

# Uncertainty Analysis in Integrated Assessment Models

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Model Uncertainty Group  
Snowmass Meeting  
July 30, 2012

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## *Why uncertainty assessment is important*

1. Classical statistical significance of projections.
2. Clarify limits of current knowledge.
3. Assessment of directions for research.
4. Value of information analysis.
5. Help reduce confusion about climate change science.
6. Diagnose differences among models.
7. Curiosity!

# *Findings of the InterAcademy Review*

The InterAcademy Review\* had only one substantive topic in its recommendations: Uncertainty.

Some key passages:

“The evolving nature of climate science, the long time scales involved, and the difficulties of predicting human impacts on and responses to climate change mean that many of the results presented in IPCC assessment reports have inherently uncertain components. To inform policy decisions properly, it is important for uncertainties to be characterized and communicated clearly and coherently.”

Recommendations on uncertainty:

- Quantitative probabilities (subjective or objective) should be assigned only to well-defined outcomes and only when there is adequate evidence in the literature and when authors have sufficient confidence in the results.
- The likelihood scale should be stated in terms of probabilities (numbers) in addition to words to improve understanding of uncertainty.
- Where practical, formal expert elicitation procedures should be used to obtain subjective probabilities for key results.

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\*InterAcademy Council, *Climate change assessments: Review of the processes and procedures of the IPCC*, 2010 (Harold Shapiro, chair)

## *Major research goals of the MUG*

1. Gather several models/modelers who can perform uncertainty experiments (August 2011)
2. Identify key uncertain variables and key output variables (Spring 2012)
3. Run and compare sensitivity experiments (2012 SM meeting)
4. Prepare Pdfs for key uncertain variables (2012-13)
5. Integrate 3 and 4.
6. Collate and compare with earlier results

# 1. Gather several models/modelers who can perform uncertainty experiments

- Initial meeting at Snowmass August 2011
- Had tentative consensus on steps in project

# *Problem of comparing different species*

Problem of how to compare vastly different models, assumptions, and structures ....



## 2. Identify key uncertain variables

Rounded up usual suspects



## 2. Identify key uncertain variables

First list at 2011 SM meeting

List for first set of sensitivity runs and comparisons:

- Add an emissions pulse of 10 Gt of CO<sub>2</sub> in 2020
- Reduce output by a one-time decline of 5% of world GDP in 2020
- Increase world GDP growth rate by 0.5 % per year 2020-2100
- Change equilibrium climate sensitivity by +1 °C
- Increase population growth by 1 % per year from 2020 to 2050
- CO2 tax from \$13 in 2010 to \$427 in 2100(per AMPERE scenario AM3ND1).

### 3. Discuss and refine sensitivity experiments at Snowmass meeting

Modelers are asked to make sensitivity analyses for SM  
2012 meeting.

Will have preliminary discussion and analysis.

## 4. Prepare distributions for key uncertain variables

- Ken will discuss a proposal for next steps at the end of today.

