# Gas, Aerosol, Cloud Drop, and Raindrop Sizes

Table 14.1.

<table>
<thead>
<tr>
<th></th>
<th>Diameter (µm)</th>
<th>Number Concentration (molec. or partic. cm⁻³)</th>
<th>Mass Concentration (µg m⁻³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas molecules</td>
<td>0.0005</td>
<td>2.45 x 10¹⁹</td>
<td>1.2 x 10⁹</td>
</tr>
<tr>
<td>Small aerosols</td>
<td>&lt; 0.2</td>
<td>10³ - 10⁶</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Medium aerosols</td>
<td>0.2 - 1.0</td>
<td>1 - 10⁴</td>
<td>&lt; 250</td>
</tr>
<tr>
<td>Large aerosols</td>
<td>1.0 - 100</td>
<td>&lt; 1 - 10</td>
<td>&lt; 250</td>
</tr>
<tr>
<td>Fog drops</td>
<td>10 - 20</td>
<td>1 - 500</td>
<td>10⁴ - 5 x 10⁵</td>
</tr>
<tr>
<td>Average cloud drops</td>
<td>10 - 200</td>
<td>&lt; 10 - 1000</td>
<td>&lt; 10⁵ - 5 x 10⁶</td>
</tr>
<tr>
<td>Large cloud drops</td>
<td>200</td>
<td>&lt; 1 - 10</td>
<td>&lt; 10⁵ - 5 x 10⁶</td>
</tr>
<tr>
<td>Drizzle</td>
<td>400</td>
<td>0.1</td>
<td>10⁵ - 5 x 10⁶</td>
</tr>
<tr>
<td>Small raindrops</td>
<td>1000</td>
<td>0.01</td>
<td>10⁵ - 5 x 10⁶</td>
</tr>
<tr>
<td>Medium raindrops</td>
<td>2000</td>
<td>0.001</td>
<td>10⁵ - 5 x 10⁶</td>
</tr>
<tr>
<td>Large raindrops</td>
<td>8000</td>
<td>&lt; 0.001</td>
<td>10⁵ - 5 x 10⁶</td>
</tr>
</tbody>
</table>
Particles and Size Distributions

Particle
    Agglomerations of molecules in the liquid and / or solid phases, suspended in air

Includes aerosols, fog drops, cloud drops, and raindrops

Example 14.1. - Idealized particle size distribution

    10,000 particles of radius between 0.05 and 0.5 µm
    100 particles of radius between 0.5 and 5.0 µm
    10 particles of radius between 5.0 and 50 µm

Example 14.2. Number of size bins needs to be limited

    $10^5$ grid cells
    100 size bins
    100 components per size bin

--> $10^9$ words = 8 gigabytes to store concentration
**Volume Ratio Size Structure**

Volume of particles in one size bin

\[ \nu_i = V_{rat} \nu_{i-1} \]  \hspace{1cm} (14.1)

\[ \nu_i = \nu_1 V_{rat}^{i-1} \]  \hspace{1cm} (14.2)

Volume-diameter relationship for spherical particles

\[ \nu_i = \pi d_i^3 / 6 \]

Fig. 14.1. Variation in particle sizes with the volume ratio size structure.
Volume Ratio Size Structure

Volume ratio of adjacent size bins

\[ V_{rat} = \left( \frac{v_{N_B}}{v_1} \right)^{(N_B - 1)} = \left( \frac{d_{N_B}}{d_1} \right)^{(N_B - 1)} \]  \hspace{1cm} (14.3)

Example 14.3.

\[ d_1 = 0.01 \, \mu m \]
\[ d_{N_B} = 1000 \, \mu m \]
\[ N_B = 30 \text{ size bins} \]

--->

\[ V_{rat} = 3.29 \]

Number of size bins

\[ N_B = 1 + \frac{\ln \left( \left( \frac{d_{N_B}}{d_1} \right)^3 \right)}{\ln V_{rat}} \]  \hspace{1cm} (14.4)

Example 14.4.

\[ d_1 = 0.01 \, \mu m \]
\[ d_{N_B} = 1000 \, \mu m \]
\[ V_{rat} = 4, \]

--->

\[ N_B = 26 \text{ size bins} \]
\[ V_{rat} = 2, \]

--->

\[ N_B = 51 \text{ size bins} \]
Volume Ratio Size Structure

Average volume in a size bin

\[ \nu_i = \frac{1}{2} (\nu_i,hi + \nu_i,lo) \]  \hspace{1cm} (14.5)

