

*Further Issues on Treatment of  
Uncertainty in IAMs*

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# Outline of lecture

1. Repeat model specification
2. Endogenous policy
3. How to represent “importance” of uncertainties
4. Treatment of risk

## Approach to Uncertainty: Monte Carlo/Subjective Probability Uncertainty Analysis

1. Begin with an abstract model. Can represent schematically as follows:

$$(1) \ y_t = H(z_t; \theta)$$

where

$y_t$  = the endogenous variables (output, emissions, etc.)

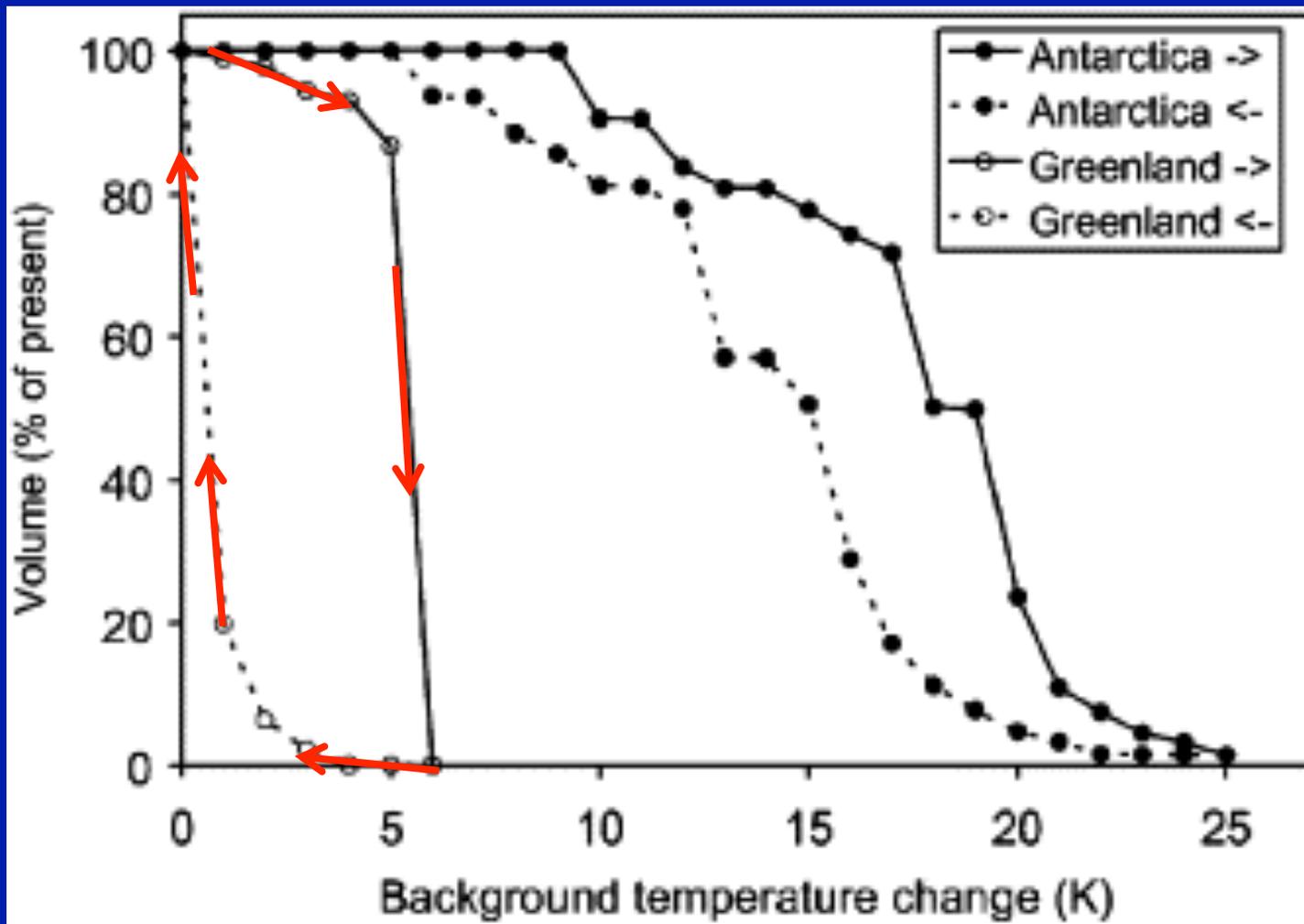
$z_t$  = exogenous, policy, and non-stochastic variables (financial meltdown, land-based emissions, etc.)