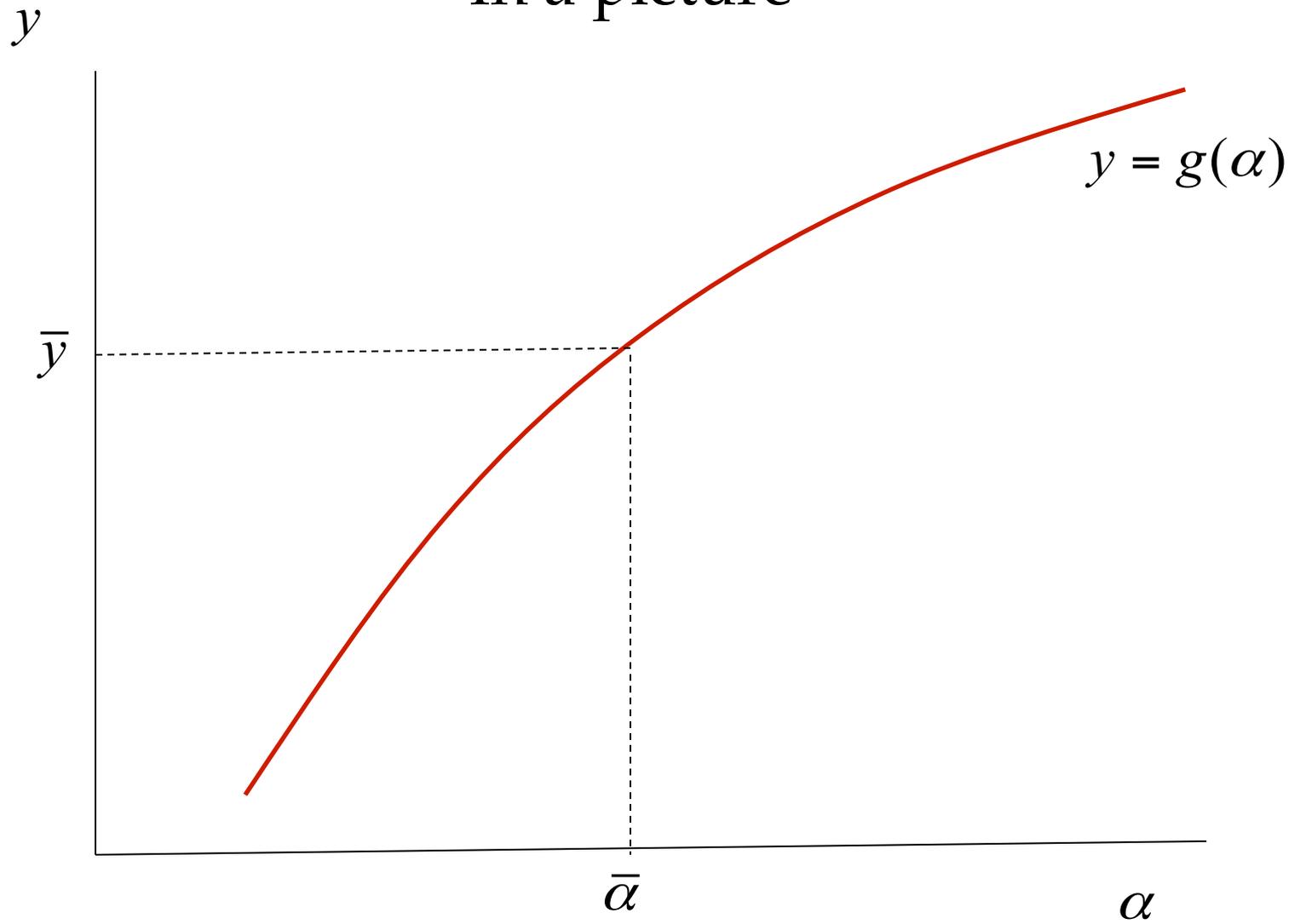
## 5. Integrate pdfs with sensitivity analyses

- **Basics:** There are multiple sources of uncertainty:
  - Model or specification uncertainty (e.g., top down v bottom-up)
  - Uncertainty of parameter values (e.g., TSC)
  - Measurement error of variables (future e.g., MER v PPP)
  - Random error (e.g., GCMs)
  - Algorithmic error (e.g., Excel Solver)
  - Coding error (e.g., OECD Green model)

