\delta)}{\cos(\beta)} \right)
\]

or

\[
\phi_s [\text{deg}] = \cos^{-1} \left( \frac{\cos(HA) \cdot \cos(\delta) \cdot \sin(LAT) - \sin(\delta) \cdot \cos(LAT)}{\cos(\beta)} \right)
\]

Unfortunately, this still leaves us in a position of having to perform a series of checks to ensure the correct equation and sign are used for each hour of the year. A much easier method that is ideal for spreadsheet applications is outlined by (112) (98). Recall that, depending upon the method used, hour angle signs before and after solar noon may be switched, so one must use caution and refer to solar AM/PM/noon.

\[
\phi_s [\text{deg}] = \begin{cases} 
- \cos^{-1} \left( \frac{\sin(\beta) \cdot \sin(LAT) - \sin(\delta)}{\cos(\beta) \cdot \cos(LAT)} \right), & \text{if } HA[\text{deg}] < 0 \text{ (solar AM)} \\
\cos^{-1} \left( \frac{\sin(\beta) \cdot \sin(LAT) - \sin(\delta)}{\cos(\beta) \cdot \cos(LAT)} \right), & \text{if } HA[\text{deg}] > 0 \text{ (solar PM)} \\
0, & \text{if } HA[\text{deg}] = 0 \text{ (solar noon)}
\end{cases}
\]

The authors call this the “ASHRAE (modified)” method, where all of ASHRAE’s process is adopted up until calculating the solar azimuth angle, at which point there is a switch to use (112).

1.C.5.A.1. Differences in methods

Having successfully calculated the solar altitude and solar azimuth angles using the three different methods discussed, the authors compare results for the equation of time, the solar
altitude angle, and solar azimuth angle using the maximum and average differences across all 8,760 hours, summarized in Table 12.

\[ \Delta[deg] = |\text{Method x Result}[deg]| - |\text{Method y Result}[deg]| \]

Table 12. Comparison of results for the equation of time, solar altitude angle, and solar azimuth angle between methods published by NOAA (N), Masters (M), and ASHRAE (A)

<table>
<thead>
<tr>
<th>Climate:</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude:</td>
<td>21.32°</td>
<td>34.30°</td>
<td>38.98°</td>
<td>41.50°</td>
<td>71.32°</td>
</tr>
<tr>
<td>Longitude:</td>
<td>-157.93°</td>
<td>-116.17°</td>
<td>-77.47°</td>
<td>-74.10°</td>
<td>-156.62°</td>
</tr>
</tbody>
</table>

| Methods compared: | N|M | N|A | M|A | N|M | N|A | M|A | N|M | N|A | M|A | N|M | N|A | M|A |
| Equation of Time | ±(min) | 0.43 | 0.43 | 0.44 | 0.44 | 0.41 | 0.44 | 0.39 | 0.44 | 0.44 | 0.39 | 0.44 | 0.39 | 0.44 | 0.42 | 0.44 |
| ±(°) | 1.18 | 1.18 | 0.27 | 1.34 | 1.34 | 0.24 | 1.24 | 1.24 | 0.23 | 1.30 | 1.31 | 0.22 | 1.54 | 1.54 | 0.09 |
| Solar Altitude Angle | ±(°) | 0.25 | 0.26 | 0.09 | 0.32 | 0.33 | 0.07 | 0.33 | 0.34 | 0.06 | 0.34 | 0.35 | 0.06 | 0.44 | 0.44 | 0.02 |
| Solar Azimuth Angle | ±(°) | 3.42 | 3.43 | 0.99 | 1.67 | 1.62 | 1.30 | 1.38 | 1.34 | 0.97 | 1.29 | 1.25 | 0.85 | 0.67 | 0.66 | 0.36 |
| Δ(°) | 0.53 | 0.53 | 0.09 | 0.35 | 0.36 | 0.11 | 0.31 | 0.31 | 0.11 | 0.29 | 0.29 | 0.11 | 0.14 | 0.15 | 0.11 |

Given the degree of accuracy achieved for the level of difficulty involved in each of the three methods, the authors conclude that the ASHRAE (modified) method is acceptable for this model’s purposes, even with the lowest effort and computational cost.

1.C.5.B. Clear-sky insolation on a collector

The authors calculate the clear-sky insolation that a solar collector might receive in order to illustrate the angles and parameters necessary to understand the calculations. This section will primarily follow Masters’ method, but any differences in the ASHRAE method are noted. The results also serve as a check on later results using TMY data for insolation, which should be lower since the TMY data is measured and accounts for cloudiness. There are three components of insolation that combine to form the total insolation striking a solar collector’s surface: direct-
beam radiation, diffuse radiation, and reflected radiation. From this point forward, the authors elect to use Masters’ sign convention for the solar azimuth angle, i.e., positive before solar noon and negative after solar noon, which, again, is opposite ASHRAE’s sign convention.

Estimates for clear-sky insolation begin with calculating the extraterrestrial solar insolation that reaches just beyond the Earth’s atmosphere. This depends upon many factors, the largest being the distance between the Earth and Sun, which is accounted for by Masters’ (114) and ASHRAE’s (115). The solar constant is an estimate of the annual average extraterrestrial insolation. The value changes with sunspots and faculae, which tend to follow an 11-year cycle. In ASHRAE’s Handbook of Fundamentals, it is reported that the World Meteorological Organization uses a value of 1,367 W m\(^{-2}\) for the solar constant (32). Masters reports that 1,377 W m\(^{-2}\) is a more commonly accepted value (33). Newer measurements from the Physikalisch-Meteorologisches Observatorium Davos / World Radiation Center adjust the scale and suggest 1,361 W m\(^{-2}\) (99).

\[
I_0 \left[ \frac{W}{m^2} \right] = \frac{SC}{m^2} \cdot \left( 1 + 0.034 \cdot \cos \left( \frac{\pi [rad]}{180 [deg]} \cdot \frac{360 [deg] \cdot n [day]}{365 [day]} \right) \right)
\]

(114)

\[
I_0 \left[ \frac{W}{m^2} \right] = \frac{SC}{m^2} \cdot \left( 1 + 0.033 \cdot \cos \left( \frac{\pi [rad]}{180 [deg]} \cdot \frac{360 [deg] \cdot (n [day] - 3 [day])}{365 [day]} \right) \right)
\]

(115)

Only a portion of this direct beam insolation actually makes it through the atmosphere and reaches the Earth’s surface. The rest is attenuated by the distance it must travel through the atmosphere’s air particles, dust, pollution, water vapor, etc. Masters presents (116) to calculate
the direct-beam insolation that reaches the Earth’s surface (on a surface normal to the Sun’s rays) where (117) calculates the apparent extraterrestrial insolation, (118) calculates a dimensionless factor for optical depth, and (119) calculates the air mass ratio, which is simply a ratio of the distance direct-beam insolation travels through the Earth’s atmosphere to the minimum distance it could travel (as if it were directly overhead). Equations (117) and (118) are approximations developed to facilitate spreadsheet calculations based on published values from ASHRAE’s Clear Day Solar Flux Model (33). Equation (116) will yield negative results, so an if-then statement is used in spreadsheet calculations to make the result equal zero for direct-beam insolation if the solar altitude angle is negative (below the horizon). ASHRAE’s Handbook of Fundamentals now uses a slightly different version of (116) that is based on pseudo optical depths ($\tau_b$ and $\tau_d$) and air mass exponents ($ab$ and $ad$) to calculate both the direct-beam and diffuse irradiance on a horizontal surface on Earth. The pseudo optical depth and air mass exponents are tabulated based on location, and users of this method must interpolate to find values for other days. Because the model is built to accept multiple locations and will later use measured values anyway, the authors omit solving for these parameters in this comparison.

\[
I_B \left[ \frac{W}{m^2} \right] = A \cdot e^{-k \cdot m} \tag{116}
\]

\[
A \left[ \frac{W}{m^2} \right] = 1,160 \left[ \frac{W}{m^2} \right] + 75 \left[ \frac{W}{m^2} \right] \cdot \sin \left( \frac{\pi [rad]}{180 [deg]} \right) \cdot \frac{360 [deg]}{365 [day]} \cdot \frac{360 [deg]}{yr} \cdot (n[day] - 275[day]) \tag{117}
\]
\[ k[-] = 0.174[-] + 0.035[-] \cdot \sin \left( \frac{\pi[rad]}{180[deg]} \cdot \frac{360[deg]}{365[day]} \cdot (n[day] - 100[day]) \right) \quad (118) \]

\[ m[-] = \frac{1}{\sin(\beta[rad])} \quad (119) \]

