**INDUSTRIAL ENERGY DEMAND:
A SIMPLE STRUCTURAL APPROACH***

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INDUSTRIAL ENERGY DEMAND

A Simple Structural Approach*

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This paper describes a simple structural model of industrial energy demand based on production-function-like concepts, with vintaging of the available capital stock. The model is designed to help users understand the past and likely future effects of variations in factor prices and rates of output growth on industrial energy demands by major industry group. The approach handles capital-stock adjustment, electric/non-electric energy competition and business cycle effects in a simple, but natural, way. A description of an interesting initial application of the model to the primary metals industry is also included. This application improves our understanding of the approach, and demonstrates its usefulness in industrial energy demand analysis.

1. Introduction

The rate of growth in the demand for energy by U.S. industry has fluctuated dramatically over the past fifteen years. Business conditions and fuel prices have changed more precipitously over this period than over any comparable period since the Second World War. In addition, many energy-intensive industries – e.g., steel, aluminium, paper, chemicals, etc. – have been subject to foreign competition to a greater degree than previously experienced [Office of Technology Assessment (1983)].

There are substantial differences of opinion about how changes in these factors have influenced industrial energy demand in the past. Uncertainty about the strength of past relationships combined with uncertainty about the future course of fuel prices and economic conditions leads to a wide range of projections of industrial energy demand over the next twenty years.

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These differences of opinion regarding future energy demand by U.S. industry are of more than academic interest to the nation's electric utilities. Given the long lead times required to construct new generation capacity, the cost of under- or over-estimating future electricity demand could be significant. If capacity turns out to be insufficient at some date in the future, short lead-time capacity – e.g., gas turbines – will have to be installed possibly dictating high fuel costs for a number of years. If, on the other hand, capacity is overbuilt, rate payers or owners will have to pay for the excess capacity.

2. Methodology

A variety of methods has been employed to try to understand past trends in industrial electricity demand and to project future ones. Numerous econometric, process engineering, and input–output techniques have been developed. These methods provide the foundations for research models like those employed in studies by Jorgenson (1984), Berndt and Wood (1984), and Hazilla and Kopp (1984), as well as for applied forecasting and analysis models like the six models employed in the recent Energy Modelling Forum (1987) study on industrial energy demand. Each of those models has been refined and enhanced over a number of years and by now includes elements of all three approaches to energy demand modeling. However, each has a specific methodology at its foundation. PURHAPS [the Purchased Heat and Power model developed by the Energy Information Administration; Werbos, (1983)] relies dominantly on econometric concepts and data-estimation techniques; ISTUM2 [the Industrial Sector Technology Use Model-2 developed by Energy and Environmental Analysis, Inc.; EEA (1982)] and AES/ISTUM1 [a model maintained by Applied Energy Services, Inc., AES; originally developed by EEA (1978)] rely on the process analysis methodology; ORIM [the Oak Ridge Industrial Model; Reister (1982)] combines the econometric and process analysis approaches; finally, the Wharton Annual Model [Wharton Econometric Forecasting Associates (1982)] and INFORUM [a model developed at the University of Maryland; Almon (1982)] rely on input–output analysis, with econometric estimation of values for parameters that reflect the adjustment of input–output coefficients to changes in the prices of inputs, as well as the composition and level of final demand.

We have built a hybrid model and have tried to keep it as simple as possible in order to make use of the available data, and to produce easily interpretable results. A more comprehensive hybrid model, currently under development, is described in Battelle–Columbus Division (1986). Notwithstanding our desire for a simple framework for the analysis of industrial energy demand, we decided that several features had to be included in our analysis framework.

2.1. Factors of production

Each industry combines capital, labor, materials, and energy inputs using particular technologies designed to produce a specific slate of products. The choice of technology used to produce a particular product depends on the relative costs of the input factors. This makes it desirable to consider explicitly the prices of all input factors in assessing the demand for energy by U.S. industry.

2.2. Electric versus non-electric energy demand

Most industrial boilers are fired by fossil fuels, and most industrial motors are driven by electricity. However, many products can be produced by two alternative technologies – one that relies predominantly on electricity, the other relying primarily on fossil fuels. The production of steel by open hearth furnaces fired by fossil fuels, and by electric arc furnaces that can produce raw steel from recycled scrap provides a clear example of this type of substitution potential. Thus, it is desirable to consider the relative prices of electric and non-electric energy in projecting future industrial electricity demand.