Relationship between high- and low-edge volume

\[ \nu_i,hi = V_{rat} \nu_i,lo \]  \hspace{1cm} (14.6)

Substitute (14.6) into (14.5) \rightarrow low edge volume

\[ \nu_i,lo = \frac{2 \nu_i}{1 + V_{rat}} \]  \hspace{1cm} (14.7)

Volume width of a size bin

\[ \Delta \nu_i = \nu_i,hi - \nu_i,lo = \frac{2 \nu_{i+1}}{1 + V_{rat}} - \frac{2 \nu_i}{1 + V_{rat}} = \frac{2 \nu_i (V_{rat} - 1)}{1 + V_{rat}} \]  \hspace{1cm} (14.8)

Diameter width of a size bin

\[ \Delta d_i = d_i,hi - d_i,lo = \left( \frac{6}{\pi} \right)^{\frac{1}{3}} \left( \nu_i,hi^{\frac{1}{3}} - \nu_i,lo^{\frac{1}{3}} \right) = d_i 2^{\frac{1}{3}} \left( \frac{V_{rat}^{\frac{1}{3}} - 1}{1 + V_{rat}} \right)^{\frac{1}{3}} \]  \hspace{1cm} (14.9)
**Particle Concentrations**

Number concentration in a size bin

\[ n_i = \frac{v_i}{\nu_i} \quad (14.10) \]

Number concentration in a size distribution

\[ N_D = \sum_{i=1}^{N_B} n_i \quad (14.11) \]

Volume concentration in a size bin

\[ v_i = \sum_{q=1}^{N_V} v_{q,i} \quad (14.12) \]

Surface area concentration in a size bin

\[ a_i = n_i 4\pi r_i^2 = n_i \pi d_i^2 \quad (14.13) \]
### Particle Concentrations

Mass concentration in a size bin

\[
m_i = \sum_{q=1}^{N_V} m_{q,i} = c_m \sum_{q=1}^{N_V} \rho_q v_{q,i} = c_m \rho_{p,i} \sum_{q=1}^{N_V} v_{q,i} = c_m \rho_{p,i} v_i
\]  

(14.14)

Volume-averaged mass density (g cm\(^{-3}\)) of particle of size \(i\)

\[
\rho_{p,i} = \frac{\sum_{q=1}^{N_V} (v_{i,q} \rho_q)}{\sum_{q=1}^{N_V} v_{i,q}}
\]  

(14.15)

Example 14.5.

\[
m_{q,i} = 3.0 \text{ µg m}^{-3} \text{ for water}
\]

\[
m_{q,i} = 2.0 \text{ µg m}^{-3} \text{ for sulfate}
\]

\[
d_i = 0.5 \text{ µm}
\]

\[
\rho_q = 1.0 \text{ g cm}^{-3} \text{ for water}
\]

\[
\rho_q = 1.83 \text{ g cm}^{-3} \text{ for sulfate}
\]

\[
\rightarrow v_{q,i} = 3 \times 10^{-12} \text{ cm}^3 \text{ cm}^{-3} \text{ for water}
\]

\[
\rightarrow v_{q,i} = 1.09 \times 10^{-12} \text{ cm}^3 \text{ cm}^{-3} \text{ for sulfate}
\]

\[
\rightarrow m_i = 5.0 \text{ µg m}^{-3}
\]

\[
\rightarrow v_i = 4.09 \times 10^{-12} \text{ cm}^3 \text{ cm}^{-3}
\]

\[
\rightarrow \nu_i = 6.54 \times 10^{-14} \text{ cm}^3
\]

\[
\rightarrow n_i = 62.5 \text{ partic. cm}^{-3}
\]

\[
\rightarrow a_i = 4.8 \times 10^{-7} \text{ cm}^2 \text{ cm}^{-3}
\]
Lognormal Distribution

Bell-curve distribution on a log scale

Geometric mean diameter
50% of area under a lognormal curve lies below it

Geometric standard deviation
68% of area under a lognormal curve lies between $+/1$ one geometric standard deviation around the mean diameter

Fig. 14.2 a. A lognormal particle volume distribution.
Lognormal Distribution

Fig. 14.2 b. The lognormal curve drawn on a linear scale.
Lognormal Parameters From Data