The incidence angle relates the amount of direct-beam insolation that reaches the Earth’s surface to the amount that actually strikes a solar collector’s surface straight on (120). Masters calculates the incidence angle as a function of the solar collector’s orientation (collector azimuth angle and tilt/slope angle) and the Sun’s position in the sky (solar altitude and azimuth angle) at any point in time (121). ASHRAE uses the same equations, with the exception of introducing a new variable, the “surface-solar azimuth angle” to define the difference between the solar azimuth and the collector azimuth found in (121) (32).

\[ I_{BC} \left[ \frac{W}{m^2} \right] = I_B \left[ \frac{W}{m^2} \right] \cdot \cos(\theta) \quad (120) \]

\[ \theta[deg] = \frac{180[deg]}{\pi[rad]} \cdot \cos^{-1}(\cos(\beta[rad]) \cdot \cos(\phi_s[rad] - \phi_c[rad]) \cdot \sin(\Sigma[rad])) + \sin(\beta[rad]) \cdot \cos(\Sigma[rad])) \quad (121) \]

Because it is possible to get negative results, one can easily filter results for (120) by making the direct-beam insolation on a collector equal to zero if the incidence angle is greater than 90°. An incidence angle of 0° would mean the Sun’s rays are perfectly normal to collector,
so incidence angles equal to or greater than 90° means the Sun’s direct rays are striking the side or back of collector and not its face.

To calculate the direct-beam insolation on a horizontal surface (i.e., the collector’s tilt is zero), use (122). It is possible to get negative values, so those must be filtered out. Negative results were already filtered out for the direct-beam insolation on the Earth (surface normal to the Sun’s rays), so any negative results now come from the sine of the altitude angle, and one must apply the same filter as previously performed on results from (116).

\[
I_{BH} \left[ \frac{W}{m^2} \right] = I_B \left[ \frac{W}{m^2} \right] \cdot \cos \left( \frac{\pi [rad]}{180 [deg]} \cdot (90 [deg] - \theta [deg]) \right) = I_B \left[ \frac{W}{m^2} \right] \cdot \sin(\theta [rad])
\]

Diffuse insolation is the result of solar radiation that gets scattered in the atmosphere and reflected by the ground, clouds, and other objects. Masters uses a model published in ASHRAE (earlier developed by Threlkeld and Jordan in 1958), that assumes the sky is isotropic, meaning the diffuse radiation is assumed equal from all directions. Using another approximation equation developed to facilitate spreadsheet calculations, Masters calculates a sky diffuse factor (123) that is then used in (124) to estimate the diffuse insolation on a horizontal surface based on the direct-beam insolation reaching Earth. Assuming isotropic skies, the amount of diffuse insolation that a solar collector will receive is directly proportional to the fraction of the sky that it is exposed to, which is a factor of the collector’s tilt angle (125). If the collector’s tilt is 0°, then it is in a horizontal position and receives 100% of the diffuse insolation on a horizontal surface. If the collector’s tilt is 90° (straight up and down) then it receives half.
ASHRAE uses a different method for calculating the diffuse insolation on a collector \((126)\). The factor, \(Y\), is a ratio of clear-sky diffuse irradiance on a vertical surface to clear-sky diffuse irradiance on a horizontal surface and is a function of the incidence angle when the vertical surface (i.e., a collector tilted to 90°) has the same azimuth as the actual collector at a given tilt angle, \(\Sigma\) \((127)\). The ASHRAE method yields slightly larger values than Masters’ method, even when using the same values for the diffuse insolation on a horizontal surface. However, ASHRAE makes a point of noting that their method is acceptable only for clear-sky analyses and should not be used for cloudy (or all-sky) analyses. The term “all-sky” refers to conditions under all atmospheric situations or measured conditions \((100)\).

\[
I_{DC} \left[ \frac{W}{m^2} \right] = \begin{cases} 
I_{DH} \left[ \frac{W}{m^2} \right] \cdot (Y[-] \cdot \sin(\Sigma[rad]) + \cos(\Sigma[rad])), & \text{if } \Sigma \leq 90^\circ \\
I_{DH} \left[ \frac{W}{m^2} \right] \cdot Y[-] \cdot \sin(\Sigma[rad]), & \text{if } \Sigma > 90^\circ 
\end{cases}
\]
\[ Y[-] = \text{Maximum of} \left\{ 0.55 + 0.437 \cdot \cos(\theta [rad]) + 0.313 \cdot (\cos(\theta [rad]))^2 \right\}^{0.45} \]  

A solar collector can receive an extra amount of insolation if the combined direct-beam and diffuse insolation striking the ground in front of the collector gets reflected onto it. Both Masters and AHSRAE offer (128) as an estimate for reflected insolation on a collector, which uses a reflectance factor, \( \rho \) (33). ASHRAE presents a table of ground reflectance of foreground surfaces with values ranging from 0.05 for new asphalt to 0.7 for an isolated, snow-covered, rural site (32). ASHRAE, Masters, and NREL all state that a value of 0.2 (typical for dry bare ground or grass) is commonly used as a default value (32–34). In fact, NREL states that their PVWatts program used a default value of 0.2 for reflectance, unless there is data in the TMY3 file’s albedo column, in which case they use the measured value (34). Here, while calculating the clear-sky insolation on a collector, the model will simply use the suggested default value. However, the model will later follow suit with NREL’s method, assuming a default of 0.2 but using measured values for albedo if available, when using TMY data for measured direct-beam and diffuse insolation.

\[ I_{RC} \left[ \frac{W}{m^2} \right] = \rho[-] \cdot \left( I_{BH} \left[ \frac{W}{m^2} \right] + I_{DH} \left[ \frac{W}{m^2} \right] \right) \cdot \left( \frac{1 + \cos(\Sigma [rad])}{2} \right) \]  

Finally, the clear-sky insolation striking a collector’s surface is found by taking the summation of the results of the direct-beam insolation on a collector (120), the diffuse radiation on a collector (125), and the reflected radiation on a collector (128), as shown by (129).

\[ I_c \left[ \frac{W}{m^2} \right] = I_{BC} \left[ \frac{W}{m^2} \right] + I_{DC} \left[ \frac{W}{m^2} \right] + I_{RC} \left[ \frac{W}{m^2} \right] \]
1.C.5.C. All-sky insolation on a collector

The next step is to select the appropriate method from the analysis above and, instead of using estimates of extraterrestrial solar radiation and a default value for reflectance, use data from the TMY weather file for the direct-beam insolation and diffuse insolation along with any measured albedo values.

In the TMY files, the direct-beam insolation is column # 8 (H) and is called the “direct normal irradiance” or DNI (130). The DNI is the “amount of solar radiation (modeled) received in a collimated beam on a surface normal to the Sun during the 60-minute period ending at the timestamp” (67).

\[
I_B \left[ \frac{W}{m^2} \right] = DNI \left[ \frac{W}{m^2} \right]
\]  

There is no change in calculating the incidence angle (121). The direct-beam insolation on a collector is calculated in the same way as previous (120), except now the model uses the TMY values for DNI (130). The direct-beam insolation on a horizontal surface (used later in calculating the reflected radiation on a collector) is calculated in the same manner as before (122).

In the TMY files, the diffuse insolation is column # 11 (K) and is called the “diffuse horizontal irradiance” or DHI (131). The DHI is the “amount of solar radiation received from the sky (excluding the solar disk) on a horizontal surface during the 60-minute period ending at the timestamp” (67).

\[
I_{DH} \left[ \frac{W}{m^2} \right] = DHI \left[ \frac{W}{m^2} \right]
\]
The diffuse insolation on a collector is calculated using the same method as described by (125).

