

Overhead Slides for
Chapter 10
of
Fundamentals of
Atmospheric Modeling

by

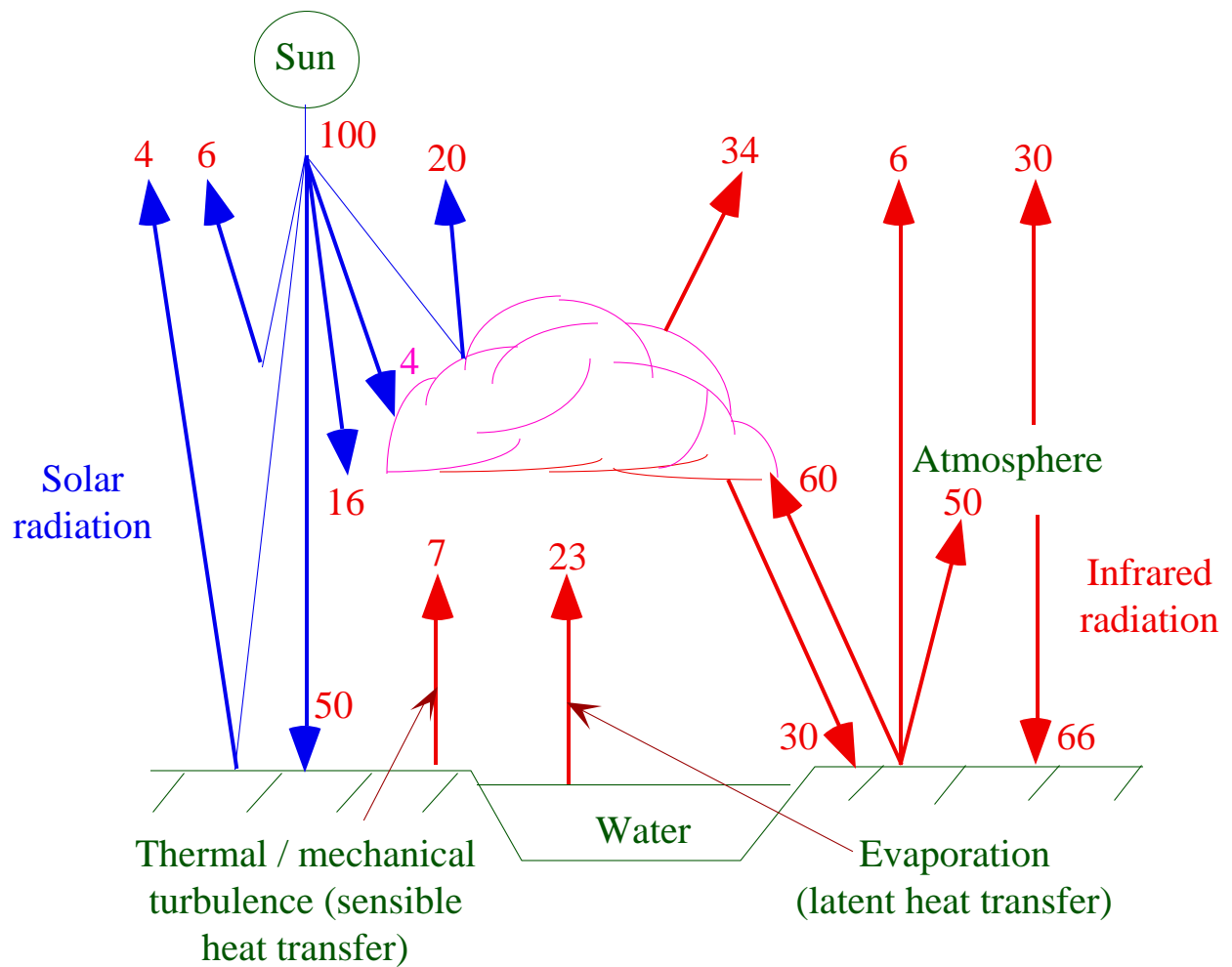
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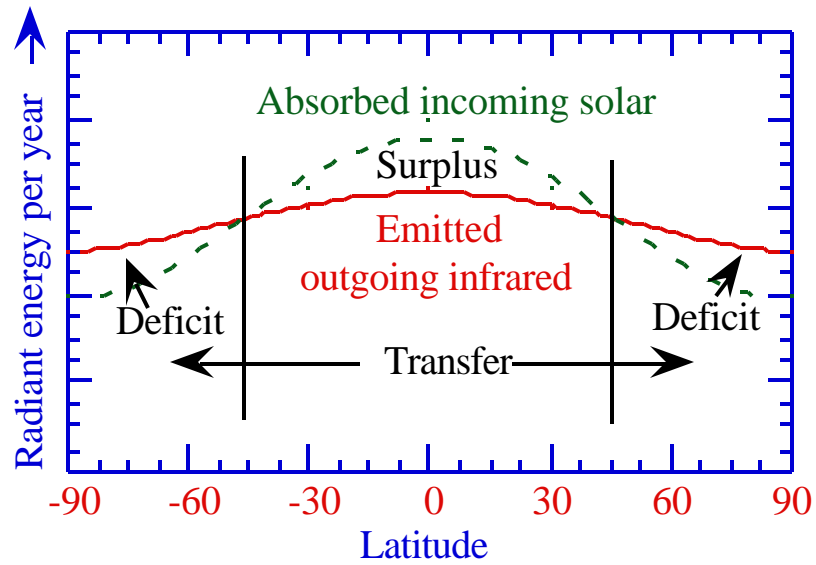
Energy Balance for Earth- Atmosphere System

Fig. 10.1.



Energy Transfer From Equator to Poles

Fig. 10.2.



Electromagnetic Spectrum

Radiation in the form of an electromagnetic wave

Wavelength

$$= \frac{c}{\nu} = \frac{1}{\tilde{\nu}} \quad (10.1)$$

Radiation in the form of a photon of energy

Energy per unit photon (J photon⁻¹)

$$E_p = h\nu = \frac{hc}{\lambda} \quad (10.3)$$

Example 10.1.

$$\begin{aligned} \text{-->} \quad E_p &= 0.5 \mu\text{m} \\ &= 3.97 \times 10^{-19} \text{ J photon}^{-1} \\ \text{-->} \quad \nu &= 5.996 \times 10^{14} \text{ s}^{-1} \\ \text{-->} \quad \tilde{\nu} &= 2 \mu\text{m}^{-1} \end{aligned}$$

$$\begin{aligned} \text{-->} \quad E_p &= 10.0 \mu\text{m} \\ &= 1.98 \times 10^{-20} \text{ J photon}^{-1} \\ \text{-->} \quad \nu &= 2.998 \times 10^{13} \text{ s}^{-1} \\ \text{-->} \quad \tilde{\nu} &= 0.1 \mu\text{m}^{-1} \end{aligned}$$

Planck's Law

Radiance

Intensity of emissions per incremental solid angle

Planck radiance ($\text{W m}^{-2} \mu\text{m}^{-1} \text{sr}^{-1}$)

$$B_{\lambda, T} = \frac{2hc^2}{5 \exp \left(\frac{hc}{k_B T} \right) - 1} \quad (10.4)$$

Radiance actually emitted by a substance

$$e_{\lambda} = \epsilon_{\lambda} B_{\lambda, T} \quad (10.5)$$

Kirchoff's law

In thermodynamic equilibrium, $a_{\lambda} = \epsilon_{\lambda}$ --> the efficiency at which a substance absorbs equals that at which it emits.

--> a perfect emitter is a perfect absorber

Table 10.1. Emissivities of different surfaces for an infrared wavelength.

Surface Type	Emissivity (fraction)	Surface Type	Emissivity (fraction)
Liquid water	1.0	Soil	0.9 - 0.98
Fresh snow	0.99	Grass	0.9 - 0.95
Old snow	0.82	Desert	0.84 - 0.91
Liq. water clouds	0.25 - 1.0	Forest	0.95 - 0.97
Cirrus clouds	0.1 - 0.9	Concrete	0.71-0.9
Ice	0.96	Urban	0.85 - 0.87

Incremental Solid Angle

Intercepted surface area on a sphere / radius squared

$$d a = \frac{dA_s}{r_s^2} \quad (10.6)$$

Incremental surface area

$$dA_s = (r_s d\theta) (r_s \sin \theta d\phi) = r_s^2 \sin \theta d\theta d\phi \quad (10.7)$$

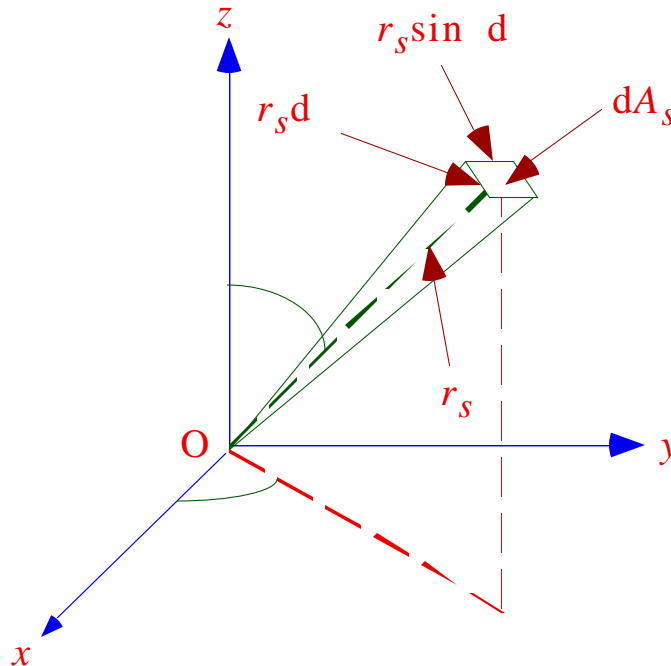
Incremental solid angle (sr)

$$d a = \sin \theta d\theta d\phi \quad (10.8)$$

Solid angle around a sphere

$$a = \int d a = \int_0^{2\pi} \int_0^\pi \sin \theta d\theta d\phi = 4 \quad (10.9)$$

Fig. 10.3. Radiance emitted from point (O) passes through incremental area dA_s at distance r_s from the point.



Spectral Actinic Flux

Integral of spectral radiance over all solid angles of a sphere

Incremental spectral actinic flux

$$dE_a = I \, d\Omega_a \quad (10.10)$$

Integral of incremental actinic flux over a sphere

$$E_a = \int_a dE_a = \int_a I \, d\Omega_a = \int_0^{2\pi} \int_0^\pi I \sin \theta \, d\theta \, d\phi \quad (10.11)$$

Isotropic spectral actinic flux

$$E_a = I \int_0^{2\pi} \int_0^\pi \sin \theta \, d\theta \, d\phi = 4\pi I \quad (10.12)$$

Spectral Irradiance

Flux of radiant energy propagating across a flat surface

Incremental spectral irradiance

$$dF_a = I \cos \theta \, d\Omega \quad (10.13)$$

Integral of incremental irradiance over hemisphere of x - y plane

$$F_a = \int_a dF_a = \int_a I \cos \theta \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} I \cos \theta \sin \theta \, d\theta \, d\phi \quad (10.14)$$

