# Overview:
# Modeling Uncertainty Project (MUP)

Snowmass 2014

July 28, 2014

# Recall Our Interest in Uncertainty

- Uncertainty is pervasive in the economics of climate change and yet is poorly quantified
  - The 2010 InterAcademy Review of the IPCC had only one substantive topic in its recommendations: *Uncertainty*
    - Quantitative probabilities should be assigned when there is adequate evidence in the literature and confidence in the results
    - Likelihood scale should be stated in terms of probabilities and words
    - Where practical, formal expert elicitations should be used for key results



- Many studies modeling the effect of uncertainty on optimal climate policy and model outputs *in a single model*:
  - Peck and Teisberg (1993), Nordhaus and Popp (1997), Pizer (1999), Webster (2002), Baker (2005), Hope (2006), Webster et al. (2012)

# Reminder of the Goals of MUP

*What we plan to accomplish:*

- A systematic study of uncertainty in a set of IAMs
  - Determine the differences among models in the uncertainties
  - Provide benchmark estimates for major parameters
  - Highlight areas where reducing uncertainties would have a high payoff

*Our focus:*

Classical statistical (forecast) errors of IAM projections

## Contributing Modeling Teams

We thank six well-known IAMs for their participation:

- MIT IGSM (John Reilly)
- JGCRI GCAM (Haewon McJeon & Jae Edmonds)
- EPRI MERGE (Geoff Blanford)
- Yale DICE (William Nordhaus, Paul Sztorc)
- Tol/Antoff FUND (David Anthoff)
- FEEM WITCH (Valentina Bosetti & Max Tavoni)

  – In earlier feasibility study: PHOENIX and PAGE

# Three Uncertain Input Variables

Modeling teams first ran a set of "feasibility runs":

- An emissions pulse, a pulse of global TFP, increase of global TFP growth, increased climate sensitivity, increased population, and a carbon tax.

Decision to focus on three that all models could handle:

- TFP growth
- Population growth
- Climate sensitivity (TSC)

For all three, a baseline and carbon tax run

# Output (Results) Variables

We choose output variables that capture key features relevant to climate change that (most) models output:

- Consumption
- Emissions
- $CO_2$ concentrations
- Global mean surface temperature
- Damages/Social cost of carbon (subset of models)

We are looking for a PDF of each of these for each model

# Methodology: Two-track Procedure

To quantify uncertainties:

1. Perform calibration runs and estimate a surface response function (SRF) for each model

2. Develop PDFs of uncertain variables

## *Schematic outline of two-track method*

Assume $y$ = endogenous variables; $u$ = exogenous or policy variables; $H^m$ = model mapping for model $m$

Steps:

1. Choose uncertain variables: TSC, TFP, Pop
2. Model calibration runs: $y = H^m(u)$. Lattice diagrams
3. Fit "surface response function," $y = R^m(u)$. Now
4. Derive Pdfs for $u$ variables, $f(u)$. After lunch
5. Then do Monte Carlo for distribution of output variables, obtaining the distribution $g^m(y)$ for output variables (this afternoon)

$$g^m(y) = \int f(u) R^m(u) du$$

# Track I: Calibration Runs and SRFs

- Calibration model runs on a 5 x 5 x 5 grid
  - The middle point of the grid is the modeler's baseline
  - The other points add and subtract from the baseline
  - Visualize results with a "lattice diagram"

- Run a baseline and carbon tax case for each grid

- Estimate the surface response functions
  - Can fit a linear surface, linear quadratic, linear quadratic with interactions, etc.

# Carbon Tax

- We use the following Carbon Tax (AMPERE AM3ND1)

| Carbon Tax (real 2005 USD per t CO2-e) | |
|---|---|
| 2005 | 0.00 |
| 2010 | 12.50 |
| 2015 | 15.21 |
| 2020 | 18.50 |
| 2025 | 22.50 |
| 2030 | 27.40 |
| 2035 | 33.30 |
| 2040 | 40.55 |
| 2045 | 49.30 |
| 2050 | 60.00 |
| 2055 | 73.00 |
| 2060 | 88.80 |
| 2065 | 108.10 |
| 2070 | 131.50 |
| 2075 | 160.00 |
| 2080 | 194.65 |
| 2085 | 236.80 |
| 2090 | 288.10 |
| 2095 | 350.55 |
| 2100 | 426.50 |

## Track II: Develop PDFs

1. Population Growth
   - Using PDFs from IIASA's demography group
   - Cross-checking with UN and Berkeley estimates

2. Total factor productivity
   - No evidence in the literature
   - Creating our own "expert survey"

3. Temperature Sensitivity
   - Basing our PDF on the literature referenced in the IPCC AR5

# Acknowledgments


- Many thanks to funding from the US DOE through the PIAMDDI grant
  - PIs: John Weyant and Karen Fisher-Vanden

- Many thanks to all of the modeling teams for joining in the effort and in particular:
  - John Reilly (MIT), Haewon McJeon and Jae Edmonds (PNNL), David Antoff (UC Berkeley), Geoff Blanford and Steve Rose (EPRI), and Valentina Bosetti (FEEM)

- Finally, thanks to Paul Sztorc and Peter Christensen for excellent RA work

# OVERVIEW SLIDES

Overview slides for the MUP presentations by the Yale team.

Snowmass EMF July 2014

Draft, preliminary and not for circulation

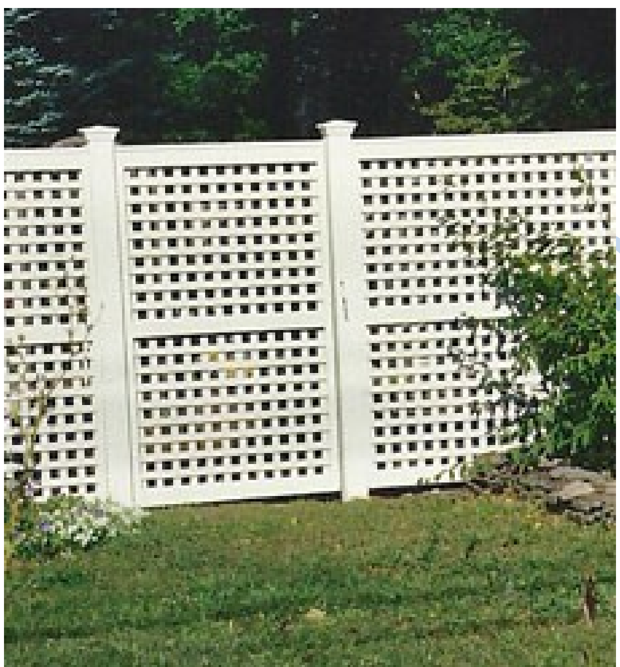
# Using Lattices to Visualize MUP Data

# Two track procedure

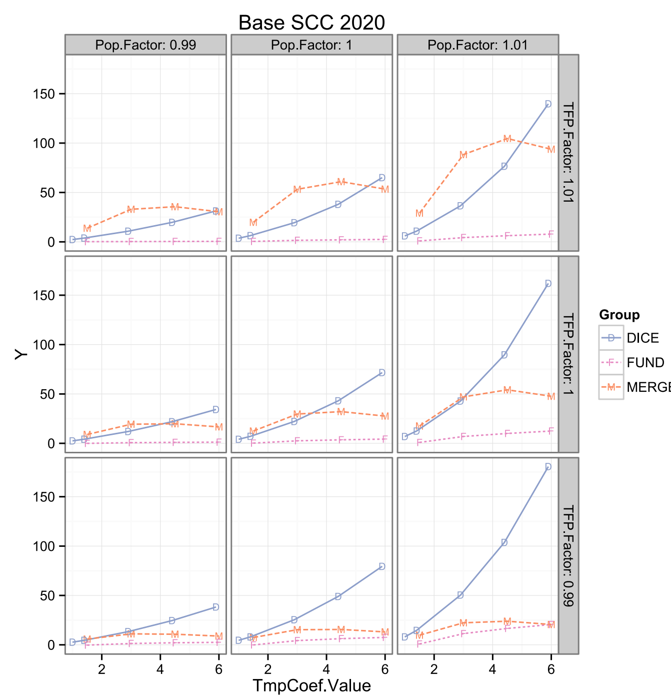
# What are Lattices?

- Lattice graphs allow us to plot high dimensional data by separating into groups
- You can thus graph an extra 1 or 2 dimensions.

Garden



MUP



# Problem: High Dimensional Data



- 6 model teams
- 3 uncertain input (TFP, Pop, TSC)
- 12 dependent vars (Temp, SCC, output…)
- 10 time periods per dependent var
- 2 modes (Base, Ampere)
- *dim[model, u1, u2, u3, dv, year, mode] = 7*

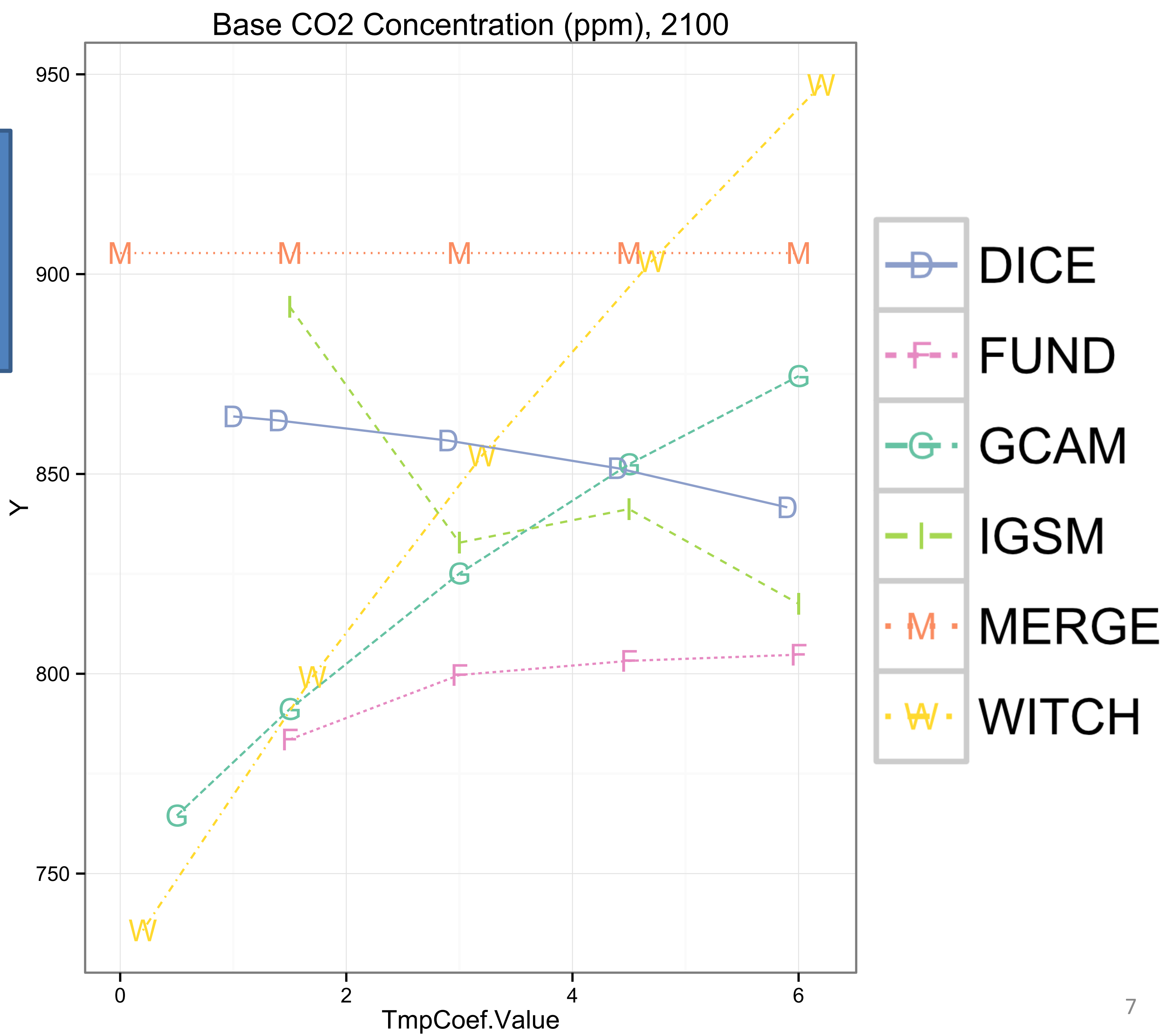
# Revisions to calibration runs

- Standardized grid for growth rate of TFP by normalizing differences of per capita GDP growth to 2100
- Compared models and suggested anomalies resulting in numerous corrections
- Still a few quirks remain… stay alert