The albedo is column # 62 (BJ) in TMY data spreadsheets and is the same as ground reflectance, i.e., “the ratio of reflected solar irradiance to global horizontal irradiance” (67). TMY files often do not have good data for albedo, in which case they record “-9900” instead. The model uses a filter to replace such instances with the default value of 0.2, the same default value as in NREL’s PVWatts program (34).

The reflected radiation on a collector is found in the same manner as before (128), except now using the measured albedo values (if available).

Reflection losses occur with PV panels, so anti-reflective coatings are often applied. However, when light travels through one medium and into another, it will bend or “refract.” Snell’s Law describes the geometric requirement that light rays remain parallel in the medium through which it passes. With the dimensionless refractive indices of air, anti-reflective (AR) coating, and glass, the model can follow NREL’s methods for calculating transmittance through “premium” PV modules (34).

Because the previously calculated angle of incidence, $\theta$, is measured normal to the collector’s surface, it is also $\theta_1$ (132) in the equation for Snell’s Law (133), which calculates the angle of refraction into the anti-reflective coating. Using Fresnel’s equation for non-reflected unpolarized radiation, the transmittance through the anti-reflective coating is calculated using (134). The same process is again repeated to find the angle of refraction (135) and transmittance (136) through the PV glass. The total transmittance through the PV cover (anti-reflective coating and glass) is given by (137). Finally, modify (129) to apply this total transmittance (a fraction)
to the beam insolation striking a collector, $I_{BC}$, and calculate the total insolation on a collector corrected for transmittance, $I_{CT}$ (138).

$$\theta_1 [rad] = \theta [rad]$$

(132)

$$\theta_2 [rad] = \sin^{-1} \left( \frac{n_{air} [-]}{n_{AR} [-]} \cdot \sin(\theta_1 [rad]) \right)$$

(133)

$$\tau_{AR} [-] = 1 - 0.5 \cdot \left( \frac{(\sin(\theta_2 [rad] - \theta_1 [rad]))^2}{(\sin(\theta_2 [rad] + \theta_1 [rad]))^2} + \frac{(\tan(\theta_2 [rad] - \theta_1 [rad]))^2}{(\tan(\theta_2 [rad] + \theta_1 [rad]))^2} \right)$$

(134)

$$\theta_3 [rad] = \sin^{-1} \left( \frac{n_{AR}}{n_{glass}} \cdot \sin(\theta_2 [rad]) \right)$$

(135)

$$\tau_{glass} [-] = 1 - 0.5 \cdot \left( \frac{(\sin(\theta_3 [rad] - \theta_2 [rad]))^2}{(\sin(\theta_3 [rad] + \theta_2 [rad]))^2} + \frac{(\tan(\theta_3 [rad] - \theta_2 [rad]))^2}{(\tan(\theta_3 [rad] + \theta_2 [rad]))^2} \right)$$

(136)

$$\tau_{cover} [-] = \tau_{AR} [-] \cdot \tau_{glass} [-]$$

(137)

$$I_{CT} \left[ \frac{W}{m^2} \right] = \tau_{cover} [-] \cdot I_{BC} \left[ \frac{W}{m^2} \right] + I_{DC} \left[ \frac{W}{m^2} \right] + I_{RC} \left[ \frac{W}{m^2} \right]$$

(138)

A PV cell’s performance also depends upon its operating temperature. As the cell temperature increases, the current will increase slightly, but the voltage will decrease more
substantially (33). Thus, as temperature increases, a PV cell’s power output will decrease. This is unfortunate because only a small fraction of insolation striking a collector is converted to electrical energy; the majority is absorbed and converted to heat energy (33). PV manufacturers publish a value for the nominal operating cell temperature (NOCT), which is the cell temperature when the ambient temperature is 20°C (68°F), insolation is 800 W m^{-2}, windspeed is 1 m s^{-1}, and mounting has an open back side (101). The authors adopt NREL’s assumption of 45°C (113°F) for the NOCT as used in the PVWatts program and estimate the PV cell’s operating temperature based on each hour’s ambient temperature and total insolation on the collector using (139) (34).

\[
T_{cell}[\degree C] = T_{db}[\degree C] + \left( \frac{NOCT[\degree C] - 20[\degree C]}{800 [W/m^2]} \right) \cdot I_{CT} [W/m^2]
\]

(139)

PV manufacturers also publish a value called the Power Temperature Coefficient. This paper uses a value of -0.35\% °C^{-1}, the same value used by NREL in PVWatts (34). This tells us that, for every degree Celsius the cell operating temperature rises above the laboratory-set 20°C, the cell’s performance will decrease by 0.35%. From the Power Temperature Coefficient, one can calculate a Cell Temperature Coefficient Correction Factor, \textit{CTCCF} (140). Should the cell temperature be less than 20°C, then the two negatives cancel and the correction factor shows an increase (positive value) in performance.

\[
CTCCF[-] = 1 + PTC \left( \frac{\%}{\degree C} \right) \cdot (T_{cell}[\degree C] - 20[\degree C])
\]

(140)

Finally, calculate the total AC power production from a FOB’s PV panels using (141). The total insolation on a collector, \(I_{CT}\) comes from (138).
The FOB’s total area of PVs, $A_{PV}$ is equal to the total area of rooftop and utility PV arrays. This model includes an option for users to declare a utility solar PV array size. However, in this paper, the authors have modeled zero utility PV arrays; all PV is installed on hut rooftops. International Fire Code includes requirements for exclusion zones below roof ridges and access zones to ridges (102). However, military buildings on contingency bases are not necessarily subject to such codes. Exclusion zones around roof edges generally depend upon the mounting system and risks due to high winds. One manufacturer states that their PV modules can be mounted as close as 20 cm from edges (103). The available rooftop space for SWA Huts with gable roofs is estimated using (142) and (143) for SIP Huts with shed roofs. For purposes here, the model simply provides a 1 ft clearance on the roof around all PV array edges.

SWA Hut designs vary with gable end roof pitches varying from 4:12 to 8:12, meaning a rise of 4 to 8 inches for every 12 inches of run. Thus, the height of the roof (excluding overhangs) can be 2 ft, 8 in to 5 ft, 4 in. In this paper, the authors assume all SWA Huts use the higher pitched roofs with 1.5 ft overhangs along the long sides and 2 ft overhangs on either end over the doors.

$$\text{(142)} \quad A_{PV,\text{Hut,SWA}}[ft^2] = (-1 + 9.6 + 1.5 - 1)[ft] \cdot (-1 + 2 + 32 + 2 - 1)[ft] = 309[ft^2]$$

Similarly, SIP Huts with their shed roofs are modeled using 1.5 ft overhangs along long walls and 2 ft overhangs on end walls over the doors.

$$\text{(143)}$$
\[ A_{PV,Hut,SIP}[ft^2] = (-1 + 16 + 1.5 - 1)[ft] \cdot (-1 + 2 + 32 + 2 - 1)[ft] = 527[ft^2] \]

Using SIP Huts with their shed roofs nearly increases the area available for rooftop PV on a FOB by over 70% without taking up additional land area for utility PV arrays. All huts are assumed oriented on FOBs to maximize solar production, i.e., with the long side and roof pitch facing solar south for locations in the Northern Hemisphere. To calculate the total FOB rooftop PV area, use (144). The total FOB PV area is the sum of the rooftop and the utility PV areas (145).

\[
A_{PV,R}[m^2] = A_{PV,Hut} \left( \frac{ft^2}{hut} \right) \cdot n[hut] \cdot \left[ \frac{0.092903m^2}{ft^2} \right]
\]

(144)

\[
A_{PV,FOB}[m^2] = A_{PV,R}[m^2] + A_{PV,U}[m^2]
\]

(145)

The efficiencies applied in (141) are for converting insolation to electricity (panel), losses in the system (system), and losses in converting from DC to AC (inverter). For panel efficiency, this paper assumes the panels are 19% efficient, which is the same as that used in NREL’s PVWatts program (34). A recent survey of 42 solar panels shows that 23 panels had maximum efficiencies over 19% (the highest being 22.8%) and five panels with average efficiencies over 19% (highest being 20.7%) (104).

