SORTING OUT FACTS AND UNCERTAINTIES IN ECONOMIC RESPONSE

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SORTING OUT FACTS AND UNCERTAINTIES IN ECONOMIC
RESPONSE TO THE PHYSICAL EFFECTS OF GLOBAL CLIMATE CHANGE

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INTRODUCTION

Conversations between researchers who concentrate upon the effects of climate change along a baseline, "best guess" trajectory of what might happen and people who are concerned about unforeseen surprises and nonlineairities which might appear as the future actually unfolds occur occasionally, but they are generally short and quite unsatisfactory. Economic efficiency requires, for example, that the marginal cost of any policy designed to abate the pace of some global change phenomenon be weighed against its marginal benefit - the marginal damage, net of adaptation but inclusive of the cost of adaptation, that would otherwise be inflicted. Presentation of work which contributes to the quantification of the (marginal) benefit side of any such policy by investigating adaptive response along a smooth, relatively likely scenario might, however, be greeted with questions about how the calculus could change if some sort of discontinuous impact were felt somewhere along the way. An economist, the author of the original work, might respond to such queries by asking for a description of a surprise event, but that response would probably lead to the fairly incredulous retort that the event in question would not be a surprise if it could be described at the outset of the exercise. Even if the surprise could be described, the economist would still want to get some idea of its relative likelihood. Only then could the damage that it might inflict and the response that it might evoke be included in a subjective expected value calculation of possible damages - the analysis required to bring the potential for surprise to bear in support of an elevated level of policy driven abatement. The
conversation might continue for a short while, in polite meetings anyway, but little headway should be expected.

The question which must be confronted, in light of the frequently encountered impasse illustrated by conversation just described, is therefore one of incorporating what little we know about nonlinearities and surprises into the analysis which supports our decision-making. If the methodology designed to overcome the impasse were sufficiently general, then there would be no need to be discouraged by how much is not known about what might happen. It would better inform current decision-making processes despite the paucity of data; it would allow new information consistently to be incorporated into future deliberations; and it could even provide insight into what type of new information would be most valuable.

This paper records some thoughts about how and why to proceed. It begins in Section I with a taxonomy of surprises which (1) recognizes the complication involved in understanding the scope of the natural and social processes which drive global change but (2) simplifies the portrait of the underlying natural and social systems in an effort uncover a practical way to proceed. Focusing upon the possibility that strongly correlated, low likelihood tails of distributions of effects might feed exaggerated impacts into nonlinear damage functions provides a means with which to capture the limit of "foreseeable surprises" which (1) meet the researchers’ need for quantification and (2) recognize the potential importance of including the damage associated with surprise into current economic analyses of various policy options designed to abate global change and thereby diminish further their relative likelihood.

Section II explores the "So what?" question. Building upon an analysis of the efficient U.S. response to the threat of greenhouse warming to her own well-being produced by William Nordhaus [1990] along a "best guess" trajectory of damage, an "uncertainty multiplier" is computed for a wide range of other, less likely scenarios. Taking the IPCC range of temperature sensitivity
to a doubling of (effective) carbon concentrations as a measure of the uncertainty which surrounds current understanding of the physical phenomenon behind global warming, the Nordhaus measure of the marginal cost associated with the efficient response for the U.S. to the effects of doubling is multiplied by as little as 0.46 and as much as 14.99. The expected value of this uncertainty multiplier is 2.64, a value which increases the efficient reduction in cumulative carbon emissions for the United States through the year 2050 from 6% to nearly 15%.

Building a method of systematically recognizing the exaggerated effect of the unfortunate coincidence of unlikely events and nonlinear damages clearly makes a difference. The precise numerical results recorded here are, of course, dependent upon the modeling and the interpretation of physical and economic data offered in their support. Some generalization of their significance, with particular reference to the broader issue of application is offered in Section III, but the implication of the actual multiplier computed here cannot be ignored. The effect of incorporating the potential cost of low-likelihood scenarios of what the future might hold is large enough to increase substantially the efficient response of the United States to the possible damage of greenhouse warming felt within her borders.

I. A TAXONOMY OF SURPRISES

Figure I.1 displays a Social Process Diagram created during the summer of 1991 to complement the Physical Climate System "wiring diagram" depicted in Figure I.2. The arrows which connect Box 7, "Global Scale Environmental Processes", to the social and psychological structures scattered throughout Figure I.1 were, in fact, intended to mimic the arrows which link the three "Human Activity" circles appearing on the right hand side of Figure I.2 with various aspects of the physical system. A common interface between social and physical processes is thus
HUMAN DIMENSIONS OF GLOBAL ENVIRONMENTAL CHANGE

Social Process Diagram

Figure I.1
defined so that the two diagrams, taken together, illustrate the complicated interaction between human activity and the sources of global environmental change.