Not a Lattice

IGSM - kink

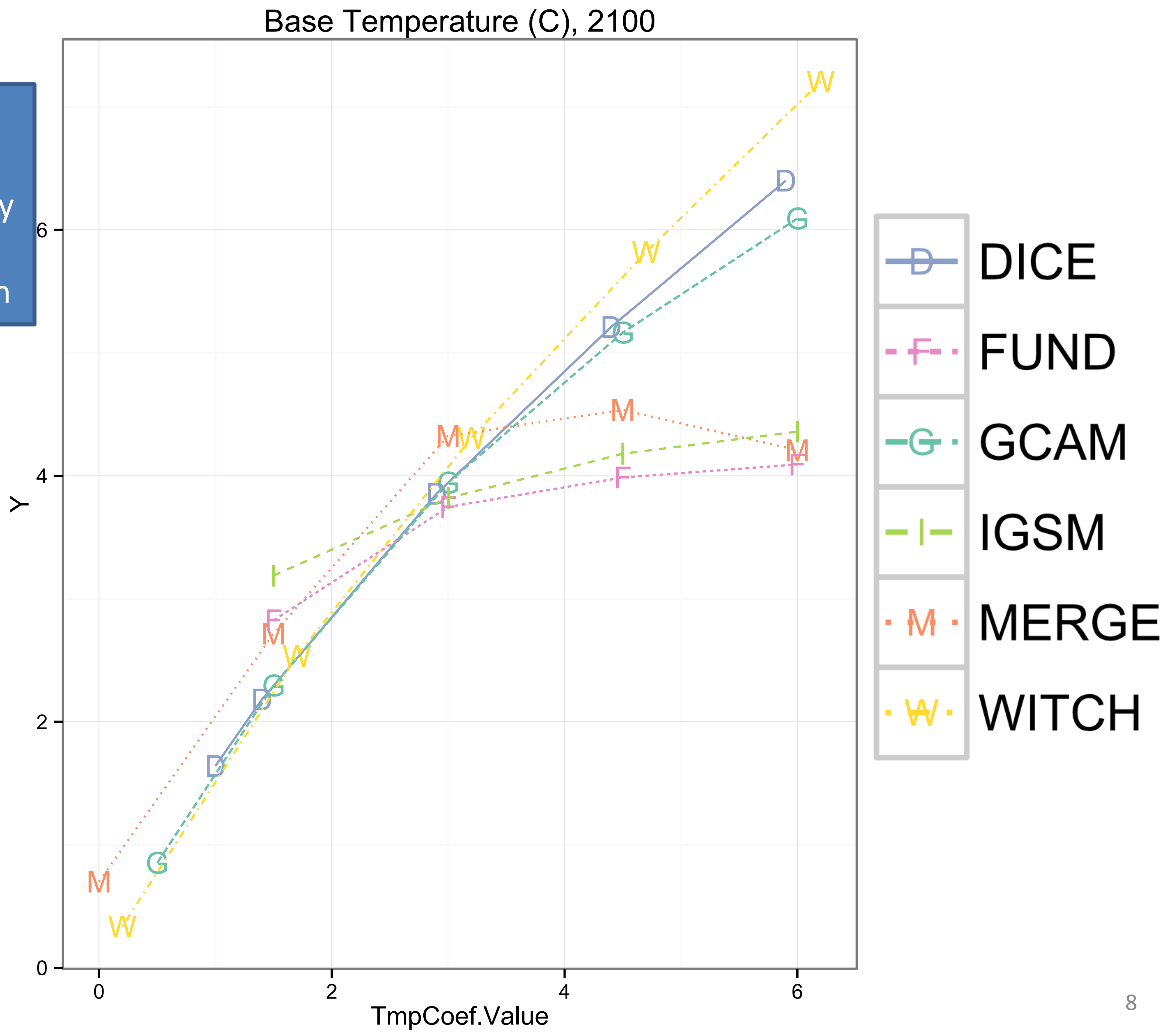
MERGE - flat



Temp 2100

central stability

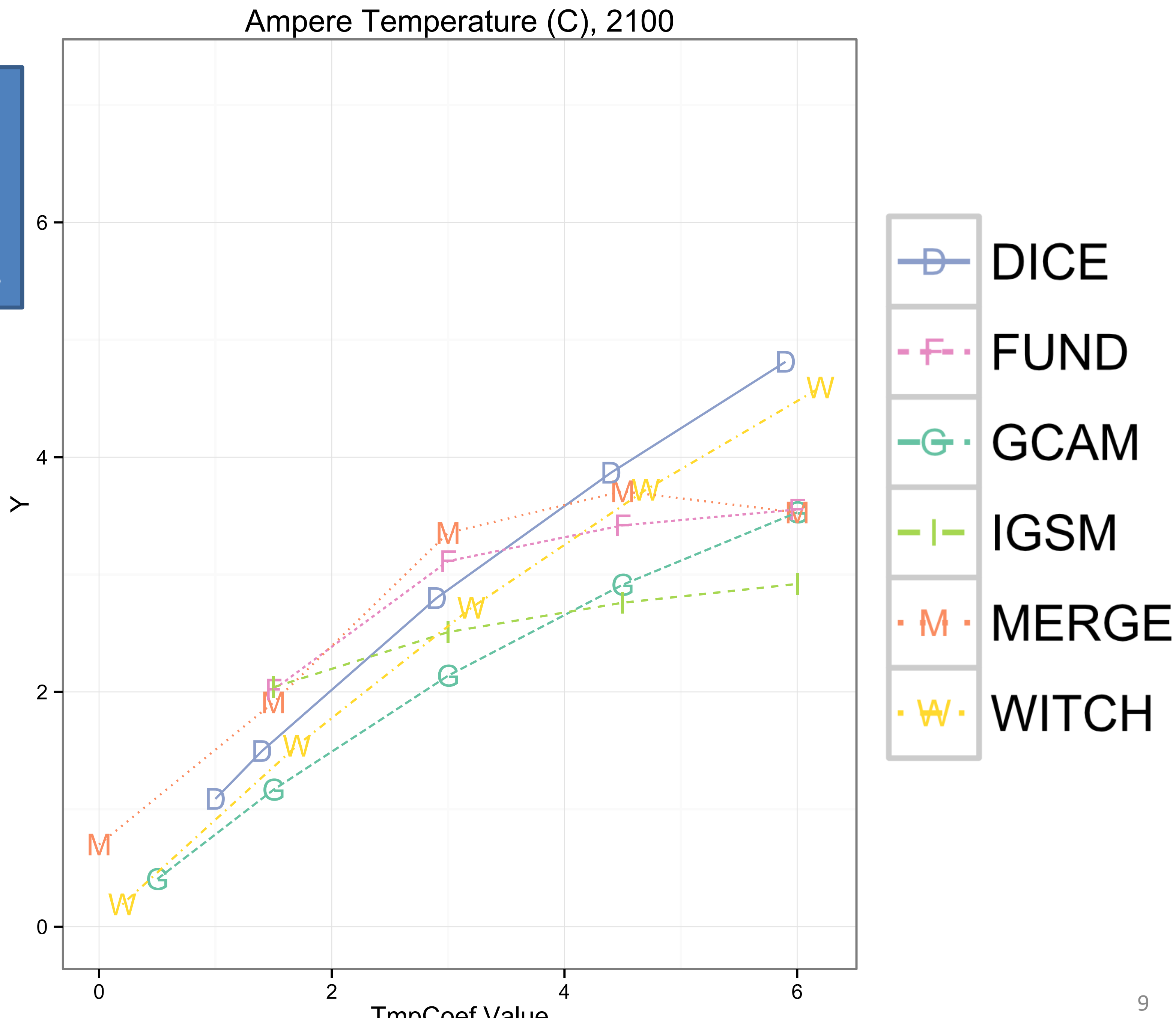
MERGE - down



Ampere

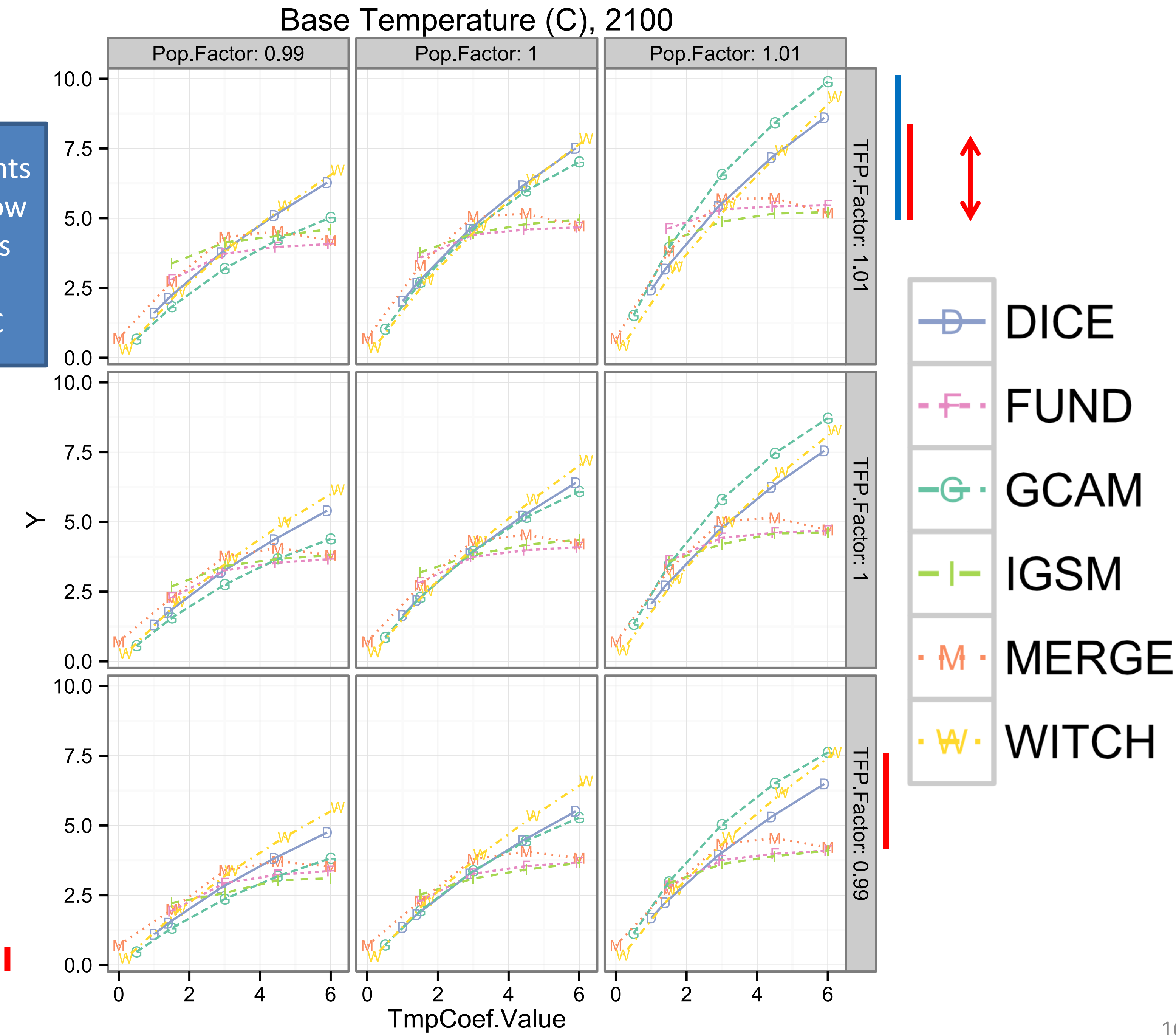
y axis

slopes/shapes

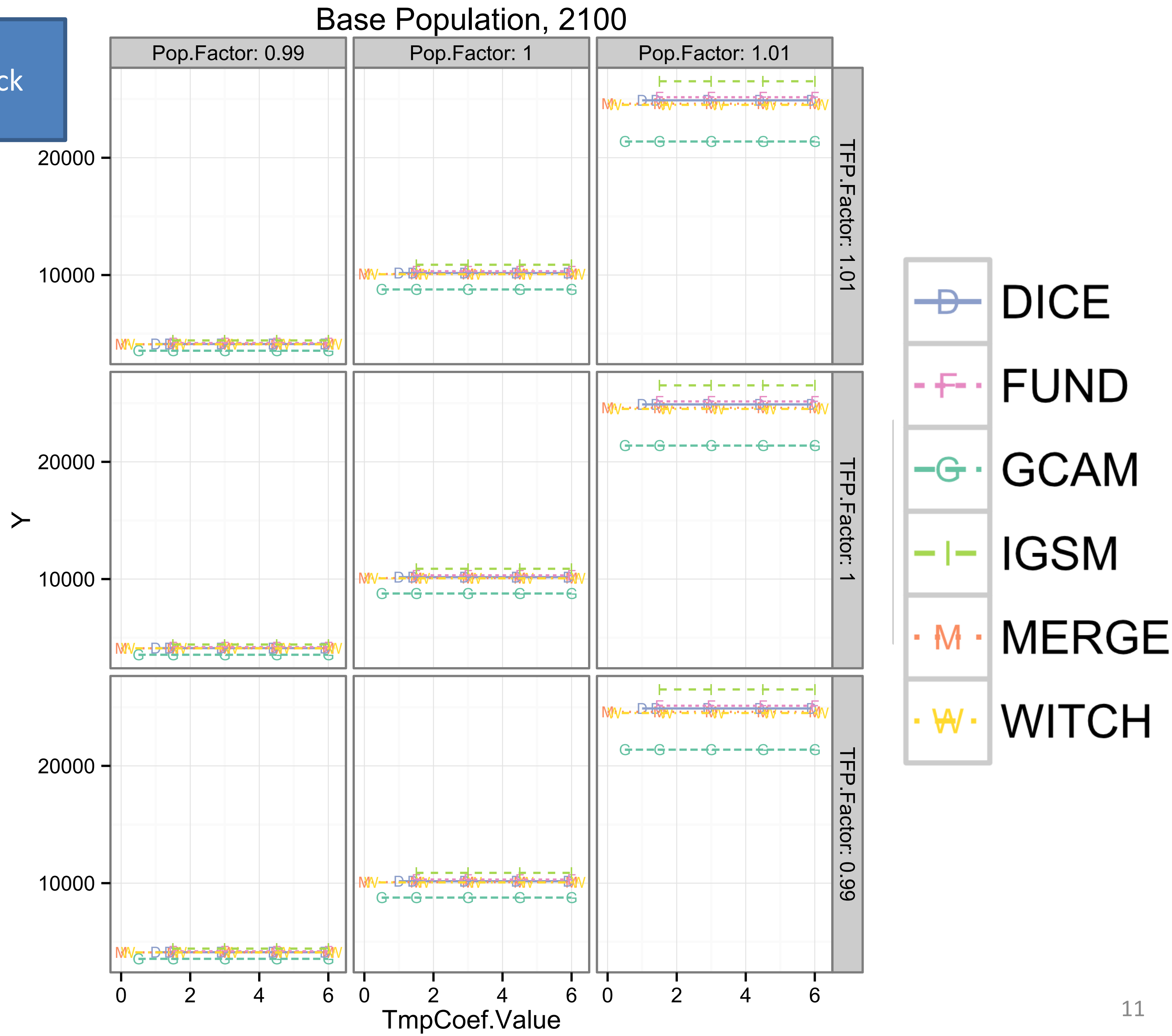


Disagreements widen/narrow at extremes

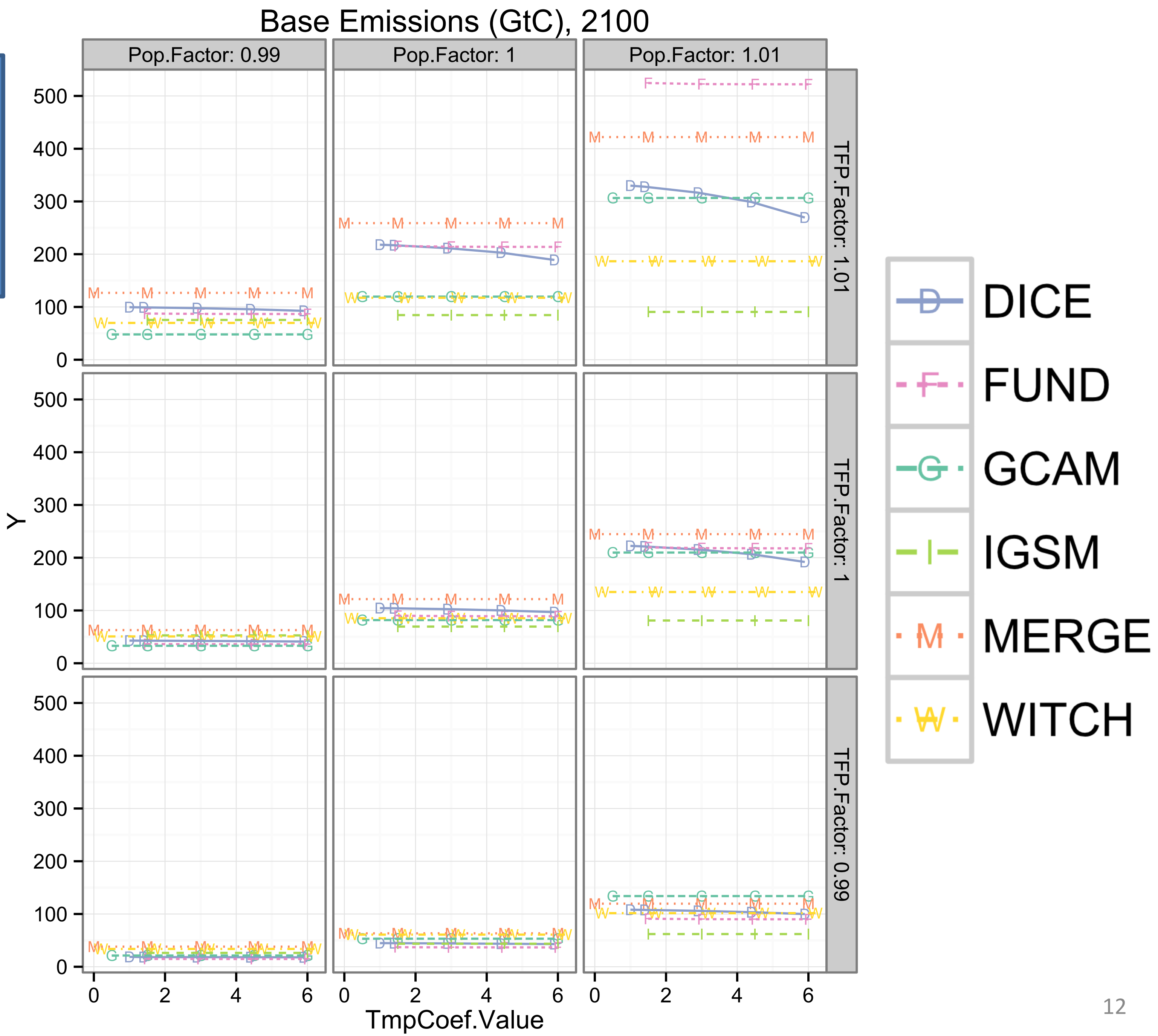
Lines: ~2 C



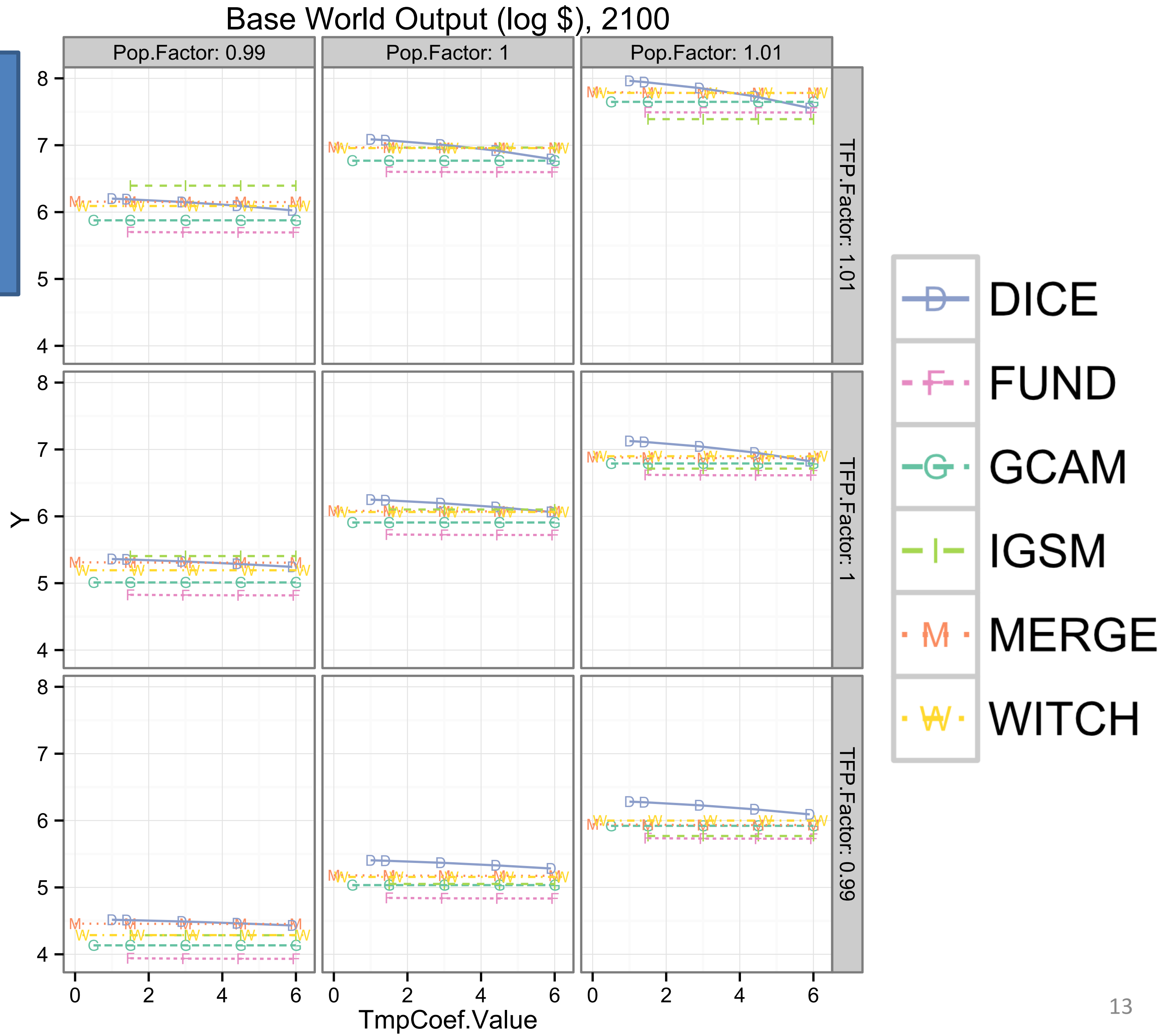
Sanity Check



More a function of Pop, TFP, than TSC

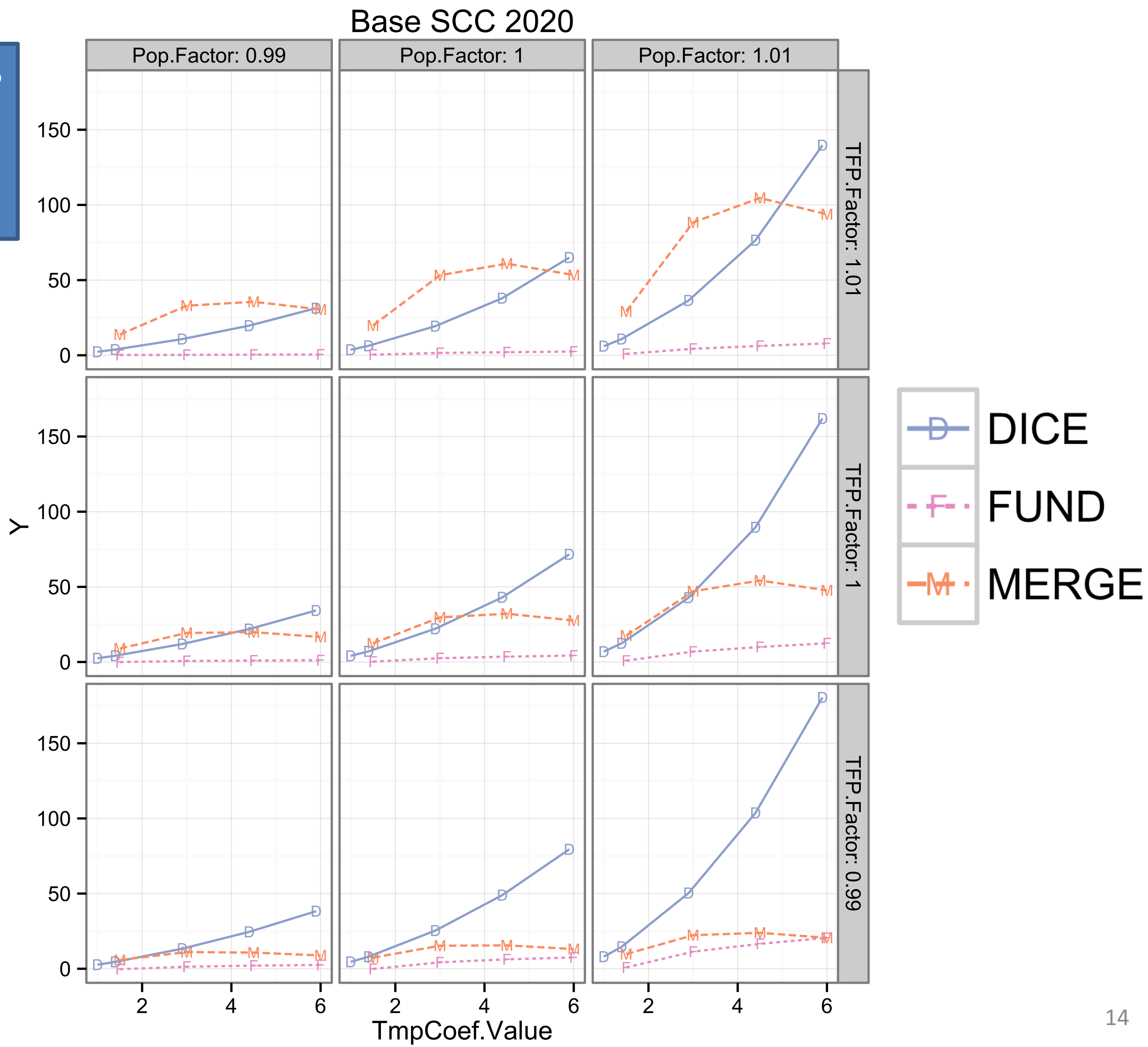


Log Output is very tame



Median= $30 ?

BIG model disagreements



# Takeaway

- Point: Lattices allow us to plot high dimensional data by **first subsetting** it and then **arranging these subsets on a grid**.
- They are highly appropriate to our needs.
- We have an entire pdf booklet of all Y variables, models, x-extremes, for years 2020, 2050, and 2100.
- Are we done with track I?

# Surface Response Functions to Represent Model Outputs in MUP

# Overview

- Major step in integrating model runs and pdfs is finding an accurate simplified ***surface response function***.
- Linear-quadratic-interaction (LQI) is our proposals.
- The results show that the LQI specification has very high reliability in representing model outputs in all tests.
- We welcome validation exercises in which modelers apply pdfs direction to determine validity of SRF.

# *Schematic outline of two-track method*

Assume *y* = endogenous variables; *u* = exogenous or policy variables; $H^m$ = model mapping for model *m*

Steps:

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# Some background

- Basic idea is to fit a continuous function to a discrete set of observations for a set of functions, $y = f(x_1, \ldots, x_n)$.
- Generally have high-dimensional and complex function.
- Applications in other areas include chemical engineering, estimating dose-response curves, manufacturing tolerances, estimating machine performance, bioconversion, earthquakes, among others.
- Deep statistical and applied literature in this area. Multiple techniques used with increased computational capacity.

# Desirable features of method

- Good fit to data.
- Good prediction of outputs
- Easily calculated and verified.
- Robust procedure.
- Avoid overfitting that can lead to poor quality predictions in tails
- Avoid data mining to prevent reverse engineering the final results.
- Methods should have unambiguous choice of variables.

CHAPTER 2

RESPONSE SURFACE EXPERIMENTS ON PROCESSES WITH HIGH VARIATION

Steven G. Gilmour and Luzia A. Trinca

The initial form of data analysis in a RS experiment of this type with $q$ factors involves fitting, and checking for lack of fit of, the second order polynomial model,

$$\mu = \beta_0 + \sum_{i=1}^{q} \beta_i x_i + \sum_{i=1}^{q} \beta_{ii} x_i^2 + \sum_{i=1}^{q-1} \sum_{j=i+1}^{q} \beta_{ij} x_i x_j, \qquad (1)$$

Comments:

- This is LQI (linear-quadratic-interaction) method discussed below.
- In many situations, this specification is trimmed by removing insignificant variables.
- We do not recommend trimming, although robustness checks should be made.