2.3. Capital stock vintaging

Since energy use by industry is so intimately tied to equipment that generally lasts for several decades, the characteristics of the existing energy-using equipment are major determinants of the amount and type of energy consumed in any given year. Each year new equipment is installed under the economic conditions that prevail in that year – the vintage year of the equipment. As conditions change, there may be some flexibility in the use of factor inputs in each vintage of equipment, but not nearly as much as there is in new equipment. In addition, some of the existing equipment may be used at less than full capacity if conditions are drastically different than when it was installed. These possibilities make it desirable to incorporate vintaging of the existing capital stock in making industrial electricity demand projections.

2.4. Methodology

The methodology employed here builds on these desiderata for industrial energy demand analysis. It allowed us to simulate the energy consumption

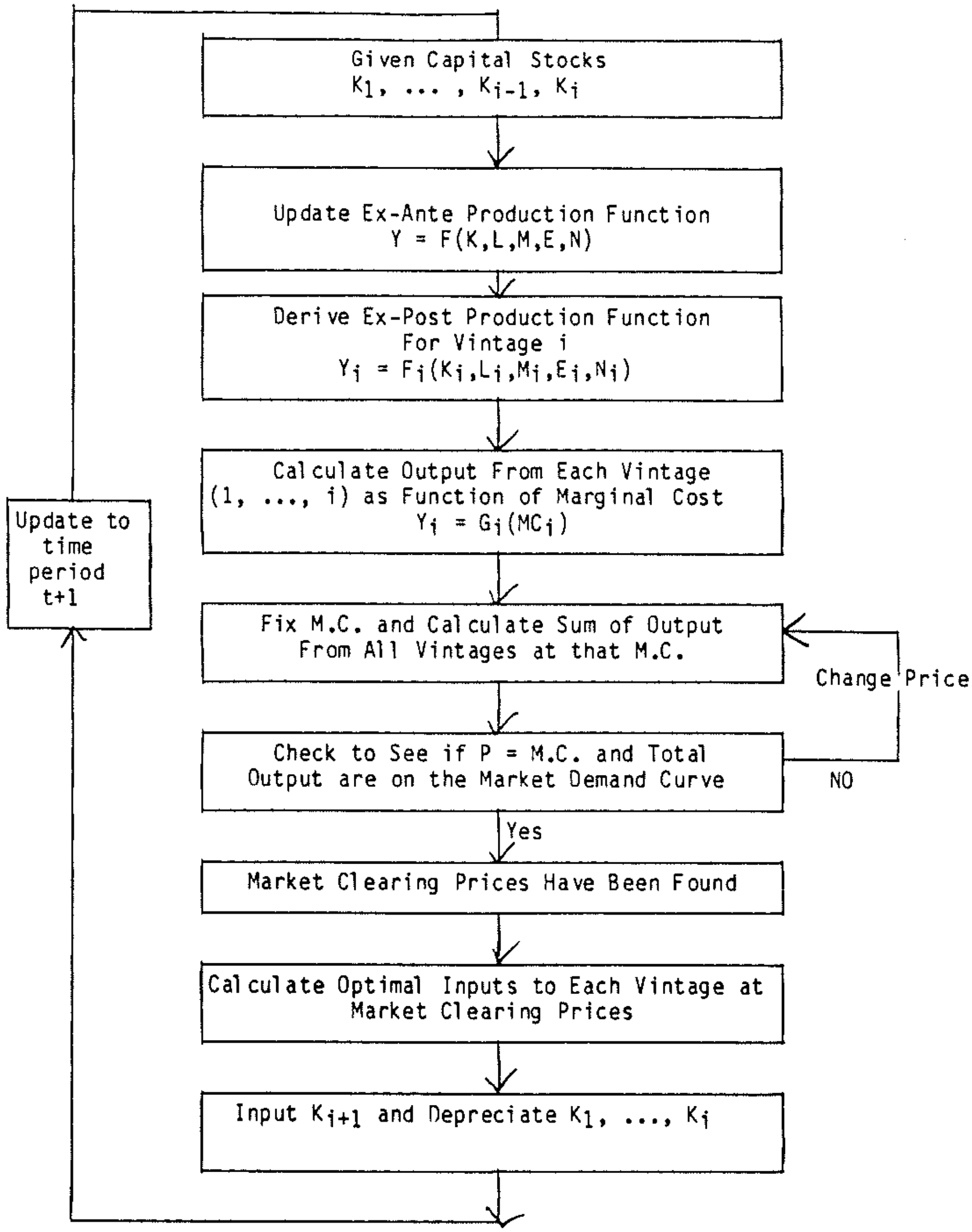

Fig. 1. Schematic diagram of model structure.

behavior of an industry or group of industries in a constructive and natural way.