Economic, political, and social institutions are, of course, all linked in Figure I.1 even without reference to global environmental process; their evolution over time should be driven by population and directed by preferences and expectations even in the absence of global scale processes. Adding global physical processes to the picture simply adds another source of stress on the system, perhaps evoking decisions designed to ameliorate their effects, to slow their progress, or both. It is, however, impossible to judge the ability of the human activity either to create global scale processes or to influence their trajectories over time without a thorough understanding of the physical interactions of Figure I.2.

Figure I.3 withdraws from the detail of the first two diagrams to offer an overly simplistic schematic of the link between global environmental change phenomena and their associated economic damage. It is drawn to focus attention on the critical interface between Figures I.1 and I.2. Despite masking the complication which clouds our understanding of the underlying physical and human processes of Figures I.1 and I.2, its stark simplicity underscores vividly the compounding of processes which must be recognized in careful consideration of the full range of possible futures. Uncoupling this compounding process provides useful insight into how sources of significant surprise might be evaluated even given current, subjective, time-dependent views of how various component parts of the future might unfold - views which attribute very little likelihood to the outlying tails of the underlying distributions.

The first link in Figure I.3, labeled A, is meant to capture the workings of the entire Physical Climate System diagram. Something drives a change in the climate (e.g., a warming driven by increased concentrations of greenhouse gases), and a vector of physical impacts is expected. Certain impacts may be larger or smaller than others, but their relative importance to
Figure I.3
human activity cannot be judged in isolation. The second link, labeled B, works toward uncovering a reasonable basis upon which to make such a judgement. It reflects a computation of the resulting economic damage, but it is a computation which can be calibrated only after the ramifications of the impacts vector have been fully considered. Each impact must, in fact, work its way through the maze of the Social Process Diagram to include any adaptive and/or ameliorating response which might be forthcoming.

The enormous complexity of Figures I.1 and I.2 notwithstanding, Figure I.3 can provide a convenient context for exploring the compounding potential of Links A and B. Figure I.4a offers a straightforward portrait of a typical "best guess" scenario. Much like the IPCC projections of sea level rise, for example, it shows a smooth, linear progression of physical change and associated impact over time for Link A in the lower right hand quadrant. It also shows a slowly increasing, linear correspondence between physical impact and economic damage in the upper left hand quadrant - a correspondence for Link B which would result from systematic response to the predictable and anticipated impacts of predictable and anticipated climate change. The result is a smooth, gradual expansion of economic damage over time in the upper right hand quadrant; this is, of course, a picture of the least troubling of the possible outcomes of global change. Notice, for future reference, that the economic damage schedule has been drawn arbitrarily so that it climbs from 0 in the year 1990 to an indexed value of 1 by the year 2050. This convention is reflected in Figure I.4a by the box drawn around the origin with arrowheads identifying the damage point for 2050 - the indicated "circle" point in the upper right hand quadrant.

If the linear progression of physical change depicted for Link A were associated with a series of "surprise" crossings of unanticipated thresholds, however, then economic damage would certainly be more sensitive to physical changes. The correspondence between impact and damage recorded for Link B would rise more quickly and perhaps nonlinearly. Figure I.4b superimposes
Figure 1.4b
this possibility over the "best guess" baseline, and produces a more severe time trajectory. A damage index of 2 is reflected in the upper right hand quadrant by the highlighted "X" point for 2050, again for reference only. The nonlinearity in the index-damage correspondence is shown producing a damage trajectory whose slope increases over time.

A similar escalation in the ultimate damage trajectory over time might also be expected if the physical system were increasingly sensitive to the climate change so that its impacts appeared more quickly. Even if they were fully anticipated so that adaptation were no more costly than before, the intertemporal damage profile would be steeper than before. Figure 1.4c displays this possibility. Only the intertemporal impact profile is changed from Figure 1.4a, but a damage index of 2 is again suggested for the year 2050, this time indicated by the highlighted "diamond" point. Notice that nonlinearity in an underlying correspondence again produces a nonlinear damage trajectory whose slope increases over time.

Neither of these two deviant cases captures the essence of the most worrisome types of potential surprise and compounding nonlinearities, however. The futures which provoke the most speculation and evoke the most concern involve rapid or sudden physical change, large and immediate impact sensitivity, and rapidly escalating damages which could be diminished only slightly, if at all, by adaptive response. Figure 1.4d reflects such a possibility simply by supplementing the nonlinear impact to damage profile of Figure 1.4b with the nonlinear intertemporal impact trajectory of Figure 1.4c; and it shows an exaggerated discrepancy from the "best guess" trajectory of Figure 1.4a. Figures 1.4b and 1.4c were drawn so that each taken alone would create a 100% increase in the cost index by the year 2050. If linearity had been preserved within the changes reflected by both cases, then the combined effect to be drawn in Figure 1.4d would have been a 200% increase in the cost index for 2050. The changes represented in the two underlying cases were, however, not linear. The nonlinearities displayed there combine to
produce a cost index for 2050 identified by the convergence of the arrowheads well in excess of 4 - an increase of more than 300%. The compounding effect shown in 1.4d is thus the result of an unfortunate coincidence of two "bad" circumstances amplified by the nonlinearities which define each.