For the system efficiency, this paper uses the same assumptions as those in NREL’s PVWatts program with the following losses: soiling (2%), shading (3%), snow (0%), mismatch (2%), wiring (2%), connections (0.5%), light-induced degradation (1.5%), nameplate rating error (1%), age (0%), and availability (3%) for a total of 14.08% losses (efficiency of 85.92%) (105).
For the inverter efficiency, the authors again adopt the default value of 96% used by NREL in PVWatts (105). A typical inverter efficiency curve suggests an average value of slightly over 95% for operations producing above 15% of design output power (106).

To find the “system size” of the FOB PV array in terms of kW\textsubscript{DC}, which is useful for comparing this model’s results to other PV analysis programs, use “1-sun” at 1 kW m\textsuperscript{-2} and (146).

\begin{equation}
    P_{PV}[\text{kW}_{\text{DC}}] = A_{PV,FOB}[\text{m}^2] \cdot 1\left[\frac{\text{kW}}{\text{m}^2}\right] \cdot \eta_{PV,panel}
\end{equation}

Finally, estimate the total cost of PVs using (147). This paper uses $4.00 \text{ W}_{\text{DC}}^{-1}$, which is the 2020 national average for residential PV installations ($2.96 \text{ W}_{\text{DC}}^{-1}$) less the 26% federal tax credit (107).

\begin{equation}
    c_{PV,FOB} [\$] = c_{PV} \left[ \frac{\$}{\text{W}_{\text{DC}}} \right] \cdot \left[ \frac{1,000 \text{W}}{1 \text{ kW}} \right] \cdot P_{PV}[\text{kW}_{\text{DC}}]
\end{equation}

1.C.6. Wind power production

There are two major types of wind turbines: HAWTs and VAWTs. Tall HAWTs can take advantage of higher wind speeds at higher elevations, but they must be spaced far apart in order to avoid aerodynamic interference from the wakes of upstream turbines (36). Large HAWTs are likely to have negative impacts on aircraft flight paths and radar, making them a poor choice for application at FOBs.

Low elevation (~10m) VAWTs, however, have the potential for higher wind farm power densities because they do not need the magnitude of spacing requirements that HAWTs require. An additional benefit of VAWTs is the fact that their swept area can be increased by making
their rotors taller, which does not impact spacing requirements like it would increasing the
diameter of HAWT rotors. Scientists are studying various VAWT placement and spacing
options, to include the possibility of counter-rotating VAWTs where the aerodynamic
interactions between turbines can actually be beneficial, mitigating the decreased performance of
downstream turbines that typically comes with reduced spacing (36,108). Although a VAWT
farm may have a higher power density than a HAWT farm, note that the land in-between
individual VAWTs is less likely to be useful for any other application, whereas the area between
HAWTs could be much greater and, therefore, more likely to support other uses.

This paper models the use of 2 kW VAWTs with the characteristics summarized in Table
13 and the power curve shown in Figure 12. The power curve includes generator efficiency and
reflects output power in AC electricity. The rotor blades can be shipped separate from the
turbine and assembled in the field, making them more efficient for transportation within shipping
containers. The turbine itself is heavy (nearly 200 lbs), which will likely require either Material
Handling Equipment (MHE) or the use of winches with a gin pole to raise. The Army does have
cranes and crane operators as well.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor height</td>
<td>2.8 m (9.2 ft)</td>
</tr>
<tr>
<td>Rotor width (“diameter”)</td>
<td>2.2 m (7.2 ft)</td>
</tr>
<tr>
<td>Horizontal swept area</td>
<td>6.16 m² (66.3 ft²)</td>
</tr>
<tr>
<td>Turbine mass (weight)</td>
<td>89 kg (196.2 lbs)</td>
</tr>
<tr>
<td>Rated output</td>
<td>2,000 W</td>
</tr>
<tr>
<td>Max output power</td>
<td>2,700 W</td>
</tr>
<tr>
<td>Cut-in wind speed</td>
<td>2.5 m s⁻¹ (5.6 mph)</td>
</tr>
<tr>
<td>Rated wind speed</td>
<td>11 m s⁻¹ (24.6 mph)</td>
</tr>
<tr>
<td>Cut-out wind speed</td>
<td>25 m s⁻¹ (55.9 mph)</td>
</tr>
<tr>
<td>Survival wind speed</td>
<td>50 m s⁻¹ (111.8 mph)</td>
</tr>
<tr>
<td>Generator efficiency</td>
<td>96%</td>
</tr>
<tr>
<td>Minimum working temperature</td>
<td>-20°C (-4°F)</td>
</tr>
<tr>
<td>Maximum working temperature</td>
<td>50°C (122°F)</td>
</tr>
</tbody>
</table>

Data from (35,109).
One VAWT manufacturer publishes a survival wind speed but not a cut-out speed. The cut-out wind speed is where the wind turbine applies a mechanical brake and stops spinning, whereas the survival wind speed is an environmental condition that the wind turbine can be expected to withstand without structural damage. In this instance, the manufacturer has developed blades that can shed wind and maintain a maximum rotation speed of 320 rpm at wind speeds of 30 m s\(^{-1}\) or even 40 m s\(^{-1}\) (35). For this paper, the authors take a more conservative approach and apply a cut-out wind speed of 25 m s\(^{-1}\) based on other, similar VAWT models. By simply updating the characteristics information and the power curve data, a user can change the model to study the application of different types of VAWTs as desired.

The estimated cost of this VAWT is ~$7,240 per turbine, which represents the summation of the costs for the turbine itself (~$4,100), controller (~$1,080), inverter (~$580),
and tower (~$1,480) (37). Consistent with most other aspects in this study, the authors do not consider transportation costs or installation/labor costs.

Using wind speed data directly from TMY3 data files involves a level of uncertainty due to discrepancies in data collection standards. Today, NOAA’s United States Climate Reference Network (USCRN) uses anemometers at a height of about 1.5 m above the surface to measure wind speed (110). However, the current TMY3 User’s Manual does not state a standard height for wind measurements, and the 1981 TMY User’s Manual makes note that “no adjustments were made in the wind velocity data due to changes in anemometer heights during the period of record” (67,111). In fact, NOAA reports that, “between 1931 and 2000, there were up to 12 significant anemometer height changes at some stations, and on average there was one change per decade at any station with more than ten years on record” with some stations as high as 23 m (75 ft) (112). Some work has been done to homogenize the elevation of wind time series data for climatological assessments (113). The authors’ search to confirm station anemometer heights resulted in being referred to the Federal Climate Complex Data Documentation for Integrated Surface Data, which states that the standard is indeed 1.5 m; however, the authors could not receive confirmation that all TMY3 data conformed to this standard (114). In this paper, the authors model the TMY3 data reflecting wind speed measurements taken at the standard 1.5 m anemometer height. Nevertheless, the model includes an option that allow users to specify an anemometer height for a station, if known, which will automatically correct for wind speed at a different height.

As estimated by (148), wind speed will increase with height due to the lessening effect of friction from ground surface roughness.
\[
\nu \left[ \frac{m}{s} \right] = \nu_0 \left[ \frac{m}{s} \right] \cdot \left( \frac{H[m]}{H_0[m]} \right)^{a^[-]}
\]

To estimate the friction coefficient, \( \alpha \), the model allows users to select the ground surface conditions and the appropriate corresponding value (33). Here, the model simply adopts the “one-seventh” rule-of-thumb for generally open terrain for all locations so as to focus the comparison of results on other factors (33). However, the model includes these as drop-down options for studying any specific location.

Published planning factors help predict the land area a FOB will require, but they do not consider the use of VAWTs (20). The model allows users to directly input the number of wind turbines used on a FOB, but the supporting land area required can lack perspective when considered independently. VAWTs could potentially be interspersed across a FOB, placed around the perimeter within standoff zones, or concentrated within dedicated wind farms. To help establish limits on the number of turbines, the model also enables users to input an allowed percent increase in the typical FOB area required to accommodate the addition of a wind farm. The authors suggest an additional 10% beyond FOB land area requirements based on planning factors for unit type and population size. However, these are all site-specific variables, so this suggestion is simply for analysis and comparison purposes.