A flow diagram for the approach is shown in fig. 1. First, a constant elasticity of substitution (CES) production function (F) – with capital (K), labor (L), materials (M), electric (E), and non-electric energy (N) inputs – is postulated for the new vintage (i) of equipment. The initial factor input ratios for the new vintage depend on the input prices that prevail when it is installed. The amount of new capital installed (K_i) is specified exogenously. Then, an ex-post production function (F_i) with less substitution potential is derived for the new vintage. We selected the CES functional form so as to be able to analytically derive a family of short-run ex-post cost functions that are consistent with the given ex-ante production function with the only difference between members of the family being that they had been built at different design prices. These ex-post production functions are used to derive factor demand equations, and short-run marginal cost functions (G_i).

Next the outputs from all existing vintages at a given marginal cost (MC) are summed to create an industry supply curve. The demand for output from the primary metals industry is represented with an accelerator type demand specification because most of its output is used in the production of capital goods. Thus, demand is more closely tied to the rate of growth of the economy than its level in any one year. Equilibrium is achieved at the marginal cost which equilibrates industry all with the demand for industry output.

Once the market-clearing output price for the industry is determined, the output quantity from each vintage and the corresponding factor inputs can be determined from the short-run marginal cost and factor demand equations. Finally, capital of all vintages is depreciated in preparation for production in the next period. The model's equations are presented in Appendix B.

3. Industrial data

The availability and quality of the existing data on industrial factor inputs and product outputs constrain the formulation and limit the accuracy of any industrial electricity demand projection methodology.

Data on labor, capital, and materials inputs and outputs by industry are maintained by the Office of Business Analysis at the U.S. Department of Commerce. That same office has recently consolidated and updated several variants of the National Energy Accounts prepared by various groups over the past ten years.

The completeness and accuracy of the National Energy Accounts have been well documented by Marlay (1983). Many data problems stem from the lack of availability of primary source data which is expensive to collect and often proprietary in nature. Although not as well documented to our knowledge, it is commonly believed that similar problems are associated with the non-energy industrial sector data.

Notwithstanding the problems with the existing data, it is good enough to do preliminary analyses. Useful insights can be gained from these preliminary analyses, and in time better data may become available.