The shapes of the curves drawn in the various versions of Figure 1.4 are critical; and their precise shapes are, of course, entirely arbitrary. They can, nonetheless, be used to suggest one means by which the potential for significant surprises might be reflected in an appropriate deliberation of how to respond now to the possibility of global environmental change sometime in the future. Much of the best current debate is informed by subjective, expected value calculations, but little attention is typically paid to how the tails of the underlying distributions my coincide. Figure 1.4d shows, however, that even a cursory consideration of the interface between the physical impacts and the economic damages heralds the need to recognize the potential for strongly correlated series of "bad" circumstances. Its fundamental message is that a cascading confluence of tails of existing subjective distributions thrust against nonlinear impact and/or damage schedules can identify surprises whose relative likelihood and potential damage can both be quantified. This sort of confluence can certainly produce interesting, "what if" scenarios to be considered, but it can, perhaps, do much more. Incorporating correlated surprises into the expected value computations which frame the current policy debate could change both the relevant mean estimates of damages and costs and distributions of the appropriate responses. Unless the ultimate distributions are symmetric and the associated damage functions are linear, consideration of what happens along the "best guess" scenario is not the same thing as consideration of what happens to define the certainty equivalent scenario computed across a distribution of possible futures.
II. CASCADING UNCERTAINTY AND THE EFFICIENT CARBON TAX

Nordhaus (1990) has produced perhaps the only attempt to quantify the efficient response of any significant nation or region to the threat of greenhouse warming. He certainly records the details an initial effort to compute a response to the threat of greenhouse warming which would equate the marginal benefits of policies designed to reduce future concentrations of greenhouse gases with their marginal cost. His results, based upon baseline damage estimates drawn for the United States by the Environmental Protection Agency, have attracted a considerable amount of attention - not only because they are firmly rooted in theoretically sound economic analysis, but also because they call for such a small response. After expanding estimated damages by a factor of nearly four to capture unmeasured and unmeasurable impacts, in fact, Nordhaus shows that efficiency criteria applied to mid-range damages of roughly 1% of GDP support only a 17% reduction in total greenhouse emissions engineered by growing a large number of trees, phasing out CFC consumption, and optimally reducing cumulative carbon emissions through 2050 by 6%. The marginal cost of this response, the shadow price of the targeted emissions reduction and thus the corresponding tax to be applied to carbon emissions, is roughly $13 per ton of carbon dioxide.

The Nordhaus work is, of course, based upon a set of "best guess" damage estimates calculated under the assumption that an effective doubling of carbon dioxide concentrations would occur around the year 2050. The major lesson of Section I is, however, that looking at the tails of distributions of possible impacts and potential damages could easily substantiate warnings of exaggerated damages and thereby push estimates of expected damages well above "best guess" trajectories. The remainder of Section II will employ the Nordhaus analytical structure to explore the degree to which elevated estimates of damages which are consistent with current and coincident subjective views of future circumstances for the United States might enlarge the theoretically justified efficient response. It will, in other words, explore the effects of recognizing
the potential for nonlinear damage functions and "foreseeable" surprises on the need to respond more vigorously to the threats of global change. Does, in short, the insight offered in Section I matter at all?

A quick review of the structure of the Nordhaus model will be followed in turn by an extension which identifies an "uncertainty multiplier" - an index which reflects the degree to which the "best guess" scenario employed by Nordhaus to compute marginal damages cause him to underestimate (or, perhaps, to overestimate) the marginal damage associated with other possible futures. A review of the existing literature provides enough information about the structural components of that multiplier to support an informed attempt to quantify a subjective density function over its range and to judge its expected value. A final subsection will relate the associated distribution of uncertainty multipliers to a set of efficient marginal cost (carbon tax) statistics that apply to across a plausible collection of possible futures. It is these statistics which are finally employed to equate the expected marginal benefit of reduced greenhouse gas concentrations (damage avoided) with the marginal cost of achieving that reduction.

II.1. The Nordhaus Model

The operative Nordhaus model begins with a simplified temperature adjustment process characterized by:

\[
\frac{dT}{dt} = a\{\mu M(t) - T(t)\}, \quad \text{with} \quad (\text{II.1})
\]

\[
\frac{dM}{dt} = bE(t) - \delta M(t). \quad (\text{II.2})
\]

Notationally, the variables T(t), M(t), and E(t) represent the driving forces behind potential global environmental change. More specifically,

(i) T(t) represents the increase in global mean temperature through time t generated by greenhouse warming since the preindustrial period of the middle of the last century;
(ii) $M(t)$ represents the atmospheric concentration of greenhouse gases at time $t$ denominated in terms of carbon dioxide equivalents; and

(iii) $E(t)$ represents the emission in time $t$ of greenhouse gases, again denominated in terms of carbon dioxide equivalents.