Isotropic spectral irradiance

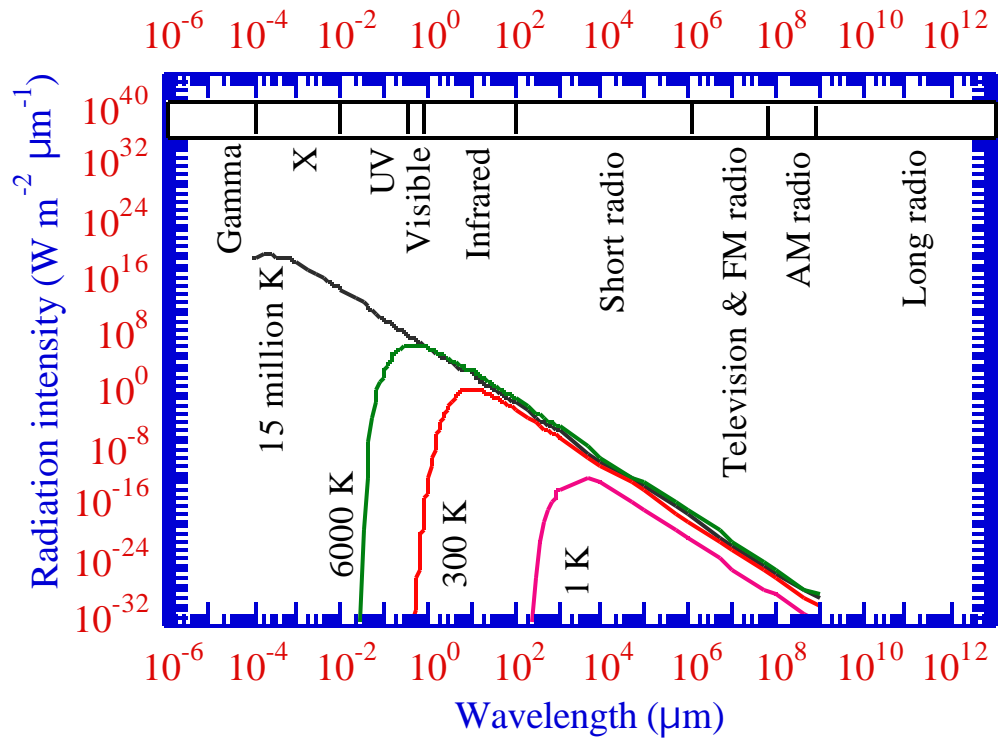
$$F_a = I \int_0^{2\pi} \int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \, d\phi = I \quad (10.15)$$

Spectral irradiance at the surface of a blackbody

$$F_a = I = B_{\lambda, T} \quad (10.16)$$

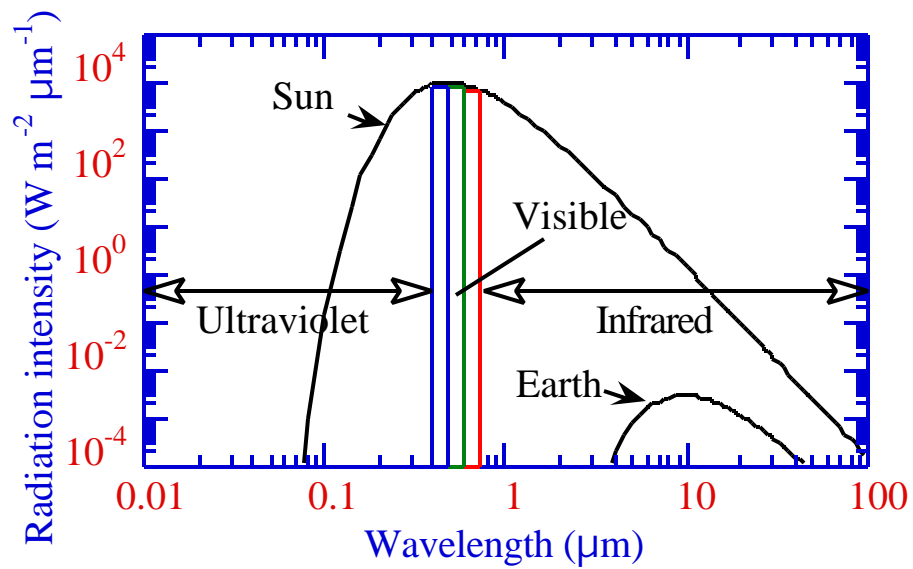
Irradiance

Fig. 10.4. Irradiance vs. wavelength and temperature resulting from Planckian emissions.



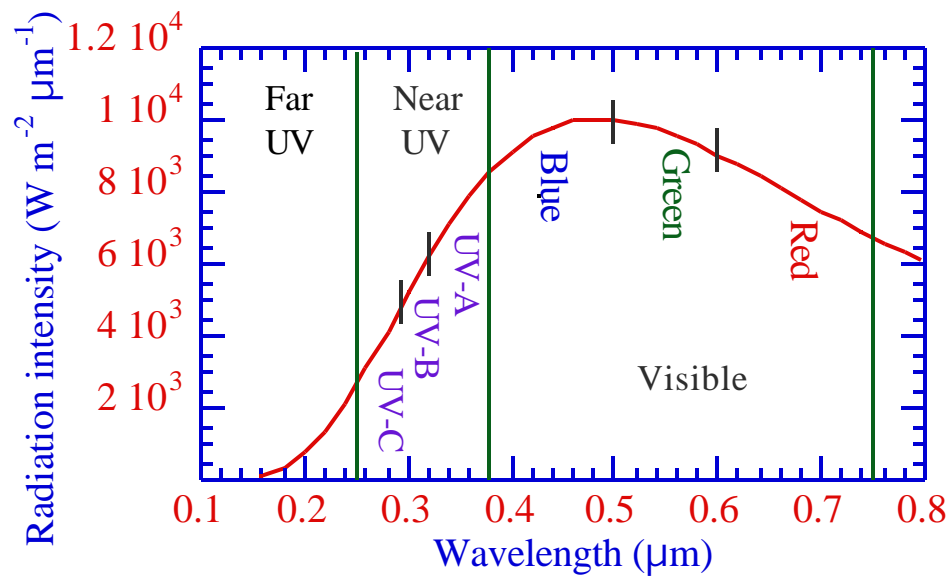
Emission Spectra of the Sun and Earth

Fig. 10.5. Irradiance emissions versus wavelength for the sun and earth when both are considered perfect emitters



Ultraviolet and Visible Portions of Solar Spectrum

Fig. 10.6. Ultraviolet and visible portions of the solar spectrum.



Wien's Displacement Law

Differentiate Planck's law with respect to wavelength at constant temperature and set result to zero

Peak wavelength of emissions from perfect emitter

$$p(\mu\text{m}) = \frac{2897}{T(\text{K})} \quad (10.17)$$

Example 10.2.

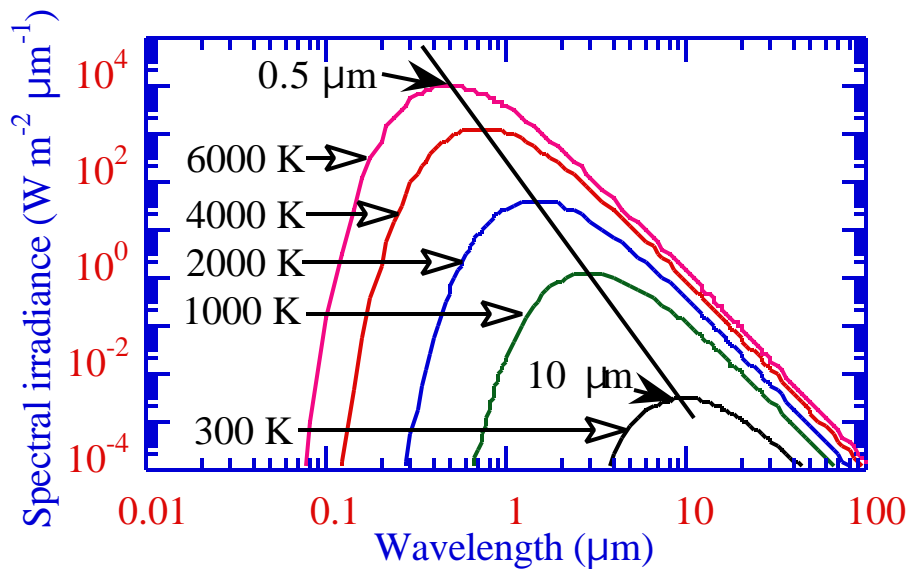
Sun's photosphere

$$p = 2897 / 5800 \text{ K} = 0.5 \mu\text{m}$$

Earth's surface

$$p = 2897 / 288 \text{ K} = 10.1 \mu\text{m}$$

Fig. 10.7. Line through peak irradiances at different temperatures.



Stefan-Boltzmann Law

Integrate Planck irradiance over all wavelength intervals

Stefan-Boltzmann law

$$F_b = \int_0^\infty B_{\lambda, T} d\lambda = \sigma T^4 \quad (10.18)$$

Stefan-Boltzmann constant

$$\sigma = \frac{2k_B^4}{15h^3c^2} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

Example 10.3.

$$\begin{aligned} \text{--->} \quad T &= 5800 \text{ K} \\ F_T &= 64 \text{ million W m}^{-2}. \end{aligned}$$

$$\begin{aligned} \text{--->} \quad T &= 288 \text{ K} \\ F_T &= 390 \text{ W m}^{-2}. \end{aligned}$$

Reflection and Refraction

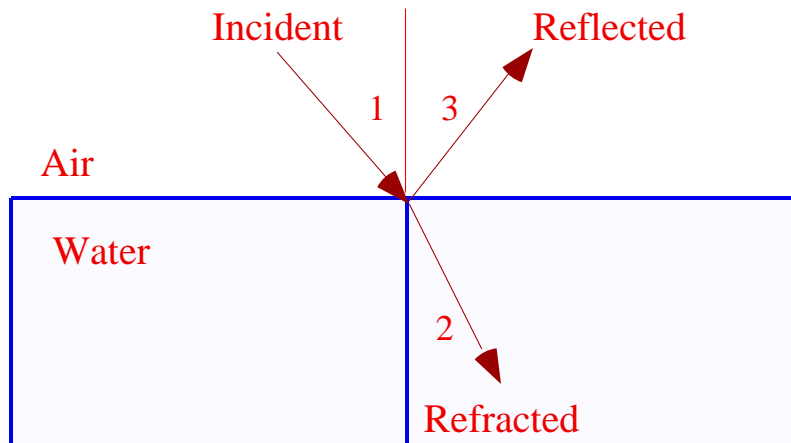
Reflection

Angle of reflection equals angle of incidence

Refraction

Angle of wave propagation relative to a surface normal changes when the density changes

Fig. 10.8. Examples of reflection and refraction.



Albedo

Albedo

Fraction of incident sunlight reflected

Table 10.2. Range of albedos in the non-UVB solar spectrum

Surface Type	Albedo (fraction)	Surface Type	Albedo (fraction)
Earth & atmosphere	0.3	Soil	0.05 - 0.2
Liquid water	0.05 - 0.2	Grass	0.16 - 0.26
Fresh snow	0.75 - 0.95	Desert	0.20 - 0.40
Old snow	0.4 - 0.7	Forest	0.10 - 0.25
Thick clouds	0.3 - 0.9	Asphalt	0.05 - 0.2
Thin clouds	0.2 - 0.7	Concrete	0.1 - 0.35
Sea Ice	0.25 - 0.4	Urban	0.1 - 0.27

Refraction

Snell's law

$$\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2} \quad (10.19)$$

Real part of the index of refraction (n_1)

Ratio of the speed of light in a vacuum to the speed of light in a given medium

$$n_1 = \frac{c}{c_1} \quad (10.20)$$

Real part of the index of refraction of air

$$n_{a, \text{ } -1} = 10^{-8} \left(8342.13 + \frac{2,406,030}{130 - \text{ }^{-2}} + \frac{15,997}{38.9 - \text{ }^{-2}} \right) \quad (10.21)$$

Example 10.4.

$$\begin{aligned} \lambda_1 &= 0.5 \text{ } \mu\text{m} \\ \theta_1 &= 45^\circ \\ \text{--->} \quad n_{\text{air}} &= 1.000279 \\ \text{--->} \quad n_{\text{water}} &= 1.335 \\ \text{--->} \quad \theta_2 &= 32^\circ \\ \text{--->} \quad c_{\text{air}} &= 2.9971 \times 10^8 \text{ m s}^{-1} \\ \text{--->} \quad c_{\text{water}} &= 2.2456 \times 10^8 \text{ m s}^{-1} \end{aligned}$$

