Fundamentals of the Value of Information

Snowmass 16

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Major Points

- The value of information:
  - Depends on decisions to be made about what to do (actions) under conditions of uncertainty
  - Depends on the character of the decision space (outcomes contingent on states of nature, irreversibility, persistence, etc…)
  - Depends on the range of possible states of nature and their distributions
  - Depends on decision-makers' priors about those distributions
  - Depends on decisions-makers' attitudes (aversion) toward risk

Examples in the Climate World

- Nordhaus, W., *Managing the Global Commons,* 1994 (Chapter 8).

- Manne, A. and Richels, R., "The Greenhouse Debate: Economic Efficiency, Burden Sharing and Hedging Strategies", *The Energy Journal,* 1995.

- Yohe, G., Strzepek, K., Pau, T. and Yohe, C., "Assessing Vulnerability in the Context of Changing Socioeconomic Conditions: A Study of Egypt" in *Climate Change, Adaptive Capacity and Development*, 2003.
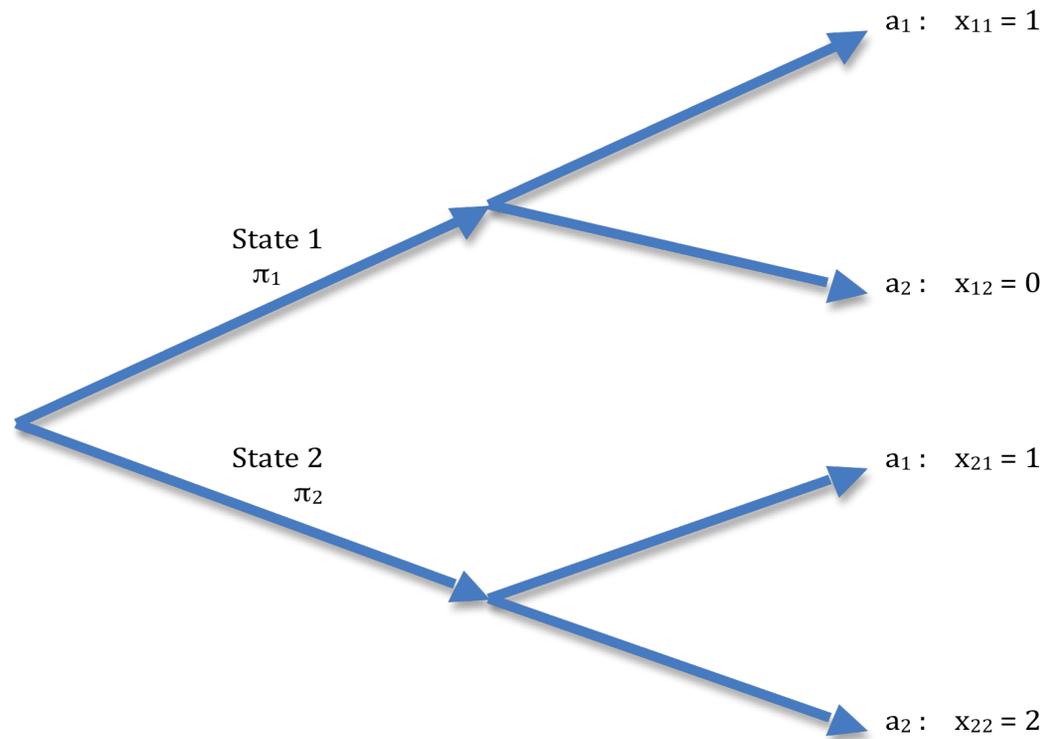
A Representative Utility Function