Proper spacing is required in order to maximize the power production of downstream wind turbines. One study on VAWT wind farm power density found that downstream VAWTs could achieve 95% of the performance of an isolated VAWT when their spacing equaled four times the VAWT diameter, \( D \) (36). Another study concluded that an optimized VAWT farm had a density of one turbine per \( 18.1D^2 \), which equates to \(~4.25D\) if horizontal and vertical spacing
are equal (115). In this paper, the authors model a VAWT spacing factor, S of 4 (Figure 13) and calculate the spacing, SD by (149).

\[ SD[m] = S[-] \cdot D[m] \]

For an analysis of wind power production for each hour of the year, begin by using (150) to calculate the corrected air density, \( \rho \) at every hour, \( h \). The molecular weight of air is 28.97 g mol\(^{-1}\), the ideal gas constant is 8.2056E-05 m\(^3\) atm K\(^{-1}\) mol\(^{-1}\), and the pressure and dry bulb temperature readings are taken directly from TMY3 data.

\[
\rho_h \left[ \frac{kg}{m^3} \right] = \frac{P_h [atm] \cdot MW_{air} \left[ \frac{g}{mol} \right]}{R \left[ \frac{m^3 \cdot atm}{K \cdot mol} \right]} \cdot \left( 273.15[^{\circ}C] + T_{db,h}[^{\circ}C] \right) [K]
\]

Next, take the recorded wind speed from the TMY3 data for each hour (measured at 1.5 m height) and estimate the wind speed at the center of the VAWT rotor height (10 m) using (148). Whereas the swept area of a HAWT rotor is circular, \( \frac{\pi b^2}{4} \), the swept area of a VAWT...
rotor is rectangular and given by the product of the rotor height (also called the blade length) and the rotor diameter (width). Now that the corrected air density, the VAWT swept area, and the corrected wind speed are known, calculate the power in the wind at every hour, \( h \) using (151).

\[
P_{\text{W,H}}[W] = \frac{1}{2} \cdot \rho \left[ \frac{kg}{m^3} \right] \cdot A_{\text{VAWT}}[m^2] \cdot (v_h \left[ \frac{m}{s} \right])^3
\]

Next, calculate the rated output power of one VAWT using (152). The output is zero if the wind speed is below the cut-in speed, zero if the wind speed is above the cut-out speed, and interpolated from the wind turbine power curve data (Figure 12) for wind speeds in-between.

\[
P_{\text{r,VAWT,h}} \left[ \frac{W}{\text{turbine}} \right] = \begin{cases} 
0, & \text{if } v_h \left[ \frac{m}{s} \right] < v_{\text{cut-in}} \left[ \frac{m}{s} \right] \\
\text{Interpolated from wind turbine power curve} \\
0, & \text{if } v_h \left[ \frac{m}{s} \right] > v_{\text{cut-out}} \left[ \frac{m}{s} \right]
\end{cases}
\]

To calculate the power produced by a FOB’s VAWT wind farm (153), apply two additional factors: a utilization factor, \( UF \) and a loss factor, \( LF \). For the utilization factor, the authors assume that each turbine will be down for maintenance a total of 48 hours per month, or 24 days a year, meaning each turbine is operational for 93% of a year. Since it is unknown when, or if, any number of turbines will be down for maintenance, the authors simply apply this reduction factor in the computation for wind power production at every hour of the year. The wind farm aerodynamic loss factor represents the loss in power production capability when moving from an isolated wind turbine to a farm of wind turbines. Professor John Dabiri uses an estimate of 10% in studies of a VAWT farm near Antelope Valley, California (36).
\[ P_{WF,h}[W] = P_{r,VAWT,h} \left( \frac{W_{turbine}}{n[turbine]} \right) \cdot UF[\%] \cdot (1 - LF[\%]) \cdot \left[ \frac{0.001kW}{1W} \right] \]

Calculate the annual wind farm energy production by taking the summation of the wind farm power (times one hour) across all 8,760 hours of the year (154).

\[ E_{WF,annual}[kWh] = \sum_{h=1}^{8,760} P_{WF,h}[kW] \cdot [1hr] \]

Next, calculate the power coefficient, \( C_p \), for a single wind turbine using (155). The power coefficient is a ratio of electric power produced by a turbine to the power in the wind passing through the turbine rotor’s swept area. For spreadsheet calculations, the power coefficient is not a number (NaN) when the hour’s power in the wind is equal to zero. If there is no wind, then no power can be extracted from the wind. Furthermore, the hourly power coefficient is also equal to zero when the numerator equals zero, e.g., if the cut-out speed had been reached and, although the wind was blowing, the turbine is not producing power.

\[ C_{p,h}[-] = \frac{P_{r,VAWT,h}[W]}{P_{WF,h}[W]} \]

The model also calculates size and performance characteristics of the FOB’s wind farm. The wind farm “size” or total rated power is given by (156). This is similar to (153), but it does not account for the utilization factor or the loss factor; it is simply in terms of turbine rated power and the number of turbines used.

\[ P_{WF,r}[kW] = n[turbines] \cdot P_{WT,r} \left( \frac{W_{turbine}}{1kW} \right) \left[ \frac{1,000W}{1,000W} \right] \]
The wind farm capacity factor, $CF$, is given by (157), which takes the annual energy produced by the wind farm (154) and divides it by the product of the wind farm total rated power (156) and 8,760 hours. A typical capacity factor for wind farms is between 25% and 35% (116).

\[
CF[\%] = \frac{E_{WF,annual}[kWh]}{P_{WF,r}[kW] \cdot 8,760[hr]}
\]

The wind farm power density, $PD_{WF}$, is given by (158) (36). The equation for wind farm power density typically does not account for a utilization factor, which is assumed will be ~93%. Therefore, in practice, the actual power density will be lower due to turbine downtime. Also, this equation includes the rated power of a single turbine, the capacity factor of the entire wind farm, the loss factor for the wind farm, and assumes that the land area required for one turbine is a circular area with a diameter equal to the wind turbine spacing, $SD$. Alternatively, (158) can be written completely in terms of the wind farm total rated power and total land area required, as in (159).

\[
PD_{WF} \left[ \frac{W}{m^2} \right] = \frac{P_{WT,r} \left[ \frac{W}{turbine} \right] \cdot CF[\%] \cdot (1 - LF[\%])}{\frac{\pi}{4} \cdot (SD[m])^2 [turbine]}
\]

\[
PD_{WF} \left[ \frac{W}{m^2} \right] = \frac{P_{WF,r}[W] \cdot CF[\%] \cdot (1 - LF[\%])}{\frac{\pi}{4} \cdot (SD[m])^2 [turbine] \cdot n[turbines]}
\]

Both (158) and (159) are accurate, but they underestimate the true land area required by excluding the area between circles (illustrated in Figure 13). A more representative value for the
required individual wind turbine and total wind farm areas may be given by (160) and (161), which can be substituted into the denominator of (158) and (159), respectively, to find a new value for the wind farm power density. This paper models the cross-stream spacing as equal to the downstream spacing, so the areas are based on square shapes. The equations could just as easily be based on rectangular shapes to accommodate different spacing in different directions.

\[
A_{WT}[m^2] = \frac{m^2}{[turbine]} = \frac{(SD[m])^2}{[turbine]}
\]

(160)

\[
A_{WF}[m^2] = \frac{(SD[m])^2}{[turbine]} \cdot n[turbines]
\]

(161)

This paper does not model wind turbines shutting down in cold weather. Although some manufacturers report a minimum working temperature, katabatic winds provide power for the Princess Elisabeth Station throughout the Antarctic winter, so there are wind turbine models capable of polar applications (117).