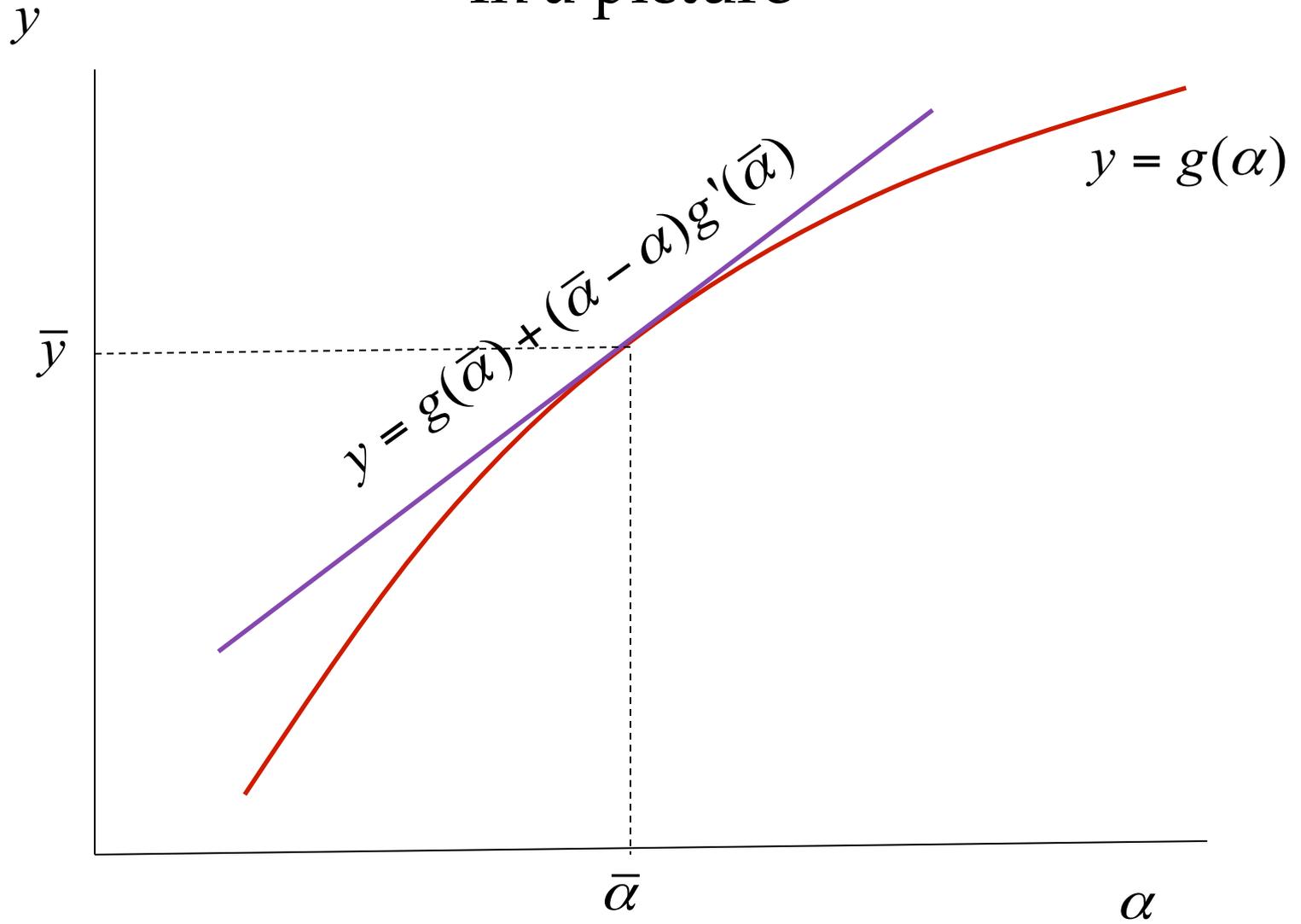
### Notes:

- Specification uncertainty is computationally intractable and underidentified because of limited observations and degrees of freedom.
- Algorithmic and coding errors are important and often ignored issues.
- Random and measurement errors probably relatively unimportant for IAMs.
- We propose to focus primarily on uncertainty of parameter values.