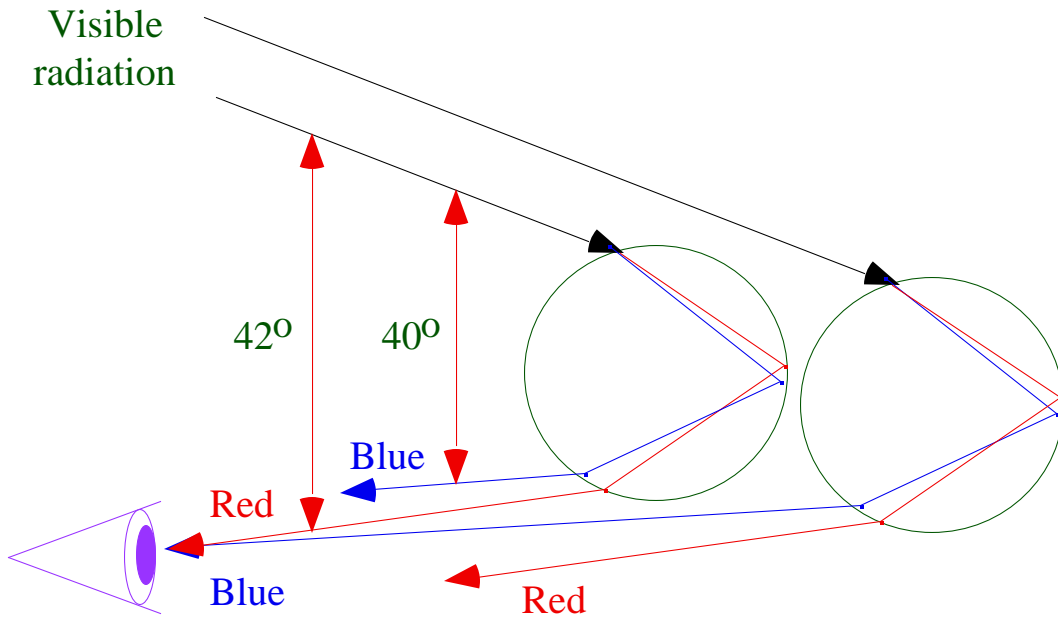
Real Indices of Refraction in Air and Water Versus Wavelength

Table 10.3.

Wavelength (μm)	Index of Refraction of Air	Index of Refraction of Water
0.2	1.000324	1.396
0.3	1.000292	1.349
0.4	1.000283	1.339
0.5	1.000279	1.335
0.6	1.000277	1.332
0.7	1.000276	1.331
1.0	1.000274	1.327
4.0	1.000273	1.351
7.0	1.000273	1.317
10.0	1.000273	1.218
20.0	1.000273	1.480

Geometry of a Primary Rainbow

Fig. 10.9.

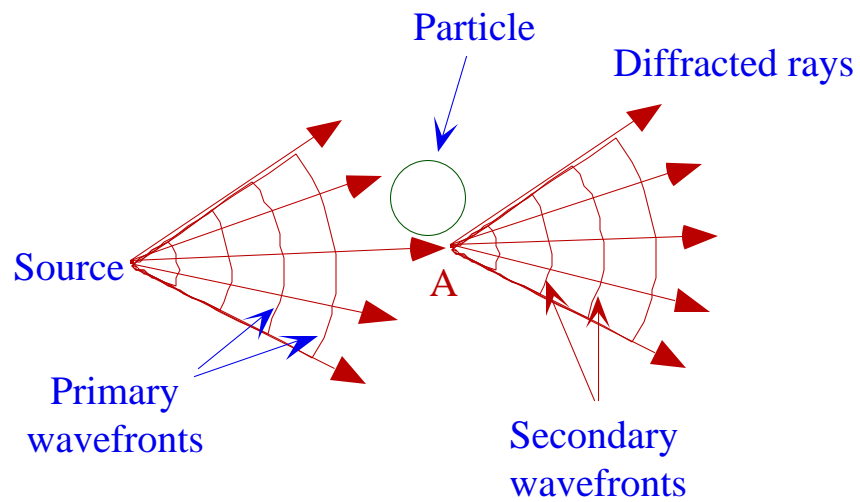


Diffraction Around A Particle

Huygens' principle

Each point of an advancing wavefront may be considered the source of a new series of secondary waves

Fig. 10.10.



Radiation Scattering by a Sphere

Fig. 10.11. Radiation scattering by a sphere.

Ray A is reflected

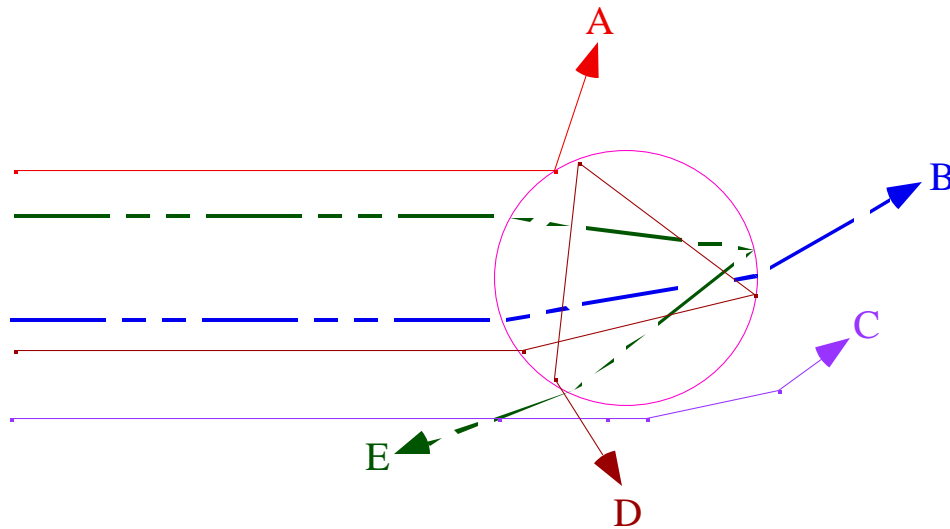
Ray B is refracted twice

Ray C is diffracted

Ray D is refracted, reflected twice, then refracted

Ray E is refracted, reflected once, and refracted

Rays A, B, C, and D scatter in the forward or sideward direction while ray E scatters in the backward direction.

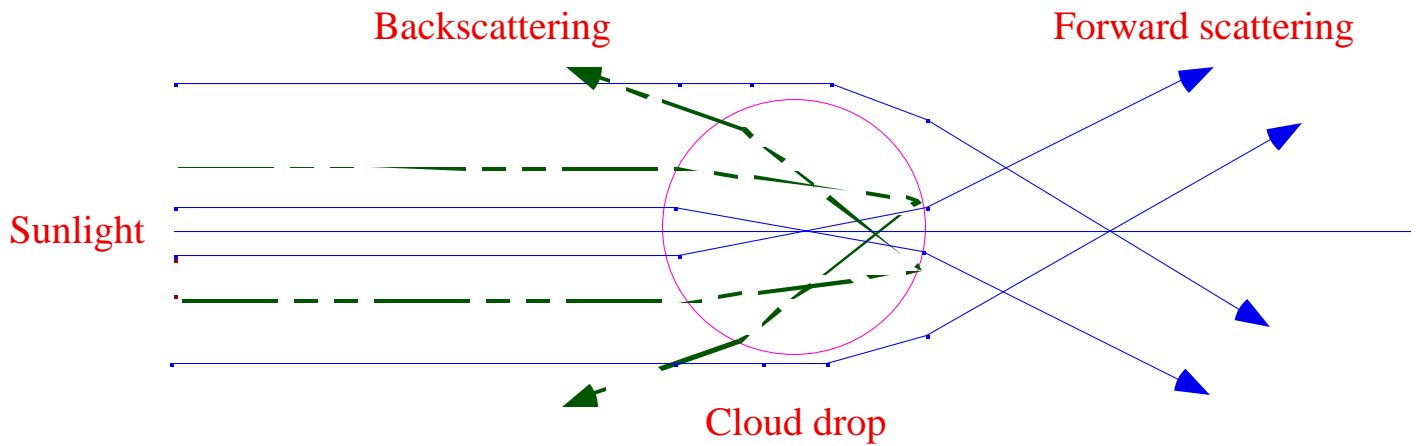


Diffraction

Process by which light bends around objects, and individual wavelengths constructively or destructively interfere with each other

Forward and Backscattering

Fig. 10.12.



Cloud droplets

Scatter primarily in the forward direction

Gas molecules

Scatter evenly in the forward and backward directions

Total Internal Reflection

Critical angle

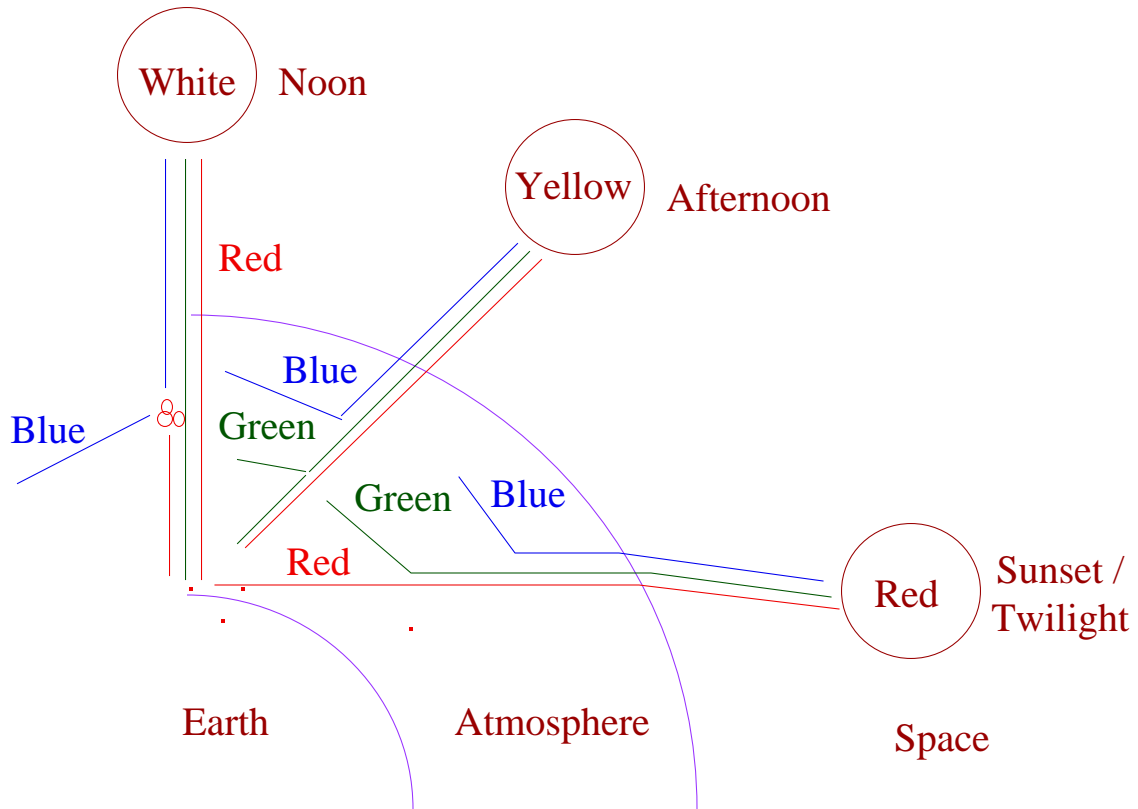
$$\theta_{c} = \sin^{-1} \frac{n_2}{n_1} \sin 90^\circ \quad (10.22)$$

Example 10.5.

$$\begin{aligned} &= 0.5 \mu\text{m} \\ \text{--->} \quad n_{\text{air}} &= 1.000279 \\ \text{--->} \quad n_{\text{water}} &= 1.335 \\ \text{--->} \quad \theta_{c} &= 48.53^\circ \end{aligned}$$

Change in Color of the Sun During the Day

Fig. 10.13.