# *Details on surface response function (SRF)*

- Will test 7 different polynomial specifications:
  - Linear (L)
  - Linear with interactions (LI)
  - Linear quadratic (LQ)
  - Linear, quadratic, linear interactions (LQI)
  - $3^{rd}$ degree polynomial, linear interactions (P3I)
  - $4^{th}$ degree poly, $2^{nd}$ degree interactions (P4I2)
  - $4^{th}$ degree poly, $4^{th}$ degree interactions, polynomial three-way interactions (P4I4S2)

# *Example for linear case*

- Assume $y_i$ = output variable i, $u_k$ = realization for uncertain variable $k$
- Surface response function in linear case is:

$$y_i = \alpha + \beta_{i,1} u_1 + \beta_{i,2} u_2 + \beta_{i,3} u_3 + \varepsilon_i$$

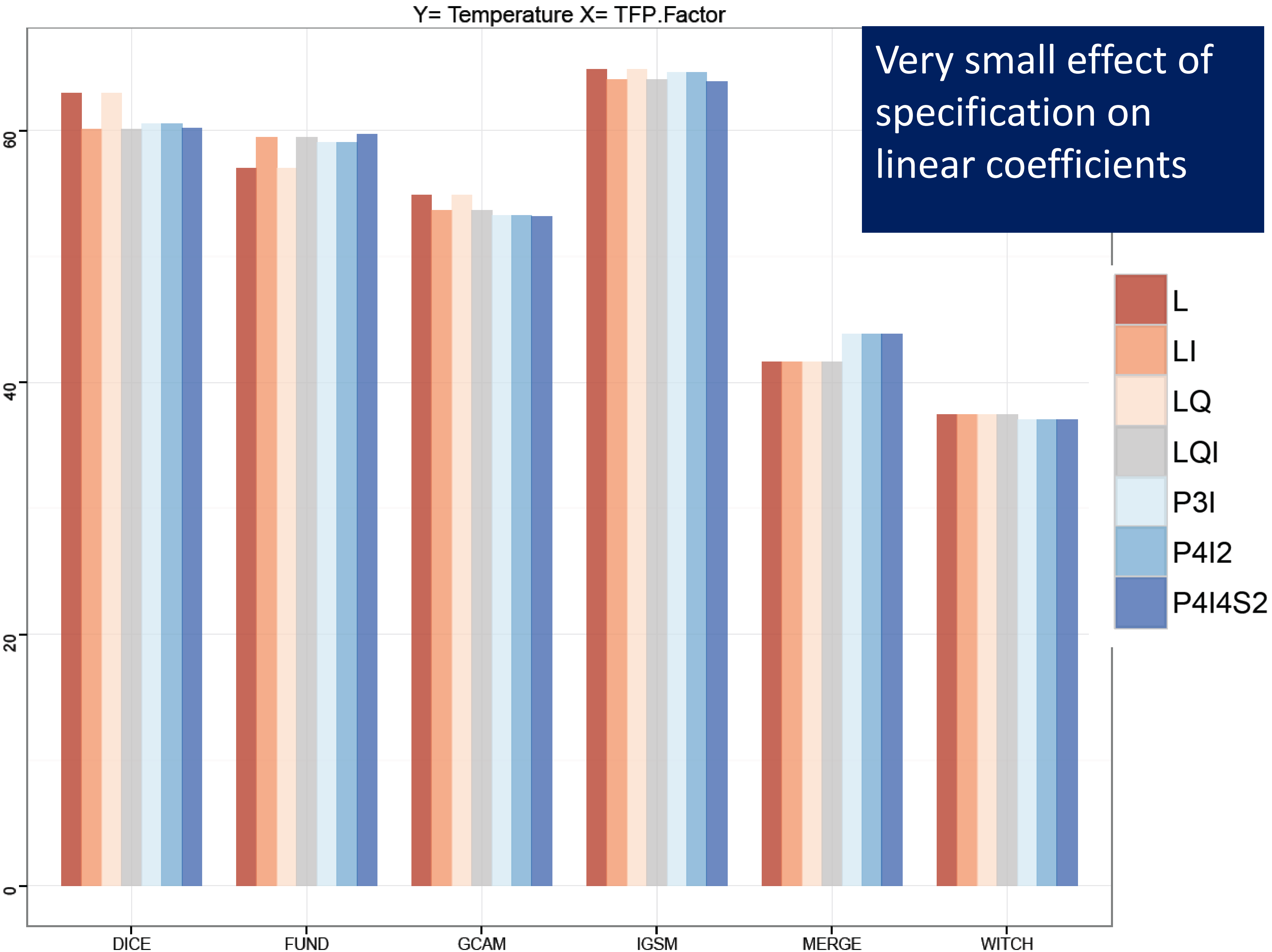
- Note that error is fitting error, not classical random disturbance.

## *Proposal for consideration*

- We propose using the LQI approach.

$$
\begin{aligned}
y_i = \alpha \; &+ \beta_{i,1} u_1 + \beta_{i,2} u_2 + \beta_{i,3} u_3 \\
&+ \gamma_{i,1} \, u_1^2 + \gamma_{i,2} \, u_2^2 + \gamma_{i,3} \, u_3^2 \\
&+ \lambda_{i,1} \, u_1 u_2 + \lambda_{i,2} \, u_2 u_3 + \lambda_{i,3} \, u_1 u_3 + \varepsilon_i
\end{aligned}
$$

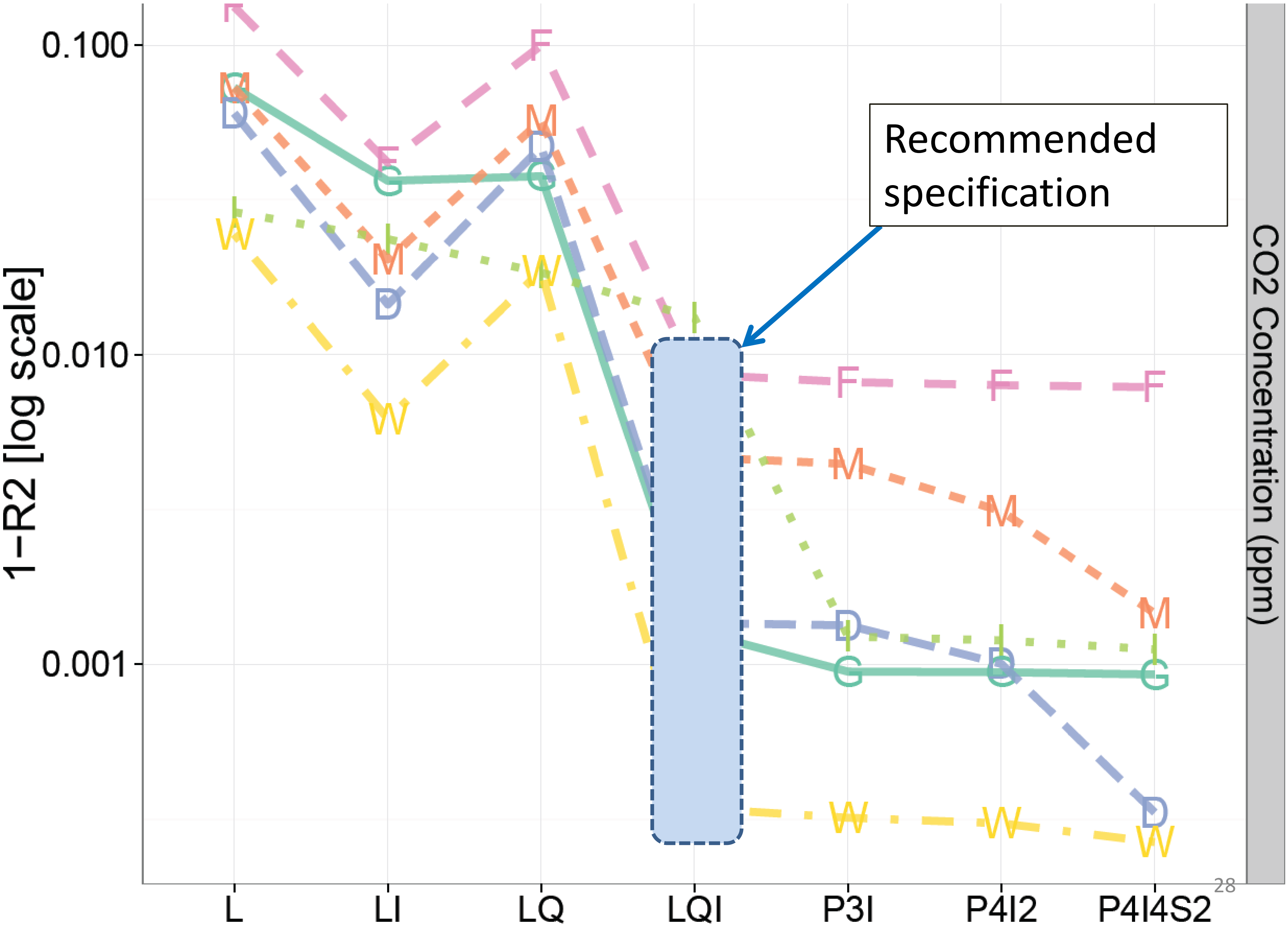
Then look at some of the estimated linear coefficients across models for different SRF specifications.

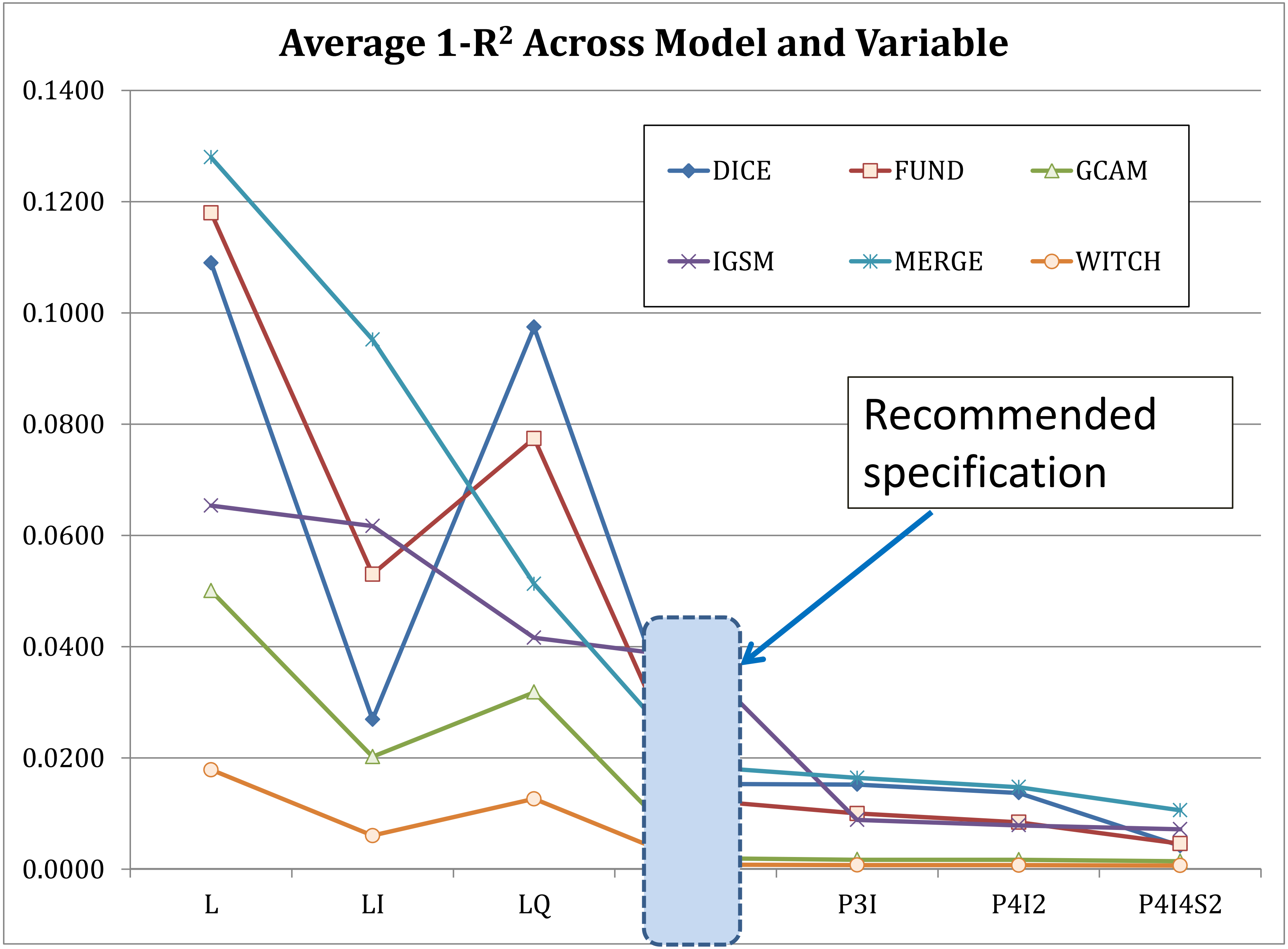



Very small effect of specification on linear coefficients

Then look at the goodness of fit of alternative specifications.

# *1-$R^2$ across models and specifications for $CO_2$ Concentration (Base)*



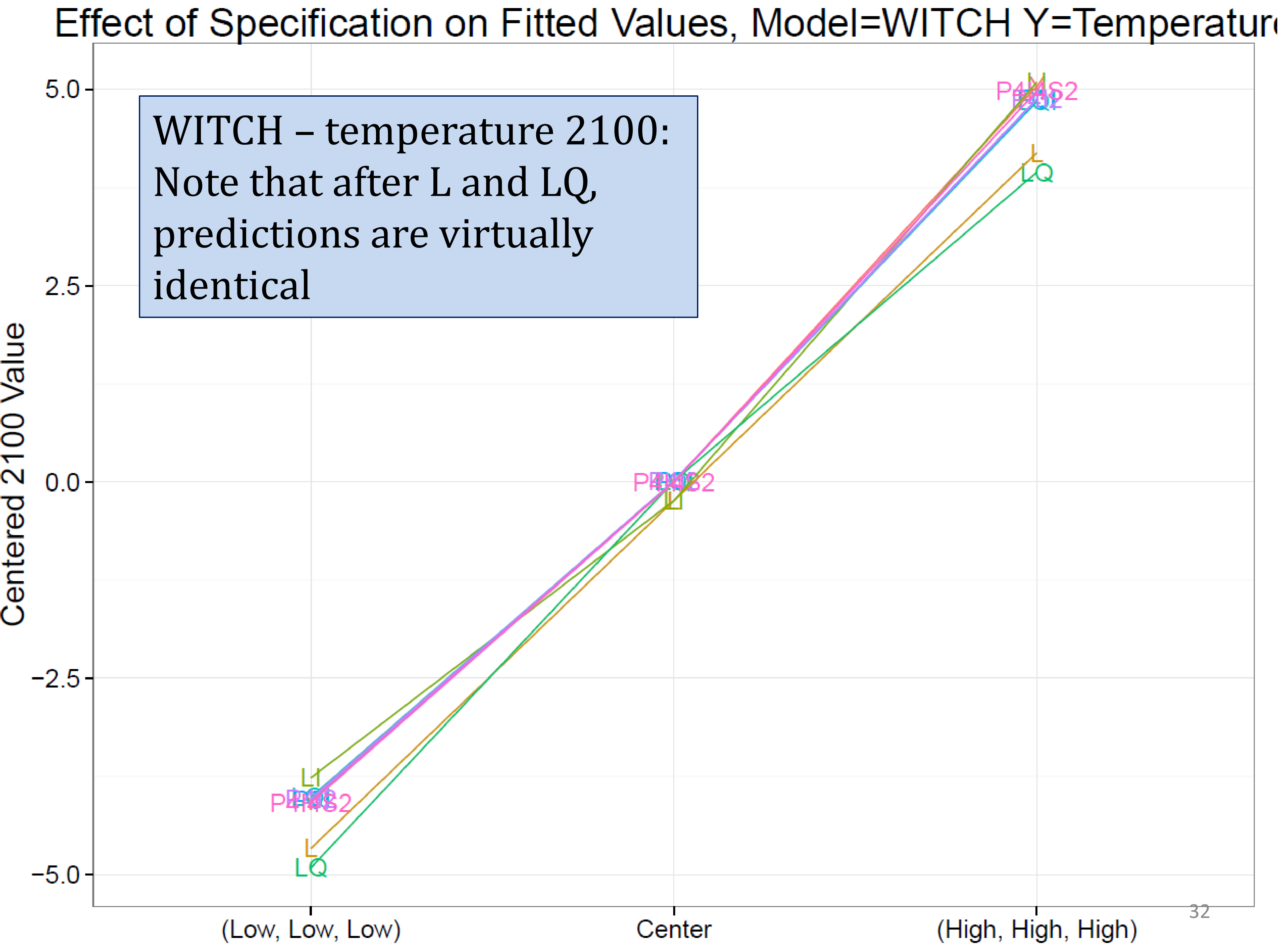


Average 1-R² Across Model and Variable
0.1400
0.1200
0.1000
0.0800
0.0600
0.0400
0.0200
0.0000
L
LI
LQ
P3I
P4I2
P4I4S2
DICE
FUND
GCAM
IGSM
MERGE
WITCH
Recommended specification

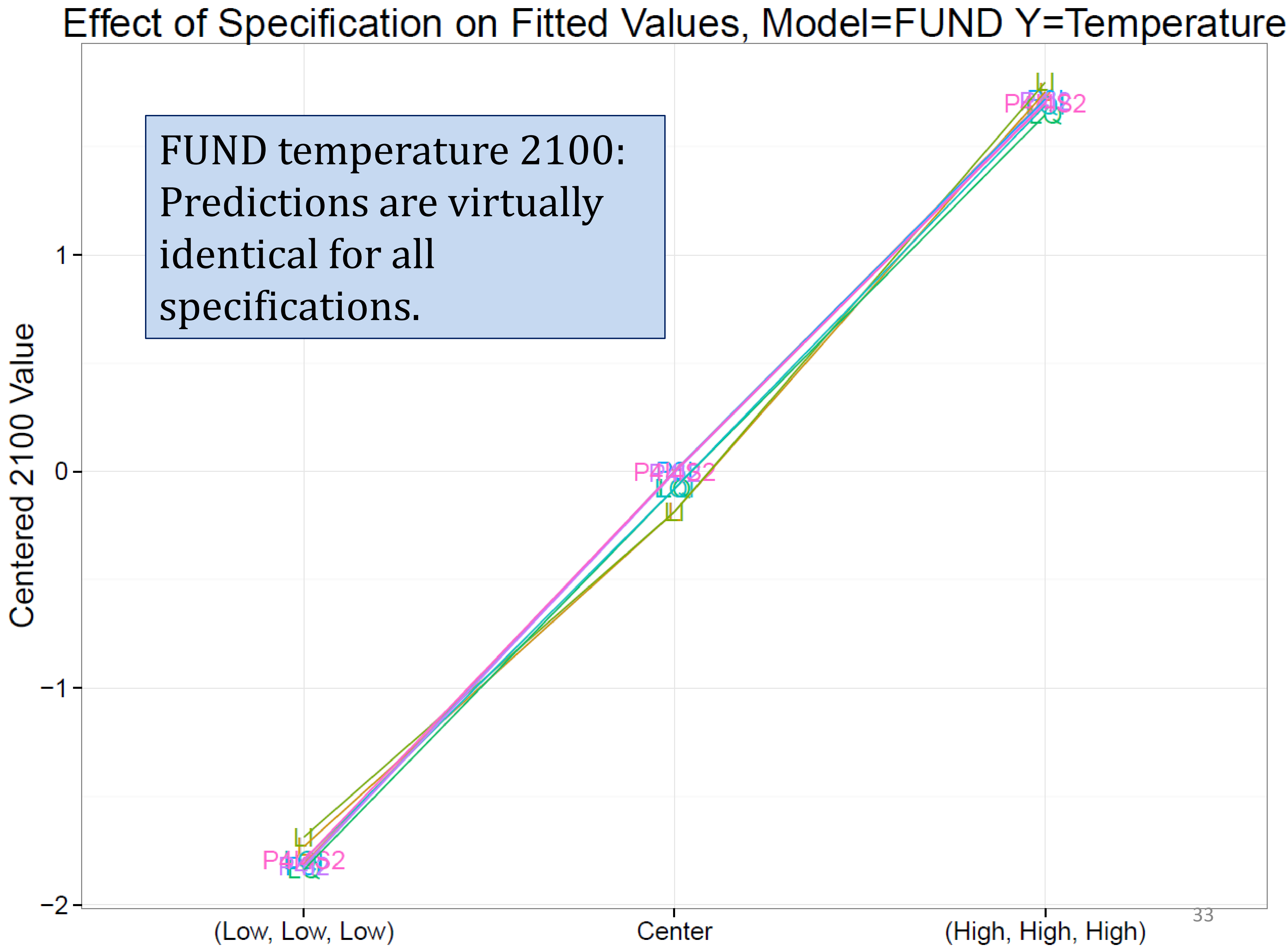
# Standard error of equation for temperature 2100 (roughly average error in prediction in degrees C)

| Spec'tion | DICE | FUND | GCAM | IGSM | MERGE | WITCH | Average |
|---|---|---|---|---|---|---|---|
| L | 0.306 | 0.249 | 0.608 | 0.137 | 0.801 | 0.393 | 0.416 |
| LI | 0.167 | 0.225 | 0.359 | 0.137 | 0.799 | 0.246 | 0.322 |
| LQ | 0.266 | 0.133 | 0.518 | 0.071 | 0.251 | 0.318 | 0.259 |
| LQI | 0.055 | 0.068 | 0.133 | 0.068 | 0.210 | 0.062 | 0.099 |
| P3I | 0.049 | 0.024 | 0.128 | 0.060 | 0.213 | 0.047 | 0.087 |
| P4I2 | 0.050 | 0.024 | 0.131 | 0.058 | 0.190 | 0.048 | 0.084 |
| P4I4S2 | 0.041 | 0.017 | 0.123 | 0.054 | 0.194 | 0.040 | 0.078 |