1.D. Optimization model

The optimization model is classified as time-series Mixed Integer Linear Programming (MILP). It includes both continuous and integer variables, with binary variables to model each diesel generator being powered on/off each hour. All variables are first power only. As previously discussed in the Methods section, the 8,760 hours of a year are broken down into 12 months and optimized by month. This significantly reduces computational effort and improves speed. A mathematical formulation of the model follows:
**Objective Function:**

\[
\begin{align*}
\text{Min } \quad & c_T \\
\text{s.t. constraints: } \quad & (E_{h}^{B,\text{out}}[\text{kWh}] \cdot \eta_h[-]) + E_{h}^D[\text{kWh}] - E_{h}^{B,\text{in}}[\text{kWh}] = E_{h}^i[\text{kWh}] + E_{h}^C[\text{kWh}] - E_{h}^{PV}[\text{kWh}] - E_{h}^W[\text{kWh}] \\
& E_{h}^{B}[\text{kWh}] = E_{h}^{B,\text{min}}[\text{kWh}] \\
& E_{h+1}[\text{kWh}] = E_{h}^{B}[\text{kWh}] + E_{h}^{B,\text{in}}[\text{kWh}] - E_{h}^{B,\text{out}}[\text{kWh}] \\
& E_{h}^{B,\text{in}}[\text{kWh}] \leq E_{h}^D[\text{kWh}] + E_{h}^{PV}[\text{kWh}] + E_{h}^W[\text{kWh}] \\
& E_{h}^{B,\text{out}}[\text{kWh}] \leq E_{h}^{B}[\text{kWh}] \\
& E_{h}^{B,\text{min}}[\text{kWh}] \leq E_{h}^D[\text{kWh}] \leq E_{h}^{B,\text{max}}[\text{kWh}] \\
\text{variable & positivity constraints: } \quad & \forall \ i \in \{0,1\}: \ y_{i,h} \ [\text{binary}] \\
& E_{h}^{B,\text{min}}[\text{kWh}] \leq E_{h}^D[\text{kWh}] \leq E_{h}^{B,\text{max}}[\text{kWh}] \\
& E_{h}^{B,\text{in}}[\text{kWh}] \leq P_{h}^{B,\text{in}}[\text{kW}] \cdot [1\text{hr}] \\
& E_{h}^{B,\text{out}}[\text{kWh}] \leq P_{h}^{B,\text{out}}[\text{kW}] \cdot [1\text{hr}] \\
& E_{h}^D[\text{kWh}] \geq 0 \\
& E_{h}^C[\text{kWh}] \geq 0 \\
\text{expressions: } \quad & E_{h}^D[\text{kWh}] = \sum_{i=0}^{n_{\text{generators}}} E_{i,h}^D[\text{kWh}] \cdot y_{i,h}[-] \\
& c_{D}[\$] = \sum_{h=0}^{n_{\text{hours}}} \sum_{i=0}^{n_{\text{generators}}} y_{i,h}[-] \cdot r_F \left[ \frac{\text{gal}}{\text{hr}} \right] \cdot c_F \left[ \frac{\$}{\text{gal}} \right] \cdot [1\text{hr}] \\
& c_{C}[\$] = \sum_{h=0}^{n_{\text{hours}}} E_{h}^C[\text{kWh}] \cdot c_C \left[ \frac{\$}{\text{kWh}} \right] \\
& c_T[\$] = c_D[\$] + c_C[\$]
\end{align*}
\]
1.D.1. Julia code

# FOB Energy Portfolio
# Original code written by Scott M. Katalenich, Emma Romack, and Susanna Struckmann, for
ENERGY291, Stanford University, March 2018
# All further modifications made by Scott M. Katalenich
# Last Updated 15 April 2020

# Initialize packages

using Pkg
# Load path to CPLEX solver and add/build package
ENV["LD_LIBRARY_PATH"] =
"/Users/scott.katalenich/Documents/Miscellaneous/Computer_Software/CPLEX_Studio129/cplex/bin"
ENV["CPLEX_STUDIO_BINARIES"] =
"/Users/scott.katalenich/Documents/Miscellaneous/Computer_Software/CPLEX_Studio129/cplex/bin"
using Libdl
push!(DL_LOAD_PATH,
"/Users/scott.katalenich/Documents/Miscellaneous/Computer_Software/CPLEX_Studio129/cplex/bin/x86-64_osx")
Pkg.add("CPLEX")
Pkg.build("CPLEX")
using CPLEX
# Add JuMP
using JuMP
# Add Microsoft Excel read/write package
Pkg.add("XLSX")

# Time conversions
hrs_in_day = 24
hrs_in_year = 8760

# Define ending Julian calendar days for each month
JANend = 31
FEBend = 59
MARend = 90
APRend = 120
MAYend = 151
JUNend = 181
JULend = 212
AUGend = 243
SEPend = 273
OCTend = 304
NOVend = 334
DECend = 365

# Read in data from pre-processing

```plaintext
import XLSX

ModelInput = XLSX.readxlsx("Model_Input.xlsx")

JulianHour = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "B2:B8761");
LoadProfile = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "C2:C8761");
SolarProduction = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "D2:D8761");
WindProduction = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "E2:E8761");
NumGenerators = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "F1");
BatteryEnergyStoredMin = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G2");
BatteryEnergyStoredMax = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G3");
ContinuousBatteryCharge = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G4");
ContinuousBatteryDischarge = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G5");
DieselUsage = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G6");
CostDiesel = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G7");
BatteryEfficiency = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G8");
MinLoadFraction = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G9");
MaxLoadFraction = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G10");
CapacityGenerator = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G11");
ExtraConstructionCost = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G12");
BatteryCost = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G13");
SolarCost = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G14");
WindCost = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G15");
PenaltyCost = XLSX.readdata("Model_Input.xlsx", "Input Data to Julia", "G16");
```

# Model

```plaintext
println("-------------------")

# Initialize reporting arrays and counters

mCosts = zeros(12)
PlottingArray = zeros(hrs_in_year,8)

for i=1:12 # since the model has too many decision variables over the course of a full year with 8,760 hours, the authors created a "for" loop to run the model for each month. This resets the battery energy storage each month, but that's acceptable for this analysis

    Month = i
    
    if Month == 1
    T1 = 1
    T2 = JANend*hrs_in_day
    elseif Month == 2
```

T1 = JANend*hrs_in_day + 1
T2 = FEBend*hrs_in_day

elseif Month == 3
    T1 = FEBend*hrs_in_day + 1
    T2 = MARend*hrs_in_day

elseif Month == 4
    T1 = MARend*hrs_in_day + 1
    T2 = APRend*hrs_in_day

elseif Month == 5
    T1 = APRend*hrs_in_day + 1
    T2 = MAYend*hrs_in_day

elseif Month == 6
    T1 = MAYend*hrs_in_day + 1
    T2 = JUNend*hrs_in_day

elseif Month == 7
    T1 = JUNend*hrs_in_day + 1
    T2 = JULend*hrs_in_day

elseif Month == 8
    T1 = JULend*hrs_in_day + 1
    T2 = AUGend*hrs_in_day

elseif Month == 9
    T1 = AUGend*hrs_in_day + 1
    T2 = SEPend*hrs_in_day

elseif Month == 10
    T1 = SEPend*hrs_in_day + 1
    T2 = OCTend*hrs_in_day

elseif Month == 11
    T1 = OCTend*hrs_in_day + 1
    T2 = NOVend*hrs_in_day

elseif Month == 12
    T1 = NOVend*hrs_in_day + 1
    T2 = DECond*hrs_in_day

End

# Build monthly vectors for hour, load, solar, and wind
Hour = JulianHour[T1:T2]
Load = LoadProfile[T1:T2]
Solar = SolarProduction[T1:T2]
Wind = WindProduction[T1:T2]

# Reset limit for hours in each monthly iteration
T = T2 - T1 + 1

# Define the model name, “m” and solver, CPLEX.
m = Model(CPLEX.Optimizer) # runs IBM's CPLEX Optimization code with no limits
m = Model(with_optimizer(CPLEX.Optimizer, CPX_PARAM_TILIM=60)) # limits computational time to a defined number of seconds per month

# Define Decision Variables (and include the Positivity/Max/Min Constraints)
# Binary on/off for diesel generators
@variable(m, DieselOnOff[1:T,1:NumGenerators], Bin) # \{0,1\} for \{Off, On\}
# Establish minimum and maximum boundaries for battery state of charge
@variable(m, BatteryEnergyStoredMin <= BatteryEnergyStored[1:T+1] <= BatteryEnergyStoredMax) # [kWh]
# Battery charge rate cannot exceed continuous charge rate (conservative assumption vs using peak rate)
@variable(m, 0 <= BatteryEnergyIn[1:T] <= ContinuousBatteryCharge*1) # [kWh]
# Battery discharge rate cannot exceed continuous discharge rate (conservative assumption vs using peak rate)
@variable(m, 0 <= BatteryEnergyOut[1:T] <= ContinuousBatteryDischarge*1) # [kWh]
# Positivity constraint for diesel energy produced
@variable(m, DieselEnergyProduced[1:T,1:NumGenerators] >= 0) # [kWh]
# Positivity constraint for curtailment (sign is later changed to negative for post-processing)
@variable(m, Curtailment[1:T] >= 0) # [kWh]