Parameters $a$, $\mu$, $b$ and $\delta$ meanwhile define the relationships, with

(i) $a$ reflecting a delay parameter which correlates a realized increase in temperature to a prior increase in radiative forcing;

(ii) $b$ indicating the fraction of carbon equivalent emissions which actually remain airborne;

(iii) $\delta$ representing a corresponding physical decay parameter for aggregated atmospheric concentrations of greenhouse gases; and

(iv) $\mu$ representing the (linearized) sensitivity of equilibrium temperature change to changes in atmospheric concentrations of greenhouse gases.

Equations II.1 and II.2 fully describe the Link A structure required to operate within the simplistic schematic of Figure I.3.

The economic side of the model, Link B in the parlance of Section I, is summarized by

$$c(t) = y(t)\{g(E^0)\cdot\phi(T^0)\} \quad \text{with}$$

$$y(t) = y^0e^{ht}. \quad \text{(II.4)}$$

Notationally,

(i) $c(t)$ represents per capital consumption at time $t$;

(ii) $y(t)$ represents per capital output growing in the absence of any emissions reduction and any deleterious effects of climate change at an annual rate of $h$;

(iii) $g(E^0)$ represents a steady state computation of the cost of reducing emissions of greenhouse gases; and
(iv) $\phi(T^*)$ represents a steady state computation of the economic damage associated with climate change.

It should be clear that temperature is being used as an index of climate change. It should be noted with equal clarity that both costs are measured in steady state after any change in radiative forcing has achieved its long run equilibrium.

The Nordhaus model operates as an exercise in long run optimization, with

$$V = \int u[c(t)]e^{\beta t}dt$$  \hspace{1cm} (II.5a)

serving as the objective function. The function $u[c(t)]$ is, of course, a utility function which allows for the possibility that the global society displays a systematic aversion to risk. For present purposes, however, including this possibility would "stack the deck" in favor of expanding uncertainty to include nonlinearities and surprises. It would hardly be news to conclude, under conditions of even moderate aversion to risk, that surprises and nonlinearities have important welfare effects which should be incorporated into the global decision-making calculus. It is, therefore, potentially far more interesting to see if recognizing the possibility of extreme events might have an effect even on risk-neutral decision-making. If so, subsequent recognition of societal risk aversion would only exacerbate the effect. A risk-neutral objective function,

$$W = \int \{c[t]\}e^{\beta t}dt$$  \hspace{1cm} (II.5b)

will thus be employed. Abstracting to a risk neutral objective function does not, of course, eliminate the need for specifying a real discount factor - the pure rate of time preference with which the present value of future consumption is computed. Risk neutrality does, however, imply that the elasticity of marginal utility with respect to per capital consumption is zero. The real rate of return on investment, denoted $r$, should therefore match the pure rate of time preference, the $\beta$ parameter in equations (II.5).
The condition which characterizes the solution of the long term optimization problem - maximize $W$ subject to the constraints imposed on the system by equations (II.1) through (II.4) - is now at hand. It states quite simply that the present value of any small change in the emissions trajectory should be zero; i.e., the immediate increase in per capita consumption associated with a small increase in emissions should be matched by a increase in the present value of the damage, denominated in reduced consumption, associated with the long run effect of those higher emissions. Nordhaus shows that this simple statement amounts to requiring that:

$$y^*g'(E^*)dE = \int [y^* e^{\delta t} \phi(T^*)dT] e^{-rt} dt.$$  \hspace{1cm} (II.6a)

Equations (II.1) and (II.2) meanwhile combine under the assumption that $\delta < a$ to define $dT(t)$ in terms of physical parameters; more specifically,

$$dT(t) = \mu b e^{-\delta t} [1-e^{-at}] dE.$$

As a result, equation (II.6a) simplifies immediately to

$$g'(E^*) = \mu b \phi(T^*) A,$$  \hspace{1cm} (II.6b)

where the last term, $A$, is given by

$$A = \frac{1}{r-h+\delta} - \frac{1}{r-h+\delta+a}.$$

It is equation (II.6b), under appropriate specification of the various physical and economic parameters, which supports the ultimate Nordhaus estimate of the efficient response to greenhouse warming; and it is an investigation of the effect of uncertainty and nonlinearities on the right hand side of equation (II.6b) which will suggest the degree to which that efficient response should be adjusted to accommodate the possibility of surprises and nonlinearities which can be quantified even now. An amended optimality condition, that the immediate increase in consumption associated with a small increase in emissions should be set equal to the present value of damage associated with the long run effect of those emissions along any scenario will produce
a distribution of efficient responses contingent upon those scenarios. Aggregating over that range
of response will then produce a second amended optimality condition, that the immediate increase
in consumption associated with a small increase in emissions should be set equal to the expected
present value of damage associated with the long run effect of those emissions.

II.2. EXPECTED MARGINAL DAMAGES OF EMISSIONS - AN UNCERTAINTY
MULTIPLIER

The marginal damage of emissions is the primary economic component of the right hand
side of equation (II.6b). As reported earlier, Nordhaus produced his point estimate of this
component by relating damage statistics offered by the Environmental Protection Agency for a
baseline warming scenario to the most vulnerable sectors of the national income accounts of the
United States. He assumed, in creating his estimate, that the most likely scenario would see an
effective doubling of carbon dioxide concentration by the middle of the next century. Generating
a series of marginal damage estimates across a range of possible future requires more than a
single baseline estimate, however; it requires, instead, a marginal damage schedule defined
throughout that range. Such schedules are few and far between, but one does exist for the
economic vulnerability of the United States to greenhouse induced sea level rise.