Next we show the effect of the specification on the model outputs

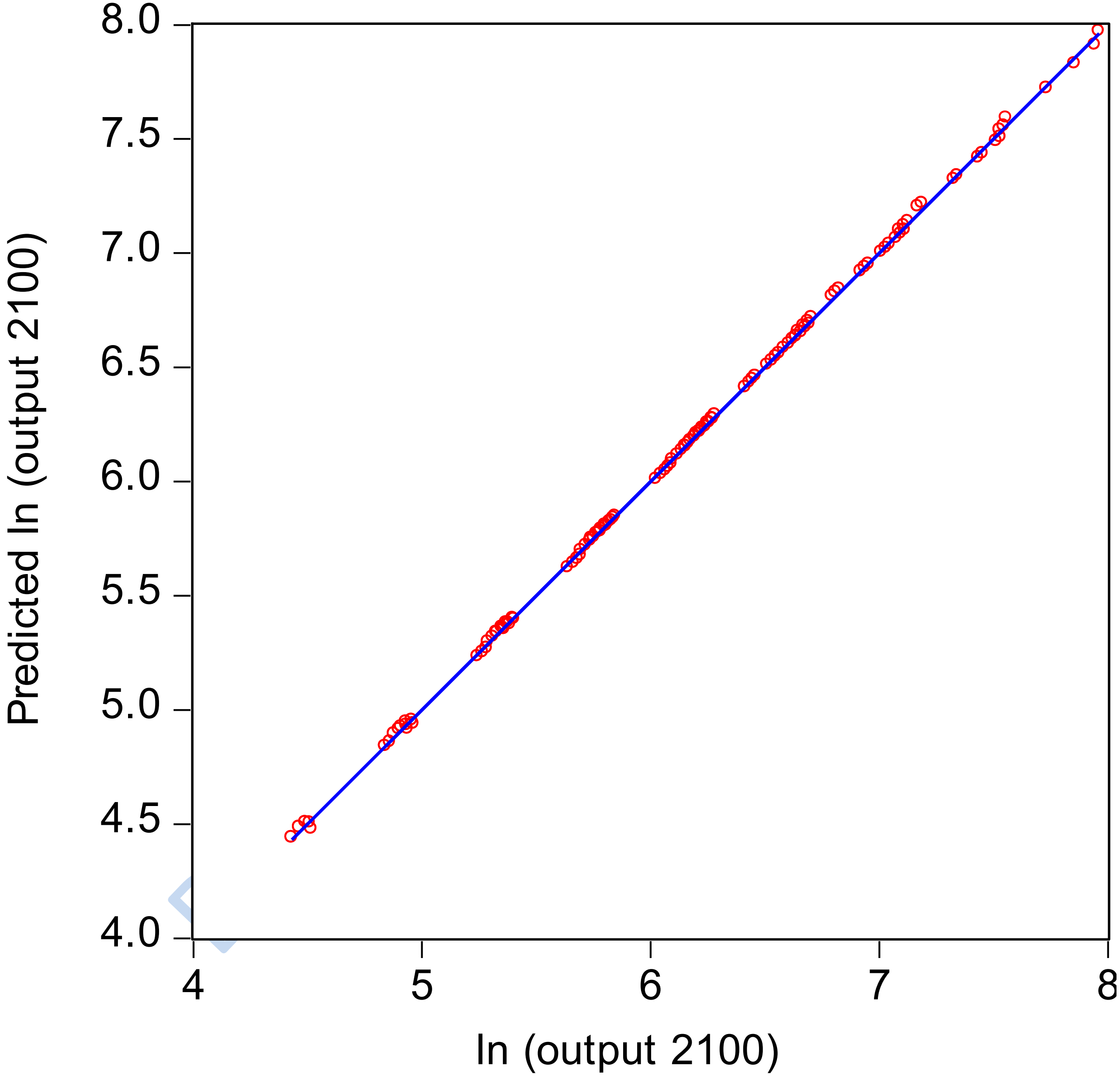



WITCH – temperature 2100: Note that after L and LQ, predictions are virtually identical

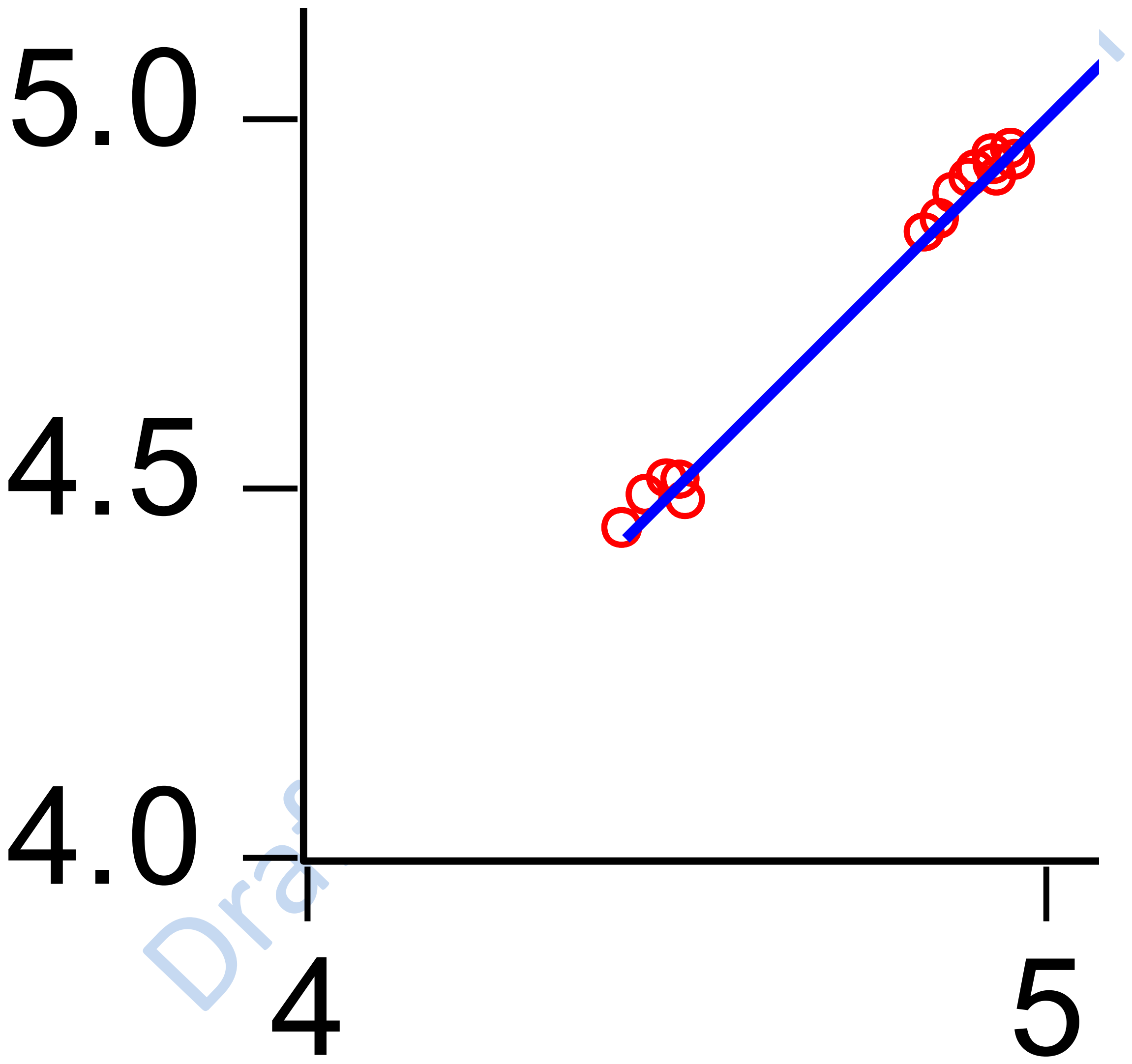

Effect of Specification on Fitted Values, Model=FUND Y=Temperature
FUND temperature 2100: Predictions are virtually identical for all specifications.
Centered 2100 Value
1
0
−1
−2
(Low, Low, Low)
Center
(High, High, High)

Next we show the effect of the specification on the model outputs for the worst fitting model

# Visuals of actual and predicted for DICE Y(2100) LQI

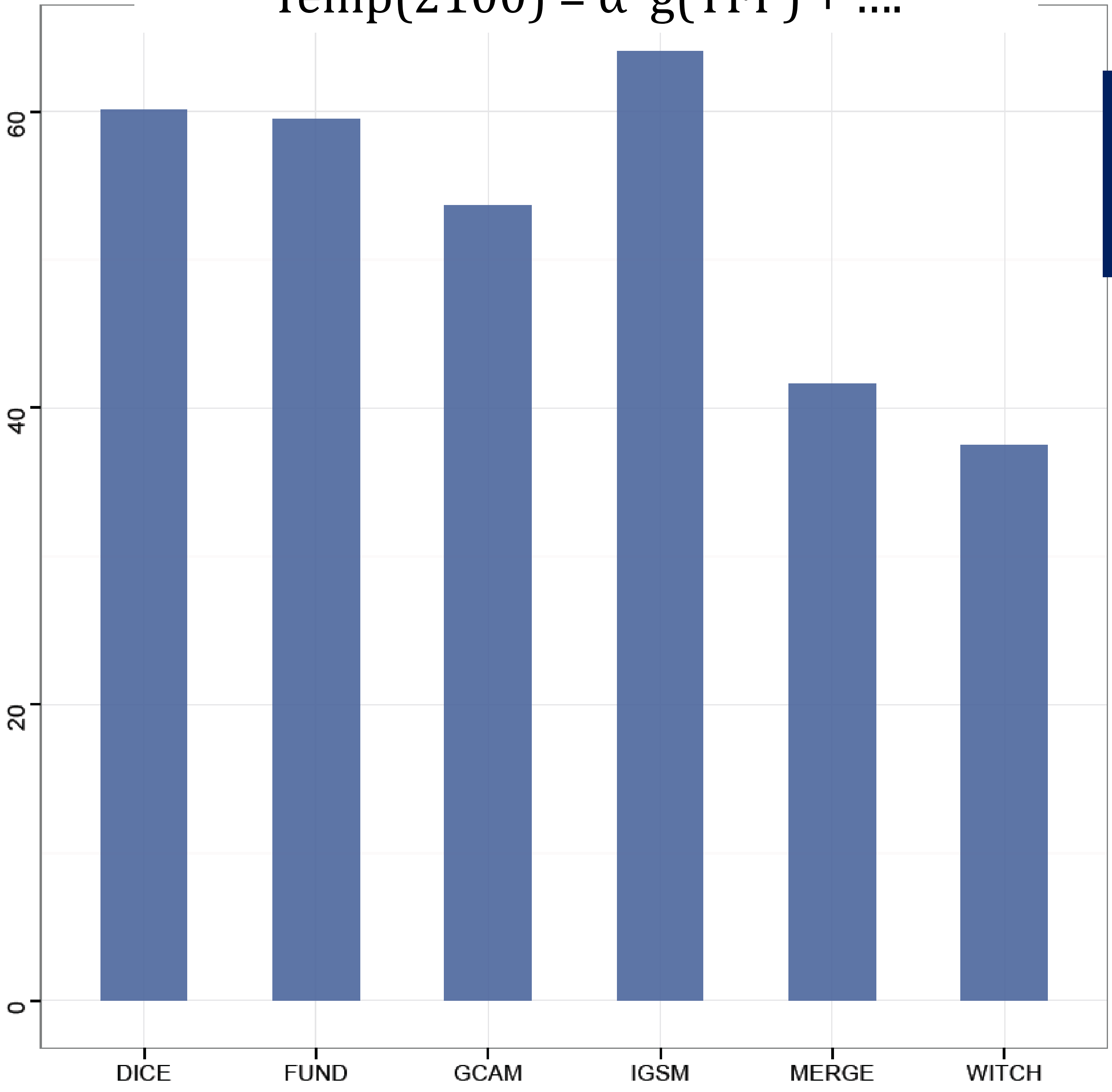


# BLOWUP for DICE Y(2100) LQI



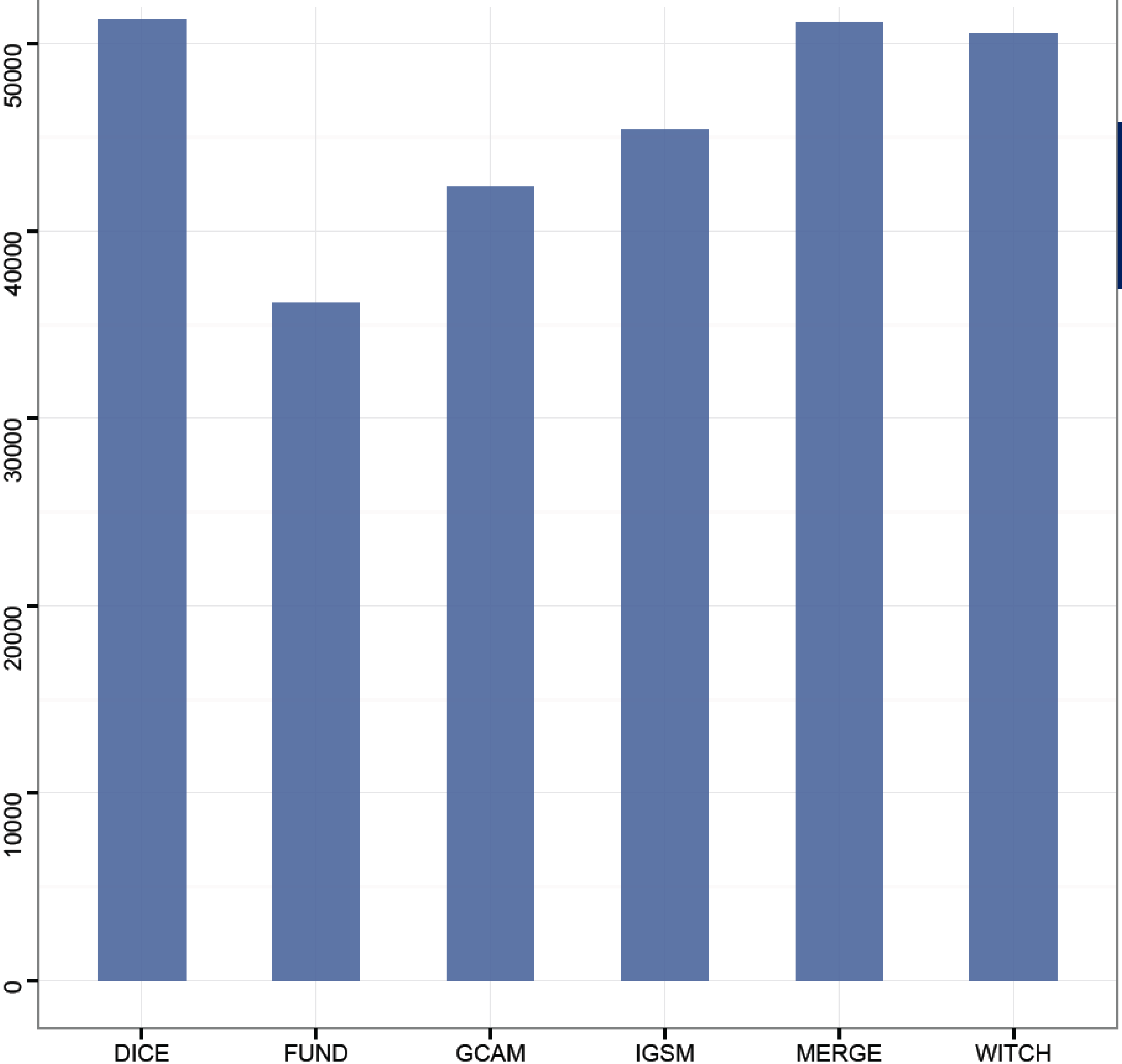
Next, the effect of specification of surface response on linear coefficients in LQI

Temp(2100) = α*g(TFP) + ....



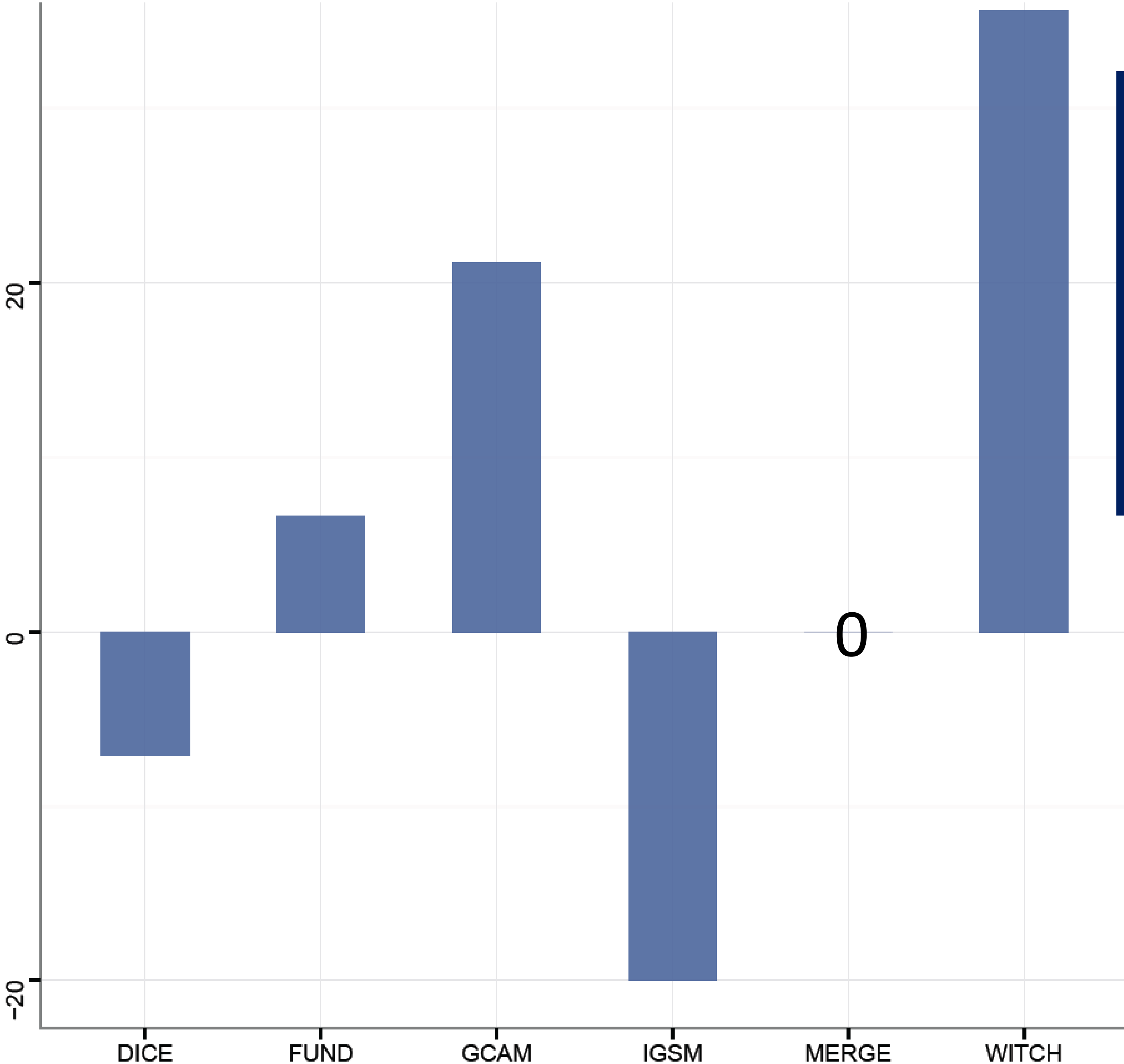

Normal model disagreement.

Output(2100) = α*g(TFP) + ....