Hering low-pressure impactor -- 7 size regimes

<table>
<thead>
<tr>
<th>Size Regime</th>
<th>Lower Limit (µm)</th>
<th>Upper Limit (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.075</td>
</tr>
<tr>
<td>0.075</td>
<td>0.075</td>
<td>0.12</td>
</tr>
<tr>
<td>0.12</td>
<td>0.12</td>
<td>0.26</td>
</tr>
<tr>
<td>0.26</td>
<td>0.26</td>
<td>0.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Natural log of geometric mean mass diameter

\[
\ln \bar{D}_M = \frac{1}{M_L} \sum_{j=1}^{7} (m_j \ln d_j) \tag{14.16}
\]

Total mass concentration of particles (µg m\(^{-3}\))

\[M_L = \sum_{j=1}^{7} m_j\]

Natural log of geometric mean volume diameter

\[
\ln \bar{D}_V = \frac{1}{V_L} \sum_{j=1}^{7} (v_j \ln d_j) \tag{14.17}
\]

Total volume concentration of particles (cm\(^3\) cm\(^{-3}\))

\[V_L = \sum_{j=1}^{7} v_j \quad v_j = \frac{m_j}{c_m \rho_j}\]
Lognormal Parameters From Data

Natural log of geometric mean area diameter

\[
\ln \bar{D}_A = \frac{1}{A_L} \sum_{j=1}^{7} (a_j \ln d_j) \tag{14.18}
\]

Total area concentration of particles (cm\(^2\) cm\(^{-3}\))

\[
A_L = \sum_{j=1}^{7} a_j \quad a_j = \frac{3m_j}{c_m \rho_j v_j}
\]

Natural log of geometric mean number diameter

\[
\ln \bar{D}_N = \frac{1}{N_L} \sum_{j=1}^{7} (n_j \ln d_j) \tag{14.19}
\]

Total number concentration of particles (partic. cm\(^{-3}\))

\[
N_L = \sum_{j=1}^{7} n_j \quad n_j = \frac{m_j}{c_m \rho_j v_j}
\]

Natural log of geometric standard deviation

\[
\ln \sigma_g = \sqrt{\frac{1}{M_L} \sum_{j=1}^{7} \left( m_j \ln^2 \frac{d_j}{D_M} \right)} = \sqrt{\frac{1}{V_L} \sum_{j=1}^{7} \left( v_j \ln^2 \frac{d_j}{D_V} \right)}
\]

\[
= \sqrt{\frac{1}{A_L} \sum_{j=1}^{7} \left( a_j \ln^2 \frac{d_j}{D_A} \right)} = \sqrt{\frac{1}{N_L} \sum_{j=1}^{7} \left( n_j \ln^2 \frac{d_j}{D_N} \right)} \tag{14.20}
\]
Redistribute Mass, etc. With Lognormal Parameters

Redistribute mass concentration in model size bin

\[
m_i = \frac{M_L \Delta d_i}{d_i \sqrt{2 \pi \ln \sigma_g}} \exp \left[ -\frac{\ln^2 \left( d_i / D_M \right)}{2 \ln^2 \sigma_g} \right] \quad (14.21)
\]

Redistribute volume concentration

\[
v_i = \frac{V_L \Delta d_i}{d_i \sqrt{2 \pi \ln \sigma_g}} \exp \left[ -\frac{\ln^2 \left( d_i / D_V \right)}{2 \ln^2 \sigma_g} \right] \quad (14.22)
\]

Redistribute area concentration

\[
a_i = \frac{A_L \Delta d_i}{d_i \sqrt{2 \pi \ln \sigma_g}} \exp \left[ -\frac{\ln^2 \left( d_i / D_A \right)}{2 \ln^2 \sigma_g} \right] \quad (14.23)
\]

Redistribute number concentration

\[
n_i = \frac{N_L \Delta d_i}{d_i \sqrt{2 \pi \ln \sigma_g}} \exp \left[ -\frac{\ln^2 \left( d_i / D_N \right)}{2 \ln^2 \sigma_g} \right] \quad (14.24)
\]

Exact volume concentration in a mode

\[
V_L = \int_0^{\infty} v_d d d = \frac{\pi}{6} \int_0^{\infty} n_d d^3 d = \frac{\pi}{6} D_N^3 \exp \left( \frac{9}{2} \ln^2 \sigma_g \right) N_L \quad (14.25)
\]
Lognormal Modes

Fig. 14.3. Number (partic. cm\(^{-3}\)), area (cm\(^2\) cm\(^{-3}\)), and volume (cm\(^3\) cm\(^{-3}\)) concentrations for a lognormal distribution.
Lognormal Parameters for Continental Particles

Table 14.2.