Table II.1 records estimates of national vulnerability for the United States as a function of
greenhouse induced sea level rise presented initially in Yohe [1991a]. It is displayed in the two
panels of Figure II.1. The data recorded there are based upon a sample of over thirty sites
systematically distributed along the entire coastline, so they take variation in the natural rate of
subsidence into account. A simple econometric fit of their trajectory suggests that

\[ \phi(SLR) = \phi_0 e^{gSLR} \]  

(II.7a)
can satisfactorily summarize their content, with \( g = .0253 \) for \( SLR < 80 \text{cm} \) and \( g = .0109 \) for
\( SLR > 80 \text{cm} \). Clearly, then,
\( \phi'(SLR) = g \phi e^{gSLR} \)  

(II.7b)

can be advanced as a possible candidate to serve as a proxy for the requisite marginal damage schedule.

The validity of using equation (II.7b) to represent overall marginal damages depends upon a number of considerations. The cost of sea level rise along unprotected coastline did, however, sum with the cost of protection to quantify the dominant source of potential damage for the United States in the Nordhaus work. Assuming that this dominance persists over the range of possible futures and that the cost of protection rises proportionately with the economic vulnerability of affected locations, advancing equation (II.7b) as a rough approximation of at least the proper form of the marginal damage component of equation (II.6b) is not totally unwarranted. Its structure is consistent, at the very least, with the notion that increasingly severe changes in climate should move the earth along nonlinear damage functions because they will be associated with increasingly frequent episodes of costly effects and adaptation.

Distributions of future sea level rise related to anticipated increases in equilibrium temperature are, in addition, required to support the structure of equation (II.7b). Given any specific expectation about the temperature sensitivity of doubling, recent work by Wilson (1988), Oerlemans (1989) and Shlyakhter & Kammen (1992) suggests that an exponential distribution of predicted sea level rise is most appropriate. A density function

\( f(SLR) = \alpha e^{-\alpha SLR} \)  

(II.8)

can thus be advance with a mean \( \mu = (1/\alpha) \) dependent upon an assumed doubling sensitivity for temperature. The IPCC Scientific Assessment meanwhile offers a best guess that an effective doubling of carbon dioxide concentrations would force a 2.5°C increase in equilibrium temperature and a 66cm increase in sea level rise by the year 2100. The low end of the temperature range reported there stands at 1.5°C, presumably associated with the low end of the
reported potential for sea level rise (33cm through 2100); the high end of temperature sensitivity
stands at 4.5°C with a 99cm sea level rise. Using \( T_o = 2.5°C \) as the basis for a temperature index,
\([T_d/T_o]\), and taking the IPCC sea level scenarios as mean estimates, a two part linear relationship
between the temperature index and the \( \alpha \) required in underlying sea level density function can
be computed:

\[
\alpha(T_d) = 0.040 - 0.04 [T_d/T_o] \quad [T_d/T_o] < 1 \\
\alpha(T_d) = 0.023 - 0.007 [T_d/T_o] \quad [T_d/T_o] \geq 1.
\] (II.9)

Equations (II.8) and (II.9) can therefore combine to relate distributions of sea level rise to
anticipated temperature increases associated with an effective doubling of carbon dioxide
concentrations.

It is most convenient, at this point, to bring equations (II.7b), (II.8), and (II.9) together to
synthesize a final representation of the expected marginal damage function. Let \( SLR_o \) represent
the expected sea level rise associated in equilibrium with the IPCC best guess doubling
temperature sensitivity of 2.5°C. A short Taylor expansion of the right hand side of (II.7b) can
then employed to produce

\[
E\{\phi'(T_d)\} = \phi_o g_{SLR_o} e^{g_{SLR_o}} \int e^{g(SLR(T_d)-SLR_o)} f(SLR | T_d) dSLR 
\] (II.10a)

for any given \( T_d \) and allowing

\[
f(SLR | T_d) = \alpha(T_d) e^{-\alpha(T_d)}.
\]

Performing the integration indicated allows a dramatic simplification of equation (II.10a):

\[
E\{\phi'(T_d)\} = \phi_o g_{SLR_o} e^{g_{SLR_o}} \frac{\alpha(T_d)}{[\alpha(T_d)-g]} e^{2g_{SLR_o}}.
\] (II.10b)

The cost structure thereby fully described, attention must now turn the other terms in the right
hand side of equation (II.6b) and their relationship with the distribution of anticipated doubling
temperature increases.
The first term appearing there is $\mu$, the sensitivity of increase in equilibrium temperature to a change in the concentration of greenhouse gases. When equilibrium has been achieved, of course, $[dT(t)/dt] = 0$, so equation (II.1) reduces to

$$T^*(t) = \mu M^*(t)$$

Recalling a commonly employed relationship between doubling temperature and concentrations, specifically that

$$T(t) = \frac{T_d}{\ln(M(t)/M(0))} \ln(2)$$

$$\approx \{T_d/[M(0)\ln(2)]\} \{M(t)-M(0)\},$$

it becomes reasonable to assert that

$$\mu \approx \{T_d/[M(0)\ln(2)]\}$$

$$= \{T_d/[M(0)\ln(2)]\} \{T_d/T_o\}.$$

It becomes essential, therefore, to explore the subjective distribution of the previously defined index of doubling temperature sensitivity; i.e., the distribution of $\{T_d/T_o\}$.