$\theta = [\theta_1, \dots, \theta_n]$  = uncertain parameters (including functional forms)

H = model structure

3. We then develop subjective probabilities for major parameters,  $f(\theta)$ .
4. Final step is Monte Carlo simulation to obtain distribution of important endogenous variables (T, Y, SCC, ...)

# Hysteresis loops for Ice Sheets and the “Tipping Point”



Frank Pattyn, “GRANTISM: Model of Greenland and Antarctica,” *Computers & Geosciences*, April 2006, Pages 316-325<sup>4</sup>

# Endogenous policy

- Work in economics, politics, war
- Very fragile
- I don't have much hope here

## *Uncertainty analysis in DICE/RICE models*

### Implementation

- Examine many (000) runs sampling from random distributions.
- Then fit a “response-surface model” that estimates major outcomes as function of uncertain parameters
- These provide information about what the uncertainties are for major variables.

*I would recommend parametric or non-parametric regression analysis to estimate importance of different variables.*

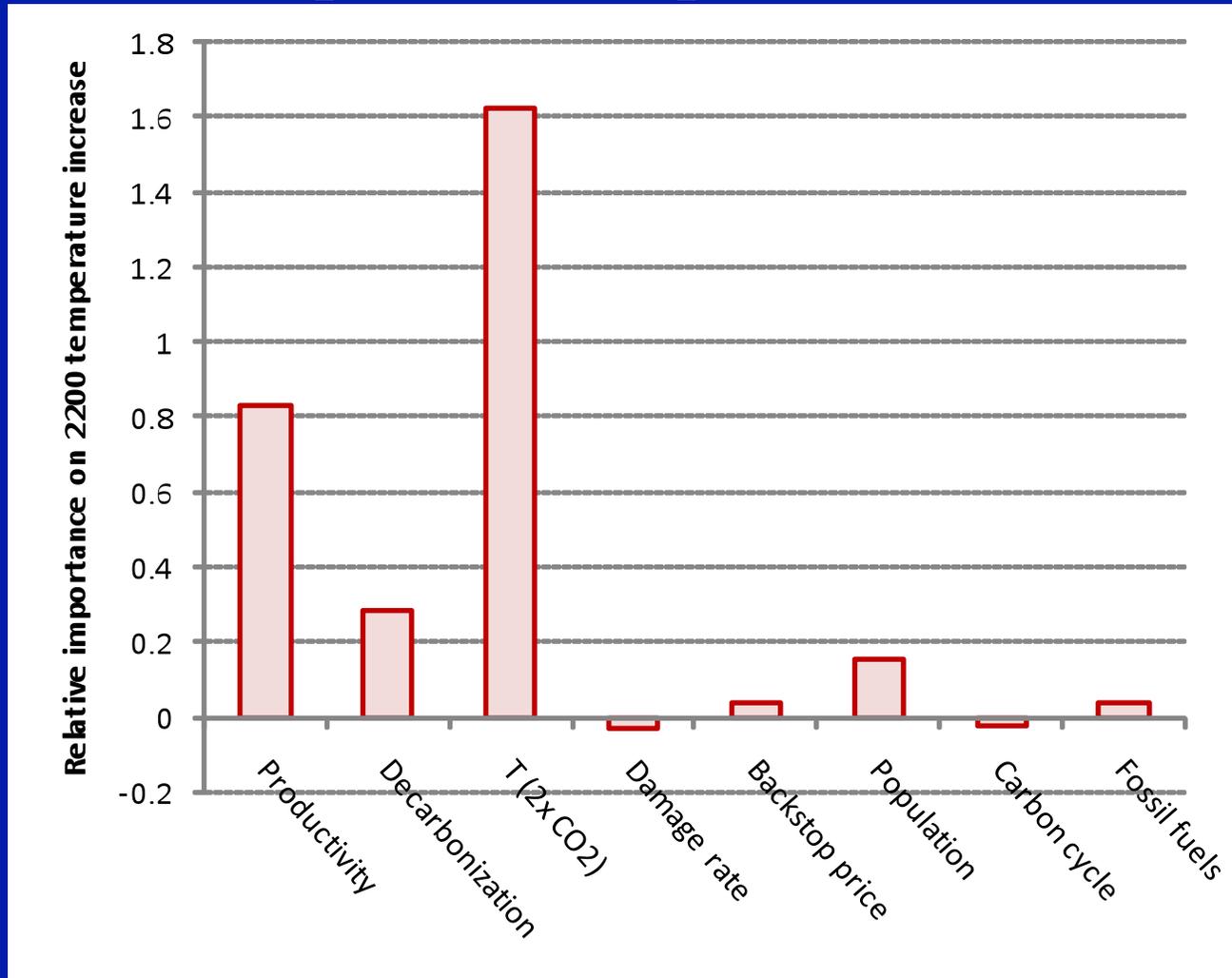
In these used only first order, but have gone to second-order as well.