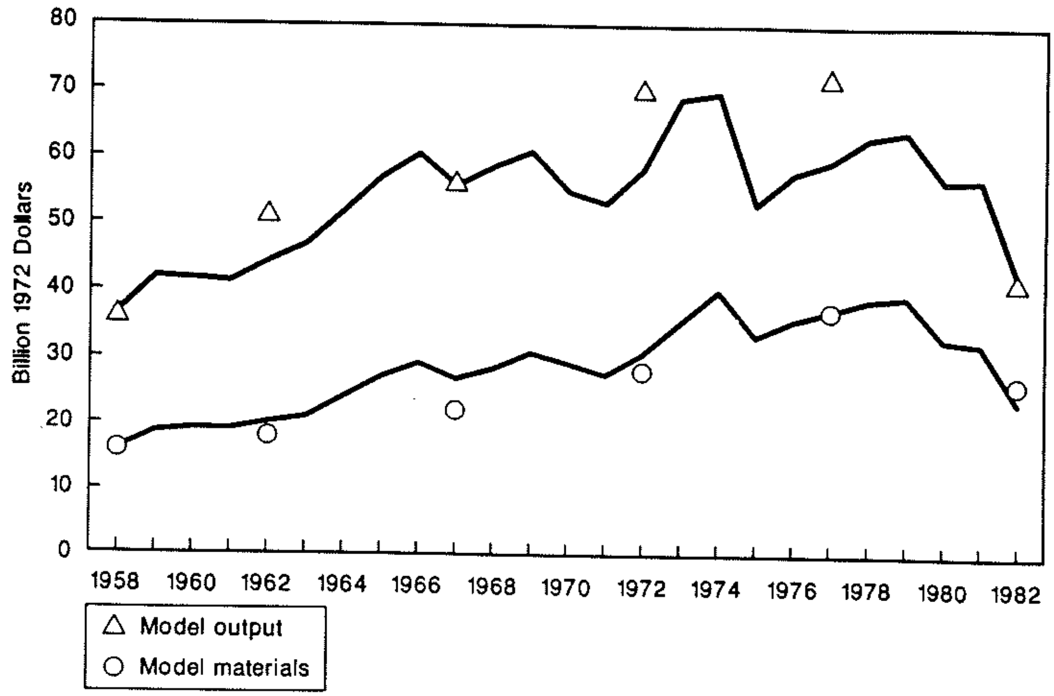

Fig. 2. Primary metals output and material use: Model results versus historical data.

4. Historical comparisons: The primary metals industry

Data from the U.S. Department of Commerce was used to exercise the model for the primary metals industry (Standard Industrial Code 33) for the period from 1958 to 1982. The input data for this test case are listed in Appendix A. The model parameter values assumed for this industry test case are recorded in table 2.

The model's projections of primary metals industry output and materials inputs are compared with actual data in fig. 2. The model is calibrated so that projected industry output and inputs match those actually observed in 1958. In general, the model under-predicts materials inputs during the early part of the sample time period and over-predicts output during the latter part. The labor input projections from the model are shown in fig. 3 and track the actual values quite well.

Although the non-energy projections are important, it is the energy – electric and non-electric – demand projections that provided the primary impetus for the development of our model. The model's projections of electric and non-electric energy demands by the primary metals industry from 1958 to 1982 are compared with the actual demands in fig. 4. The differences in projected and actual factor input average annual percentage growth rates are shown in table 1. In general, the model under-predicts electricity demand, particularly in the post 1973–74 oil embargo era.

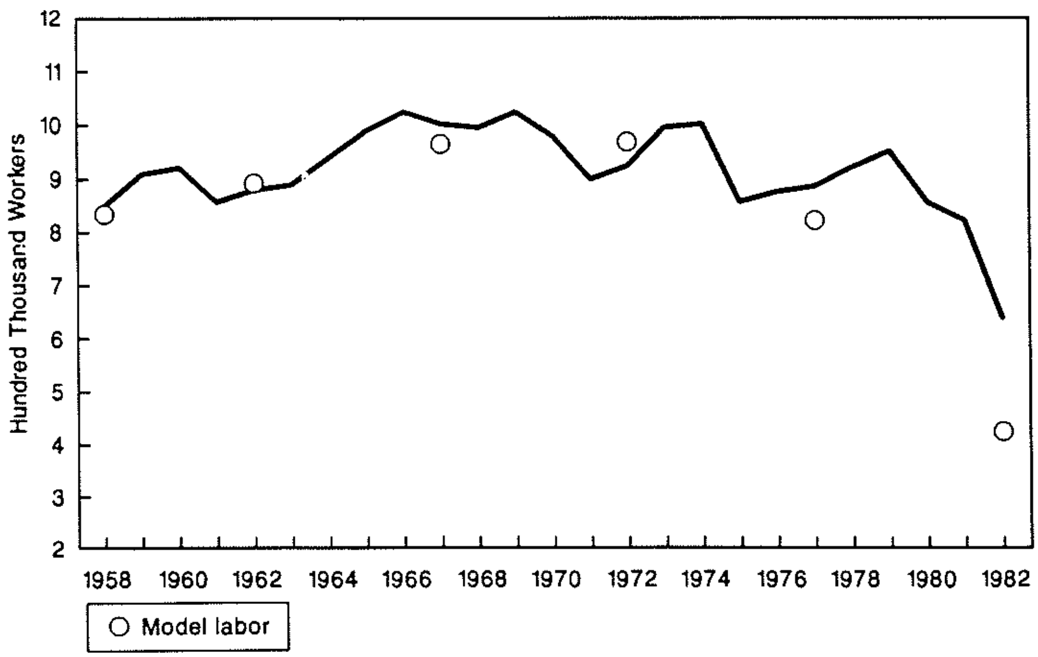

Fig. 3. Primary metals labor input: Model results versus historical data.