The IPCC Scientific Assessment [1990] provides some insight into that distribution, but not very much. It records, in Table 3.2(a), a series of recent studies which report equilibrium doubling temperature increases between 1.9°C and 5.2°C. The authors weigh this evidence against modeling results provided to the IPCC from independent researchers to conclude that:

...the sensitivity of global mean surface temperature to doubling (effective) carbon dioxide (concentrations) is unlikely to lie outside the range of 1.5 to 4.5°C. There is no compelling evidence to suggest in what part of this range the correct value is most likely to lie. There is no particular virtue in choosing the middle of the range, and both the sensitivity and the observational evidence neglecting factors other than the greenhouse effect indicate that a value in the lower part of the range may be more likely. Most scientists decline to give a single number, but for the purpose of illustrating IPCC scenarios, a value of 2.5°C is considered to be the "best guess" in the light of current knowledge [page 139].

Setting 2.5°C as the benchmark "best guess" $T_o$, this passage suggests that the doubling temperature increase index $\{T_d/T_o\}$ could be as low as 0.6 or as high as 1.8. Placing equal weight
on the likelihood that 0.6, 1.2, and 1.8 will turn out to be the correct value (to reflect the "no compelling evidence to suggest in what part of this range the correct value is most likely to lie" phrase) yields an implicit variance for a representative subjective distribution over index number range of 0.24. Imposing a gamma distribution over the range to capture the notion that the "best guess" index number is 1 (for a doubling temperature of 2.5°C) then suggests that \( f_{T_d}(T_d) = \Gamma(5;6) \) might be a reasonable density function. Table II.2 displays the resulting relative frequency weights across the prescribed range of possible values, showing a modal value of 1 lying just below the median; Figure II.2 illustrates the truncated distribution across the temperature sensitivity interval [0.6,1.8].

The other terms in the right hand side of equation (II.6b) are less problematical and less important. The discount parameter \( A \) depends, for the most part, upon economic parameters which are best handled with sensitivity analysis. The one exception is \( \delta \), the atmospheric decay parameter which is small relative to \((r-h)\) and negatively correlated with \( b \), the airborne fraction parameter. The value assumed by this airborne fraction is generally uncorrelated with the doubling sensitivity of the climate, since it effects a process which occurs before radiative forcing occurs. It is therefore unlikely that either \( b \) or \( A \) will systematically influence either the \( \{T_d/T_o\} \) index or the marginal damages associated with a particular climate change effect, so uncertainty in both will be ignored.

It is now possible to express the expected value of the right hand side of equation (I.6b) in terms which are familiar and easily interpretable. Allowing \( b \), \( A \), \( r \), \( h \) and \( \delta \) now to represent either the expected values of underlying random variables which are uncorrelated with anticipated doubling sensitivities or specific values for exogenous economic variables which frame the overall
\[ y = 3.5655 + 0.02529x \quad R = 0.97593 \]

Figure II.1a
growth context of the greenhouse problem, that expected value can now be written as

$$E(\text{RHS}) = Ab \int \frac{T_o}{\ln(2)} \left[ \frac{\alpha(T_d)}{[\alpha(T_d)-g]} \right] c^{gSLR_o} f_{T_d}[T_d] dT_d$$

$$= Ab \phi_o g SLR_o c^{gSLR_o} \int \frac{\alpha(T_d)}{[\alpha(T_d)-g]} \frac{\alpha(T_d)}{[T_d/T_o]} c^{gSLR_o} f_{T_d}[T_d] dT_d$$

Everything to the left of the integral sign is captured in the Nordhaus baseline estimate of marginal damage. Everything to the right, therefore, is an uncertainty index which can exaggerate or diminish the original baseline statistic. The key to producing an understanding of the degree to which uncertainty would cause the expected marginal damage of increased emissions to exceed the baseline estimate therefore lies in understanding the degree to which this uncertainty index,

$$\pi = \int \frac{\alpha(T_d)}{[\alpha(T_d)-g]} c^{gSLR_o} f_{T_d}[T_d] dT_d$$

$$= \int \frac{\alpha(T_d)}{[\alpha(T_d)-g]} \frac{\alpha(T_d)}{[T_d/T_o]} D[T_d] f_{T_d}[T_d] dT_d$$

$$= \int \pi[T_d/T_o] f_{T_d}(T_d) dT_d,$$

exceeds unity. The key to producing a range of possible marginal damage statistics contingent upon specific temperature sensitivities within the quoted IPCC range similarly lies in investigating the range of values which might be assumed by the various $\pi[T_d/T_o]$ - the weighted marginal damage multipliers defined implicitly by equation (II.11).