(Whitby, 1978)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nuclei</th>
<th>Accumulation</th>
<th>Coarse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_g$</td>
<td>1.7</td>
<td>2.03</td>
<td>2.15</td>
</tr>
<tr>
<td>Number (partic. cm$^{-3}$)</td>
<td>$7.7 \times 10^4$</td>
<td>$1.3 \times 10^4$</td>
<td>4.2</td>
</tr>
<tr>
<td>$\bar{D}_N$ (µm)</td>
<td>0.013</td>
<td>0.069</td>
<td>0.97</td>
</tr>
<tr>
<td>Surface (µm$^2$ cm$^{-3}$)</td>
<td>74</td>
<td>535</td>
<td>41</td>
</tr>
<tr>
<td>$\bar{D}_A$ (µm)</td>
<td>0.023</td>
<td>0.19</td>
<td>3.1</td>
</tr>
<tr>
<td>Volume (µm$^3$ cm$^{-3}$)</td>
<td>0.33</td>
<td>22</td>
<td>29</td>
</tr>
<tr>
<td>$\bar{D}_V$ (µm)</td>
<td>0.031</td>
<td>0.31</td>
<td>5.7</td>
</tr>
</tbody>
</table>
Quadramodal Size Distribution

Quadramodal lognormal distribution

<table>
<thead>
<tr>
<th>Mode</th>
<th>Diameter Range or Peak (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nucleation</td>
<td>&lt; 0.1</td>
</tr>
<tr>
<td>Accumulation</td>
<td>0.1 - 2</td>
</tr>
<tr>
<td>submode 1</td>
<td>~ 0.2</td>
</tr>
<tr>
<td>submode 2</td>
<td>~ 0.5 - 0.7</td>
</tr>
<tr>
<td>Coarse</td>
<td>&gt; 2</td>
</tr>
</tbody>
</table>

Fig. 14.4. Size distribution at Claremont, California, on the morning of August 27, 1987.
Marshall-Palmer Distribution

Raindrop number concentration between $d_i$ and $d_i + \Delta d_i$

\[
n_i = \Delta d_i n_0 e^{-\lambda_r d_i}
\]  \hspace{1cm} (14.30)

$\Delta d_i n_0$ = value of $n_i$ at $d_i = 0$

$n_0$ = $8.0 \times 10^{-6}$ partic. cm$^{-3}$ µm$^{-1}$

$\lambda_r$ = $4.1 \times 10^{-3} R^{-0.21}$ µm$^{-1}$

$R$ = rainfall rate (1 - 25 mm hr$^{-1}$)

Total number concentration and liquid water content

\[
n_T = n_0 / \lambda_r \hspace{1cm} w_L = 10^{-6} \rho_w \pi n_0 / \lambda_r^4
\]

Example 14.6.

\[
\begin{align*}
R & = 5 \text{ mm hr}^{-1} \\
 d_i & = 1.0 \text{ mm} \\
 d_i + \Delta d_i & = 2.0 \text{ mm} \\
\rightarrow n_i & = 0.00043 \text{ partic. cm}^{-3} \\
\rightarrow n_T & = 0.0027 \text{ partic. cm}^{-3} \\
\rightarrow w_L & = 0.34 \text{ g m}^{-3}
\end{align*}
\]
Modified Gamma Distribution

Number concentration of drops (partic. cm\(^{-3}\)) in size bin \(i\)

\[
n_i = \Delta d_i A_g r_i \alpha_g \exp \left[ -\frac{\alpha_g \left( \frac{r_i}{r_{c,g}} \right)^\gamma_g}{\gamma_g} \right]
\]

(14.31)

Table 14.3. Modified gamma size distribution parameters.