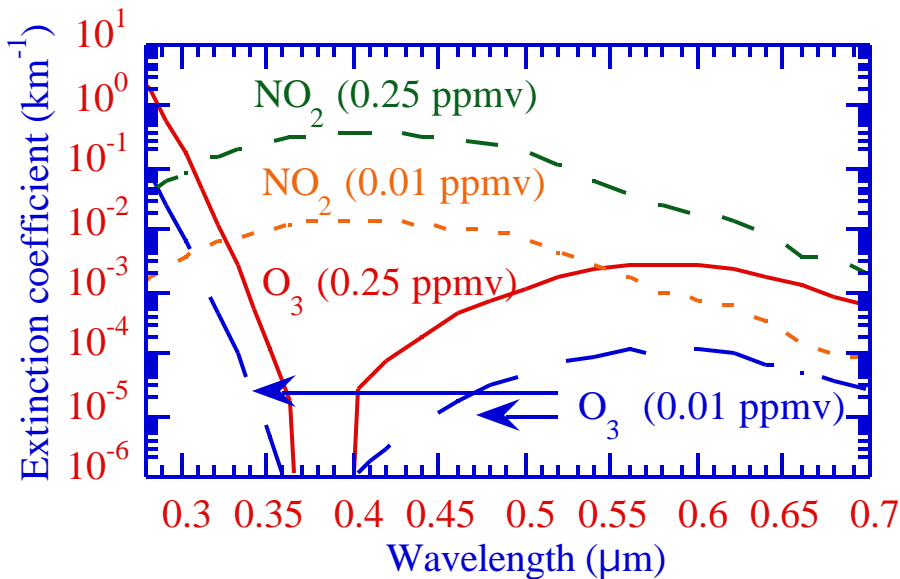


Gas Absorption

Table 10.4. Wavelengths of gas absorption in the solar spectrum.

Gas	Absorption Wavelengths (μm)	Gas	Absorption Wavelengths (μm)
N_2	< 0.1	N_2O_5	< 0.38
O_2	< 0.245	HNO_3	< 0.33
O_3	0.17 - 0.35, 0.45 - 0.75	HO_2NO_2	< 0.33
CO_2	< 0.21	HCHO	0.25 - 0.36
H_2O	< 0.21	CH_3CHO	< 0.345
H_2O_2	< 0.35	$\text{CH}_3\text{CO}_2\text{NO}_2$	< 0.3
NO_2	< 0.71	HCl	< 0.22
N_2O	< 0.24	CFCL_3	< 0.23
NO_3	0.41 - 0.67	CF_2CL_2	< 0.23
HONO	< 0.4	CH_3Cl	< 0.22

Fig. 10.15. Extinction coefficient due to NO_2 and O_3 absorption.



Infrared Absorption by Gases

Table 10.5. Peak absorption wavelength bands (μm) of several greenhouse gases. Data from (Liou, 1992)

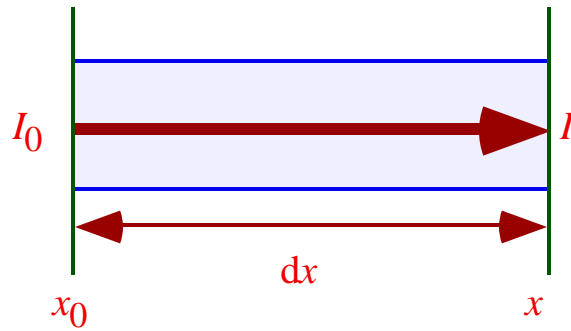
H ₂ O	CO ₂	O ₃	N ₂ O	CH ₄	CF ₂ Cl ₂	CFCl ₃	CH ₃ Cl
0.72	1.4	9.6	4.5	3.31	8.6	11.8	11.8
0.82	1.6	14.27	7.78	3.43	9.13	9.2	
0.94	2.0		17.0	6.55			
1.1	2.7			7.65			
1.38	4.3						
1.87	4.8						
2.7	5.2						
3.2	15.0						
6.25							
> 12.0							

Efficiency of absorption compared to CO₂ (IPCC, 1990)

- CH₄ = 21 x more efficient
- N₂O = 206 x more efficient
- CFCl₃ = 12,400 times more efficient

Extinction Coefficient

Fig. 10.14. Attenuation of incident radiance, I_0 , due to absorption as it travels through a column of gas.



Extinction coefficient () (cm^{-1} , m^{-1} , or km^{-1})

A measure of the loss of radiation per unit distance

Example

Reduction in radiance with distance through a gas

$$\frac{dI}{dx} = -N_q b_{a,g,q} I = -a_{g,q} I \quad (10.23)$$

Integrate

$$I = I_0 e^{-N_q b_{a,g,q} (x-x_0)} = I_0 e^{-a_{g,q} (x-x_0)} \quad (10.24)$$

Extinction Coefficient

Extinction coefficient due gas absorption

$$a_{g, T} = \sum_{q=1}^{N_{ag}} b_{a,g,q, T} = \sum_{q=1}^{N_{ag}} a_{g,q, T} \quad (10.25)$$

Meteorological range (Koschmieder equation)

$$x = \frac{3.912}{ext,}$$

Table 10.6. Extinction coefficients () and meteorological ranges (x) due to NO₂ absorption and meteorological range due to Rayleigh scattering (x_{sg}).

		NO ₂ (ppmv)				
		<--0.01-->		<--0.25-->		
	<i>b</i>	<i>a_g</i>	<i>x_{a,g}</i>	<i>a_g</i>	<i>x_{a,g}</i>	<i>x_{s,g}</i>
μm	10 ⁻¹⁹ cm ²	10 ⁸ cm ⁻¹	km	10 ⁻⁸ cm ⁻¹	km	km
0.42	5.39	13.2	296	330	11.8	112
0.45	4.65	11.4	343	285	13.7	148
0.50	2.48	6.10	641	153	25.6	227
0.55	0.999	2.46	1590	61.5	63.6	334
0.60	0.292	0.72	5430	18.0	217	481
0.65	0.121	0.30	13,000	7.5	520	664

Gas Scattering

Rayleigh scatterer: $2r/\lambda \ll 1$

Extinction coefficient due to Rayleigh scattering

$$\sigma_{s,g} = N_a b_{s,g} \quad (10.27)$$

Scattering cross section of a typical air molecule (cm^2)

$$b_{s,g} = \frac{8}{3} \frac{\pi^3 (n_{a,0}^2 - 1)^2}{4N_{a,0}^2} f(\lambda) = \frac{32}{3} \frac{\pi^3 (n_{a,0}^2 - 1)^2}{4N_{a,0}^2} f(\lambda) \quad (10.28)$$

Anisotropic correction factor

$$f(\lambda) = \frac{6 + 3\lambda}{6 - 7\lambda} \approx 1.05 \quad (10.29)$$

Rayleigh Scattering Example

Example 10.6.

$$= 0.5 \mu\text{m}$$

$$p = 1 \text{ atm (sea level)}$$

$$T = 288 \text{ K}$$

$$\text{---> } s, g, = 1.72 \times 10^{-7} \text{ cm}^{-1}$$

$$\text{---> } x = 227 \text{ km}$$

$$= 55 \mu\text{m}$$

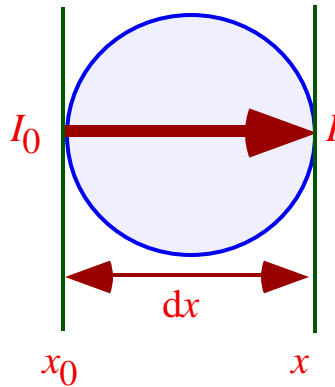
$$\text{---> } s, g, = 1.17 \times 10^{-7} \text{ cm}^{-1}$$

$$\text{---> } x = 334 \text{ km}$$

Imaginary Index of Refraction

Determines extent to which a substance absorbs radiation

Fig. 10.16. Attenuation of incident radiance, I_0 , due to absorption as it travels through a particle.



Attenuation of radiation due to particle absorption

$$\frac{dI}{dx} = -\frac{4}{\lambda} I \quad (10.30)$$

Integrate

$$I = I_0 e^{-4 \frac{(x-x_0)}{\lambda}} \quad (10.31)$$

Complex Index of Refraction

$$m = n - i k \quad (10.32)$$

Table 10.7. Real and imaginary indices of refraction for some substances at $\lambda = 0.50$ and $10.0 \mu\text{m}$.

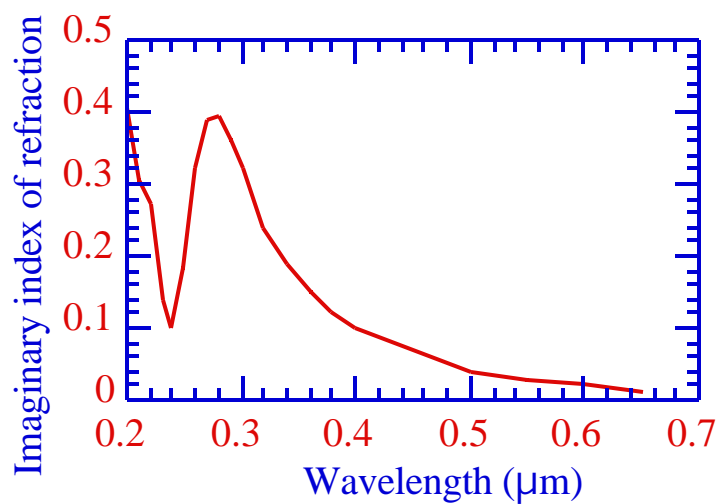
	0.5 μm		10 μm	
	Real (n)	Imaginary (k)	Real (n)	Imaginary (k)
H ₂ O (aq)	1.34	1.0×10^{-9}	1.22	5.0×10^{-2}
Elemental C (s)	1.82	7.4×10^{-1}	2.40	1.0×10^0
Organic C (s)	1.45	1.0×10^{-3}	1.77	1.2×10^{-1}
H ₂ SO ₄ (aq)	1.43	1.0×10^{-8}	1.89	4.6×10^{-1}
(NH ₄) ₂ SO ₄ (s)	1.52	5.0×10^{-4}	2.15	2.0×10^{-2}
NaCl(s)	1.45	1.5×10^{-4}	1.53	5.3×10^{-2}

Table 10.8. Light transmission through particles at $\lambda = 0.50 \mu\text{m}$.

Substance and α	Particle Diameter (μm)	Transmission (I/I_0)
Elemental carbon ($\alpha = 0.74$)	0.1	0.16
	1.0	8.0×10^{-9}
	10.0	0
Water ($\alpha = 10^{-9}$)	0.1	0.999999997
	1.0	0.999999997
	10.0	0.99999997

Imaginary Index of Refraction of Liquid Nitrobenzene

Fig. 10.17



Particle Extinction Coefficients

Particle absorption and scattering extinction coefficients

$$a_{a,i} = \sum_{i=1}^{N_B} n_i b_{a,a,i}, \quad (10.33)$$

$$s_{s,i} = \sum_{i=1}^{N_B} n_i b_{s,a,i}, \quad (10.33)$$

Absorption and scattering cross sections of a particle (cm²)

$$b_{a,a,i} = r_i^2 Q_{a,i}, \quad (10.34)$$

$$b_{s,a,i} = r_i^2 Q_{s,i}, \quad (10.34)$$

Size parameter

$$x_i = 2 r_i / \lambda \quad (10.35)$$

Tyndall Absorption / Scattering

Rayleigh regime ($r_i \ll 0.1$ or $k_i r_i < 0.1$)

Single particle Tyndall absorption efficiency r_i / λ

$$Q_{a,i} = -4 \frac{2 r_i}{\lambda} \operatorname{Im} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 = \frac{2 r_i}{\lambda} \frac{24n}{(n^2 + 2)^2 + 4(n^2 - 2 + 1)} \quad (10.36)$$

$$0 \rightarrow Q_{a,i} = \frac{2 r_i}{\lambda} \frac{24n}{(n^2 + 2)^2} \rightarrow \text{linear with } \quad (10.37)$$

Single particle Tyndall scattering efficiency $(r_i / \lambda)^4$

$$Q_{s,i} = \frac{8}{3} \frac{2 r_i}{\lambda} \left| \frac{m^2 - 1}{m^2 + 2} \right|^4 \quad (10.38)$$

Example 10.7. (liquid water)

	r_i	$= 0.5 \mu\text{m}$		r_i	$= 0.01 \mu\text{m}$
---	n	$= 1.34$	---	r_i	$= 1.0 \times 10^{-9}$
---	$Q_{s,i}$	$= 2.92 \times 10^{-5}$			
---	$Q_{a,i}$	$= 2.8 \times 10^{-10}$			