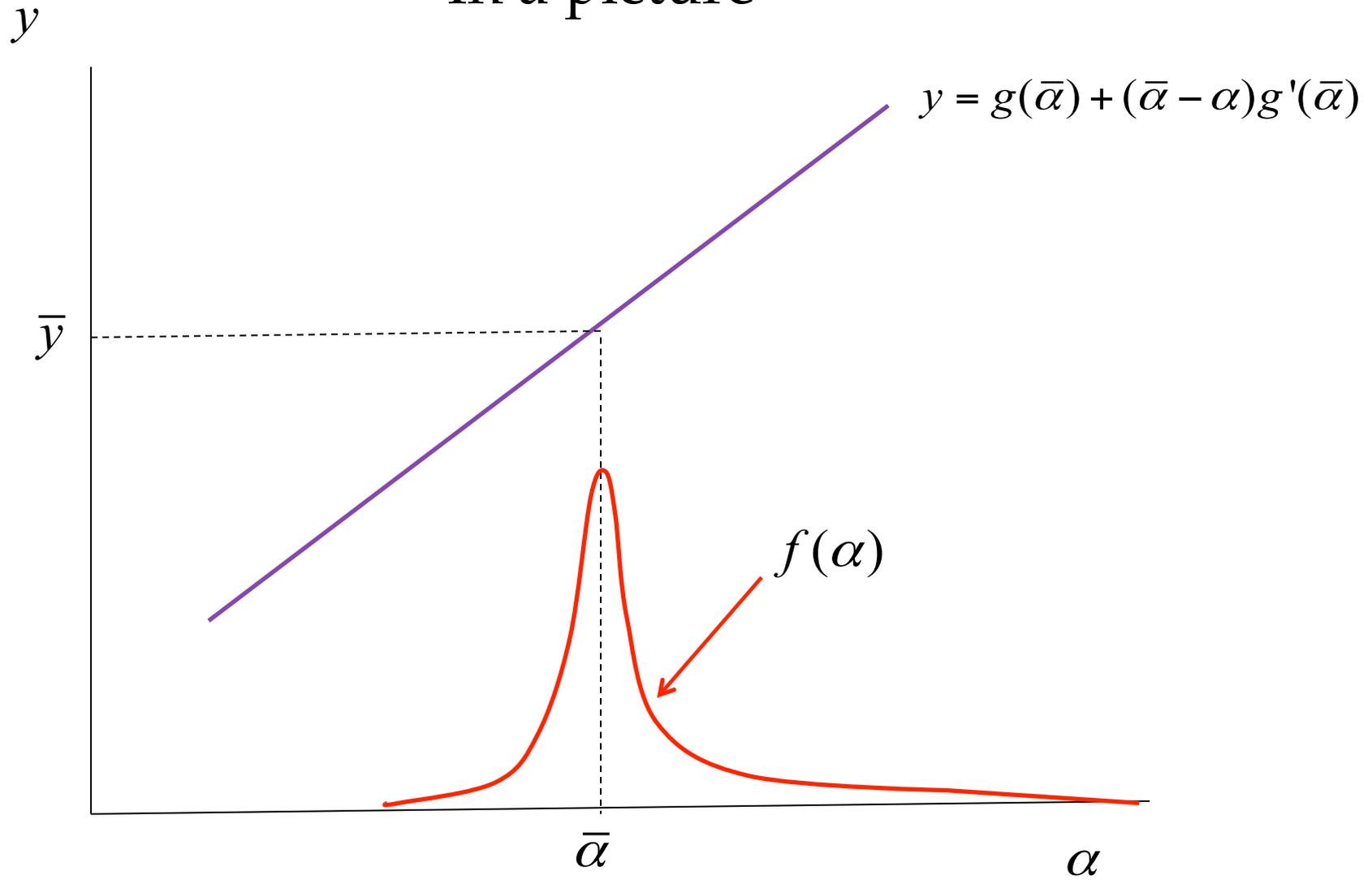
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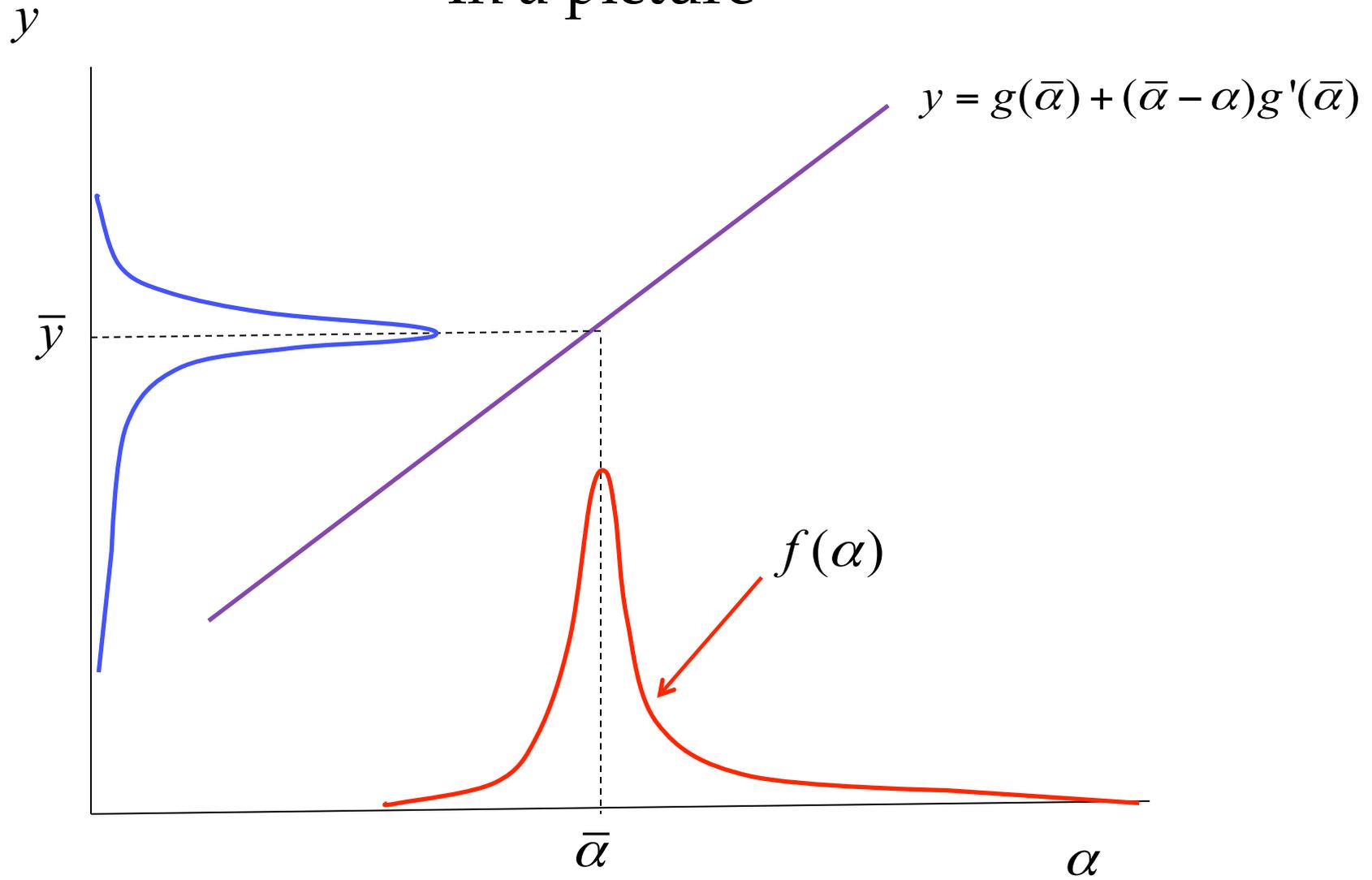
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