# Define Expressions
# Calculate the hourly energy produced by all generators
@expression(m, TotalHourlyDieselEnergy[t=1:T], sum(DieselEnergyProduced[t,x] for x=1:NumGenerators)) # [kW]
# Calculate hourly fuel cost of diesel generators
@expression(m, TotalDieselCost, sum(sum(DieselOnOff[t,x]*DieselUsage*CostDiesel for x = 1:NumGenerators) for t=1:T)) # [$]
# Calculate the total penalty for any curtailment
@expression(m, Penalty, sum(Curtailment[t]*PenaltyCost for t=1:T))
# Calculate total fuel cost + penalty for optimization
@expression(m, TotalCost, TotalDieselCost+Penalty)

# Define Constraints
# Overall FOB energy balance
@constraint(m, [t=1:T], (BatteryEnergyOut[t]*BatteryEfficiency) + TotalHourlyDieselEnergy[t] - BatteryEnergyIn[t] == Load[t] - Solar[t] - Wind[t] + Curtailment[t]) # [kWh]
# Initial storage is minimum storage level across all batteries [kWh]
@constraint(m, BatteryEnergyStored[1] == BatteryEnergyStoredMin) # [kWh]
# Battery energy balance
@constraint(m, [t=1:T], BatteryEnergyStored[t+1] == BatteryEnergyStored[t] + BatteryEnergyIn[t] - BatteryEnergyOut[t]) # [kWh]
# Battery energy stored in an hour cannot exceed the actual energy produced by diesel generators, solar, and wind in that hour (i.e., can't use battery energy to store battery energy)
@constraint(m, [t=1:T], (BatteryEnergyIn[t] <= TotalHourlyDieselEnergy[t]+Solar[t]+Wind[t]))
# Battery energy out in an hour cannot exceed the battery energy stored at the beginning of that hour

@constraint(m, [t=1:T], (BatteryEnergyOut[t] <= BatteryEnergyStored[t]))

# Generators, if on, can run between minimum- and maximum- defined load; otherwise wet stacking occurs and has significant negative impacts on maintenance and generator longevity

@constraint(m, [t=1:T, x=1:NumGenerators], MinLoadFraction * DieselOnOff[t,x] * CapacityGenerator <= DieselEnergyProduced[t,x])

@constraint(m, [t=1:T, x=1:NumGenerators], DieselEnergyProduced[t,x] <= MaxLoadFraction * CapacityGenerator * DieselOnOff[t,x])

# Define the Objective Function

@objective(m, Min, TotalCost)

optimize!(m)

println("Month ", Month, " Optimal cost = \$, JuMP.value.(TotalDieselCost))

mCosts[Month] = JuMP.value.(TotalDieselCost)

PlottingArray[T1:T2, 1] = Hour[1:T]
PlottingArray[T1:T2, 2] = Load[1:T]
PlottingArray[T1:T2, 3] = Solar[1:T]
PlottingArray[T1:T2, 4] = Wind[1:T]
PlottingArray[T1:T2, 5] = JuMP.value.(TotalHourlyDieselEnergy[1:T])
PlottingArray[T1:T2, 6] = JuMP.value.(BatteryEnergyStored[1:T])
PlottingArray[T1:T2, 7] = JuMP.value.(BatteryEnergyIn[1:T])
PlottingArray[T1:T2, 8] = -1*JuMP.value.(BatteryEnergyOut[1:T])
PlottingArray[T1:T2, 9] = EnergyStoredMax
PlottingArray[T1:T2, 10] = -1*JuMP.value.(Curtailment[1:T])

# Output results

# Print results in Julia

println("-------------------------------------------------------------------")
println("Operations Costs:")
println("Annual Diesel Consumption = sum(mCosts)/CostDiesel")
println("Annual Diesel Consumption = ", AnnualDieselConsumption," gallons")

AnnualFuelCost = sum(mCosts)
println("Annual Fuel Cost = \
$", AnnualFuelCost)
println("")

# Write data to a new blank Excel spreadsheet

columns = Vector()
push!(columns, PlottingArray[:,1])
push!(columns, PlottingArray[:,2])
push!(columns, PlottingArray[:,3])
push!(columns, PlottingArray[:,4])
push!(columns, PlottingArray[:,5])
push!(columns, PlottingArray[:,6])
push!(columns, PlottingArray[:,7])
push!(columns, PlottingArray[:,8])
push!(columns, PlottingArray[:,9])
push!(columns, PlottingArray[:,10])

labels = ["Hour (hr)", "Load (kW)", "Solar (kW)", "Wind (kW)", "Diesel (kW)", "Battery SOC at beginning of hour (kWh)", "Battery energy in (kWh)", "Battery energy out (kWh)", "Max battery storage (kWh)", "Curtailment (kWh)"

XLSX.openxlsx("Model_Output_Results.xlsx", mode="w") do xf
    sheet = xf[1]
    XLSX.rename!(sheet, "Julia Output")
    XLSX.writetable!(sheet, columns, labels, anchor_cell=XLSX.CellRef("A5"))
    sheet["K2"] = "Maximum"
    sheet["K3"] = "Average"
    sheet["K4"] = "Minimum"
    sheet["L6"] = "Summary values and checks on results"
    sheet["L7"] = AnnualDieselConsumption
    sheet["M7"] = "Annual diesel consumption (gal)"
    sheet["L8"] = AnnualFuelCost
    sheet["M8"] = "Annual fuel cost (\$)"
    sheet["L9"] = ContinuousBatteryCharge
    sheet["M9"] = "Continuous battery charge rate (kW)"
    sheet["L10"] = ContinuousBatteryDischarge
    sheet["M10"] = "Continuous battery discharge rate (kW)"
    sheet["L11"] = ExtraConstructionCost
    sheet["M11"] = "Extra building efficiency cost (\$)"
    sheet["L12"] = BatteryCost
    sheet["M12"] = "Battery cost (\$)"
    sheet["L13"] = SolarCost
    sheet["M13"] = "Solar PV array cost (\$)"
    sheet["L14"] = WindCost
    sheet["M14"] = "Wind farm cost (\$)"
end
2. Supplementary analysis

To further expand upon these findings, the authors have quantified results not just in terms of potential energy resilience to be gained, but also in terms of casualties, air pollution, carbon emissions, and costs to be avoided.

2.A. Casualties

Casualties, both Wounded In Action (WIA) and Killed In Action (KIA) are tracked by the Defense Causality Analysis System (DCAS) maintained by the Defense Manpower Data Center (DMDC) (118). Recent overseas contingency operations, Operation Enduring Freedom (OEF), Operation Iraqi Freedom (OIF), Operation New Dawn (OND), Operation Inherent Resolve (OIR), and Operation Freedom’s Sentinel (OFS), have resulted in 60,260 casualties, combined (119).