Mie Absorption / Scattering

Mie regime ($r_i \sim$ or $0.1 < i, < 100$)

Single particle Mie scattering efficiency

$$Q_{s,i} = \frac{2}{i} \sum_{k=1} (2k+1) (|a_k|^2 + |b_k|^2) \quad (10.39)$$

Single particle Mie absorption efficiency

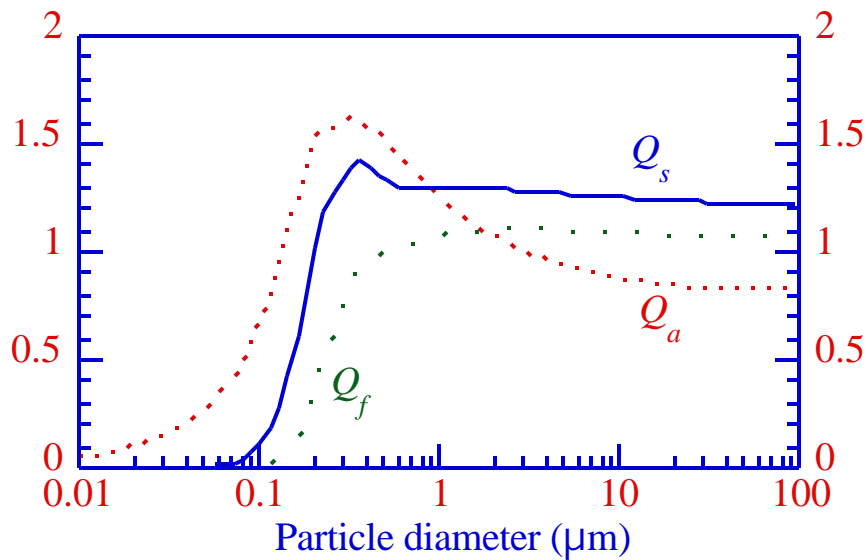
$$Q_{a,i} = Q_{e,i} - Q_{s,i}$$

Single particle total extinction coefficient

$$Q_{e,i} = \frac{2}{i} \sum_{k=1} (2k+1) \text{Re}(a_k + b_k) \quad (10.40)$$

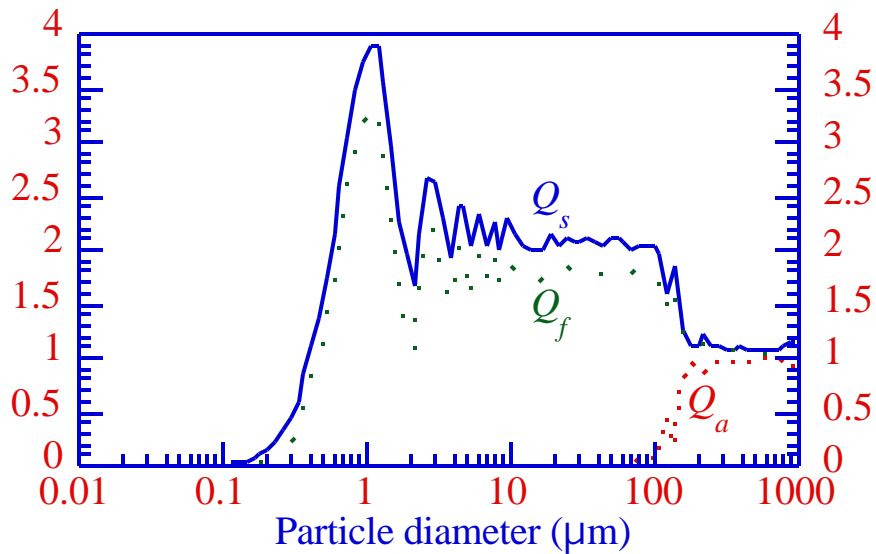
Single Particle Absorption / Scattering Efficiency by Soot

Fig. 10.18. Q_a , Q_s , and Q_f of elemental carbon (soot) particles at $\lambda = 0.50 \mu\text{m}$ ($n = 1.94$, $k = 0.66$). The efficiency for soot at any other visible wavelength λ_1 and diameter d_1 is the efficiency at diameter, $d = 0.5\mu\text{m} \times d_1 / \lambda_1$.



Single Particle Absorption / Scattering Efficiency by Water

Fig. 10.19. Q_a , Q_s , and Q_f of liquid water drops of different sizes at a wavelength of $\lambda = 0.50 \mu\text{m}$ ($n = 1.34$, $k = 0$). The efficiency for water at any other visible wavelength λ_1 and diameter d_1 is the efficiency at diameter, $d = 0.5\mu\text{m} \times d_1 / \lambda_1$.



Geometrical Scattering / Absorption

Geometrical regime ($k_i r \gg 1$) ---> significant diffraction

In the limit ($k_i r \gg 1$), scattering efficiency is constant

$$\lim_{k_i r \gg 1} Q_{s,i} = 1 + \left| \frac{m - 1}{m + 2} \right| \quad (10.41)$$

Example 10.8.

$$r = 0.5 \mu\text{m}$$

---> $n = 1.34$ for liquid water

---> $Q_{s,i} = 1.1$ as $k_i r \gg 1$, from (10.41)

---> $Q_{a,i} = 1.1$ as $k_i r \gg 1$, from fig (10.18)

Also, as $k_i r \gg 1$, $Q_{s,i}$, $Q_{a,i}$, regardless of how weak the imaginary index of refraction is.

Volume-averaged effective index of refraction

$$n = \sum_{q=1}^{N_V} \frac{v_{q,i}}{v_i} n_{,q} \quad (10.42)$$

Predicted Extinction Coeff. Profiles

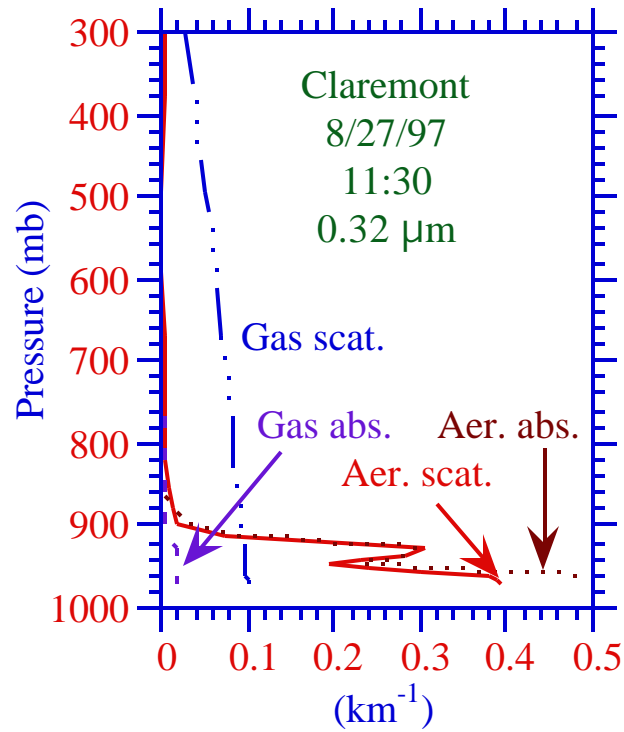


Fig. 10.20 a.

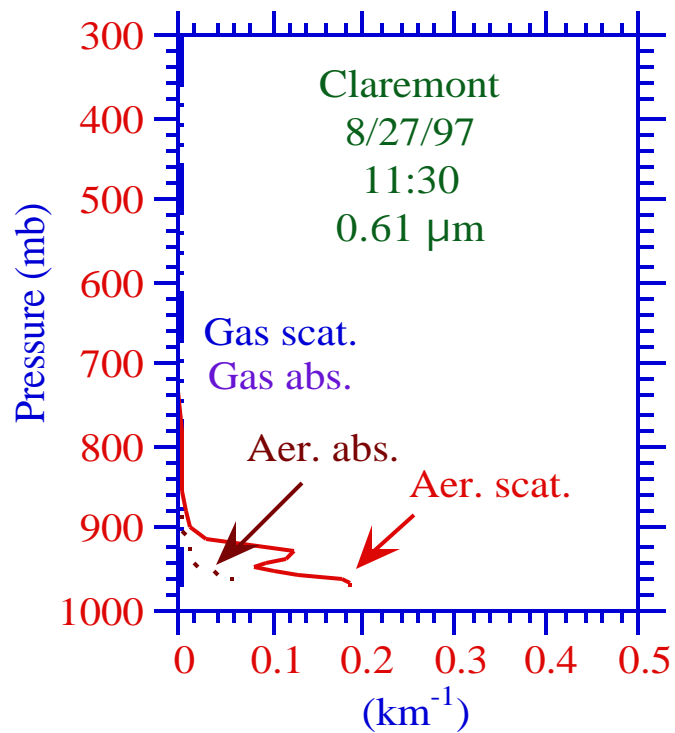


Fig. 10.20 b.

Visibility Definitions

Meteorological range

Distance from an ideal dark object at which the object has a 0.02 liminal contrast ratio against a white background

Liminal contrast ratio

Lowest visually perceptible brightness contrast a person can see

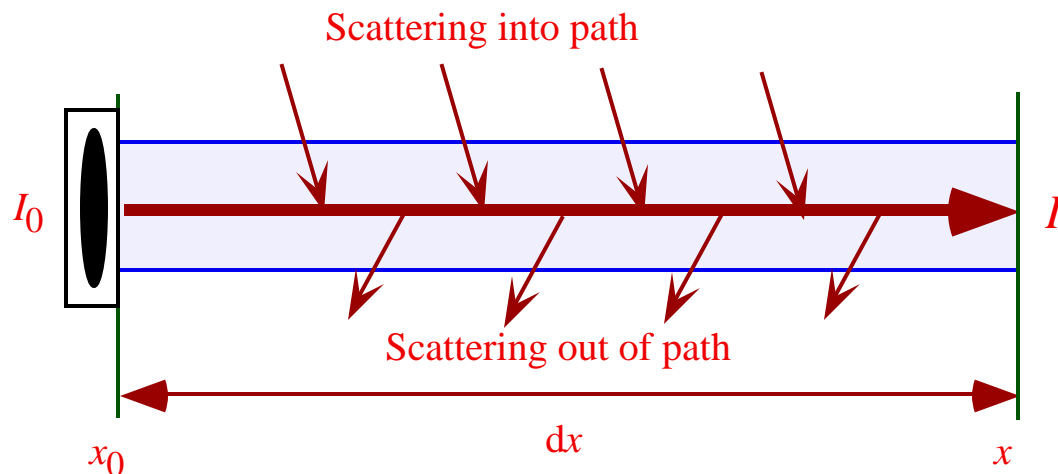
Visual range

Actual distance at which a person can discern an ideal dark object against the horizon sky

Prevailing visibility

Greatest visual range a person can see around a 180° horizontal arc, but not necessarily in continuous sectors around the arc

Fig. 10.21. At point x , the intensity of the radiation originating from the direction of an object has increased to the background intensity.



Meteorological Range

Change in object intensity along path of radiation

$$\frac{dI}{dx} = bI_B - extI \quad (10.43)$$

I_B = constant background intensity (radiance) of light,

bI_B = constant scattering of background light back into the line of vision along the path

ext = extinction coefficient due to absorption and scattering along the path

Change in background intensity

$$\frac{dI_B}{dx} = bI_B - extI_B = 0 \quad (10.44)$$

Substitute $b = ext$ into (10.43)

$$\frac{dI}{I_B - I} = ext dx. \quad (10.45)$$

Integrate from $I_0 = 0$ to I and $x_0 = 0$ to x

$$C_{ratio} = \frac{I_B - I}{I_B} = e^{-extx} \quad (10.46)$$

Meteorological range (Koschmieder equation)

$x = \frac{3.912}{ext} \quad (10.47)$

Meteorological Range

Table 10.9. Meteorological ranges due to scattering by gases (sg), absorption by gases (ag), scattering by particles (sp), absorption by particles (ap), and total extinction (ext) at $\lambda = 0.55 \mu\text{m}$ in Los Angeles. (Larson, 1984).

