Optimal emission-extraction policy in a world of irreversibility and scarcity

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Motivation

Design of climate and resource management policy.

Exhaustible resources: fossil fuels available in limited amounts.

Accumulation of CO_2: climate change and environmental damage.

Irreversible pollution:

- Assimilative capacity of ecosystems changes in response to the increasing concentration of CO_2.
- Above a critical threshold, regeneration may cease.

Literature I: management of exhaustible resources

Withagen (1994), Tahvonen (1997): optimal extraction policy under pollution damage from fossil fuel consumption.

$y \geq 0$: extraction rate/consumption; $U(y)$, utility function

x: stock of fossil fuels, $\dot{x} = -y$, $x(0) = x_0$ given,

z: stock of CO_2, $\dot{z} = y - \alpha z$, $z(0) = z_0$ given; $D(z)$, damage function

$\Rightarrow$ Pollution problem has no influence on the total amount of resource extracted; resource depletion in finite time.

$\Rightarrow$ Characterization of optimal extraction and pollution trajectories.

But, linear decay $\Rightarrow$ IGNORE IRREVERSIBILITY

Evidences of irreversibility

Local pollutant problems (Dasgupta and Mäler, 2003)

- Use of fertilizers and the **eutrophication** of shallow lakes,
- **Salinification** of soils, **loss of biodiversity** because of land use.

Global environmental threats: climate change and positive feedbacks (IPCC, 2008)

- Oceans and terrestrial ecosystems' ability to store carbon.
- Natural reservoirs will likely **switch from sinks to sources**.

Literature II: control of pollution and irreversibility

Assumption of linear decay is too simplistic!

Alternative: **as soon as pollution exceeds some critical threshold $\bar{z}$, assimilation vanishes**

$$\dot{z} = \begin{cases} y - \alpha(z) & \text{if } z \leq \bar{z} \\ y & \text{else} \end{cases}$$

- Tahvonen & Withagen (1996): optimal control of pollution under irreversibility

Same model except that inverted U-shaped decay function

$\Rightarrow$ Multiple optimality candidates, *i.e.*, paths satisfying the necessary optimality conditions.

But, no resource $\Rightarrow$ IGNORE EXHAUSTIBILITY

Approach and Results

Extension of the classical exhaustible resource/stock pollution problem with irreversibility of decay.

What is the impact of irreversibility on the optimal management of resource?

Not necessarily optimal to deplete the resource in finite time.

Should we really care about irreversibility because the source of pollution is finite?

Yes! In general, multiplicity of candidates, reversible and irreversible.

Conditions under which the unique optimum is reversible or not.

Outline of the talk

Two remarks

Nature of the solution

- Reversible *versus* irreversible policies,
- Full *versus* only partial exhaustion of the resource,
- With *versus* without a period of time staying at the threshold.

$\Rightarrow$ Lot (2^3) of possible combinations!

Ecological catastrophe due to the abrupt vanishing of the decay

$$\dot{z} = \begin{cases} y - \alpha z & \text{if } z \leq \bar{z} \\ y & \text{else} \end{cases}$$

$\Rightarrow$ Discontinuity in the decay ($\neq$ Tahvonen & Withagen, 1996)

Reversible policies

$$\max_{\{y\}} W = \int_0^\infty [U(y) - D(z)]\, e^{-\delta t} dt$$

subject to,

$$\begin{aligned} &\dot{z} = y - \alpha z,\ z(0) = z_0 < \bar{z} \text{ given} \\ &\dot{x} = -y, x(0) = x_0 \text{ given} \\ &y(t) \geq 0, \quad x(t) \geq 0, \quad z(t) \leq \bar{z} \quad \forall t \end{aligned}$$

Set of necessary optimality conditions (+transversality condition):

$$\begin{aligned} &U'(y) \leq \lambda + \mu,\ y \geq 0, y[U'(y) - \lambda - \mu] = 0 \\ &\dot{\mu} = \delta\mu \\ &\dot{\lambda} = (\delta + \alpha)\lambda - D'(z) - \kappa, \quad \kappa \geq 0, \quad \kappa(\bar{z} - z) = 0 \\ &\dot{z} = y - \alpha z \\ &\dot{x} = -y, x \geq 0 \end{aligned}$$

Irreversible policies I

Problem 1. T_E date when entering the irreversible region

$$\max \int_{T_E}^{\infty} [U(y) - D(z)] e^{-\delta t} dt$$

s.t. $\dot{z} = y$, $z(T_E) = \bar{z}$, $\dot{x} = -y$, $y \geq 0$ and $x(T_E) = \bar{x} > 0$.

Problem 2. T_H date when hitting the threshold

$$\max \int_{0}^{T_H} [U(y) - D(z)]\, e^{-\delta t} dt$$

s.t. $\dot{z} = y - \alpha z$, $z(0) = z_0 < \bar{z}$, $z(T_H) = \bar{z}$, $\dot{x} = -y$, $y \geq 0$, $x(0) = x_0$ and $x(T_H) = \underline{x} > 0$.

Irreversible policies II

Either $T_H < T_E$: stage at the threshold,

Or $T_H = T_E$: immediate switching.