<table>
<thead>
<tr>
<th>Cloud Type</th>
<th>(A_g)</th>
<th>(\alpha_g)</th>
<th>(\gamma_g)</th>
<th>(r_{c,g}) ((\mu m))</th>
<th>Liquid Water Content (g m(^{-3}))</th>
<th>Number Conc. (partic. cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratocumulus base</td>
<td>0.2823</td>
<td>5.0</td>
<td>1.19</td>
<td>5.33</td>
<td>0.141</td>
<td>100</td>
</tr>
<tr>
<td>Stratocumulus top</td>
<td>0.19779</td>
<td>2.0</td>
<td>2.46</td>
<td>10.19</td>
<td>0.796</td>
<td>100</td>
</tr>
<tr>
<td>Stratus base</td>
<td>0.97923</td>
<td>5.0</td>
<td>1.05</td>
<td>4.70</td>
<td>0.114</td>
<td>100</td>
</tr>
<tr>
<td>Stratus top</td>
<td>0.38180</td>
<td>3.0</td>
<td>1.3</td>
<td>6.75</td>
<td>0.379</td>
<td>100</td>
</tr>
<tr>
<td>Nimbostratus base</td>
<td>0.08061</td>
<td>5.0</td>
<td>1.24</td>
<td>6.41</td>
<td>0.235</td>
<td>100</td>
</tr>
<tr>
<td>Nimbostratus top</td>
<td>1.0969</td>
<td>1.0</td>
<td>2.41</td>
<td>9.67</td>
<td>1.034</td>
<td>100</td>
</tr>
<tr>
<td>Cumulus congestus</td>
<td>0.5481</td>
<td>4.0</td>
<td>1.0</td>
<td>6.0</td>
<td>0.297</td>
<td>100</td>
</tr>
<tr>
<td>Light rain</td>
<td>4.97x10(^{-8})</td>
<td>2.0</td>
<td>0.5</td>
<td>70.0</td>
<td>1.17</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Example 14.7.
Find number concentration of droplets between 14 and 16 \(\mu m\) in diameter at base of a stratus cloud.

\[
\rightarrow \quad r_{c,g} = 7.5 \mu m
\]

\[
\rightarrow \quad \Delta d_i = 2 \mu m \quad \rightarrow \quad n_i = 19.46 \text{ partic. cm}^{-3}
\]
**Full-Stationary Size Structure**

Average volume in size bin ($v_i$) stays constant. When growth occurs, $n_i$ changes.

Advantages:
- Covers wide range in diameter space with few bins
- Nucleation, emissions, transport treated realistically

Disadvantages:
- When growth occurs, information about the original composition of the growing particle is lost.
- Growth leads to numerical diffusion

**Fig. 14.5.** Demonstration of a problem with the full-stationary size bin structure.
Full-Moving Structure

Number concentration \( (n_i) \) of particles in a size bin does not change during growth; instead, volume \( (\nu_i) \) changes

Advantages:
• Core volume preserved during growth
• No numerical diffusion during growth

Disadvantages:
• Nucleation, emissions, transport treated unrealistically.
• Reordering of size bins required for coagulation

Fig. 14.6. Preservation of aerosol material upon growth and evaporation in a moving structure.

Fig. 14.7. Particle size bin reordering for coagulation
**Quasistationary Structure**

Same as full-stationary structure, except particle volumes fluctuate during growth. Adjusted particle volumes are fit back onto a stationary grid.

**Advantages and disadvantages:**
- Same as for full stationary structure

**Example:**
- After growth, particles in bin $i$ have volume $\nu_i'$, where $\nu_j \leq \nu_i' < \nu_k$
- Partition $i$ particles between bins $j$ and $k$,

Conserve particle number concentration

$$n_i = n_j + n_k$$

Conserve particle volume concentration

$$n_i \nu_i' = n_j \nu_j + n_k \nu_k$$

Solution to this set of two equations and two unknowns

$$n_j = n_i \frac{\nu_k - \nu_i'}{\nu_k - \nu_j}$$

$$n_k = n_i \frac{\nu_i' - \nu_j}{\nu_k - \nu_j}$$  \hspace{1cm} (14.32)
Moving-Center Structure

Particle volume ($\nu_i$) is varies between $\nu_{i,hi}$ and $\nu_{i,lo}$ during growth, but $\nu_{i,hi}$, $\nu_{i,lo}$, and $d\nu_i$ remain fixed.

Advantages:

- Covers wide range in diameter space with few bins
- Little numerical diffusion during growth
- Nucleation, emissions, transport treated realistically.

Disadvantages:

- When growth occurs, information about the original composition of the growing particle is lost.

Fig. 14.8. Comparison of moving-center, full-moving, and quasi stationary size structure after growth of water onto aerosols to form cloud-sized drops.