	Meteorological Range (km)				
	x_{sg} ,	x_{ag} ,	x_{sp} ,	x_{ap} ,	x_{ext} ,
Clean day (4 / 7 / 83)	352	326	151	421	67.1
Polluted day (8 / 25 / 83)	366	130	9.59	49.7	7.42

Optical Depth

Total extinction coefficient

$$= \mu_{s,g} + \mu_{a,g} + \mu_{s,a} + \mu_{a,a} + \mu_{s,c} + \mu_{a,c}, \quad (10.48)$$

Incremental distance vs. incremental path length

$$dz = \cos s \, dS_b = \mu_s \, dS_b \quad (10.49)$$

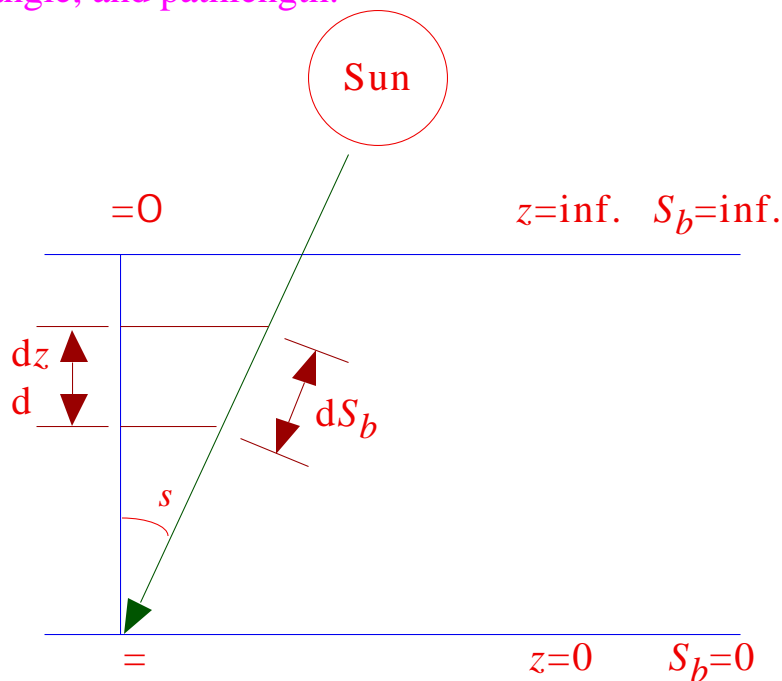
Incremental optical depth

$$d\tau = - \mu_s \, dz = - \mu_s \, dS_b \quad (10.51)$$

Optical depth as a function of altitude

$$\tau = \int_z^{\infty} \mu_s \, dS_b \quad (10.52)$$

Fig. 10.22. Relationship between optical depth, altitude, solar zenith angle, and pathlength.



Solar Zenith Angle

Cosine of solar zenith angle

$$\cos \theta_s = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H_a \quad (10.53)$$

Solar declination angle (δ)

Angle between the equator and the north or south latitude of the subsolar point

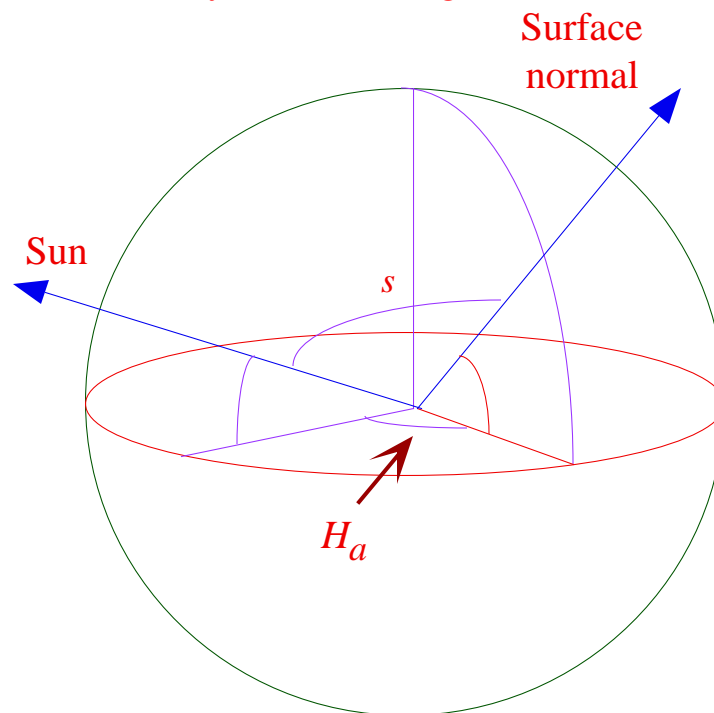
Subsolar point

Point at which the sun is directly overhead

Local hour angle (H_a)

Angle, measured westward, between longitude of subsolar point and longitude of location of interest.

Figs. 10.23 b. Geometry for zenith angle calculations.



Solar Declination Angle

Solar declination angle

$$= \sin^{-1}(\sin ob \sin ec) \quad (10.54)$$

Obliquity of the ecliptic

Angle between the plane of the earth's equator and the plane of the ecliptic, which is the mean plane of the earth's orbit around the sun.

$$ob = 23^{\circ}.439 - 0^{\circ}.0000004N_{JD} \quad (10.55)$$

Ecliptic longitude of earth

$$ec = L_M + 1^{\circ}.915 \sin g_M + 0^{\circ}.020 \sin 2g_M \quad (10.57)$$

Mean longitude of the sun

$$L_M = 280^{\circ}.460 + 0^{\circ}.9856474N_{JD} \quad (10.58)$$

Mean anomaly of the sun

$$g_M = 357^{\circ}.528 + 0^{\circ}.9856003N_{JD} \quad (10.58)$$

Solar Zenith Angle

Local hour angle

$$H_a = \frac{2 t_s}{86,400} \quad (10.59)$$

Example.

At noon, when sun is directly overhead, $H_a = 0$ --->

$$\cos \theta_s = \sin \phi \sin \delta + \cos \phi \cos \delta \cos H_a$$

When the sun is over the equator, the $\delta = 0$ --->

$$\cos \theta_s = \cos \phi \cos H_a$$

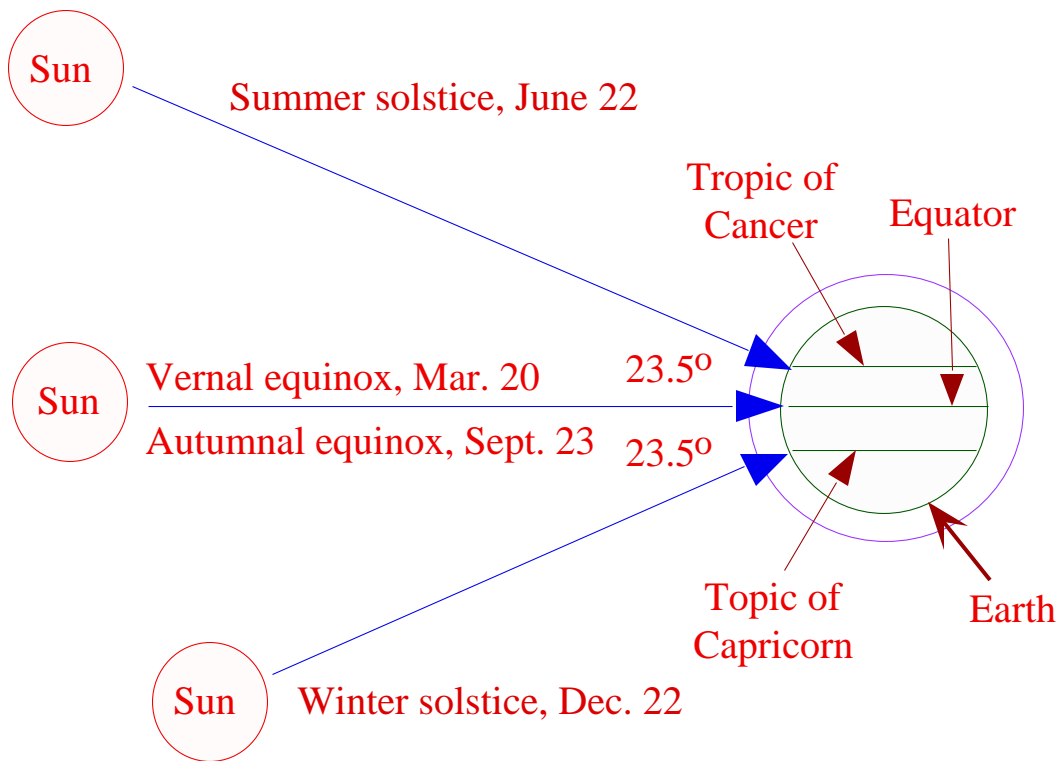
Example 10.9.

1:00 p.m., PST, Feb. 27, 1994, $\phi = 35^\circ \text{N}$.

$$\begin{aligned} \text{--->} \quad D_J &= 58 \\ \text{--->} \quad N_{JD} &= -2134.5 \\ \text{--->} \quad g_m &= -1746.23^\circ \\ \text{--->} \quad L_m &= -1823.40^\circ \\ \text{--->} \quad ce &= -1821.87^\circ \\ \text{--->} \quad ob &= 23.4399^\circ \\ \text{--->} &= -8.52^\circ \\ \text{--->} \quad H_a &= 15.0^\circ \\ \text{--->} \quad \cos \theta_s &= \sin(35^\circ) \sin(-8.52^\circ) + \cos(35^\circ) \cos(-8.52^\circ) \cos(15.0^\circ) \\ \text{--->} \quad \theta_s &= 45.8^\circ \end{aligned}$$

Solstices and Equinoxes

Fig. 10.24. Solar declinations during solstices and equinoxes. The earth-sun distance is greatest at the summer solstice.



Radiative Transfer Equation

Change in radiance / irradiance along a beam of interest

Change in radiance along incremental path length

$dI = -dI_{so} - dI_{ao} + dI_{si} + dI_{Si} + dI_{ei} \quad (10.60)$

Scattering of radiation out of the beam

$$dI_{so} = I_s dS_b \quad (10.61)$$

Absorption of radiation along the beam

$$dI_{ao} = I_a dS_b \quad (10.62)$$

Multiple scattering of diffuse radiation into the beam

$$dI_{si} = \sum_k \frac{\omega_{s,k}}{4} \int_0^{\pi} \int_{-1}^1 I_{s,\mu} P_{s,k}(\mu, -\mu) \mu \, d\mu \, dS_b \quad (10.63)$$

Single scattering of direct solar radiation into the beam

$$dI_{Si} = \sum_k \frac{\omega_{s,k}}{4} P_{s,k}(\mu, -\mu_s) \mu_s F_{s,\mu_s} e^{-\mu_s} dS_b \quad (10.64)$$