An Illustrative Utility Function for a Risk Averse Decision-maker
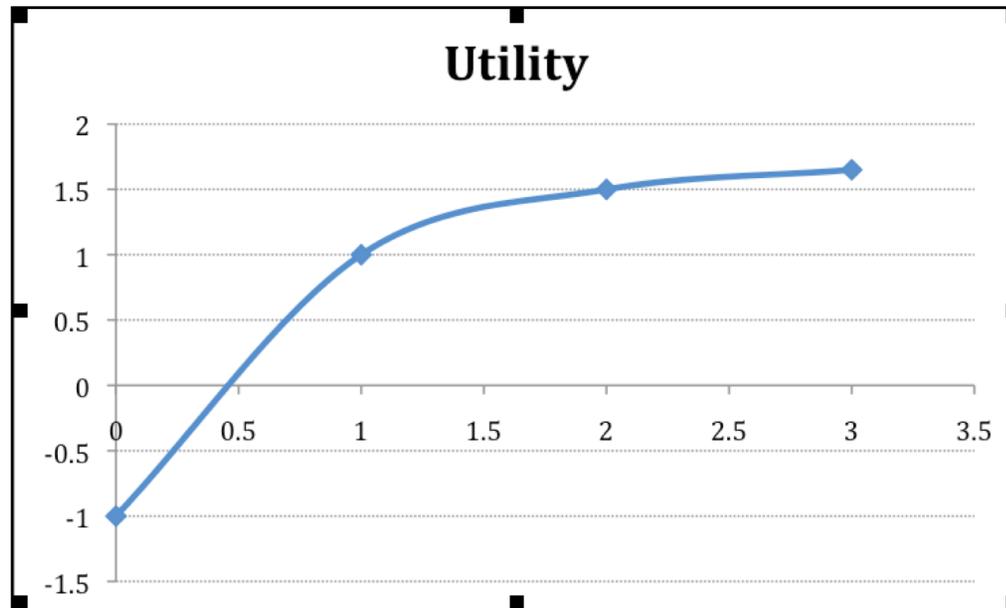
An Illustrative Decision Tree

$a_1 :\quad x_{11} = 1$

$a_2 :\quad x_{12} = 0$

State 1
π_1

State 2
π_2

$a_1 :\quad x_{21} = 1$

$a_2 :\quad x_{22} = 2$

Some Important Concepts

- Expected outcome: $EX = \Sigma\, \pi_i\, x_i$.

- Expected Utility: $EU = \Sigma\, \pi_i\, U(x_i)$.

- Certainty equivalent: X_{ce} solves $U(X_{ce}) = EU$.

- VOI can be measured as changes in expected outcome if decision-makers are risk neutral; otherwise, use differences in certainty equivalents.

Some Analysis

- With perfect information:
  - Do a_1 in state 1 so X = 1 and U(x) = 1
  - Do a_2 in state 1 so X = 2 and U(x) = 1.5
  - EX = $\pi_1(1) + (1-\pi_1)(2) = 2 - \pi_1$
  - EU = $\pi_1(1) + (1-\pi_1)(1.5) = 1.5 - .5\pi_1$
- Without any more information:
  - Do a_1 means X = 1 and U(x) = 1 in either state
  - Do a_2 means in X = 0 and U(x) = -1 in state 1 and X = 2 with U(x) = 1.5 in state 2
  - EX = $\pi_1(0) + (1-\pi_1)(2) = 2 - 2\pi_1$
  - EU = $\pi_1(-1) + (1-\pi_1)(1.5) = 1.5 - 2.5\pi_1$

Some Analysis – Ranking Options

- Ranking using expected outcome:
  - Do a_1 if $1 > (2 - 2\pi_1)$; i.e., if $\pi_1 > 0.5$

- Ranking using expected utility:
  - Do a_1 if $1 > (1.5 - 2.5\pi_1)$; i.e., if $\pi_1 > 0.2$

- Prior on the probability of state 1 determines the ranking of actions, and the threshold depends on criterion used; there is a range of ambiguity $-.5 > \pi_1 > 0.2$ within which expected outcome favors a_2 and expected utility favors a_1.