One important reason is that have statistical interpretation.

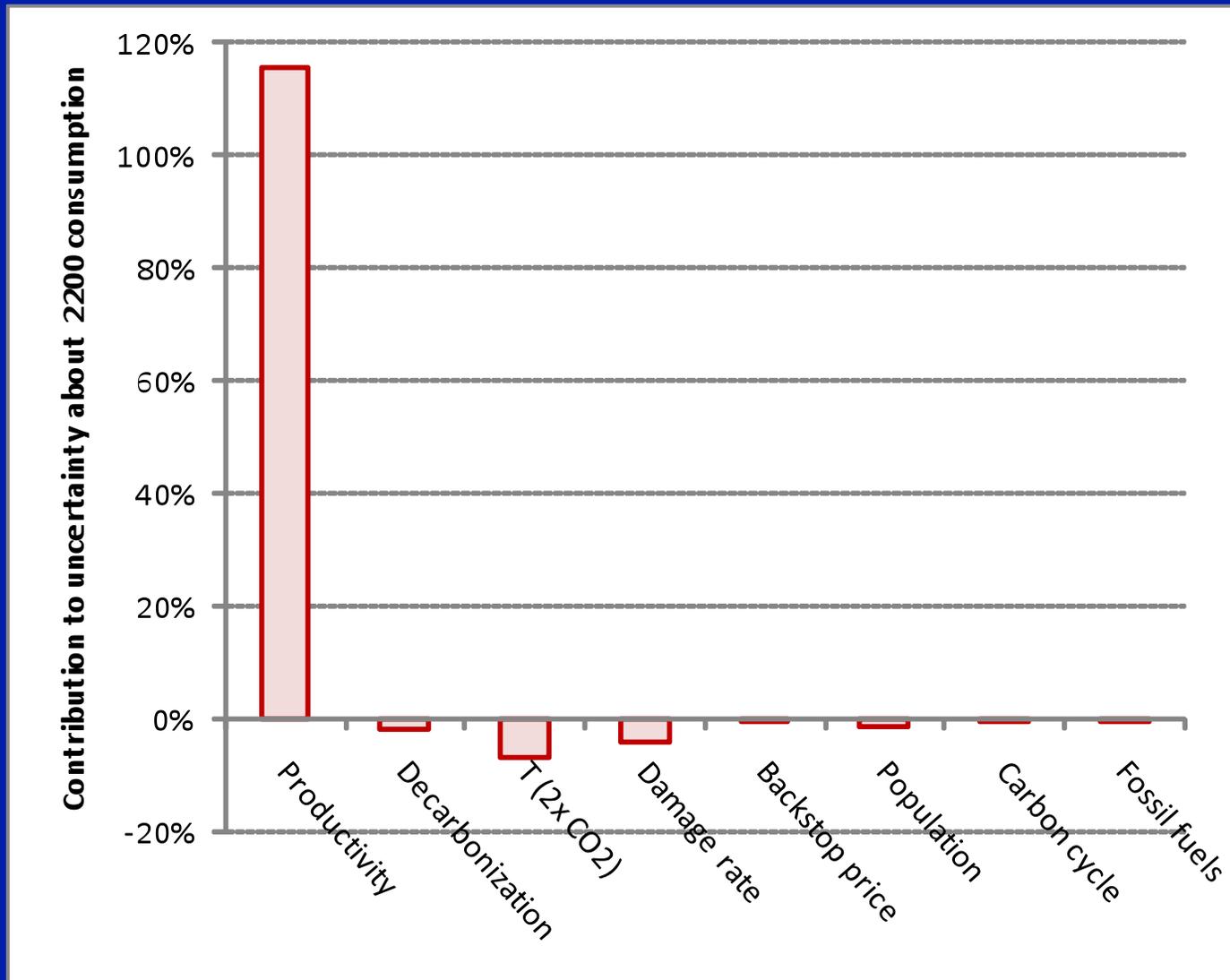
## Major uncertain variables

Variable	Mean	Standard deviation
Rate of growth of total factor productivity (% per year)	0.0092	0.004
Rate of decarbonization (% per year)	-0.0070	0.002
Equilibrium temperature-sensitivity coefficient (°C per CO <sub>2</sub> doubling)	3.00	1.11
Damage parameter (intercept of damage equation)	0.0028	0.0013
Price of backstop technology (\$ per ton C removed at 100 % removal)	1,170	468
Asymptotic global population (millions)	8,600	1,892
Transfer coefficient in carbon cycle	0.189	0.017
Total resources of fossil fuels (billions of tons of carbon)	6,000	1,200