II.3. QUANTIFYING THE UNCERTAINTY MULTIPLIER

Table II.3 displays the critical results, given the specifics of the modeling extension described above. The second column records the marginal damage multiplier for each $[T_d/T_o]$ index - the
\[ y = 4.6689 + 0.010875x \quad R = 0.99392 \]

Figure II.1b
Figure II.3b
$D[T_d]$ parameter implicitly defined in equation (II.11) as

$$D[T_d] = \frac{\alpha(T_d)}{[\alpha(T_d) - g]} e^{gS_LR_0}.$$

Column (3) records the corresponding weighted marginal damage multiplier - the $\pi[T_d/T_o]$ computed according to equation II.11 as the product of the index value of Column (1) and the damage multiplier shown in Column (2). Notice that these uncertainty multipliers run from a low of 0.45, for a doubling temperature index of 0.6 (doubling associated with 1.5°C), to a high of 14.99 for a temperature index of 1.8 (an equilibrium doubling temperature of 4.5°C). These values represent the expected damage multiplier contingent upon the indicated value of $[T_d/T_o]$. For $[T_d/T_o] = 0.6$, for example, $T_d = 1.5^\circ C$ and $\pi[T_d/T_o] = 0.46$ so that the expected marginal damage estimate given a doubling temperature of $1.5^\circ C$ is $5.84$. For $[T_d/T_o] = 1.0$, $T_d = 2.5^\circ C$, $\pi[T_d/T_o] = 1.42$ is the resulting expected damage multiplier and $18.03$ represents the contingent estimate of expected marginal damage. Notice that uncertainty in our understanding of possible sea level rise trajectories even given the best guess temperature estimate increases marginal damage by 42%. On the opposite extreme, the contingent estimate of expected marginal damage is $190.50$ when $T_d = 4.5^\circ C$ so that $[T_d/T_o] = 1.8$ and the expected damage multiplier is 15.00. The expected value calculation prescribed by equation (II.11) yields a mean of 2.64 over the entire range - a value roughly matching the 70th percentile of the $\pi[T_d/T_o]$ distribution.

Recall that the multipliers of Column (3) are translated into marginal damage estimates in Column (4) by multiplying them by the middle value reported by Nordhaus (i.e., $12.70$). A mean of $33.53$ lies above the median of a distribution stretching from $5.84$ on the low end to $190.37$ on the high side. Column (5) finally relates these marginal damage statistics to the marginal cost of reducing carbon emissions, thereby suggesting a range of emission reductions which could
prove to be efficient. The marginal cost figures used to support these reduction percentages are
drawn, once again, from the original Nordhaus work. His Figure 5, in particular, displays a
composite marginal cost curve which is the result of a log-linear regression of emissions
reductions against estimated cost run on data recorded in published long run carbon emissions
scenarios.\textsuperscript{9} Emissions reductions supported by an efficiency criteria which equates their marginal
cost with the expected marginal damage of allowed emissions run from a 3% reduction in
cumulative emissions through 2050 (if a 1.5°C increase in the global mean temperature were
associated with an effective doubling of carbon concentrations) to a 61% reduction in cumulative
emissions (if doubling were to cause a 4.5°C increase). Figure II.3 graphically displays the
relationship between the \([T_d/T_o]\) index and efficient cumulative reductions in carbon emissions.
The mean percentage reduction, roughly equal to 14%, lies slightly below the 15% reduction
supported by the mean damage multiplier.

The specific numbers recorded in Table II.3 are certainly the product of the underlying
structure. They are, however, quite insensitive to changes in the distribution of the doubling
temperature which preserve the general gamma shape displayed in Table II.2; i.e., fairly uniform
density functions with some increased weight given to the lower half of the 1.5°C to 4.5°C range.
Not surprisingly, however, the range of marginal damage estimates is extremely sensitive to the
specific nonlinearity of the conditional damage function - the \(\phi(SLR)\) function characterized in
equation (II.7a). If that function were linear in sea level rise, as an extreme example, then
expected marginal damages would be a mere 23% higher than the Nordhaus estimate and support
only a 7% emissions reduction.
III. CONCLUSIONS

Incorporating the subjective distributions of the uncertainty with which the future effects of global change phenomena are currently viewed is a difficult process even before consideration of nonlinear impacts and dramatic surprises is added to the calculus. Proper evaluation across a range of possible futures requires, at the very least, some understanding of how a schedule of impacts and potential damage (net of adaptation but including the cost of adaptation) might be constructed over a range of foreseeable outcomes. Extending such a schedule to include events which lie well outside the realm of experience and far away from the best guess expectation requires expensive research into the consequences of unlikely possibilities. These schedules have, for the most part, not yet been constructed, because they require an extraordinary amount of research into "what if" scenarios which are presently deemed "not very likely". The exaggerated effects and potential damages associated with the lightly weighted but perhaps highly correlated tails of existing subjective distributions suggest, however, that devoting scarce research resources to that end might pay dividends over the long run.