Normal model disagreement

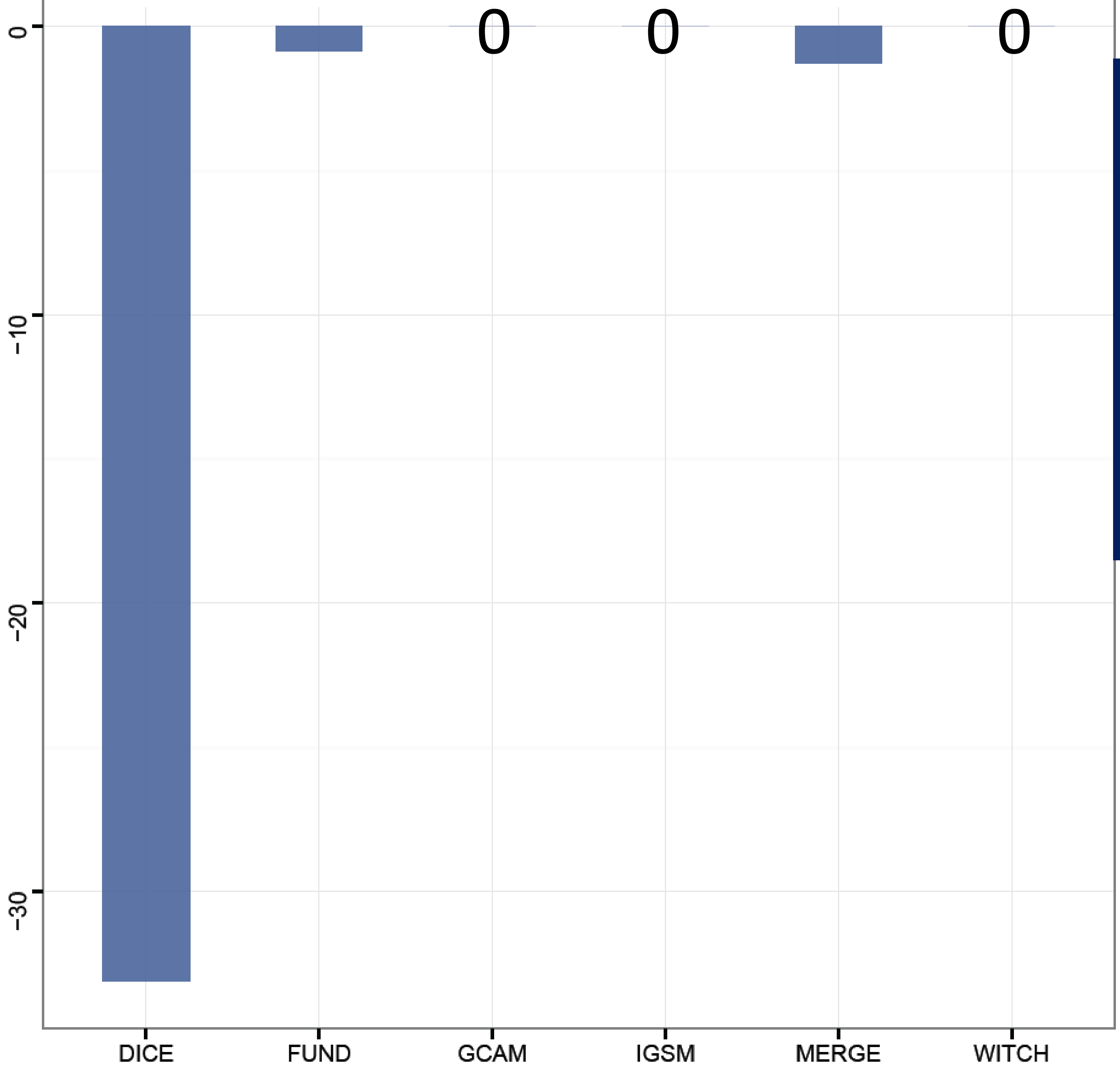
# CO2 Conc(2100) = α*TSC + ….




Model linear effect of TSC on CO2:

Note sign difference!

Output(2100) = α*TSC + ....




Model linear effect of TSC on output:

Some models have no feedback. Major differences among models.

# Alternative approaches

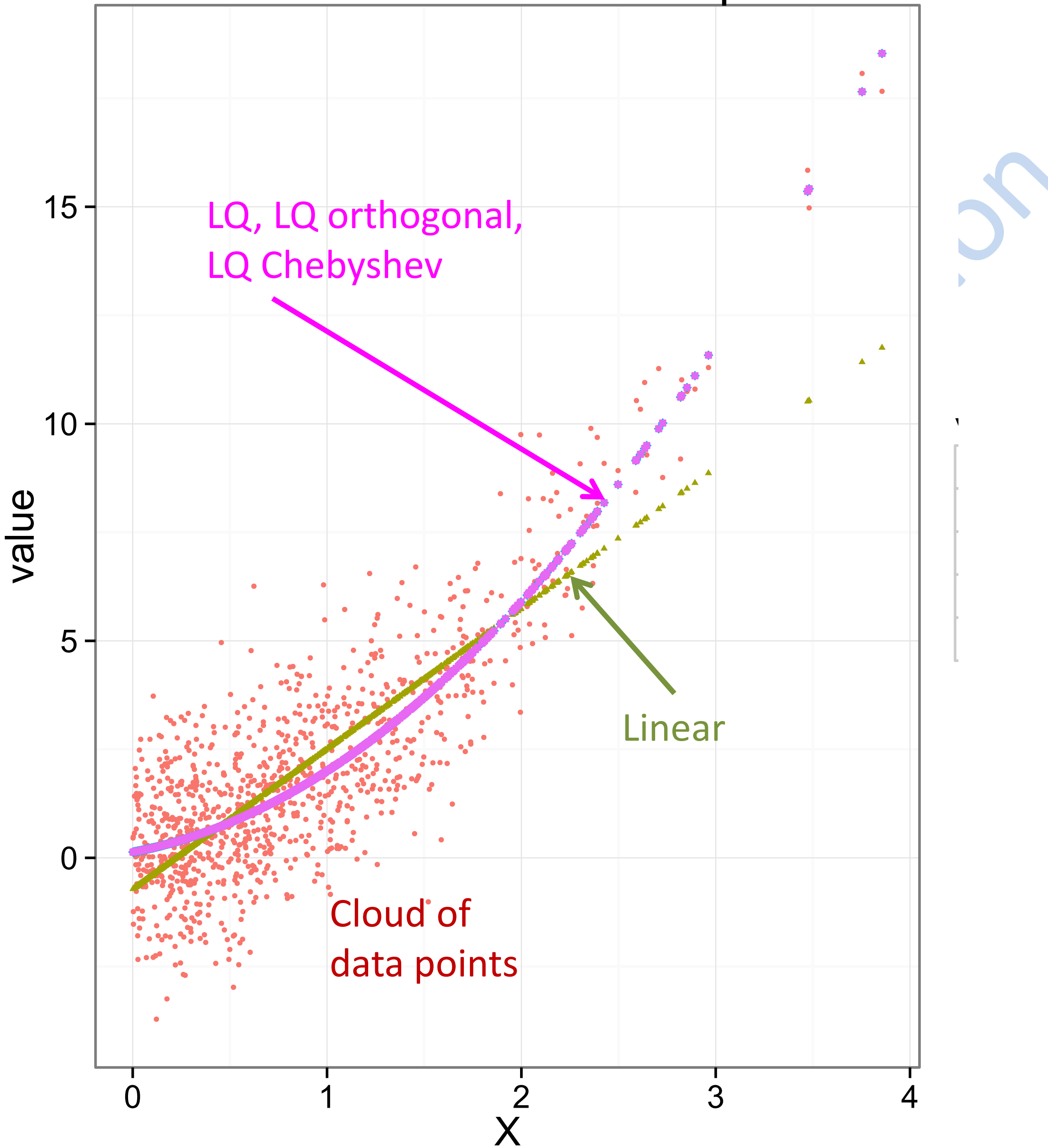
Since last year, we have received two other suggestions for functional forms:

1. Chebyshev Polynomials:
2. Basis Splines

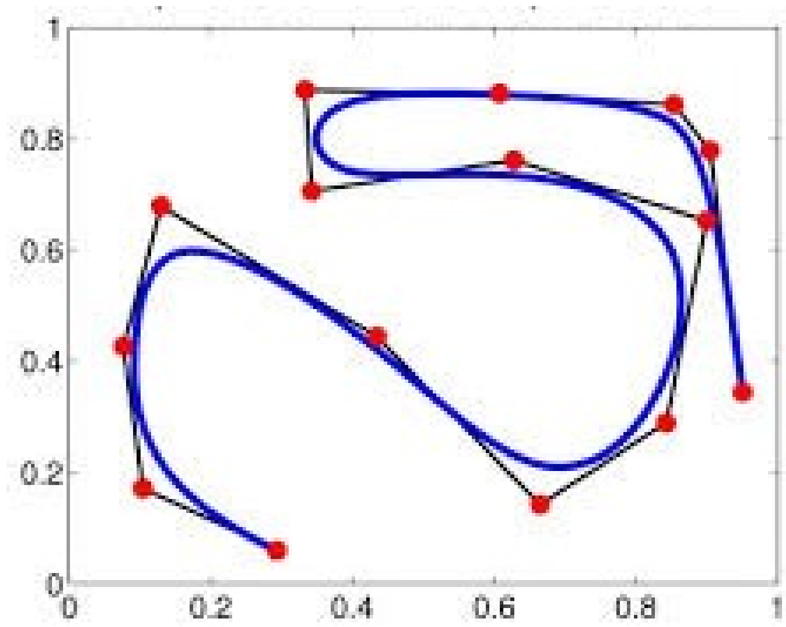
# Chebyshev Polynomials:

- These are polynomials that have useful orthogonality and boundedness properties
- They are identical to (appropriate) polynomial interpolation as shown on the next slide

# All Quadratics Seem Equal



# Basis Splines (B-splines):



# Basis Splines (B-splines):

- Basis splines are piecewise polynomial functions of degree k in a variable x.
- So it might be LQI in *m x m* partitions of the space (instead of 1 x 1 partition in our proposal).

BUT…

- Has arbitrary selection of both m and position of knots
- And doesn’t make any difference (see next+1 slide)
- For LQI++, B-spline **adds 135 coefficients**

# Comparison of LQI and B-spline for temperature 2100

| Specification | Model | Adj.R2 | df |
|---|---|---|---|
| LQI, DICE | No Spline | .8926 | 240 |
| LQI, DICE | B-Spline | .8554 | 181 |
| LQI++, WITCH | No Spline | .8384 | 227 |
| LQI++, WITCH | B-Spline | .7301 | 92 |



Result: B-splines do worse because of increase in number of fitted parameters. (P.S., even B- Spline *unadjusted* R2 is hardly better.)

# Conclusions

- Major step in integrating model runs and pdfs is finding an accurate simplified ***surface response function***.
- Issue arises in many fields (clinical trials, engineering,…).
- Linear-quadratic-interaction (LQI) is our central proposals.
- The results show that the LQI specification has very high reliability in representing model outputs in all tests.
- We welcome validation exercises in which modelers apply pdfs direction to determine validity of SRF.

# *Distribution for Population for MUP analysis*

# *Basic approach*

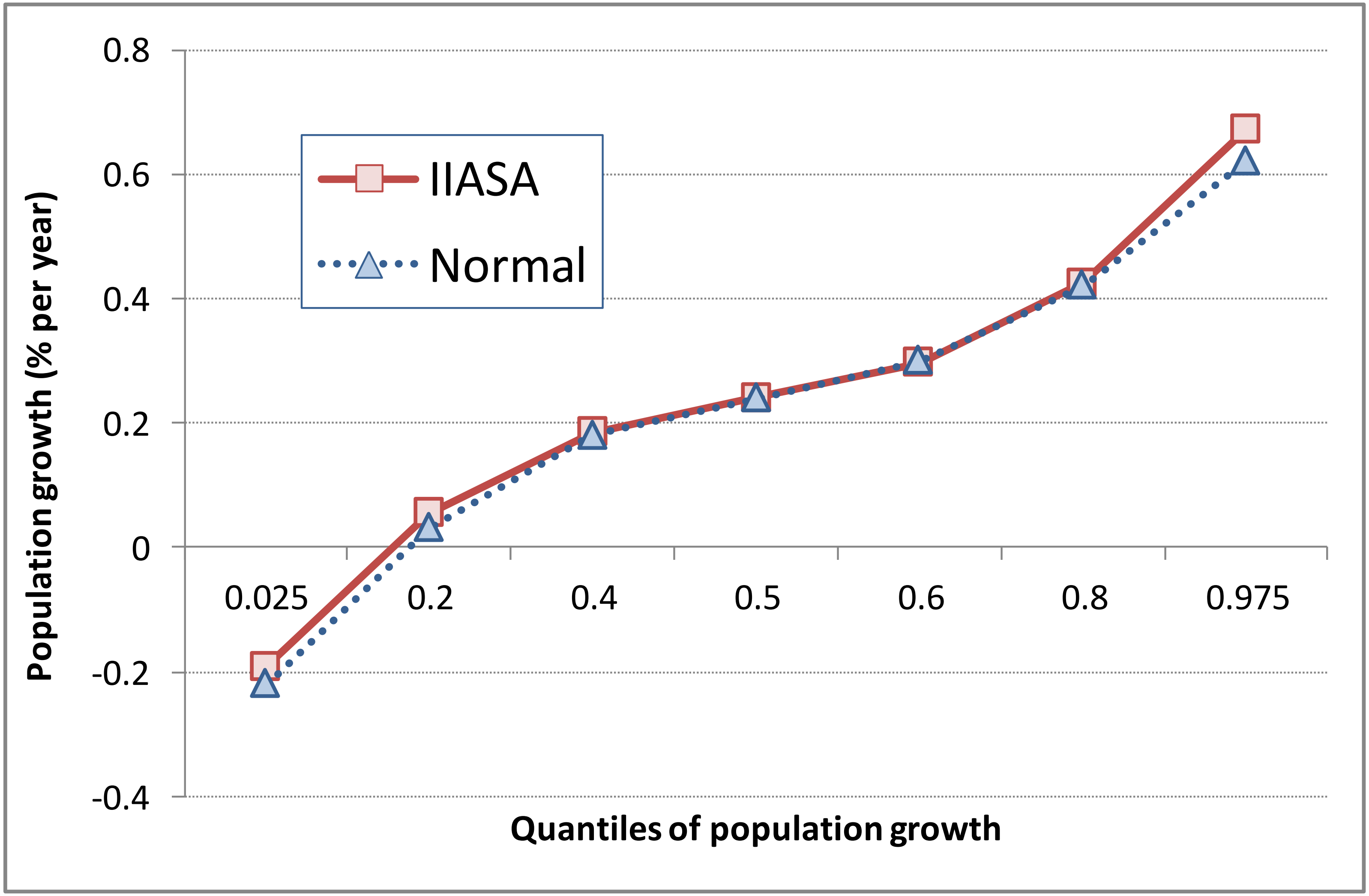
- We reviewed existing uncertainty modeling
- Major long-term efforts at Berkeley, UN, IIASA
- IIASA appeared to be closest to the spirit of MUP
- Checked by looking at historical changes

# IIASA methodology



- IASA's projections are based explicitly on the results of discussions of a group of experts convened for the purpose of producing scenarios for these vital rates.
- The latest projections (2013) are an update to the previous projections (2007).
- 13 world regions.
- The forecasts [are] the distribution of the results of 2000 different cohort component projections.

# IIASA distribution v. normal



# Tests of distributions of growth rates for IIASA*

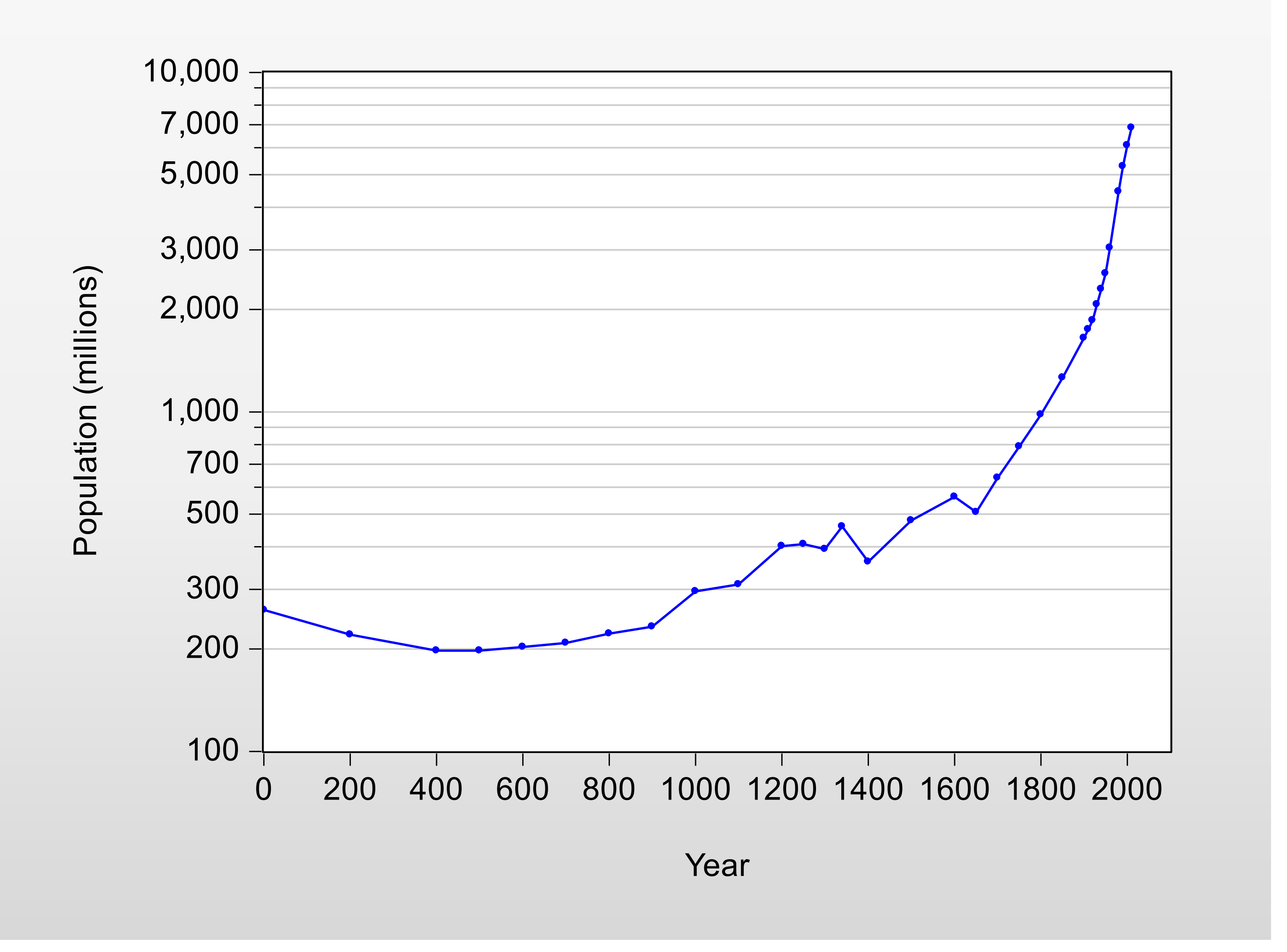
| Distribution | logL | AIC | BIC | Chisq (value) | AD(value) | H(AD) | KS(value) |
|---|---|---|---|---|---|---|---|
| Normal | ***32.0094*** | ***-60.0189*** | ***-60.1271*** | 0.1595 | ***0.1587*** | Not rejected | ***0.1453*** |
| Exponential | -7.0157 | 16.0314 | 15.9773 | 761.7996 | 3.1954 | Rejected | 0.6305 |
| Chi-square | -8.3333 | 18.6666 | 18.6125 | 919.6466 | 2.8419 | Null | 0.5716 |
| Uniform | Null | Null | Null | Null | Inf | Null | 0.1641 |
| Gamma | 32.0081 | -60.0162 | -60.1244 | 0.1581 | 0.1590 | Not rejected | 0.1457 |
| Lognormal | 32.0074 | -60.0149 | -60.1230 | ***0.1580*** | 0.1592 | Not rejected | 0.1458 |
| Weibull | 31.8757 | -59.7515 | -59.8597 | 0.3502 | 0.1834 | Not rejected | 0.1562 |
| F | 29.9355 | -55.8709 | -55.9791 | 2.0391 | 1.9084 | Null | 0.4197 |

= preferred.