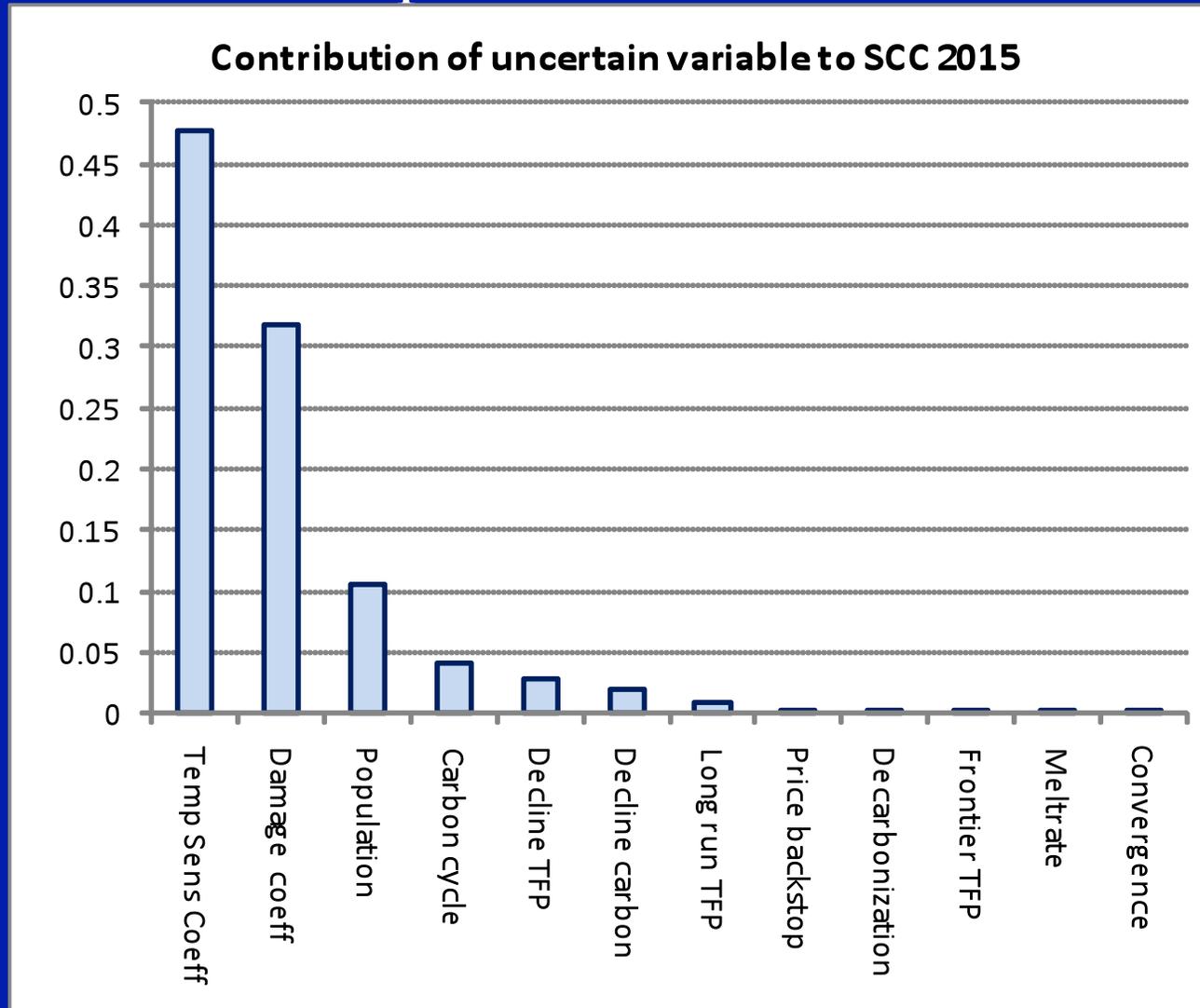
# Normalized contribution of uncertain variable: impact on temperature, 2200



# Normalized contribution of uncertain variable: impact on per capita consumption, 2200



# Normalized contribution of uncertain variable: Impact on SCC 2015\*



\*Note that elasticity of consumption and rate of time preference are not included.

## *4. The macro risk question: theory*

Should we be paying a large risk premium on high-climate scenarios? The idea is that we would invest more than the certainty equivalent to prevent the high-climate outcome.

Earlier studies generally assume yes. These approaches applied a risk-averse utility function to the damages. (This was the approach in Nordhaus-Boyer 2000 and Stern 2007.)

This is a partial-equilibrium approach. The correct approach would be to apply modern risk analysis in the consumption capital asset pricing model.

In this framework, want to pay a risk premium  $\leftrightarrow$  the high-climate outcome is positively correlated with high MU consumption (or negatively correlated with consumption).

Relevant economics is CCAPM model (Samuelson-Merton, see Mehra, Handbook on Equity Premium).