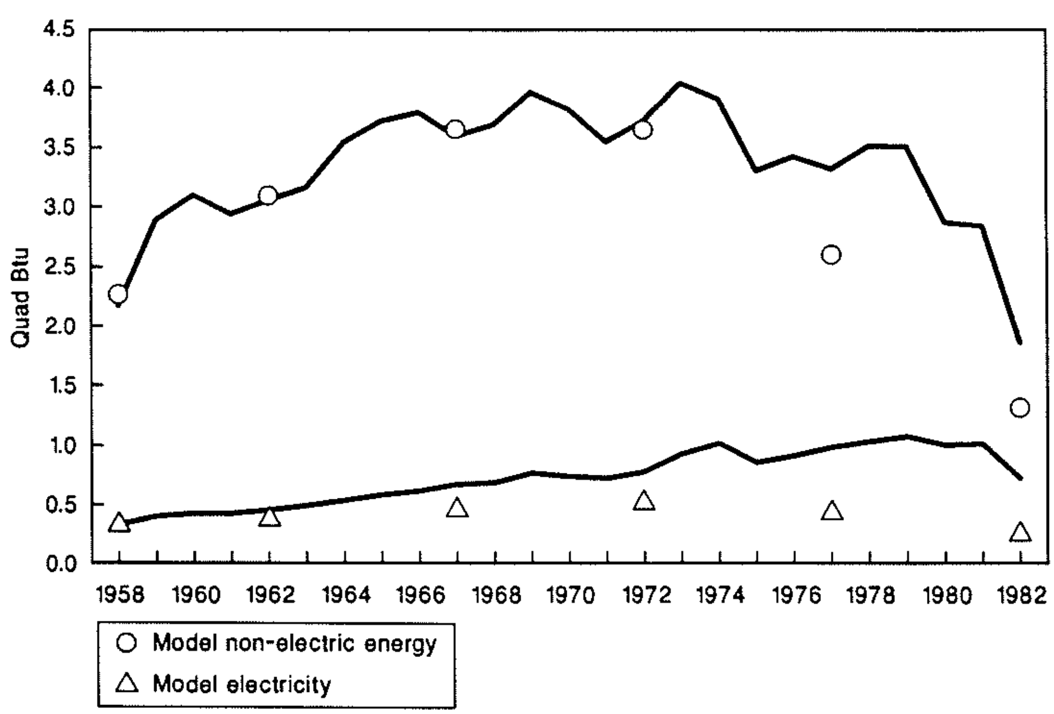

Fig. 4. Primary metals energy inputs: Model results versus historical data.

Table 1

Comparison of model projections and historical growth rates for the primary metals industry (percentage annual growth rates: 1958–1982).

	Historical	Model projections	Absolute error
Output	0..6%	0.6%	0
Labor	−1.2%	−2.7%	1.5%
Materials	1.6%	2.1%	0.5%
Electricity	3.3%	−1.2%	4.5%
Non-electric energy	1.6%	2.1%	0.5%

A somewhat unique technological trend in this particularly industry makes matching history with a simple aggregate model like that proposed here a difficult challenge. Conventional steel making technologies utilize fossil fuel intensive processes like a blast furnace in conjunction with either an open hearth or direct reduction process to produce virgin steel ingots. However, it is also possible to use newer electric-arc furnaces to convert scrap steel directly to ingots. The favorable economics of this technology coupled with the availability of sufficient scrap has led to a dramatic increase in steel production by this method over the past 10 to 15 years. This trend has resulted in a marked shift from fossil fuel to electricity inputs in the primary metals industry that may be difficult to discern from aggregate fuel use data. In the current implementation of the model, technical change is assumed to be Hicks-neutral and is represented by an exponential factor that affects all relevant production function parameters equally. Inclusion in the model of factor specific rates of technological change that favored electricity could probably be used to produce a better fit between model projections and actual electricity demands by the primary metals industry.

5. Projections for the primary metals industry

The model can also be used to project future energy demands by the primary metals industry. The parameter values employed for the historical comparisons shown at the top of the first column of table 2, and the baseline factor price, and capital stock growth rates shown at the bottom of the first column are used to produce a reference projection. The output and materials projections for this case are shown in fig. 5 and show about a one percent per year growth rate. As shown in fig. 6, non-electric energy demands are projected to decline at about 0.25 percent per year from 1982 to 2017, while electricity demands are projected to increase by 2.1 percent per annum. Finally, labor requirements are projected to decline gradually as shown in fig. 7. The sharp drop in inputs to – and output from – the primary metals industry in 1982 and the return to trend growth by 1987 reflects the effects of

Table 2

Parameter assumptions.