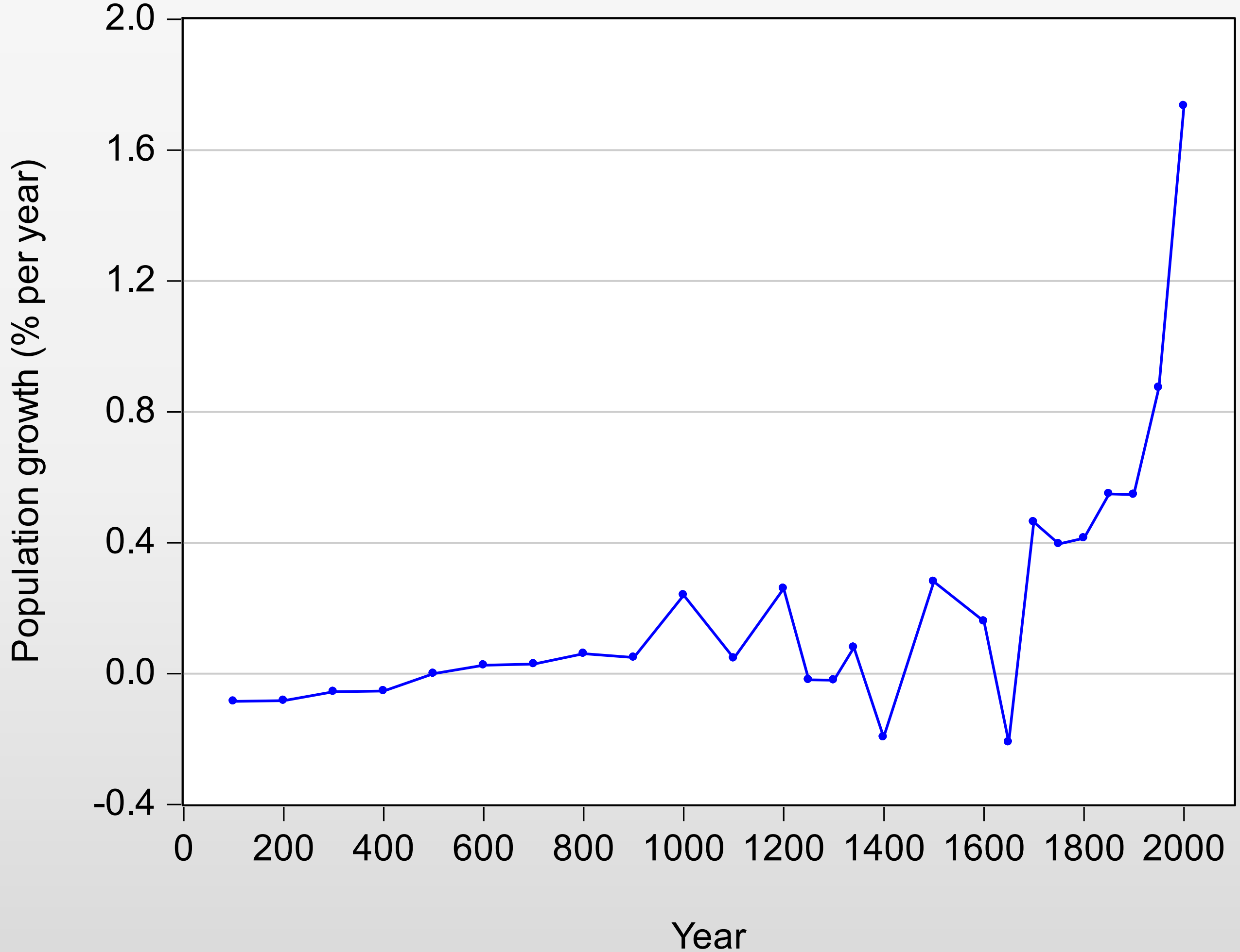
Note that the variable is (1+g), so log-normal is essentially normal.

# Next we compare IIASA with historical data

# Historical population



# Historical population growth: clearly non-stationary



# Variation across models

For 6 MUP models:

| | |
|---|---|
| Mean growth rate 2010-2100: | 0.043 % per year |
| Standard deviation: | 0.086 % per year |

UN (low, medium, high) projections 2000-2090:

| | |
|---|---|
| - If take as equally likely: | 0.41 % per year |
| - If take as (.25, .5, .25): | 0.34 % per year |

# 100-year forecast errors at different horizons

| Method | Forecast or projection error |
|---|---|
| In sample | |
| 50 year | 0.21% |
| 100 year | 0.14% |
| Out of sample | |
| 50 year | 0.43% |
| 100 year | 0.67% |
| UN long run | 0.34% - 0.41% |
| MUP models | 0.09% |
| NRC (50 yr) | < IIASA range |
| IIASA | 0.22% |

-Note that error is linear in time, not in time$^{0.5}$

# Summary again

Recommendation:

- Normal distribution
- Annual sigma of 0.22% per year
- Sigma is constant at different horizons
- Worry about model uncertainty?

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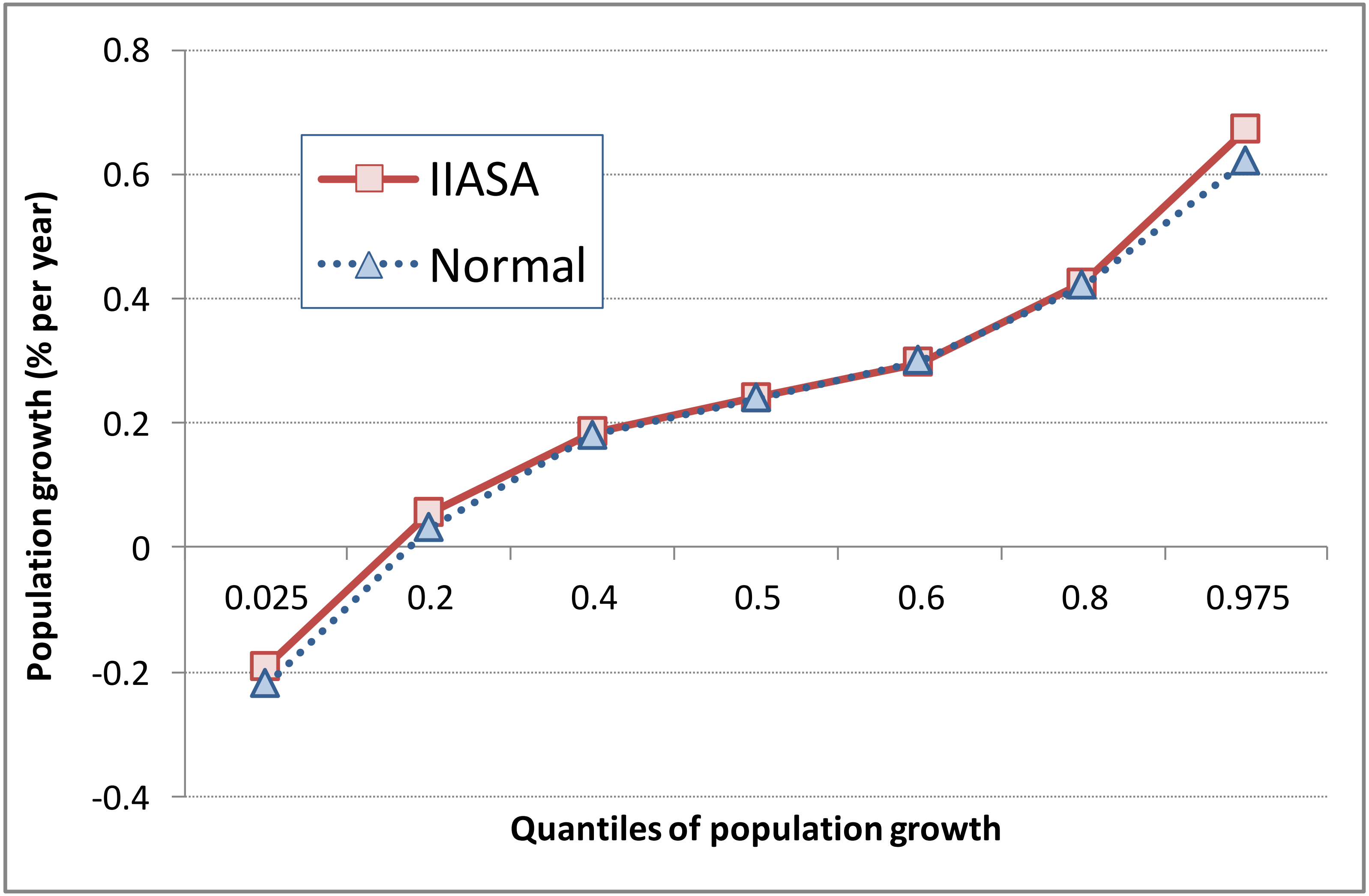
- The forecasts [are] the distribution of the results of 2000 different cohort component projections.

# IIASA distribution v. normal



# Tests of distributions of growth rates for IIASA*

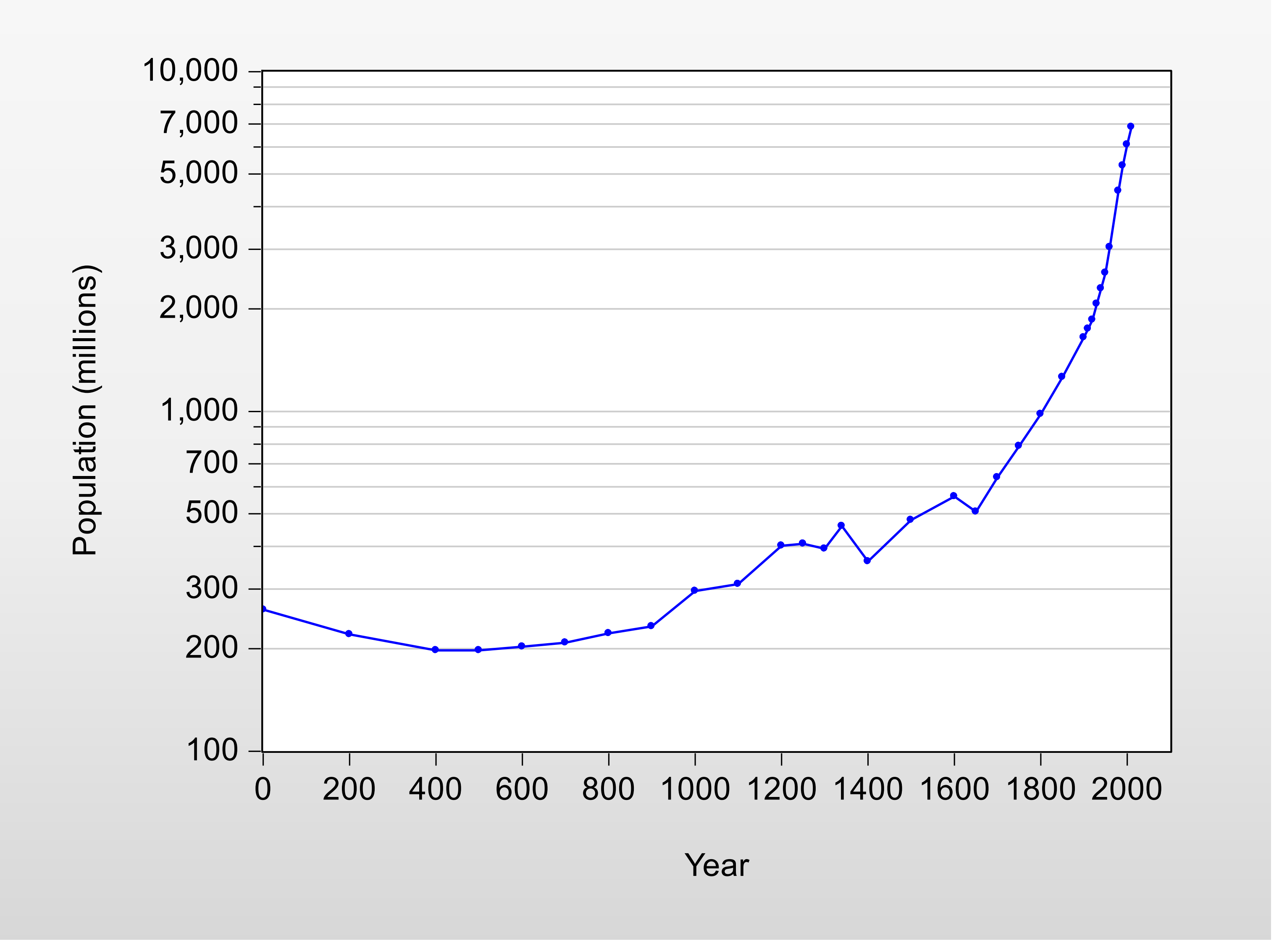
| Distribution | logL | AIC | BIC | Chisq (value) | AD(value) | H(AD) | KS(value) |
|---|---|---|---|---|---|---|---|
| Normal | ***32.0094*** | ***-60.0189*** | ***-60.1271*** | 0.1595 | ***0.1587*** | Not rejected | ***0.1453*** |
| Exponential | -7.0157 | 16.0314 | 15.9773 | 761.7996 | 3.1954 | Rejected | 0.6305 |
| Chi-square | -8.3333 | 18.6666 | 18.6125 | 919.6466 | 2.8419 | Null | 0.5716 |
| Uniform | Null | Null | Null | Null | Inf | Null | 0.1641 |
| Gamma | 32.0081 | -60.0162 | -60.1244 | 0.1581 | 0.1590 | Not rejected | 0.1457 |
| Lognormal | 32.0074 | -60.0149 | -60.1230 | ***0.1580*** | 0.1592 | Not rejected | 0.1458 |
| Weibull | 31.8757 | -59.7515 | -59.8597 | 0.3502 | 0.1834 | Not rejected | 0.1562 |
| F | 29.9355 | -55.8709 | -55.9791 | 2.0391 | 1.9084 | Null | 0.4197 |

= preferred.

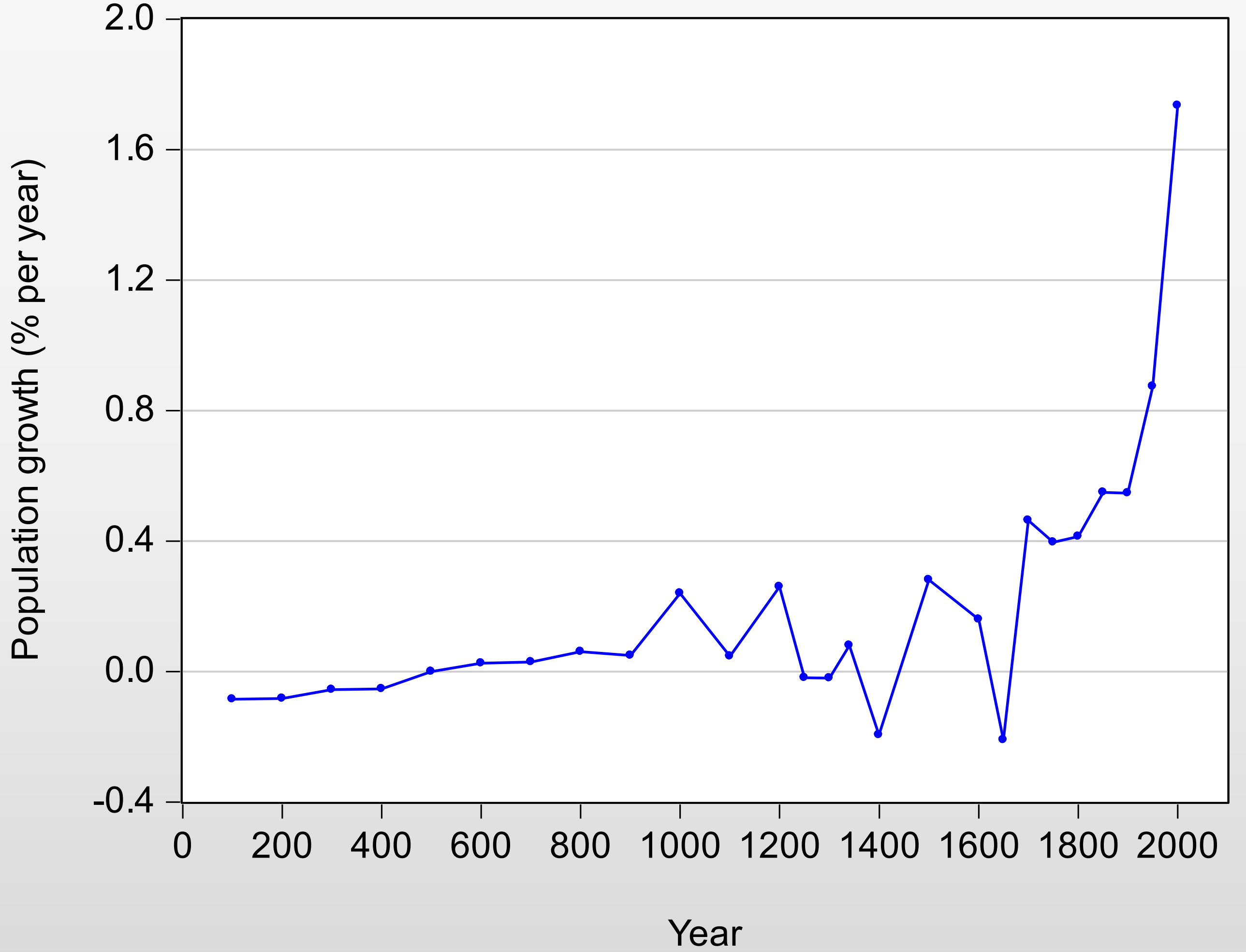
Note that the variable is (1+g), so log-normal is essentially normal.

# Next we compare IIASA with historical data

# Historical population



# Historical population growth: clearly non-stationary



# Variation across models



For 6 MUP models:

| | |
|---|---|
| Mean growth rate 2010-2100: | 0.043 % per year |
| Standard deviation: | 0.086 % per year |

UN (low, medium, high) projections 2000-2090:

| | |
|---|---|
| - If take as equally likely: | 0.41 % per year |
| - If take as (.25, .5, .25): | 0.34 % per year |

# 100-year forecast errors at different horizons

| Method | Forecast or projection error |
|---|---|
| In sample | |
| 50 year | 0.21% |
| 100 year | 0.14% |
| Out of sample | |
| 50 year | 0.43% |
| 100 year | 0.67% |
| UN long run | 0.34% - 0.41% |
| MUP models | 0.09% |
| NRC (50 yr) | < IIASA range |
| IIASA | 0.22% |

-Note that error is linear in time, not in $time^{0.5}$

# Summary again

Recommendation:

- Normal distribution
- Annual sigma of 0.22% per year
- Sigma is constant at different horizons
- Worry about model uncertainty?

# Preliminary results of Monte Carlo integration of tracks I and II

# *Schematic outline of two-track method*

Assume $y$ = endogenous variables; $u$ = exogenous or policy variables; $H^m$ = model mapping for model $m$

Steps:

1. Choose uncertain variables: TSC, TFP, Pop
2. Model calibration runs: $y = H^m(u)$.
3. Fit "surface response function," $y = R^m(u)$.
4. Derive Pdfs for $u$ variables, $f(u)$.
5. Then do Monte Carlo for distribution of output variables, obtaining the distribution $g^m(y)$ for output variables

$$g^m(y) = \int f(u) R^m(u) du$$

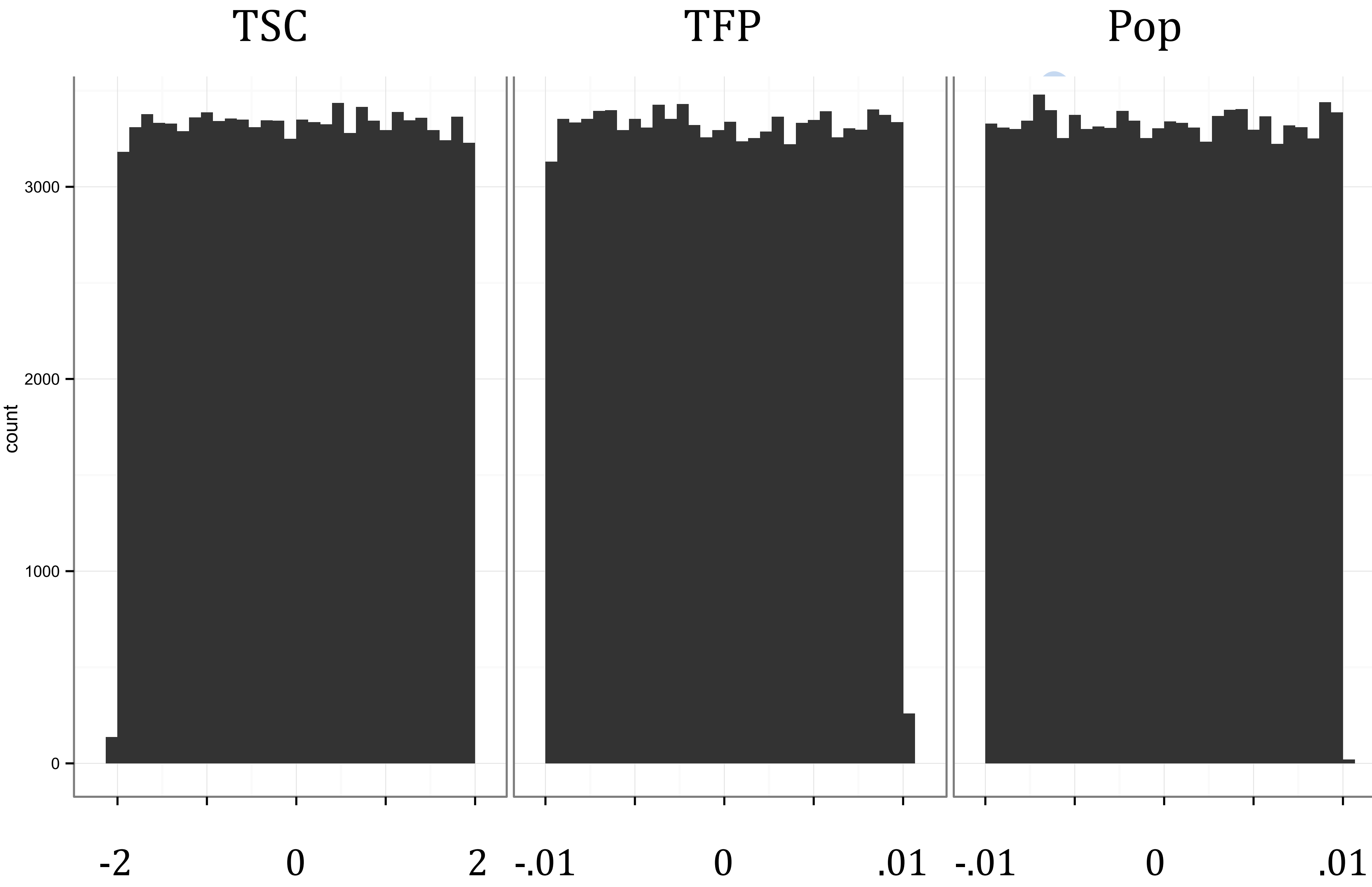
# Overview

- Integration of tracks I and II is straightforward: Monte Carlo sampling from the pdfs and associated SRFs.
- Easily replicated and validated.
- Get distributions of all variables and models
- Errors in the procedure:
- - sampling error (can be shown to be small)
- - SRF error (shown to be low from SRF presentation)
- Would be useful to have validation from modeling and pdfs.