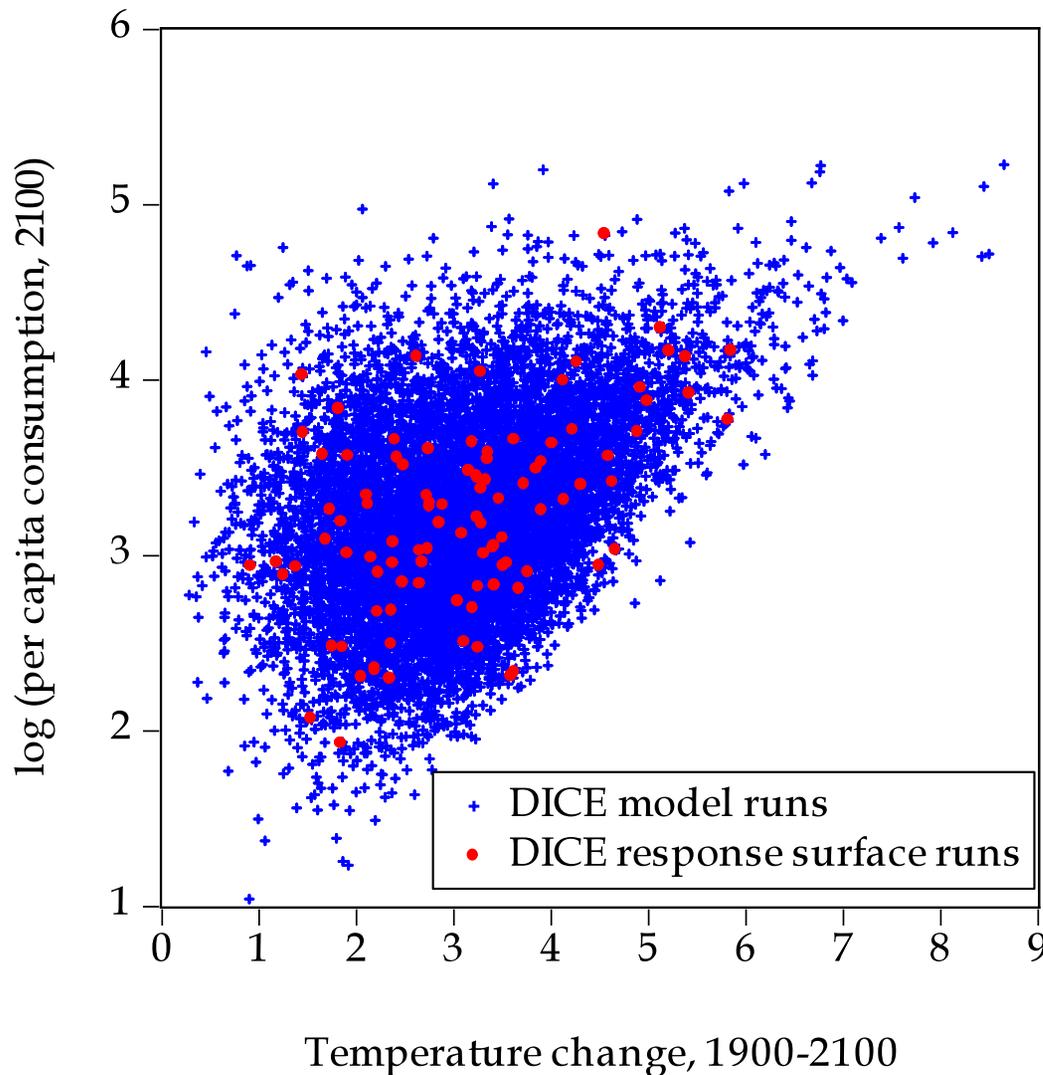
Social risk comes where have low payoffs in situations where MU of consumption is high (c is low).

Begin with a utility function,  $U(C)$ . Examine the expected utility in year  $t$  (say 2100). This is:

$$E(U) = \int_{\text{SOW } i} p^i U[C(\boldsymbol{\theta}^i)]$$

In this,  $p^i$  are probabilities, and  $\boldsymbol{\theta}^i = [\theta_1^i, \dots, \theta_8^i]$  are uncertain parameters, and  $i$  are uncertain states of the world (SOW). In general, we apply risk premia to SOWs where  $U'[C(\boldsymbol{\theta}^i)]$  or the MU of consumption is high. The question is, when do these occur?