	Base case[a]	Range for sensitivity analysis
Production function		
Productivity growth factor (α)	1.0	0.5 – 1.5
Growth rate of productivity growth factor (g_α)	0.15%	0.0% – 2.0%
Exp. of L.R. CES function (ρ)[b]	0.5	0.01 – 2.0
Exp. of S.R. CES function (γ)[b]	5.0	3.0 – 11.0
Factor price growth rates		
Capital	0..0%	−3.0% – +3.0%
Labor	+1.3%	0.0% – +2.0%
Electricity	0.0%	−3.0% – +3.0%
Non-electric energy		
1987–1992	+8.0%	+4.0% – +12.0%
1992–2017	+4.0%	+2.0% – +6.0%
Materials	0.0%	−3.0% – +3.0%
Demand equation parameters		
Demand income elasticity (v)	1.0	0.9 – 1.1
Demand price elasticity (j)	−1.0	−0.5 – −1.5
Demand growth rate (g_I)	1.0%	0.5% – 1.5%
Primary metals stock depreciation rate (Δ)	10%	5% – 15%
Capital stock parameters		
Capital depreciation rate (d)	10%	5% – 15%
Capital stock growth rate (g)	+2.0%	+1.0% – +3.0%

[a]The base case value for g_α was derived from Jorgenson (1984); the base case growth rates for capital, labor and materials prices, as well as for Δ, d and g, are historical averages over the past two decades. Other parameters were assigned values consistent with theory and in logical relationship to one another.

[b]It was assumed that the short-run elasticity of substitution equals $\frac{1}{4}$ times the long-run elasticity of substitution. So as the long-run elasticity varies from $\frac{1}{3}$ to $\frac{2}{3}$ to 1, the short-run elasticity varies from $\frac{1}{12}$ to $\frac{1}{6}$ to $\frac{1}{4}$.

the recession of 1981–82 which had a particularly devastating impact on this industry.

5.1. Sensitivity analysis

Since parameter estimation is not exact, projecting future factor – especially energy – prices is difficult, and the model did not track past electricity demand well, it would be foolhardy to rely on a single projection of energy demands. In fact, the sensitivity of the projections to plausible variations in parameter and input variable assumptions often proves more useful than baseline projections.

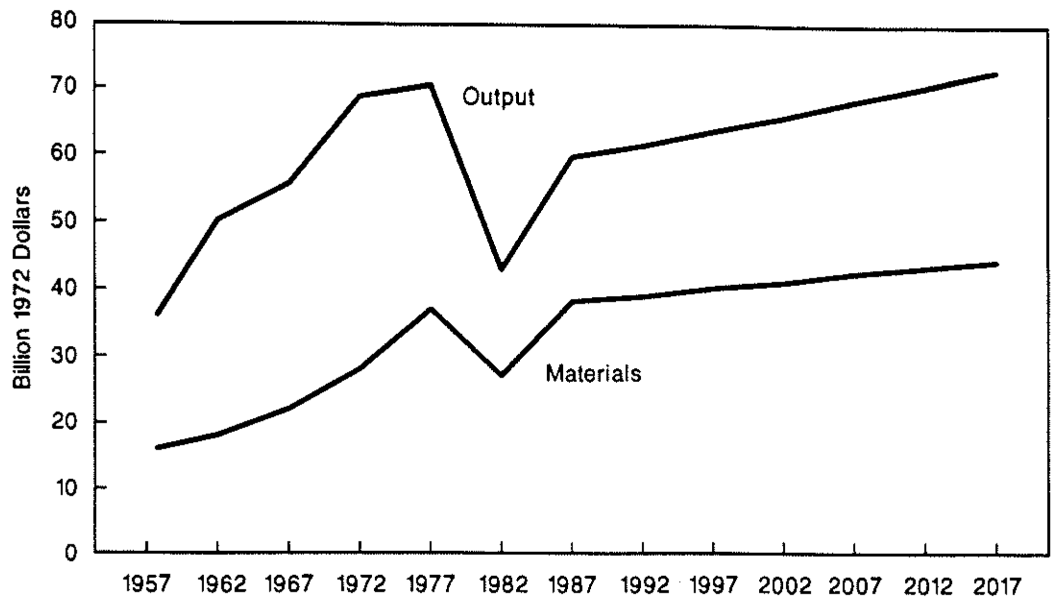

Fig. 5. Projections of primary metals output and material use.