# Preliminary and illustrative results

- Today's results are illustrative to show how the method works and determine useful outputs.
- Use the LQI estimated surface response functions
- Assume the pdfs are uniform over the range of the uncertain variables in the calibration runs:
  - Population growth uniform over (-1%, +1%)
  - TFP growth uniform over (-1%, +1%)
  - TSC uniform over (-2 $^0$C, +2 $^0$C)
- N = 100,000

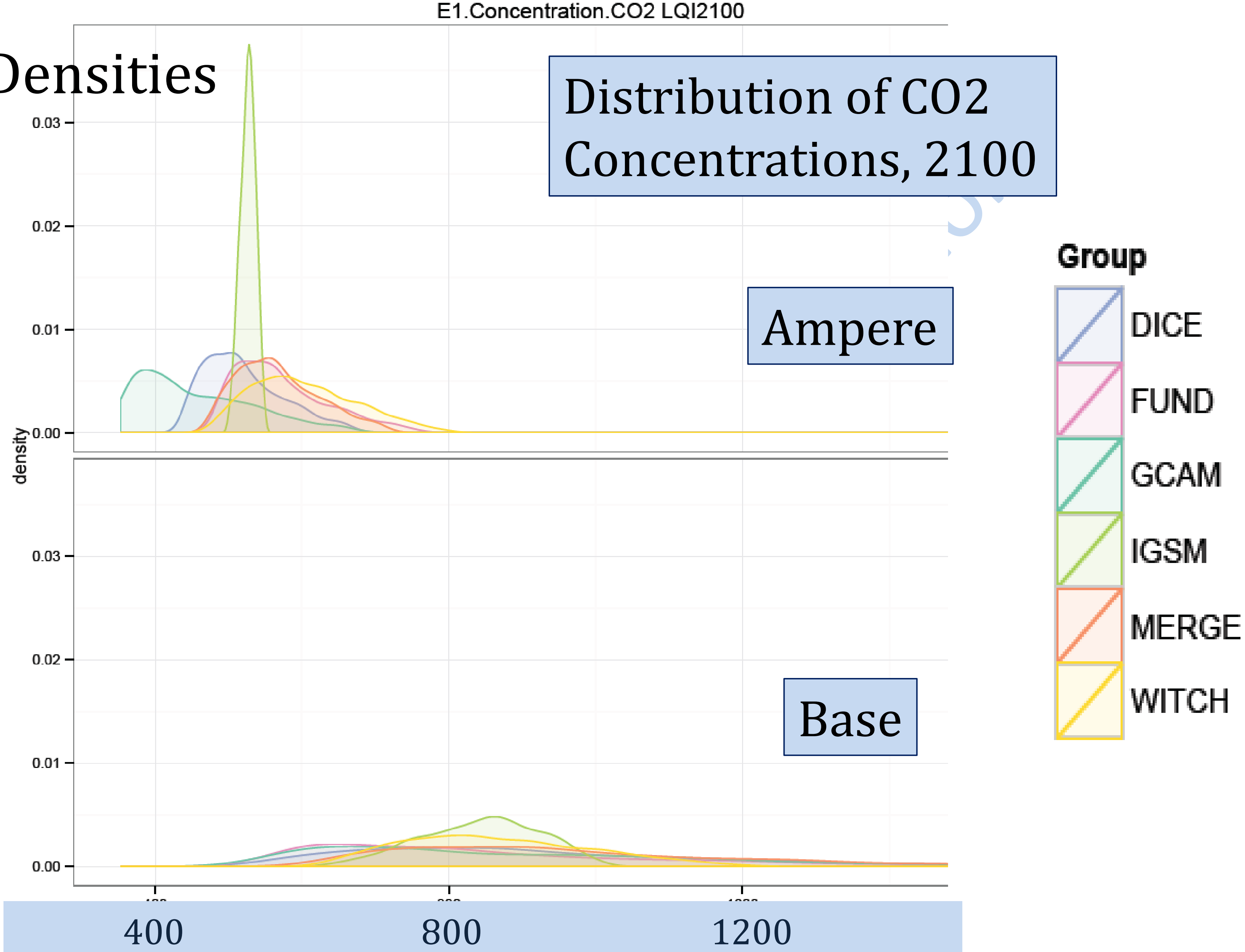
# Distribution of runs in sample

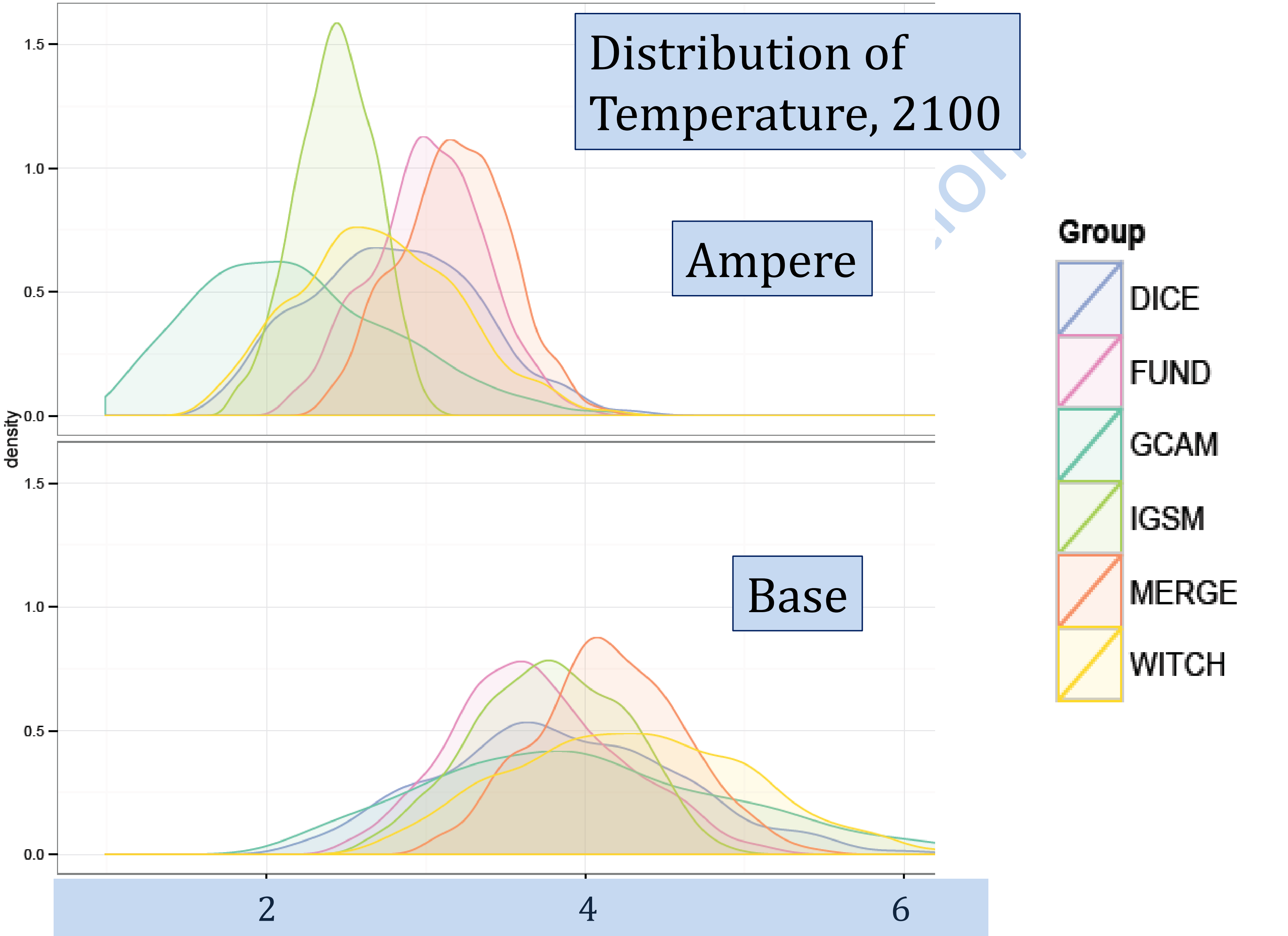


# Results Aggregated and Plotted

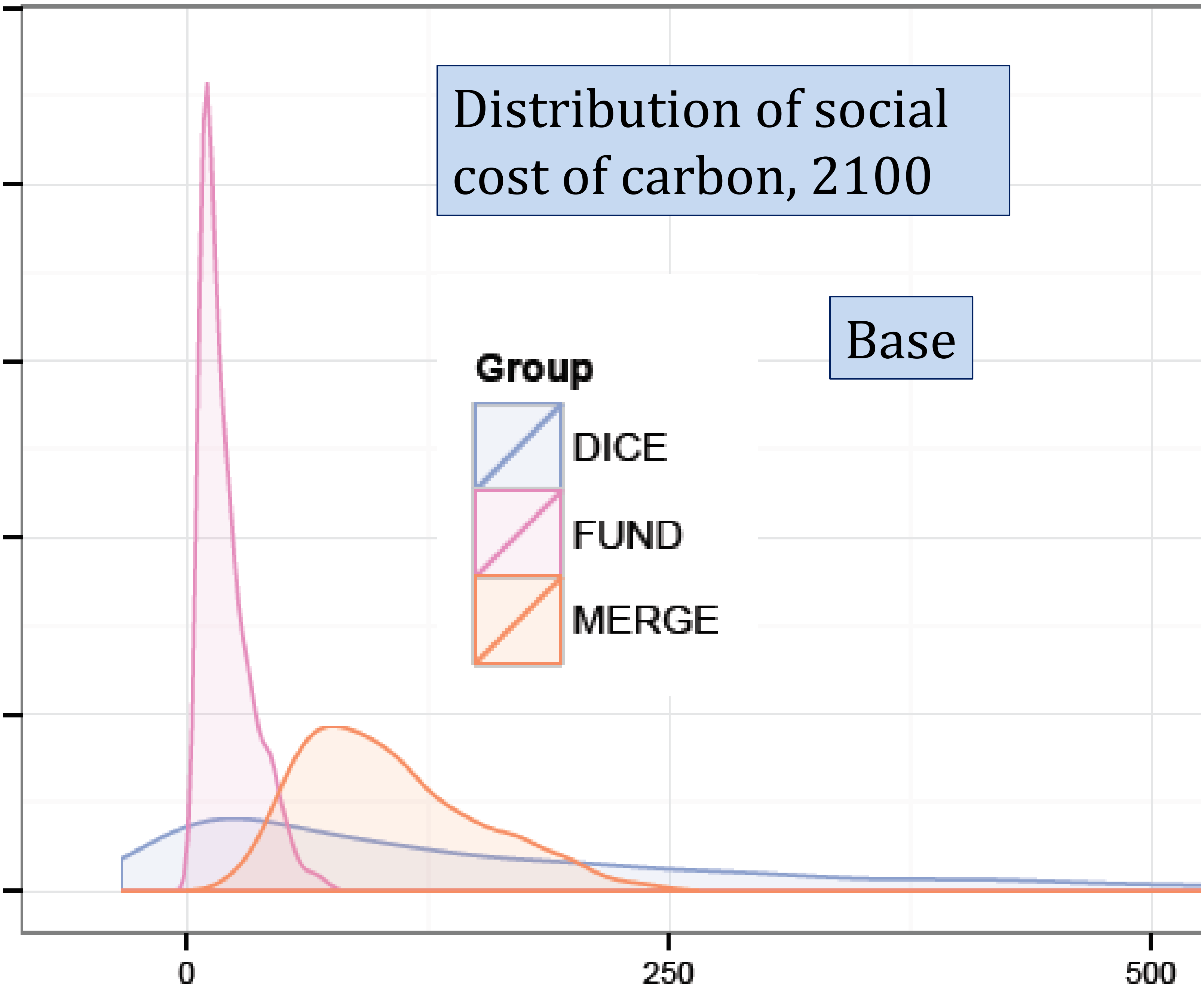
- Again, these are purely illustrative results.

# Densities


E1.Concentration.CO2 LQI2100
Distribution of CO2 Concentrations, 2100
Ampere
Base
density
0.03
0.02
0.01
0.00
400
800
1200
Group
DICE
FUND
GCAM
IGSM
MERGE
WITCH



Distribution of Temperature, 2100
Ampere
Base
density
1.5
1.0
0.5
0.0
2
4
6
Group
DICE
FUND
GCAM
IGSM
MERGE
WITCH



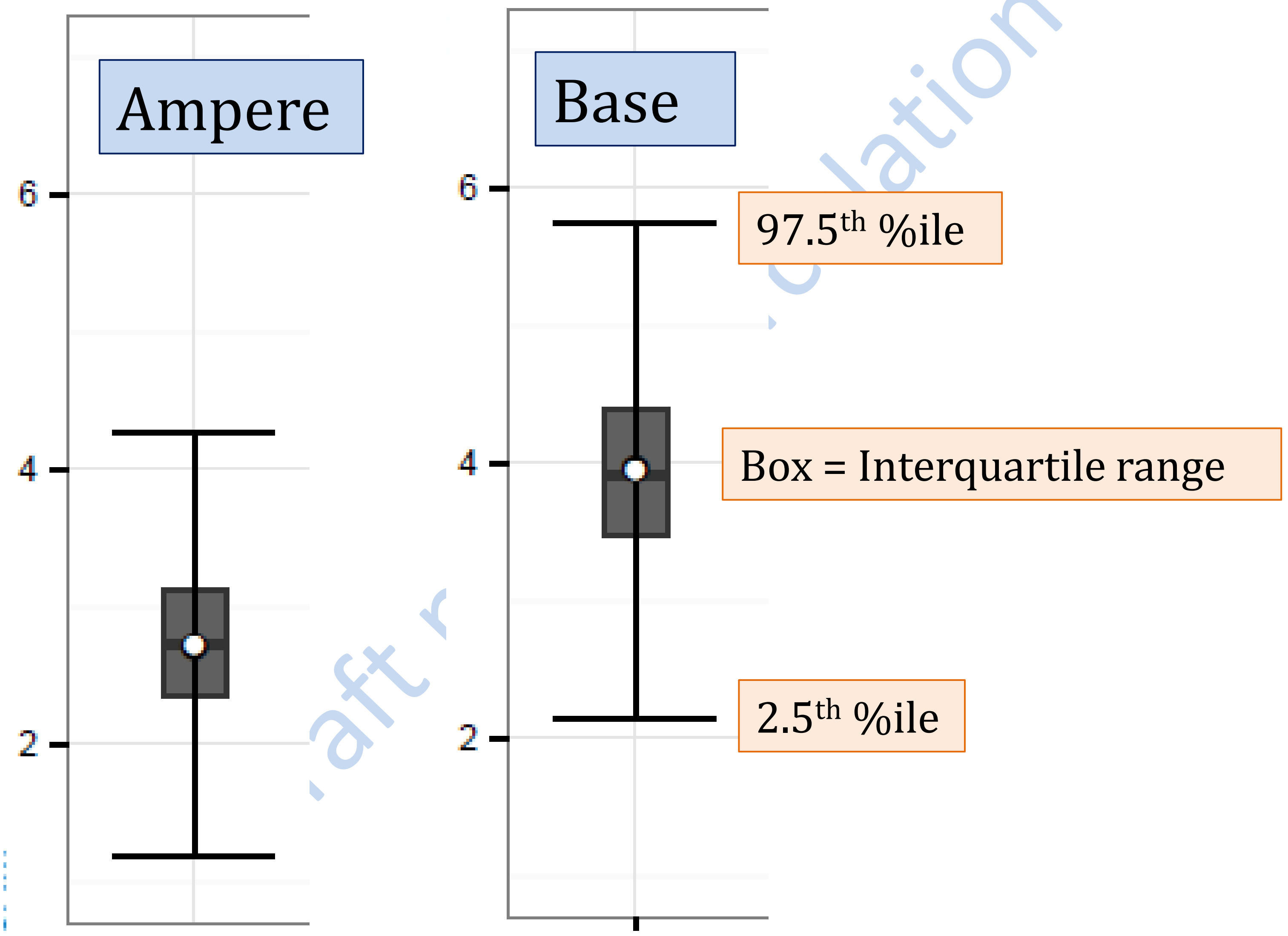
Distribution of social cost of carbon, 2100
Base
Group
DICE
FUND
MERGE
0
250
500

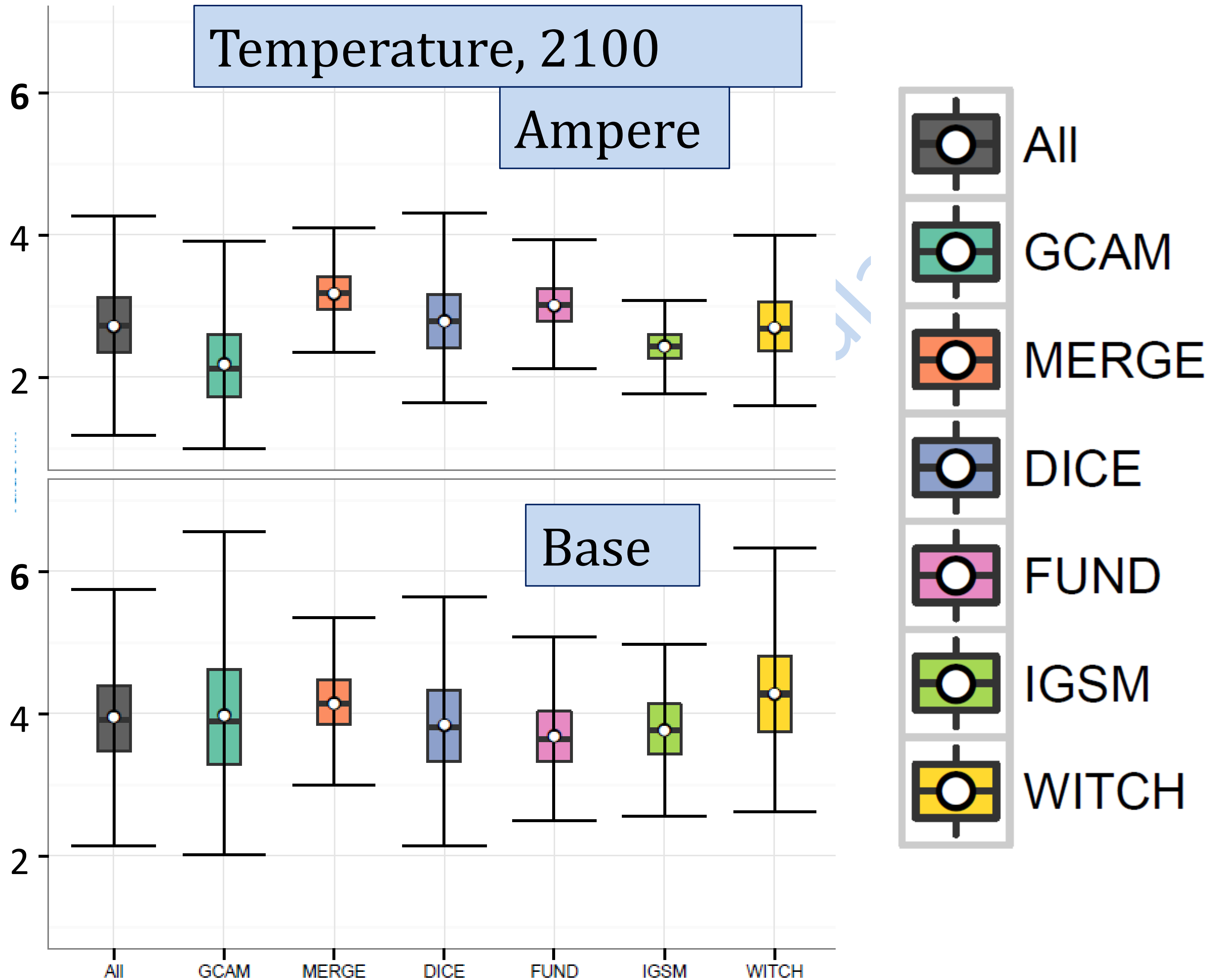
# Summary results

Summary results show box plots for:

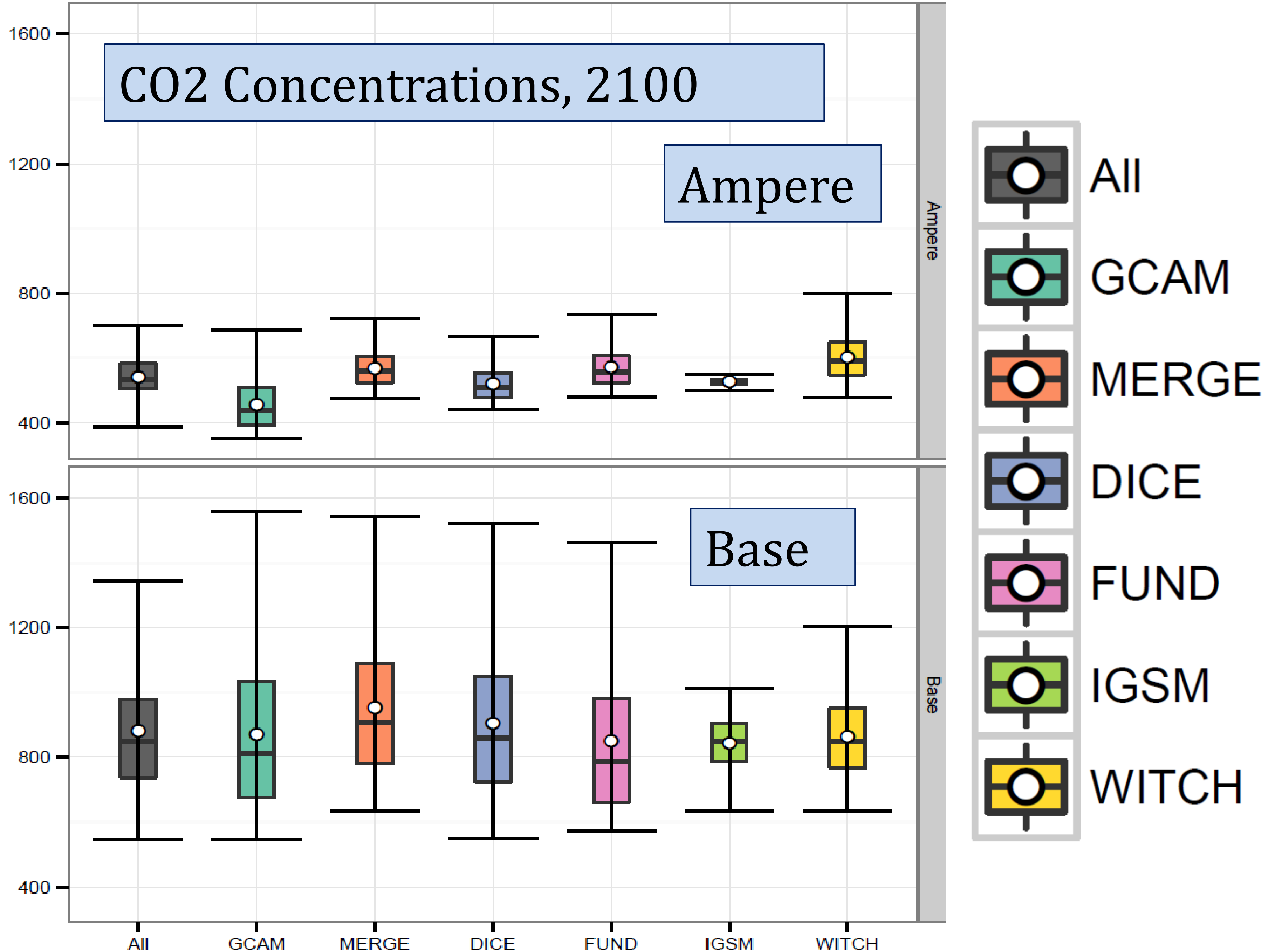
- Pooled models
- Individual models

# Temperature 2100





Temperature, 2100
Ampere
Base
6
4
2
6
4
2
All
GCAM
MERGE
DICE
FUND
IGSM
WITCH
All
GCAM
MERGE
DICE
FUND
IGSM
WITCH



CO2 Concentrations, 2100
Ampere
Base
1600
1200
800
400
All
GCAM
MERGE
DICE
FUND
IGSM
WITCH

# Further tests

***Clear track ahead for:***

1. Introduce actual Pdfs
2. Separate model uncertainty from parameter uncertainty
3. Calculate marginal contributions of different variables
4. Compare results with pseudo-distributions such as from IPCC.
5. Decompose uncertainties within models
6. Analyze impact of policies

***More distant destinations:***

7. Produce model estimates with harmonized inputs (not just harmonized uncertainties)
8. Do validation of Monte Carlos with individual models
9. Compare with others (AMPERE)

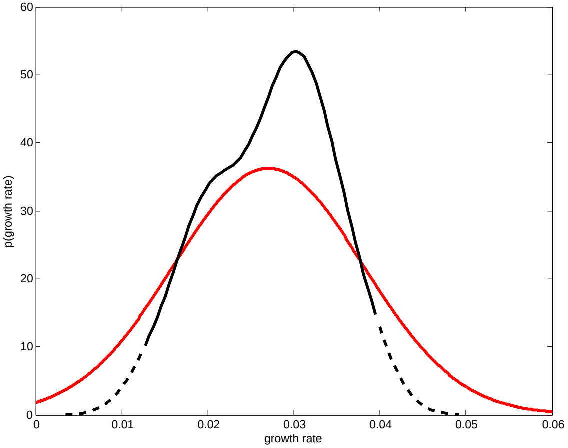
# Expert Survey on Long-Run Economic Growth

Modeling Uncertainty Project

EMF, Snowmass 2014

July 25, 2014

# GDP Growth Rates (2010-2050): Normal and Nonparametric PDFs



# Existing Treatment of Uncertainty in Long-Run Productivity Growth

- Simulations using Historical Data (Webster and Cho, 2006)
  - may not be sufficiently informative re- future uncertainty
  - limited historical data and measurement error
- Expert Surveys
  - Nordhaus 1991 (unpublished)
  - Webster et al 2002
- Reduced-Form Forecasts
  - PwC: “World in 2050” (PWC), “World in 2050” (HSBC)

# Expert Survey: Design

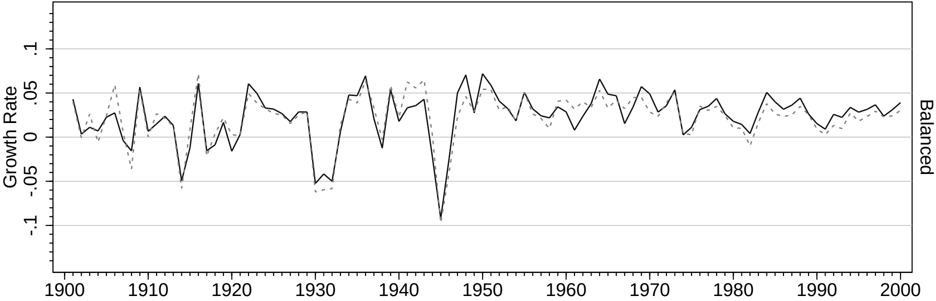
- GDP per capita, average annual rates
- 10th, 25th, 50th, 75th, 90th percentiles
- World and major regions
- Self-reported expertise

# Survey Design

Q2. Please provide estimates for the 10th, 25th, 50th, 75th, and 90th percentile rates of growth of per capita real GDP (average percent per year) for the following regions...

| | 2010-2050 | | | | |
|---|---|---|---|---|---|
| | 10th | 25th | 50th | 75th | 90th |
| World | | | | | |
| United States | | | | | |
| China | | | | | |
| High Income (Includes US) | | | | | |
| Middle Income | | | | | |
| Low Income (includes China and India) | | | | | |

# Data Provided: Historical Growth Rates (1900-2000)




Solid Line - Barro-Ursua (2010)
Dashed Line - Maddison (2003)

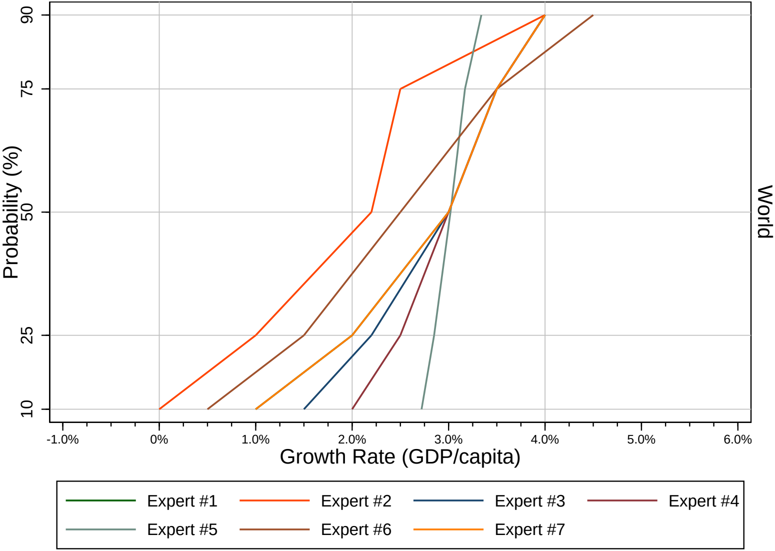
# Self-Reported Expertise

*7.4.*

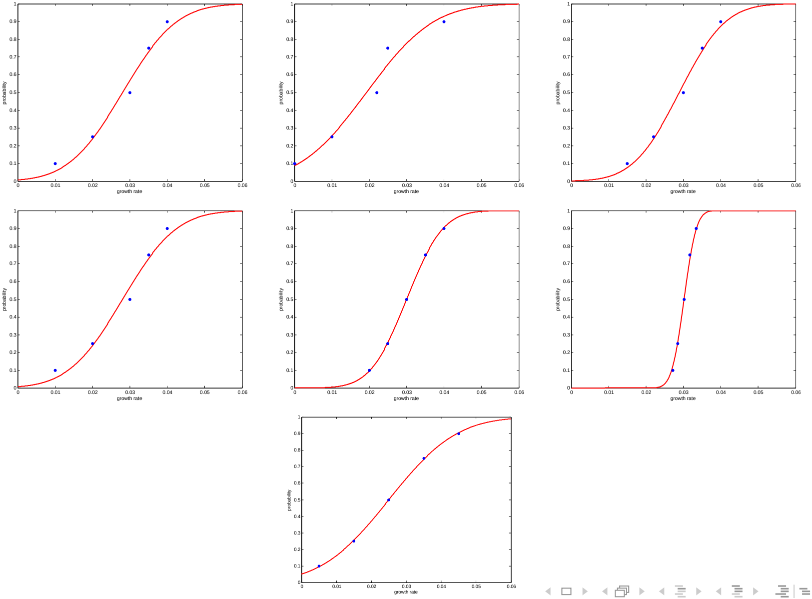
Please rank your own expertise on the different regions by checking a box on the following scale of 1-10. A rating of 1 indicates little expertise, a 5 indicates the expertise of someone who has studied the subject but is not a specialist, and 10 indicates an expertise that is among the leading experts.

| | Expertise on Economic Growth | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| World | ○ | ○ | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ |
| United States | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ | ○ | ○ |
| China | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ | ○ | ○ |
| High Income | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ | ○ | ○ |
| Middle Income | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ | ○ | ○ |
| Low Income | ○ | ○ | ○ | ○ | ○ | ○ | ● | ○ | ○ | ○ |

# Sample of Expert Growth Rates Projections: 2010-2050



# Best Fitting Normal Distribution (CDFs)

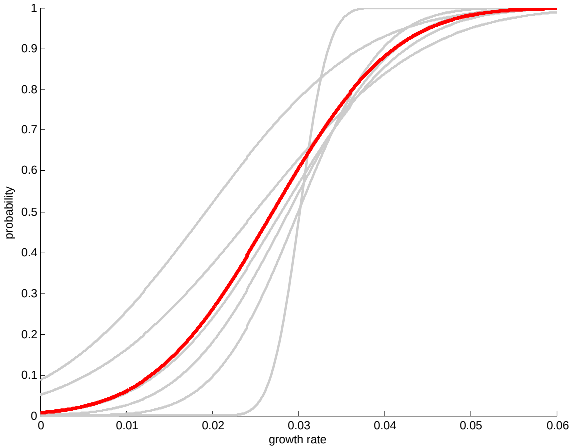


## Estimated Parameters and Kolmogorov-Smirnov Test

| | E1 | E2 | E3 | E4 | E5 | E6 | E7 |
|---|---|---|---|---|---|---|---|
| $\mu$ | 0.028 | 0.019 | 0.029 | 0.028 | 0.030 | 0.030 | 0.025 |
| $\sigma$ | 0.011 | 0.014 | 0.0097 | 0.011 | 0.0077 | 0.0026 | 0.015 |
| K-S | 0.17 | 0.18 | 0.16 | 0.17 | 0.14 | 0.14 | 0.14 |
| K-S(p) | 0.995 | 0.990 | 0.997 | 0.995 | 0.999 | 0.999 | 0.999 |

Notes: One sample K-S test compares empirical distribution function for each expert to the cdf of the normal distribution; H(0): the empirical cdf is drawn from the normal cdf.

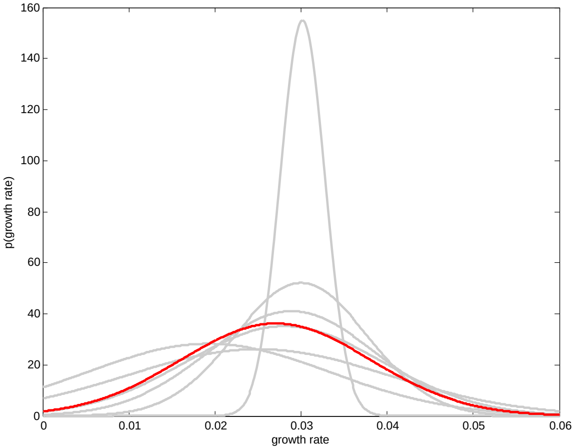
# Marginal and Combined (Normal) CDFs: 2010-2050




Notes:

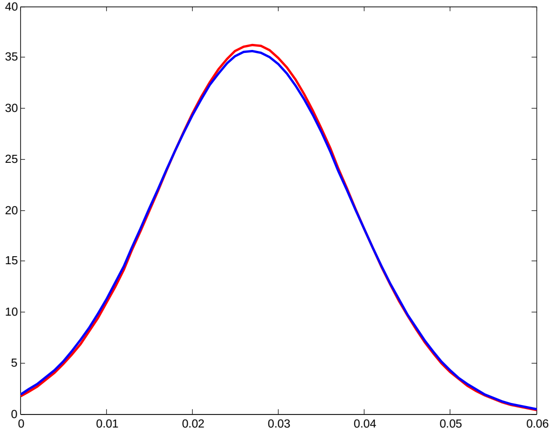
(i) Combined Normal Distribution: $F(x; \mu^*, \sigma^*)$, where $\mu^* = \frac{1}{n}\sum_{i=1}^{n} \mu_i$ and $\sigma^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n} \sigma_i^2}$. This d.f. assumes that estimates are iid.

# Marginal and Combined (Normal) PDFs 2010-2050



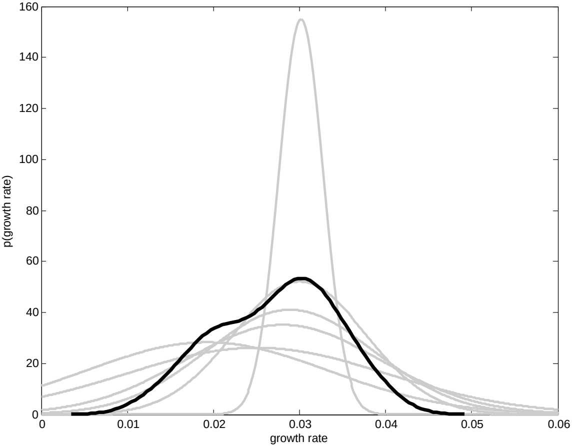

Notes:
(i) Combined Normal Distribution: $F(x;\mu^*,\sigma^*)$, where $\mu^* = \frac{1}{n}\sum_{i=1}^{n}\mu_i$ and $\sigma^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\sigma_i^2}$. This d.f. assumes that estimates are iid.

# Weighted vs. Unweighted Combined (Normal) PDFs 2010-2050



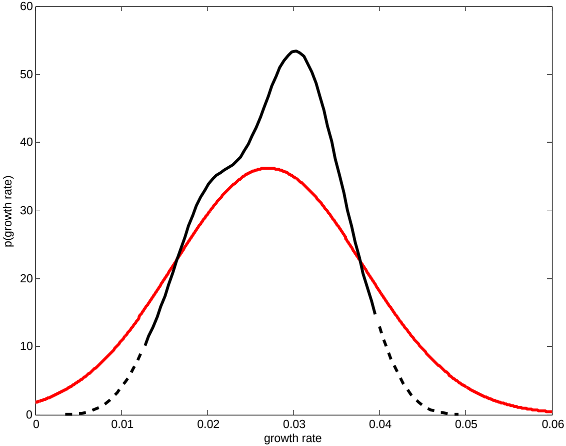

Notes:
(i) Weighted Normal Distribution: $F(x;\mu^*,\sigma^*)$, where $\mu^* = \frac{1}{n}\sum_{i=1}^{n}(W)\mu_i$ and $\sigma^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(W)\sigma_i^2}$. W is a vector of weights reflecting self-reported expertise. This d.f. assumes that estimates are iid.

# Nonparametric Aggregation




Notes:
(i) Combined Nonparametric Distribution: $F_i(x) = F(x; \theta_{i,p=10}, \theta_{i,p=25}, \theta_{i,p=50}, \theta_{i,p=75}, \theta_{i,90})$, where $F_i$ is a kernel density function. Combined estimates $(F(\theta^*))$ are constructed using quantile aggregation: $F(\theta^*) = \frac{1}{n}\sum_{i=1}^{n} \theta_i, p$. See Thomas and Ross (1980) for original formulation. This d.f. assumes that estimates are iid.

# Normal and Nonparametric PDFs 2010-2050




Notes:

(i) Combined Normal Distribution: $F(x;\mu^*,\sigma^*)$, where $\mu^* = \frac{1}{n}\sum_{i=1}^{n}\mu_i$ and $\sigma^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\sigma_i^2}$

(ii) Combined Nonparametric Distribution: $F_i(x) = F(x;\theta_{i,p=10},\theta_{i,p=25},\theta_{i,p=50},\theta_{i,p=75},\theta_{i,90})$, where $F_i$ is a kernel density function. Aggregate estimates $(F(\theta^*)$ are given by: $F(\theta_p^*) = \frac{1}{n}\sum_{i=1}^{n}\theta_i, p$.

(iii) These d.f.s assume that estimates are iid.

# Take-Home Messages

- Uncertainty in productivity growth will be modeled using expert projections – augmented by historical simulations
- We have a sample of 8 pdfs to date
- Experts estimates express divergent views but distributions are overlapping
- Distributions are approximately normal (maybe life is simple!)
- $\sigma$'s for World region 2010-2050 are reasonable relative to historical processes: $\sigma^* = 1.1\%$ vs. $\sigma_{4,25} = 1.2\%$

# Timeline

1. Design
    - February 2013 - Decision to undertake TFP Elicitation
    - April 2013 - Draft #1 of Elicitation
2. Development
    - July 2013: Round #1 Pilot: Graduate Students
    - August 2013: Round #2 Pilot: Broader Research Community
    - Remainder of 2013: Pilot #3, Pilot #4, Pilot #5
3. Administer Survey
    - Early 2014: Finalize Instrument and Sample
    - March-August 2014: Rounds 1-4
4. Analyze Responses and Develop Aggregation Method
    - **June-September: Collect Responses – reliability, dependence, functional form**