Emissions of infrared Planckian radiation into the beam

$$dI_{ei} = \epsilon_a B_{\lambda,T} dS_b \quad (10.65)$$

Extinction Coefficients

Extinction due to total scattering only

$$s_t = s_{g,t} + s_{a,t} + s_{c,t}, \quad (10.66)$$

Extinction due to total absorption only

$$a_t = a_{g,t} + a_{a,t} + a_{c,t}, \quad (10.66)$$

Total extinction due to scattering plus absorption

$$= s_t + a_t, \quad (10.67)$$

Scattering Phase Function

Gives angular distribution of scattered energy vs. direction

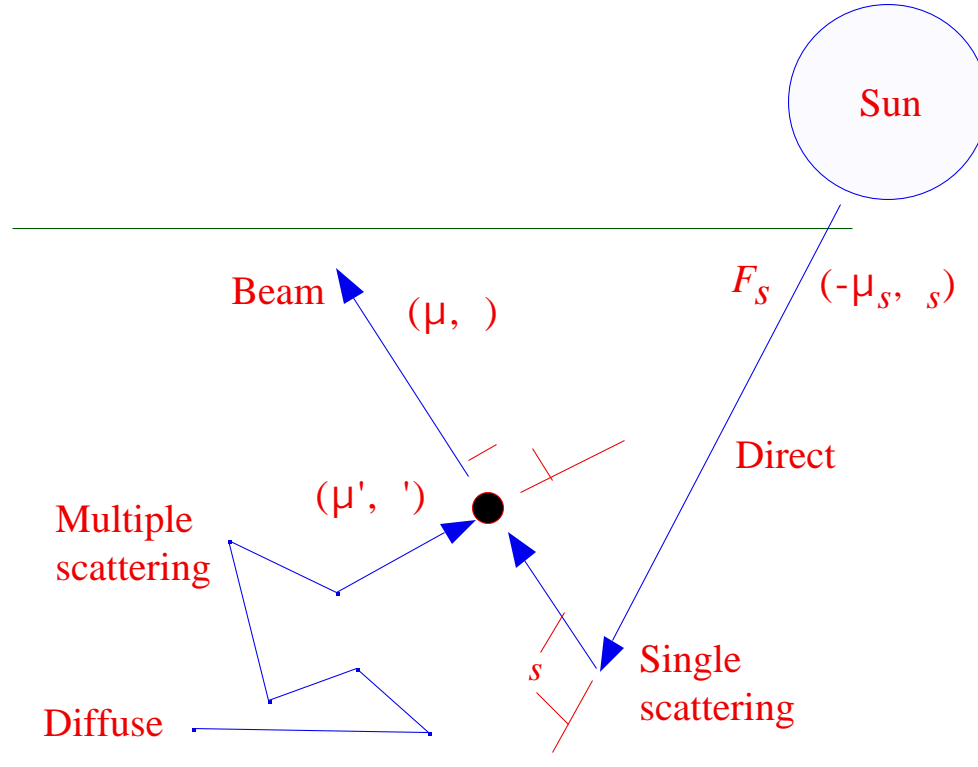
Scattering phase function for diffuse radiation

$P_{s,k}(\mu_s, \mu, \mu_s, \mu)$ redirects diffuse radiation from μ_s to μ ,

Scattering phase function for direct radiation

$P_{s,k}(\mu_s, -\mu_s, \mu, \mu_s)$ redirects direct solar radiation from $-\mu_s$ to μ ,

Fig. 10.25. Single scattering of direct solar radiation and multiple scattering of diffuse radiation.



Scattering Phase Function

Scattering phase function defined such that

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi P_{s,k}(\theta) \sin \theta d\theta d\phi = 1 \quad (10.68)$$

θ = angle between directions μ_0 and μ_s ,

Substitute $\theta = \arccos(\mu_0 \mu_s) \rightarrow$

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi P_{s,k}(\theta) \sin \theta d\theta d\phi = 1 \quad (10.69)$$

Phase function for isotropic scattering

$$P_{s,k}(\theta) = 1 \quad (10.70)$$

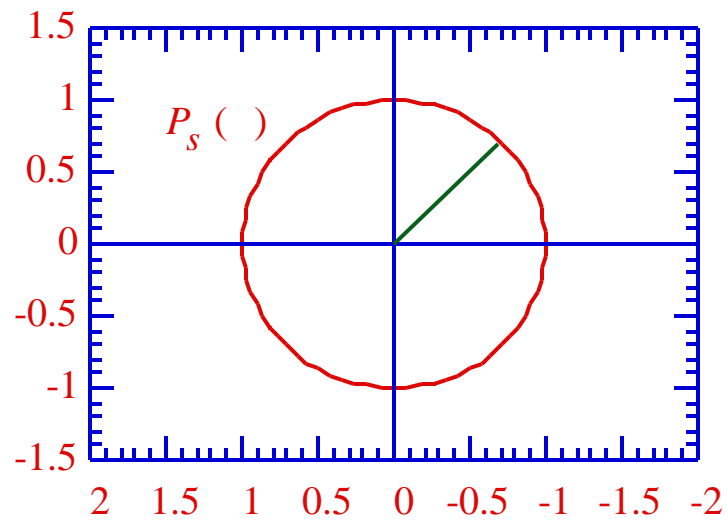
Phase function for Rayleigh scattering

$$P_{s,k}(\theta) = \frac{3}{4} (1 + \cos^2 \theta) \quad (10.71)$$

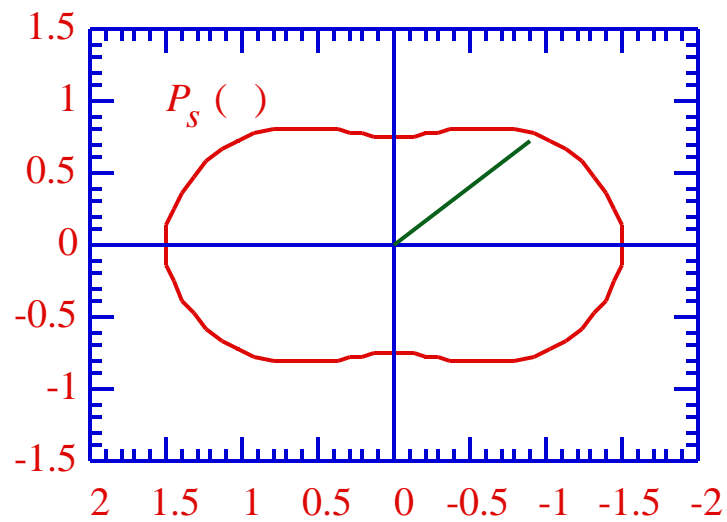
Scattering Phase Functions

Figs. 10.26 a and b.

Scattering phase functions for (a) isotropic and (b) Rayleigh scattering.



(a)



(b)

Asymmetry Factor

First moment of phase function -- gives relative direction of scattering by particles or gases

In general,

$$\begin{aligned}
 > 0 & \quad \text{forward (Mie) scattering} \\
 g_{a,k} & = 0 \quad \text{isotropic or Rayleigh scattering} \\
 < 0 & \quad \text{backward scattering}
 \end{aligned}
 \tag{10.73}$$

$$g_{a,k} = \frac{1}{4} \int_0^\pi P_{s,k}(\theta) \cos \theta \sin^2 \theta \, d\theta
 \tag{10.72}$$

Expand with $\theta = \sin^{-1}(\mu)$ -->

$$g_{a,k} = \frac{1}{4} \int_0^1 P_{s,k}(\mu) \cos \mu \sin^2 \mu \, d\mu
 \tag{10.74}$$

Isotropic Scattering ---> $P_{s,k}(\mu) = 1$ --->

$$g_{a,k} = \frac{1}{4} \int_0^1 \cos \mu \sin^2 \mu \, d\mu = -\frac{1}{2} \int_1^{-1} \mu^{-1} \, d\mu = 0
 \tag{10.75}$$

Rayleigh scattering --->

$$\begin{aligned}
 g_{a,k} & = \frac{1}{4} \int_0^1 \frac{3}{4} (1 + \cos^2 \mu) \cos \mu \sin^2 \mu \, d\mu \\
 & = -\frac{3}{8} \int_1^{-1} (\mu + \mu^3) \, d\mu = 0
 \end{aligned}
 \tag{10.76}$$

Mie scattering --->

$$g_{a,k} = Q_{f,i} / Q_{s,i}
 \tag{10.78}$$

Incident Solar Flux

Cumulative irradiance (W m^{-2}) emitted by sun's photosphere

$$F_p = \frac{L_p}{4 R_p^2} = \sigma T_p^4 \quad (10.79)$$

L_p = emissions from sun's photosphere = 3.9×10^{26} W

R_p = radius from sun center to photosphere = 6.96×10^8 m

T_p = temperature of photosphere = $T_p = 5796$ K

Example 10.10.

$$\begin{aligned} & T_p = 5796 \text{ K} \\ \text{--->} & F_p = 6.4 \times 10^7 \text{ W m}^{-2} \end{aligned}$$

Solar Constant

Mean cumulative irradiance at top of earth's atmosphere

$$\bar{F}_s = \frac{R_p}{R_{es}}^2 F_p = \frac{R_p}{R_{es}}^2 BT_p^4 \quad (10.80)$$

Calculated	\bar{F}_s	1379 W m ⁻²
Observed	\bar{F}_s	1365 W m ⁻²

Varies by +/- 1 W m⁻² over each 11 year sunspot cycle

Daily cumulative irradiance depends on earth-sun distance

$$F_s = \frac{R_{es}}{R_{es}}^2 \bar{F}_s \quad (10.81)$$

Empirical formula

$$\frac{R_{es}}{R_{es}}^2 = 1.00011 + 0.034221 \cos J + 0.00128 \sin J + 0.000719 \cos 2 J + 0.000077 \sin 2 J \quad (10.82)$$

$$J = 2 \pi D_J / D_Y \quad (10.83)$$

Incident Solar Flux

Example 10.11.

December 22

$$\text{---> } F_s = 1365 \times 1.034 = 1411 \text{ W m}^{-2}$$

June 22

$$\text{---> } F_s = 1365 \times 0.967 = 1321 \text{ W m}^{-2}$$

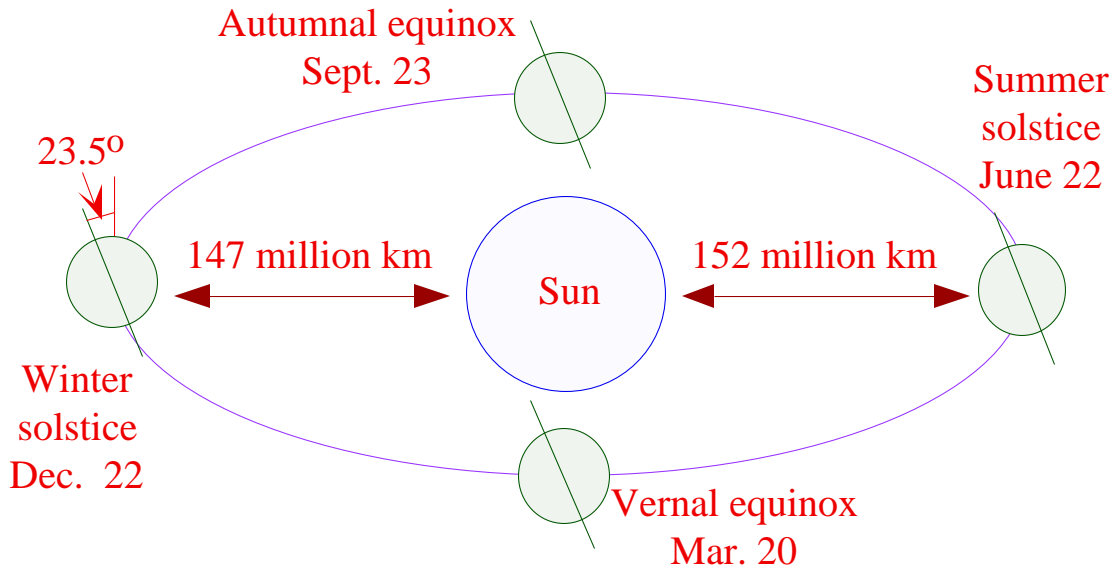
Irradiance varies by 90 W m^{-2} (6.6%) between Dec. and June.

Cumulative solar irradiance as sum of spectral irradiances

$$F_s = \left(F_{s, \lambda} \right) = \frac{R_{es}}{\bar{R}_{es}} \quad \left(\bar{F}_s \right) = \frac{R_{es}}{\bar{R}_{es}} \bar{F}_s \quad (10.83)$$

Seasons

Fig. 10.27. Relationship between the sun and earth during the solstices and equinoxes.