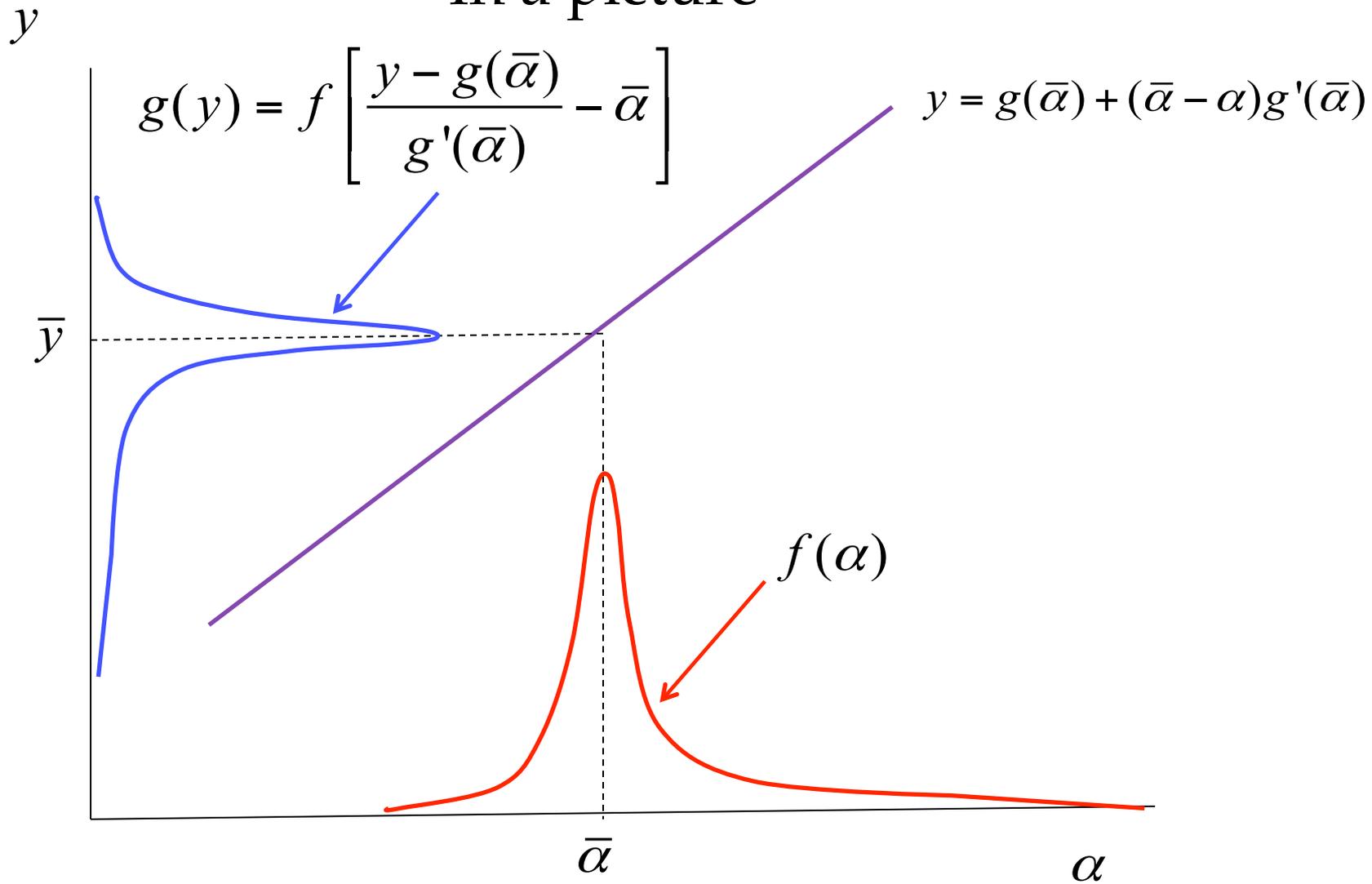
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## 5. Integrate pdfs with sensitivity analyses (cont.)