Some Analysis – Value of (perfect) Information Depends on Priors; let it be $\pi_1 = 0.4$

- VOI using Expected Outcome:
  - With perfect information, X = 1 in state 1 and X = 2 in state 2; as a result, EX = .4(1) + .6(2) = 1.6.
  - Without perfect information, do a_2 and see what happens: X = 0 in state 1 and X = 2 in state 2 so EX = .4(0) + .6(2) = 1.2
  - VOI = 1.6 – 1.2 = 0.4.
- VOI using Expected Utility:
  - With perfect information, X = 1 in state 1 and X = 2 in state 2; as a result, EU = .4(1) + .6(1.5) = 1.3. From the graph, $X_{ce} \approx 1.35$
  - Without perfect information, do a_1 and see what happens: X = 1 in state both states 1 so EU = 1 and X_{ce} = 1.
  - VOI ≈ 1.35 – 1 = 0.35.

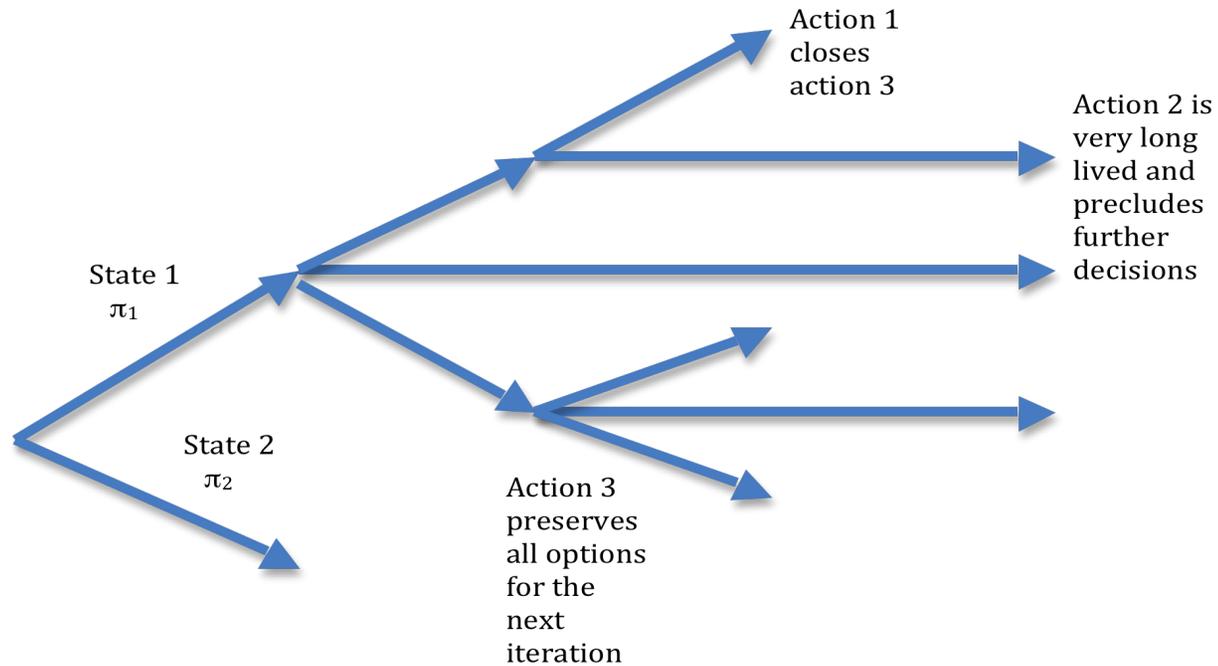# Some Analysis – Value of (perfect) Information Depends on Priors; let it be $\pi_1 = 0.8$

- VOI using Expected Outcome:
  - With perfect information, X = 1 in state 1 and X = 2 in state 2; as a result, EX = .8(1) + .2(2) = 1.2.
  - Without perfect information, do a_1 and see what happens: X = 1 in both states state 1 so EX = 1.
  - VOI = 1.2 – 1 = 0.2.
- VOI using Expected Utility:
  - With perfect information, X = 1 in state 1 and X = 2 in state 2; as a result, EU = .8(1) + .2(1.5) = 1.1. From the graph, $X_{ce} \approx 1.12$
  - Without perfect information, do a_1 and see what happens: X = 1 in state both states 1 so EU = 1 and X_{ce} = 1.
  - VOI ≈ 1.12 – 1 = 0.12.

Some Insights

- Using expected outcome (risk neutrality or efficient insurance available): calculate the difference in expected outcome with and without the new information.