Suppose $T_H = T_E$ and let $\hat{V}(T_E)$ and $V(T_E)$ be the present values of problem 1 and 2, evaluated in the optimum, then

$$\frac{\partial V(T_E)}{\partial T_E} + \frac{\partial \hat{V}(T_E)}{\partial T_E} = 0.$$

$\Rightarrow$ **Upward discontinuity** in y at T_z.

Existence, multiplicity of solutions I

Suppose $U'(0) \leq D'(\bar{z})/\delta$,

And/or α **large enough,**

And/or the initial resource stock lower than a critical value $\hat{x}_0$.

$\Rightarrow$ For any (z_0, x_0), a reversible policy exists and corresponds to the unique optimum.

The resource stock is depleted eventually, pollution strictly remains in the reversible region.

Opposite situation: several optimality candidates.

Existence, multiplicity of solutions II

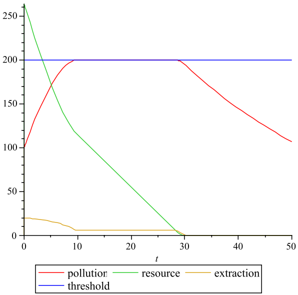
- One reversible policy featuring an interval of time along which the pollution is at the threshold.

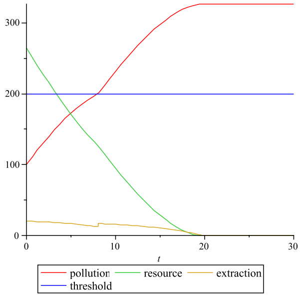
- Two irreversible policies with full exhaustion:

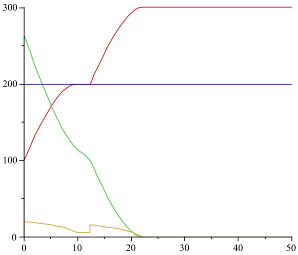
One directly reaches the irreversible region whereas along the other, the economy stays at threshold for a period of time.

- One irreversible candidate with some amount of resource left in the ground in the long run and with no period of time spent at the threshold.

Illustration of reversible and irreversible policies







Questions

Suppose fundamentals are such that irreversibility is possible.

When does multiplicity of optimality candidates arise? What role for initial conditions?

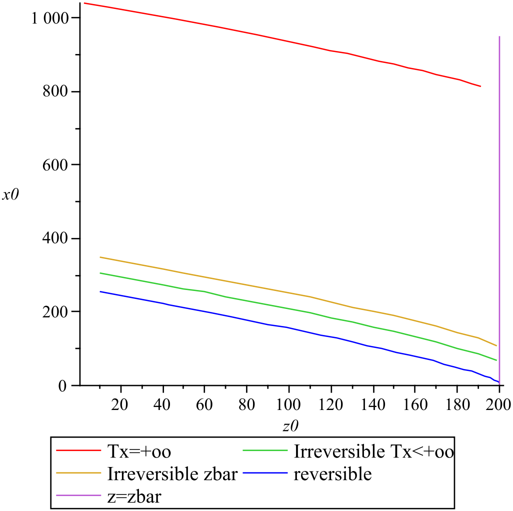
In case of multiplicity, what is the best policy? Is it reversible or irreversible?

Numerical example with functional forms:

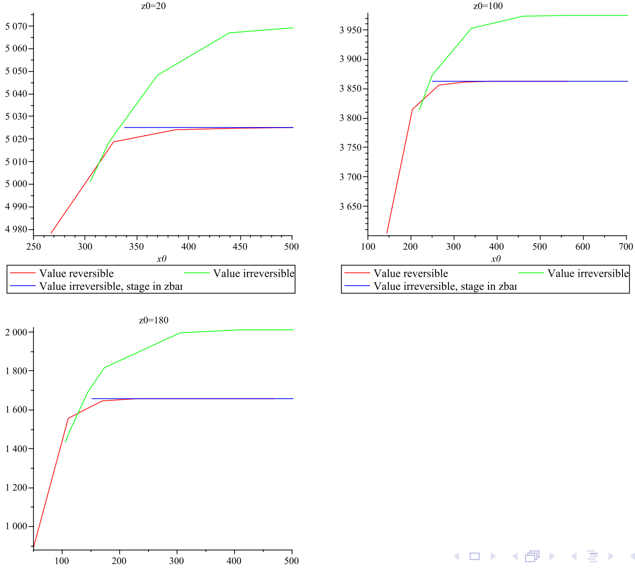
$$\begin{cases} U(y) = \theta y(\bar{y} - y) \\ D(z) = \frac{\gamma z^2}{2} \\ \text{with } \theta, \gamma, \bar{y} \geq 0 \end{cases}$$

Set of baseline parameters:
$\{\delta, \alpha, \gamma, \theta, \bar{y}, \bar{z}\} = \{0.2; 0.003; 0.02; 2; 50; 200\}$.

Multiplicity: frontiers



Optimality: multiplicity and dominance



Further developments

Exhaustibility and **quadratic decay**: in a sense simpler because not profitable to remain at the threshold.

Discussion about the shape of the decay function and consequences on the best policy.

Add **backstop**: optimal timing of adoption, optimal combination of "technologies".

Add **uncertainty**: about the level of available fossil fuels and proximity of environmental thresholds.

Exhaustibility and irreversibility are relevant issues from policy perspective: **optimal carbon tax**.