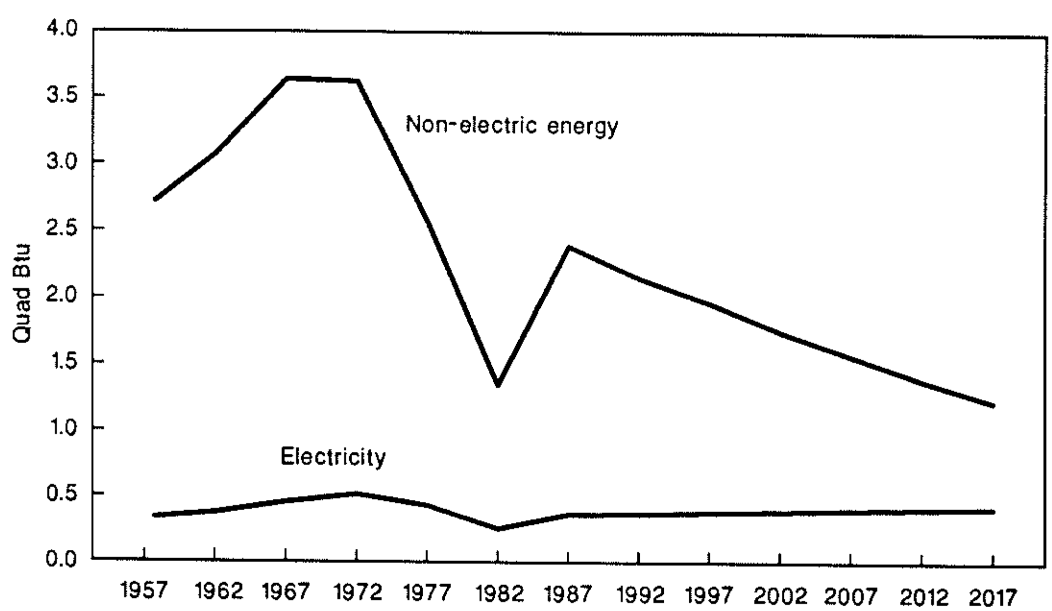

Fig. 6. Projections of primary metals energy inputs.

A systematic sensitivity analysis of parameter values and input assumptions was conducted on the simple primary metals model. The second and third columns of table 2 show the range assumed for each parameter and input assumption. Although the variation considered for each sensitivity is somewhat arbitrary, the ranges shown here seem plausible and reflect our

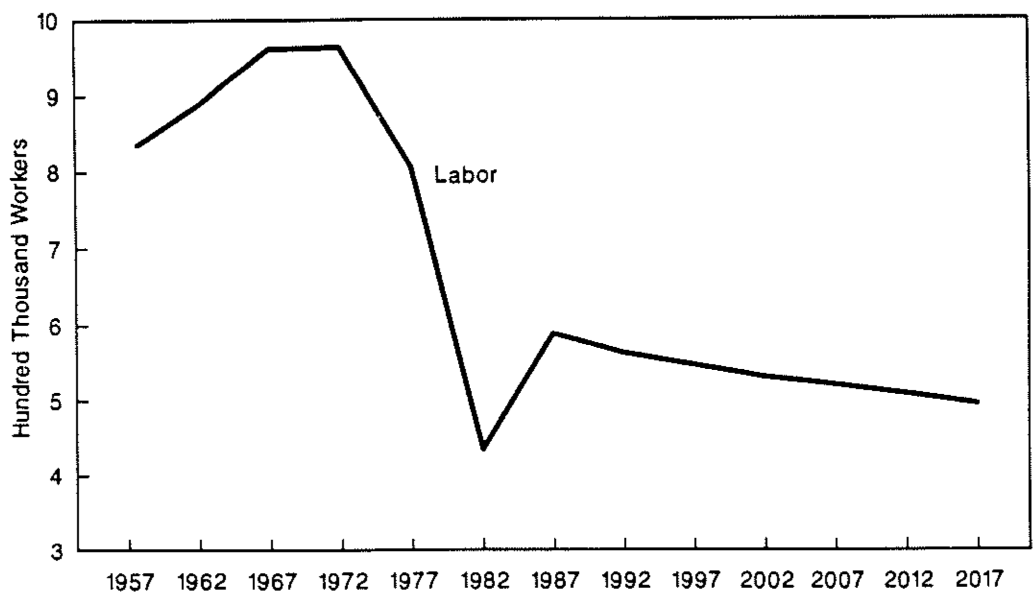

Fig. 7. Projection of primary metals labor input.