# Consumption and 2100 Temperature

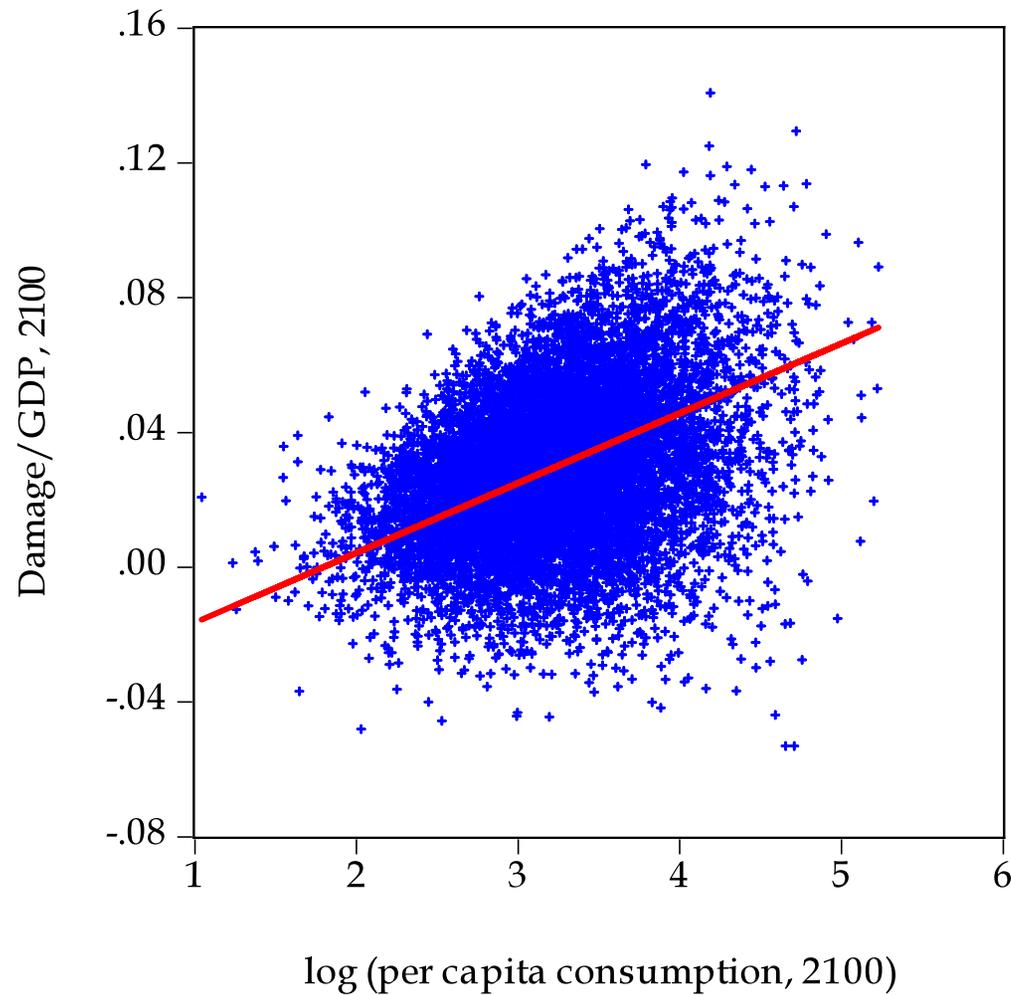


The red dots are 100 runs of DICE model with random draw of uncertain parameters

The blue dots are 10,000 runs of response-surface model.

Key result is that per capita consumption is positively correlated with temperature.