- Example of analysis of first-order parameter uncertainty

Assume a structure:

$$y = g(z, \alpha)$$

where  $y$  = endogenous,  $z$  = exogenous variables,  $\alpha = n$  uncertain parameters,  
 $g$  = model structure.

We take the first-order Taylor expansion and remove means to get:

$$y - \bar{y} = \frac{1}{2} \sum_{j=1}^n \lambda_{ij} (\alpha_i - \bar{\alpha}_i)(\alpha_j - \bar{\alpha}_j) + o^3(\alpha_j), \text{ where } \lambda_{ij} = \partial^2 g / \partial \alpha_i \partial \alpha_j \text{ evaluated at } \bar{y}.$$

Assume that we have the var-cov matrix for  $\alpha$ . We can then calculate the variance of  $y$  as:

$$V(y) = \sum_{i,j} \lambda_{ij} \text{cov}(\alpha_i, \alpha_j)$$

Or with independence:

$$V(y) = \sum_i \lambda_i^2 \sigma^2(\alpha_i)$$

- This is clearly very parsimonious and requires  $n \leq k \leq n^2$  calibration runs and between  $n \leq m \leq n(n-1)r$  parameters for the uncertainty pdfs.
- For non-linear estimates, much more computationally burdensome.

## 6. Collate and compare with earlier results

- Do over next year.
- Plan for final meeting Snowmass 2013.

# Agenda for presentatoin

- Remind people briefly about the key features of your model (brief because they are generally well known to the audience).
- It will be helpful to explain how you implemented the uncertainties. For example, what parameters were changed to implement the increase in output growth? Were there modeling problems?
- Also, you might note where there are important anomalies or problems with specific uncertainties or parameters. Which variables were poor choices, or badly defined? Are there others that should be added?
- Any analytical thoughts you have about the uncertainty analysis?
- Thoughts on next steps on generating the pdfs and integrating those with the sensitivity analyses?