The present paper supports this contention by exploring the degree to which extrapolating the basic form of an available schedule of economic vulnerability to greenhouse induced sea level rise might alter the efficient abatement response to the threat of global warming. Correlated subjective distributions of temperature sensitivity and associated sea level rise were employed to reflect both the potential for nonlinear damage and the possibility that uncertainties might cascade to add more weight to the unfortunate coincidence of extreme events. Quantification of both the correlated distributions impacts and the nonlinearities of their consequences combined to increase the baseline marginal damage estimate of Nordhaus [1990] by more than 160%, and thereby increase the corresponding efficient reduction in cumulative carbon emissions for the United States from 6% to roughly 15%. Coupled with a complete phase-out of CFC
consumption and a 1% reduction in carbon emissions produced by carbon sequestering in managed forests, adding uncertainty to the calculus brought to almost 30% the efficient cumulative reduction in the emission of greenhouse gases through the year 2050. It must be noted, however, that even this higher response falls well short of the 10% or 20% reductions in emissions relative to 1990 levels which were discussed in the negotiations prior to UNCED in June of 1992. Stabilizing emissions of carbon dioxide at 1990 levels would, for example, be consistent with a 50% reduction in cumulative emissions through 2050.

Besides adding some weight to the claim that more substantial emissions reductions should be supported by even the United States, the results suggest something more fundamental for the conduct of research into issues of global change. Analyses of these sorts of issues are typically so involved that careful attention can be paid to only a very limited number of possible futures. Best guess scenarios have typically been selected, especially when time and resources reduce this number to one, but that might be a mistake. Taken qualitatively, the results reported above suggest that focusing on a scenario which describes something around the 75th percentile of potential economic damage might be a better choice - a better reflection of the potential significance of expected damage computed to include the coincidence of "bad-news" tails. At the very least, the construction of an uncertainty multiplier which measures the added expected cost of a wide range of extreme outcomes, is shown to be a productive tactic with which to surround a "best guess" or "70th percentile" trajectory with some illustrative measure of the uncertainty with which the future is viewed.

It should be noted, of course, that all of the analysis presented here was built upon the house-of-cards which is over-simplification. The original Nordhaus paper abstracts dramatically from the complexity of the natural and social process which drive global change - Figure I.3 is a better portrait of its structure than the larger synthesis of Figures I.1 and I.2. It also assumes that
the composition of the United States economic in the year 2050 will look like the composition of 1981. It ignores investment possibilities, even as it looks at the tradeoff between current and future consumption, and it ignores other market failures which might increase or reduce the degree of efficient response to greenhouse warming. The extension presented in Section II.2 adds to these oversimplifications by extrapolating the shape of the aggregate damage function from the shape of the function relating sea level rise to economic vulnerability - an expansion of the consistent composition assumption. It also linearizes some complicated structures and assumes risk neutrality in the social objective function.

Allowing more structural flexibility would certainly reduce damages, but adding risk aversion would certainly increase their welfare cost. The net effect of all of this simplification is likely to be significant, but the direction in which the uncertainty multipliers would move if more complete reflections of what might happen were included is unknown. The lesson that systematic inclusion of possible surprise events and nonlinear damages should make us more cautious in the protection of our health and well-being should not, therefore, be dismissed as the consequence of oversimplification.
REFERENCES


ENDNOTES

1. Figure I.1 was prepared at a workshop hosted by the Aspen Global Change Institute in the summer of 1991 under the sponsorship of CEISIN. A complete description of Figure I.2 can be found in Volume 19 of MOSAIC [1988].

2. The IPCC Scientific Assessment [1990] shows, for example, a linear best guess sea level rise trajectory with a slope of 6cm per decade surrounded by high and low scenarios characterized by slopes of 9cm and 3cm per decade, respectively. See page xxx in the Policymakers Summary or Chapter 9 for more detailed descriptions.

3. Estimates of economic vulnerability of the United States show a nonlinear correspondence with greenhouse induced sea level rise, primarily because inundation thresholds for economically valuable properties scattered around the coastline are crossed with increasing frequency as the seas rise even along a linear trajectory. See Yohe [1991] for details.

4. The two sources cited by Nordhaus to support his empirical application are both reports to Congress, one issued in 1988 and the other issued in 1989. See EPA [1988] and EPA [1989] for details.

5. See Nordhaus [1990] for a complete description of the model outlined briefly here and extended to incorporate uncertainty in the next section.

6. See page 338 in Changing Climate [1983], for example.

7. This value is computed, by definition, as the middle value reported by Nordhaus ($12.70) times the multiplier 0.46.

8. See footnote 10 in Nordhaus [1990] for a more complete description of this regression.

9. Yohe [1991b] suggests a method by which this sort of information about the distribution of future trajectories can be used to identify useful, "interesting" scenarios.