With regard to connecting combat casualties to fuel-related convoys, a 2009 study by the Army Environmental Policy Institute (AEPI) reported that, in FY07 alone, there were 132 casualties for 5,133 fuel-related convoys in Iraq and 38 casualties for 897 fuel-related convoys in Afghanistan (54). Combining stats for both Iraq and Afghanistan yields 170 casualties on 6,030 fuel-related convoys for a factor of 0.0282 casualties per convoy (162).

\[
\text{Casualty factor} = \frac{132 + 38}{5,133 + 897} \text{casualties per convoy} = 0.0282 \text{ casualties per convoy}
\]

(162)

The AEPI report further estimated that there was, on average, 97,818 gallons of fuel per convoy carried by 16 supply trucks (54). Inverting the volume of fuel per convoy estimate, the casualty factor in (162) can be reframed to that in (163), which can also be reported as one casualty for every 3.5 million gallons of diesel resupplied.
Casualty factor = 0.0282 \left[ \frac{\text{casualties}}{\text{convoy}} \right] \cdot \frac{1}{97,818} \left[ \frac{\text{convoy}}{\text{gallons}} \right] = 2.9 \times 10^{-7} \left[ \frac{\text{casualties}}{\text{gallon}} \right]

Therefore, the potential for avoided casualties resulting from offset energy demand at FOBs can be estimated using (164).

\begin{equation}
\text{Casualties avoided} = 2.9 \times 10^{-7} \left[ \frac{\text{casualties}}{\text{gallon}} \right] \cdot x[\text{gallons diesel offset}]
\end{equation}

2.B. Air pollution

There are numerous pollutants from internal combustion engines, but those primarily tracked are non-methane hydrocarbons (NMHC), nitrogen oxides (NO\textsubscript{X}); carbon monoxide (CO); and particulate matter (PM) (56,120,121). NMHCs are also classified under total organic compounds (TOC). NO\textsubscript{X} emissions are the result of high pressure and temperature during the combustion process, whereas NMHC, CO, and PM are mostly due to incomplete combustion (a factor that makes the appropriate loading and phase balancing of generators even more important) (120). CO is a colorless, odorless, mostly inert gas formed when there is insufficient oxygen, insufficient temperature, and/or insufficient residence time in the cylinder to create CO\textsubscript{2} (120). PM includes both visible smoke and nonvisible emissions. Smoke can be white (from liquid particulates), blue (from partially burned oil), or black (from soot) (120).

Sulfur oxides (SO\textsubscript{X}), mostly sulfur dioxide (SO\textsubscript{2}), are also emitted, their levels being directly related to the sulfur content of the fuel itself and not factors related to the engine or combustion process (120). Low sulfur diesel (LSD) has a sulfur content of fewer than 500 ppm (0.05% wt.) and ultra-low sulfur diesel (ULSD) has a sulfur content of fewer than 15 ppm (0.0015% wt.) (122). Further oxidation of SO\textsubscript{2} in the atmosphere leads to sulfur trioxide (SO\textsubscript{3}),
which can react with water to form sulfuric acid (H\textsubscript{2}SO\textsubscript{4}), which contributes to acid rain and reacts with other particles to yield even further PM and visibility degradation (120). The EPA reports a SO\textsubscript{x} emission factor for large stationary diesel engines according to (165) and assumes that all sulfur in the fuel is converted to SO\textsubscript{2} (120). The conversion factors (specific here for #2 diesel with 137,381 BTU gal\textsuperscript{-1} and a generator producing 13.44 kWh gal\textsuperscript{-1} diesel) to go from lb MMBTU\textsuperscript{-1} to g gal\textsuperscript{-1} and g kWh\textsuperscript{-1} are given by (166) and (167), respectively.

\begin{equation}
1.01 \cdot (\% \text{ wt. sulfur in fuel}) \left(\frac{\text{lb}}{\text{MMBTU}}\right)
\end{equation}

\begin{equation}
\frac{0.137381[\text{MMBTU}]}{1[\text{gal diesel}]} \cdot \frac{0.453592[\text{kg}]}{1[\text{lb}]} \cdot \frac{1,000[\text{g}]}{1[\text{kg}]} = 62.3149 \left(\frac{\text{g}}{\text{gal}}\right) \left(\frac{\text{MMBTU}}{\text{lb}}\right)
\end{equation}

\begin{equation}
\frac{0.137381[\text{MMBTU}]}{1[\text{gal diesel}]} \cdot \frac{0.453592[\text{kg}]}{1[\text{lb}]} \cdot \frac{1,000[\text{g}]}{1[\text{kg}]} \cdot \frac{1[\text{gal diesel}]}{13.44[\text{kWh}]} = 4.63653 \left(\frac{\text{g}}{\text{kWh}}\right) \left(\frac{\text{MMBTU}}{\text{lb}}\right)
\end{equation}

Table 14 summarizes the major emissions factors for diesel generators from three relevant sources: the US Environmental Protection Agency’s (EPA’s) Compilation of Air Pollutant Emissions Factors (AP-42), a study done specifically on small military generators, and the most recent applicable EPA exhaust emission standards (56,120,121). The specific generators used in this model are relatively new. The 60 kW AMMPS reportedly meets Tier IV “interim compliance,” and it is assumed that the recently fielded 840 kW generator also meets Tier IV standards (39). In practice, however, the military often relies on inefficient generators procured on the local economy that may not meet such standards (66). To estimate the air pollution produced by diesel generators, the authors stay consistent with the types of generators
modeled and assume they just meet EPA’s Tier IV emissions standards for generators larger than 560 kW and use (168).

**Table 14. Survey of published emissions factors (g kWh\(^{-1}\)) for diesel generators**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tier I standards</td>
</tr>
<tr>
<td>NMHC</td>
<td>0.4</td>
<td>N/A</td>
<td>1.3</td>
</tr>
<tr>
<td>NO(_X)</td>
<td>14.8</td>
<td>7.3</td>
<td>9.2</td>
</tr>
<tr>
<td>CO</td>
<td>3.94</td>
<td>4.0</td>
<td>N/A</td>
</tr>
<tr>
<td>PM</td>
<td>0.46</td>
<td>0.3</td>
<td>N/A</td>
</tr>
</tbody>
</table>

(168)

**2.C. Carbon emissions**

Similar to calculating avoided air pollution, avoided carbon emissions can be calculated based on the reduction in annual diesel consumption from the base case scenario. This paper uses the 100-year Global Warming Potential (GWP) of greenhouse gases published by the Intergovernmental Panel on Climate Change (IPCC) (Table 15) along with the EPA’s published emissions for #2 diesel in stationary (non-transport) combustion engines (Table 16) to calculate avoided carbon emissions in (169).

**Table 15. 100-year global warming potential of greenhouse gases**

| Carbon dioxide, CO\(_2\) | 1 |
| Methane, CH\(_4\)         | 25 |
| Nitrous Oxide, N\(_2\)O   | 298 |

Data from (59).
Table 16. Combustion emission factors of greenhouse gases by gallon distillate fuel oil #2

<table>
<thead>
<tr>
<th>Greenhouse gas</th>
<th>Emissions (g gal⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon dioxide, CO₂</td>
<td>10,210</td>
</tr>
<tr>
<td>Methane, CH₄</td>
<td>0.41</td>
</tr>
<tr>
<td>Nitrous Oxide, N₂O</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Data from (58).

\[
\text{Carbon reduction} = \frac{\text{tonne CO}_2 \text{e}}{\text{yr}} = \text{Reduced energy demand} \cdot \text{GWP} \cdot \frac{\text{grams CO}_2 \text{e}}{\text{grams GHG}} \cdot \text{CEF} \cdot \frac{\text{grams GHG}}{\text{gal diesel}} \cdot 1 \times 10^{-6} \frac{\text{tonne}}{\text{gram}}
\]

(169)

2.D. Costs

Fuel costs and source references were previously discussed in Section 1.C.4.A, specifically drawing attention to the fact that offset costs can vary widely depending upon a FOB’s location, economic conditions, transportation requirements, and force protection requirements. To illustrate how much these costs can vary, a low cost is calculated using a factor of $2.66 FY20 USD gal⁻¹, which is the DLA’s forecasted standard purchase price for the fiscal year. In this paper, the authors follow Congressionally-mandated requirements to consider the fully burdened cost of fuel and include costs for transportation, sustainment, and ground force protection based on FY07 OIF data adjusted using the CPI for inflation. The authors do not include costs for personnel and force protection using aircraft for two reasons. First, although it could be argued that fewer personnel might have to be deployed in the first place, it is presumed that the military personnel would be there anyway getting paid whether they were conducting a fuel convoy or not. To be consistent with other costs where labor is not included, the authors also do not include personnel costs here. Second, not all fuel convoys receive specific coverage from air assets for force protection. Here again, it could be argued that overall costs for
“controlling the skies” could be proportioned to supporting fuel convoys, but the authors have not attempted to do this. Finally, for the high estimate, the authors do include personnel and force protection from aircraft using the FY07 OIF data, again adjusted for inflation, which equates to 32.55 FY20 USD gal\(^{-1}\).

Although not included in this study, further work can relate avoided air pollution and avoided carbon emissions directly to avoided mortality and health related costs and avoided social costs of carbon, respectively.
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