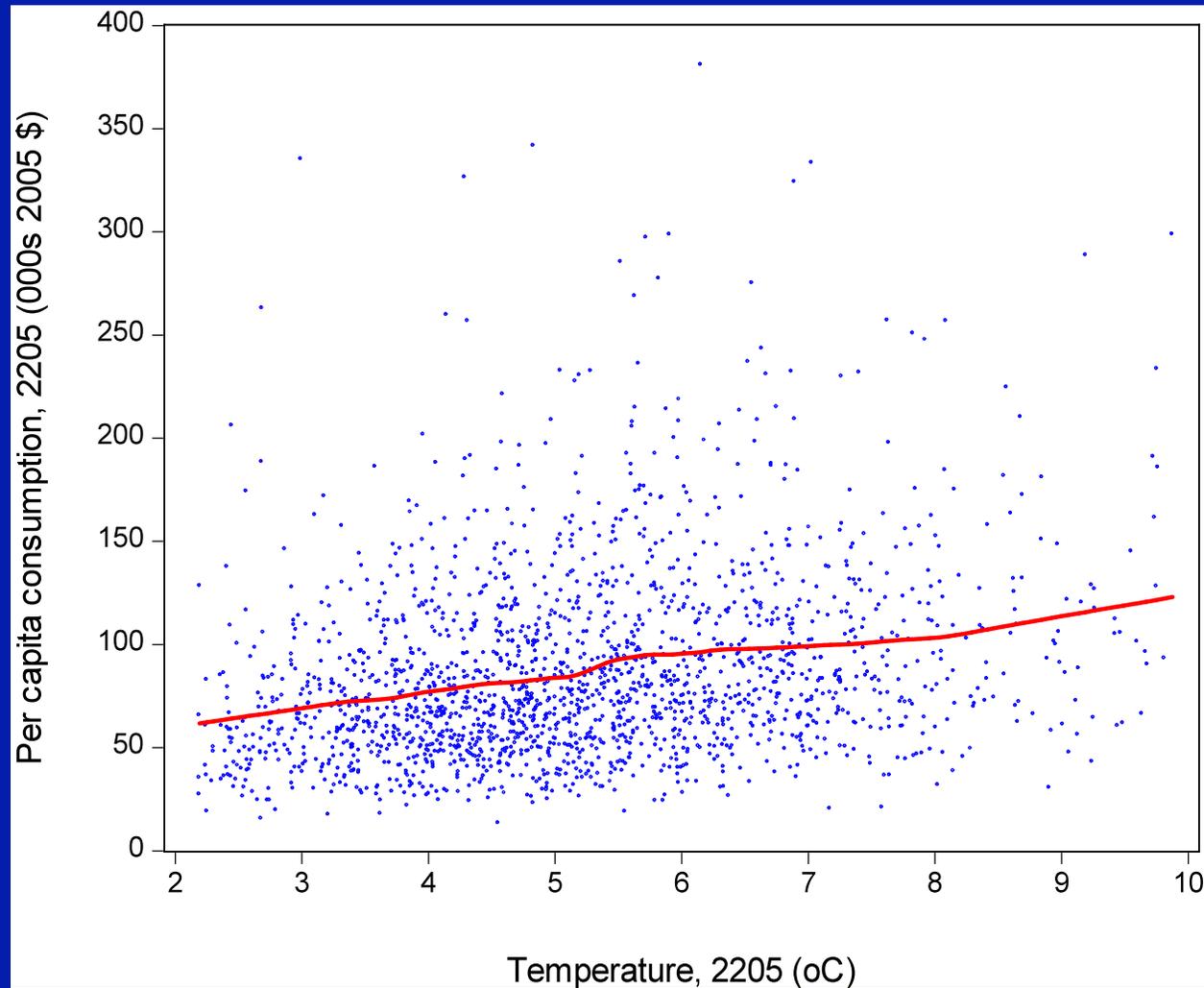
# Consumption and damage ratio



The blue dots are 10,000 runs of response-surface model.

Key result is that damage ratio is high when per capita consumption is high.

# Temperature and p.c. consumption (RICE-2011)



Update using  
RICE-2011  
model

## *Implications for Risk Premium*

Study suggests that the most important uncertainty in long-run is growth in productivity. High climate damages are associated with high growth rates of TFP.

- So good economic news = bad climate news in baseline.
- Example of beach houses of hedge fund owners and hurricanes.

But this also means that the worst climate cases are ones in which the world is *rich*, which is a situation where we are more likely to be able to afford more costly climate abatement.

This analysis applies as well to social cost of carbon with equity weights.