Equilibrium Earth Temperature

Energy (W) absorbed by the earth-atmosphere system

$$E_{\text{in}} = \bar{F}_s (1 - A_{e,0}) \left(R_e^2 \right) \quad (10.84)$$

Energy emitted by the earth's surface

$$E_{\text{out}} = \epsilon_{e,0} \sigma T_e^4 \left(4 R_e^2 \right) \quad (10.85)$$

Equate --> temperature without greenhouse effect

$T_e = \frac{\bar{F}_s (1 - A_{e,0})}{4 \epsilon_{e,0} \sigma}^{1/4} \quad (10.86)$

Example 10.12.

$$\begin{aligned} \bar{F}_s &= 1365 \text{ W m}^{-2} \\ A_{e,0} &= 0.3 \\ \text{--->} \quad T_e &= 254.8 \text{ K} \end{aligned}$$

Actual average surface temperature on earth 288 K -->
difference due to absorption by greenhouse gases

Radiative Transfer Equation

$$\frac{dI_{s,\mu}}{dS_b} = -I_{s,\mu} (\mu_s + a_s) + \int_0^2 \frac{s_{s,k}}{4} P_{s,k}(\mu, \mu_s) I_{s,\mu_s} d\mu_s + F_{s,e} e^{-\tau_s} \int_0^2 \frac{s_{s,k}}{4} P_{s,k}(\mu, -\mu_s) I_{s,\mu_s} d\mu_s + a_s B_{s,T} \quad (10.87)$$

Single scattering albedo (10.88)

$$\omega_s = \frac{s_s}{s_s + a_s} = \frac{s_{s,g} + s_{s,a} + s_{s,c}}{s_{s,g} + a_{s,g} + s_{s,a} + a_{s,a} + s_{s,c} + a_{s,c}}$$

Total extinction coefficient

$$\mu_{t,s} = \mu_{s,s} + a_s$$

Optical depth

$$d\tau_s = -\mu_{s,s} dS_b$$

Radiative Transfer Equation

Rewrite radiative transfer equation

$$\mu \frac{dI_{\mu}}{ds} = I_{\mu} - J_{\mu}^{diffuse} - J_{\mu}^{direct} - J_{\mu}^{emis} \quad (10.89)$$

where

$$J_{\mu}^{diffuse} = \frac{1}{4} \int_{-1}^1 \int_0^{2\pi} P_{s,k}(\mu, \mu') I_{\mu'} d\mu' d\phi \quad (10.90)$$

$$J_{\mu}^{direct} = \frac{1}{4} F_s e^{-|\mu_s|} \int_{-1}^1 P_{s,k}(\mu, -\mu_s) d\mu_s \quad (10.91)$$

$$J_{\mu}^{emis} = (1 - \epsilon_s) B_{\mu, T} \quad (10.92)$$

Beer's Law

Consider only absorption in downward direction

$$-\mu \frac{dI_{\lambda, -\mu}}{d a_{\lambda}} = I_{\lambda, -\mu} \quad (10.93)$$

Solution

$$I_{\lambda, -\mu} (a_{\lambda}) = I_{\lambda, -\mu} (a_{\lambda, t}) e^{-(a_{\lambda} - a_{\lambda, t})/\mu} \quad (10.95)$$

Schwartzchild's Equation

Consider absorption and infrared emissions

$$-\mu \frac{dI_{\lambda, -\mu}}{d a_{\lambda}} = I_{\lambda, -\mu} - B_{\lambda, T} \quad (10.96)$$

Solution

$$I_{\lambda, -\mu} (a_{\lambda}) = I_{\lambda, -\mu} (a_{\lambda, t}) e^{-(a_{\lambda} - a_{\lambda, t})/\mu} + \frac{1}{\mu} \int_{a_{\lambda, t}}^{a_{\lambda}} B_{\lambda, T} e^{-(a_{\lambda} - a_{\lambda, t})/\mu} d a_{\lambda} \quad (10.98)$$

Two-Stream Method

Divide phase function into upward (+) and downward component

$$\frac{1}{4} \int_0^2 \int_{-1}^1 I_{s,\mu} P_{s,k}(\mu,\mu) d\mu d$$

$$\frac{(1+g_{a,k})}{2} I_+ + \frac{(1-g_{a,k})}{2} I_- \quad \text{upward}$$

$$\frac{(1+g_{a,k})}{2} I_- + \frac{(1-g_{a,k})}{2} I_+ \quad \text{downward}$$

(10.99)

Substitute (10.99) into (10.90)

$$\frac{1}{4} \int_0^2 \int_{-1}^1 I_{s,\mu} P_{s,k}(\mu,\mu) d\mu d$$

$$s_s (1-b) I_+ + s_s b I_-$$

$$s_s (1-b) I_- + s_s b I_+$$

(10.100)

Integrated fraction of forward scattered energy

$$1 - b = \frac{1 + g_a}{2} \quad (10.101)$$

Integrated fraction of backscattered energy

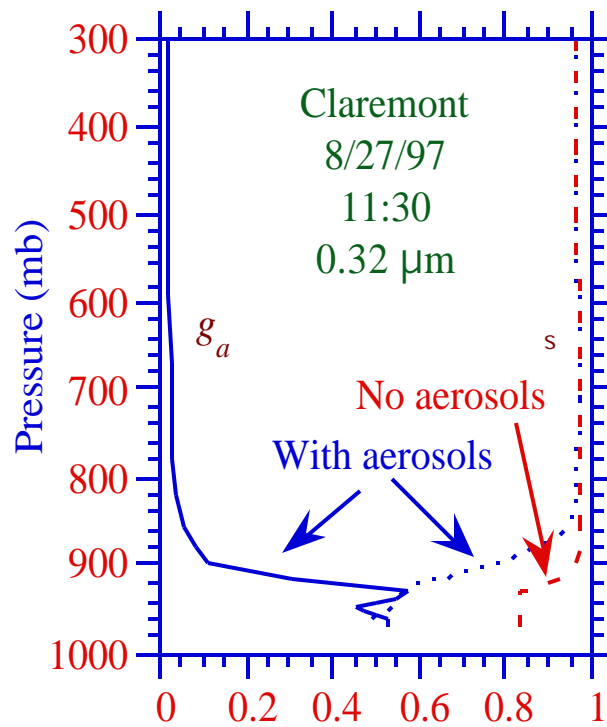
$$b = \frac{1 - g_a}{2} \quad (10.101)$$

Effective asymmetry parameter

$$g_a = \frac{s_{s,a} g_{a,a} + s_{s,c} g_{a,c}}{s_{s,g} + s_{s,a} + s_{s,c}} \quad (10.102)$$

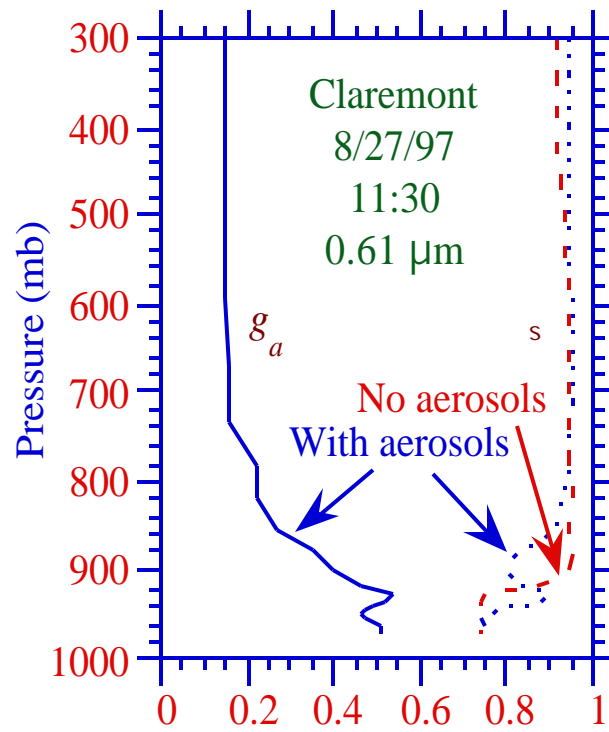
Predicted Asymmetry Parameter and Single-Scattering Albedo

Fig. 10.28 a



Predicted Asymmetry Parameter and Single-Scattering Albedo

Fig. 10.28 b



Two-Stream Approximation

Upward radiance equation (10.104)

$$\mu_1 \frac{dI_+}{d} = I_+ - \tau_s (1 - b) I_+ - \tau_s b I_- - \frac{\tau_s}{4} (1 - 3g_a \mu_1 \mu_s) F_s e^{-\tau_s / \mu_s}$$

Downward radiance equation (10.105)

$$-\mu_1 \frac{dI_-}{d} = I_- - \tau_s (1 - b) I_- - \tau_s b I_+ - \frac{\tau_s}{4} (1 + 3g_a \mu_1 \mu_s) F_s e^{-\tau_s / \mu_s}$$

Upward and downward irradiances in terms of radiance for two-stream approximation

$$F_+ = 2 \mu_1 I_+$$

$$F_- = 2 \mu_1 I_-$$

Substitute irradiances and generalize for different approximations to phase function

Solar irradiance

$$\frac{dF_+}{d} = -\mu_1 F_+ - \mu_2 F_- - 3 \tau_s F_s e^{-\tau_s / \mu_s} \quad (10.106)$$

$$\frac{dF_-}{d} = -\mu_1 F_- + \mu_2 F_+ + (1 - 3) \tau_s F_s e^{-\tau_s / \mu_s} \quad (10.106)$$

Surface boundary condition (10.108)

$$F_{+, N_L + 1/2} = A_e F_{-, N_L + 1/2} + \frac{A_e \mu_s F_s e^{-\tau_{N_L + 1/2} / \mu_s}}{B_T} \quad \begin{array}{l} \text{solar} \\ \text{infrared} \end{array}$$

Two-Stream Approximation

Table 10.10. Coefficients for two stream approximations using two techniques.

Approximation	1	2	3
Quadrature	$\frac{1 - \mu_1(1 + g_a)/2}{\mu_1}$	$\frac{\mu_1(1 - g_a)}{2\mu_1}$	$\frac{1 - 3g_a\mu_1\mu_s}{2}$
Eddington	$\frac{7 - \mu_1(4 + 3g_a)}{4}$	$-\frac{1 - \mu_1(4 - 3g_a)}{4}$	$\frac{2 - 3g_a\mu_s}{4}$

Infrared irradiance

$$\frac{dF_+}{d} = \mu_1 F_+ - \mu_2 F_- - 2(1 - \mu_s)B_T \quad (10.107)$$

$$\frac{dF_-}{d} = -\mu_1 F_- + \mu_2 F_+ + 2(1 - \mu_s)B_T \quad (10.107)$$

Delta Functions

Quadrature and Eddington solutions underpredict forward scattering because expansion of phase function is too simple to obtain the strong peak in scattering efficiency.

--> adjust terms with delta functions

$$g_a = \frac{g_a}{1 + g_a} \quad (10.109)$$

$$s = \frac{(1 - g_a^2) s}{1 - s g_a^2} \quad (10.109)$$

$$= (1 - s g_a^2) \quad (10.109)$$

Heating Rates

Net flux divergence equation

$$\frac{dT}{dt} = \frac{1}{c_{p,m}} \left(\frac{dQ_{solar}}{dt} + \frac{dQ_{ir}}{dt} \right) = \frac{1}{c_{p,m}} \frac{F_n}{\Delta z} \quad (10.110)$$

Net downward minus upward radiative flux

$$F_n = \int_0^{\Delta z} (F_{-,z} - F_{+,z}) dz$$

Partial derivative term

$$\frac{F_{n,k}}{\Delta z} = \frac{\left[(F_{-,z,k-1/2} - F_{+,z,k-1/2}) - (F_{-,z,k+1/2} - F_{+,z,k+1/2}) \right]}{z_{k-1/2} - z_{k+1/2}}$$

(10.111)

Temperature change

$$T_k = \frac{1}{c_{p,m}} \frac{F_n}{\Delta z} h \quad (10.112)$$

Photolysis Coefficients

Photolysis rate (s^{-1}) at bottom of layer k

$J_{q,p,k+1/2} = \int_0^4 I_{p, \lambda, k+1/2} b_{a,g,q, \lambda} Y_{q,p, \lambda} d\lambda \quad (10.113)$
--

Radiance at bottom of layer k (photons $cm^{-2} \mu m sr^{-1} s^{-1}$)

$$I_{p, \lambda, k+1/2} = \left(I_{-, \lambda, k+1/2} - I_{+, \lambda, k+1/2} \right) 10^{-8} \frac{1}{hc} \quad (10.114)$$

Example 10.14.

$I = 12 \text{ W m}^{-2}$ in band $0.495 \mu m < \lambda < 0.505 \mu m$

---> mean $\lambda = 0.5 \mu m$

---> $I_p = 3.02 \times 10^{15} \text{ photons cm}^{-2} \text{ s}^{-1}$