- Using expected utility so that risk aversion comes into play: calculate the difference in certainty equivalents with and without the new information.

- Priors matter in both cases, but VOI varies with risk aversion (in ways that depend on priors).

Character of Decisions Can Matter

Character of Available Options Matters

(Reversibility; Duration)

Action 1 closes action 3

Action 2 is very long lived and precludes further decisions

State 1
π_1

State 2
π_2

Action 3 preserves all options for the next iteration

Timing can be a factor in the Value of Information

- Infrastructure project to protect an urban area in Boston. Significant cost up front and then committed to maintenance.

- Whether or when to undertake the project given sea level rise implications on the character of coastal storms (stochastic with mean changing with sea level rise).

- Use IRR as the criterion – the discount rate for which discounted benefits equal discounted costs at time of investment; IRR ≥ interest rate means ok.

Distributions of Damages from Coastal Storms over Time

Distributions of Damages over 100 Runs - Urban 0.6 meters

Distributions of Damages over 100 Runs - Urban 1 meter
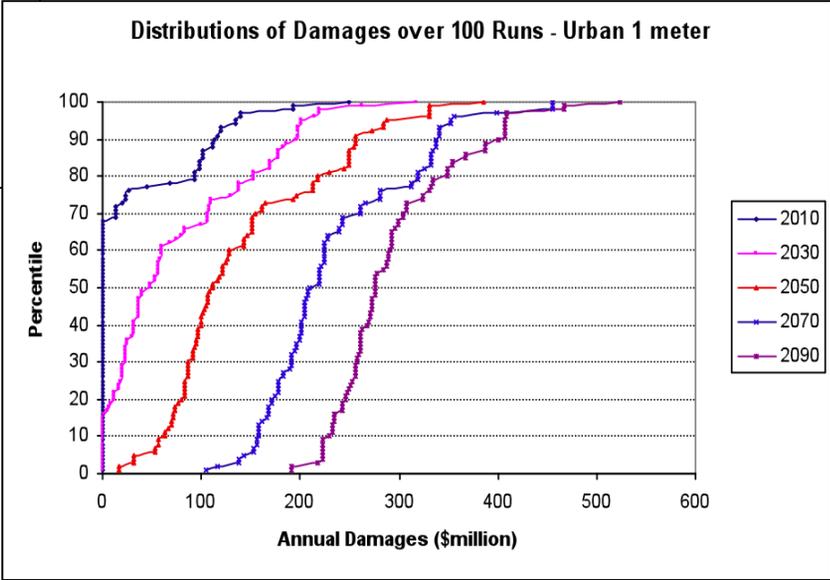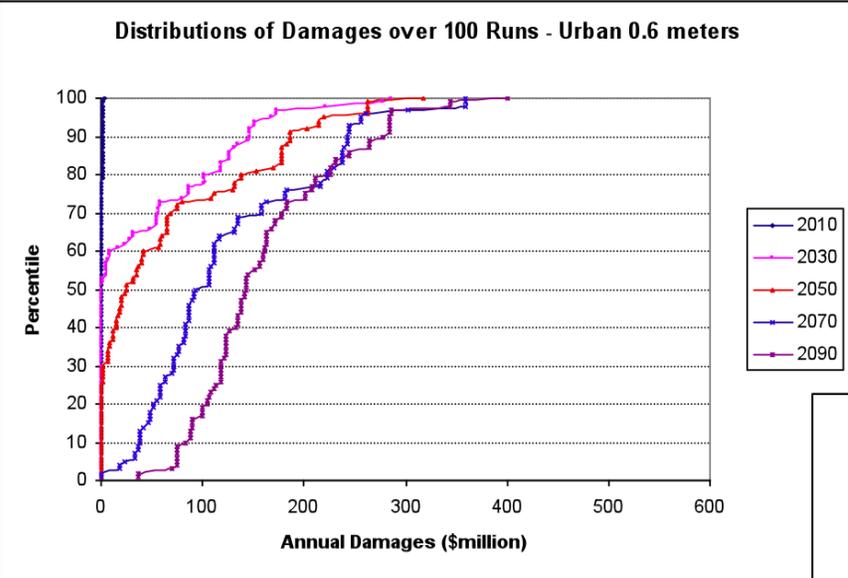
Threshold of 4% means wait to 2045 along a 0.6 m scenario with RRA > 1.5; it means begin in 2015 along a 1.0 m scenario
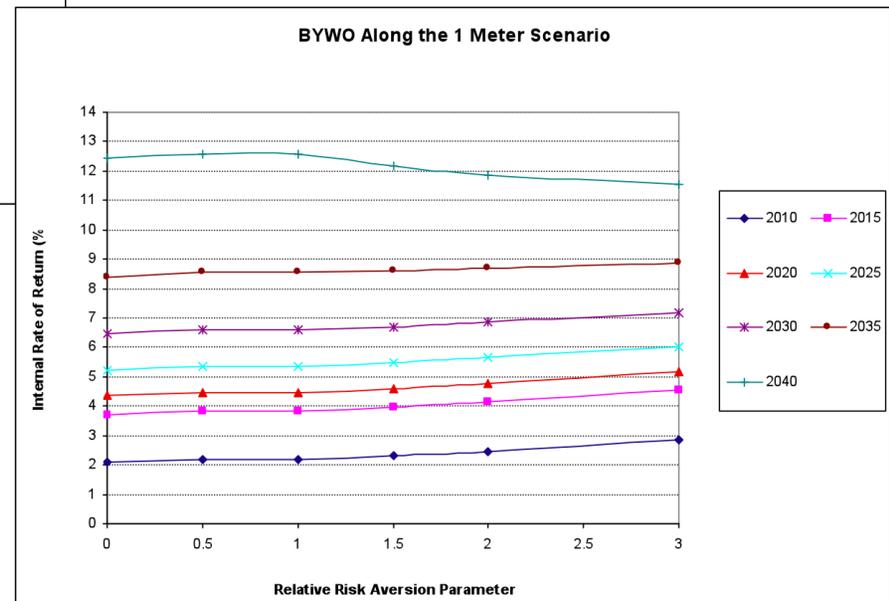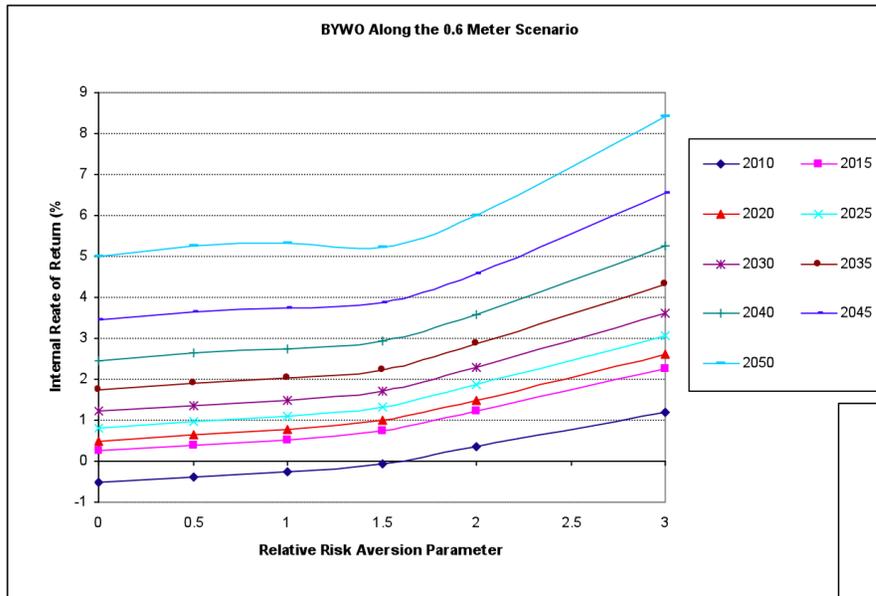
- Subtext – VOI is a p oint of entry for the new decision-supporting science advocated by America's Climate Choices and the Ramanathan Report from the National Academies of Science