degree of uncertainty about the underlying elements of the projection system. The impact of the sensitivities on projected electric and non-electric energy demands are shown in figs. 8(a) and 8(b), respectively. The variation in values for each variable listed at the bottom of the figure are shown on the bars above it. The bar indicates the variation in energy demand projection growth rates corresponding to the variation in that parameter or input assumption.

The sensitivities are ordered so that the inputs with the strongest impacts on the demand projections are shown on the left of the figures and those with the weakest impacts are shown on the right. For example, projected electricity demand is most sensitive to the assumed electricity price, and least sensitive to the assumption about the annual change in the rate of technological progress. Similarly, the non-electric energy demand projections are most sensitive to the assumed income elasticity and also least sensitive to the assumed yearly change in the rate of technological progress.

5.2. Model experiments

Once constructed, a model can be used to explore interesting controlled variations in conditions that have not been experienced in the real world. Such exercises can reinforce our confidence in the model, and yield insights about the behavior of the system being analyzed. To understand the results of the simulation experiments we have found the following decomposition to be very useful.

The average intensity of electricity use at time period t, I_t, can be

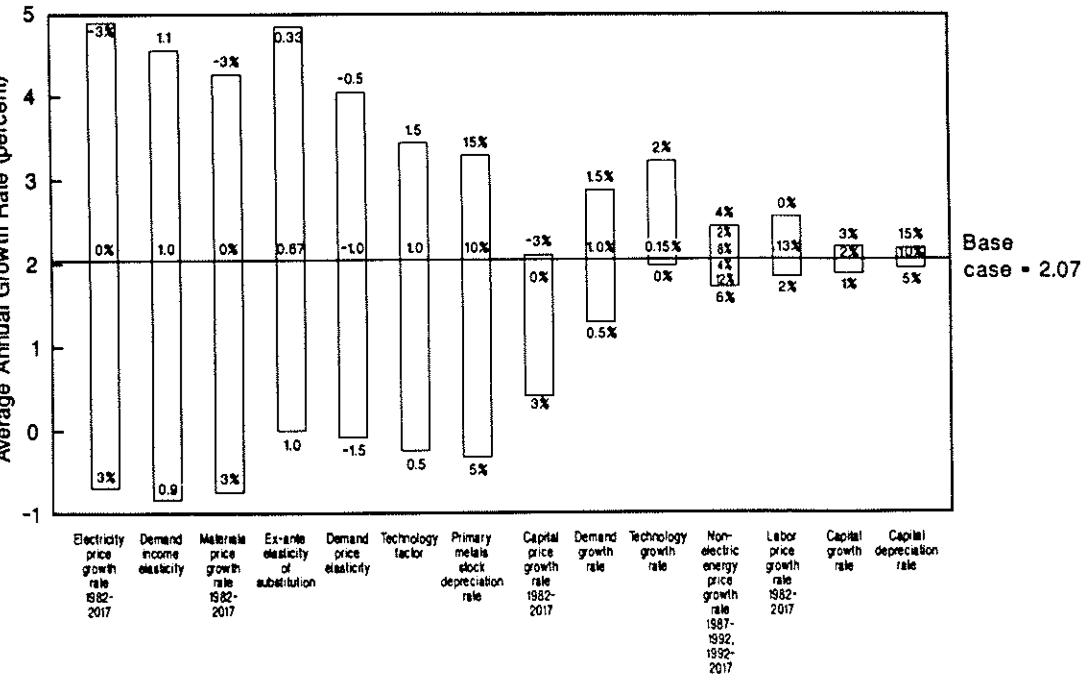

Fig. 8a. Sensitivity analysis for electricity demand growth rate: 1982–2017.

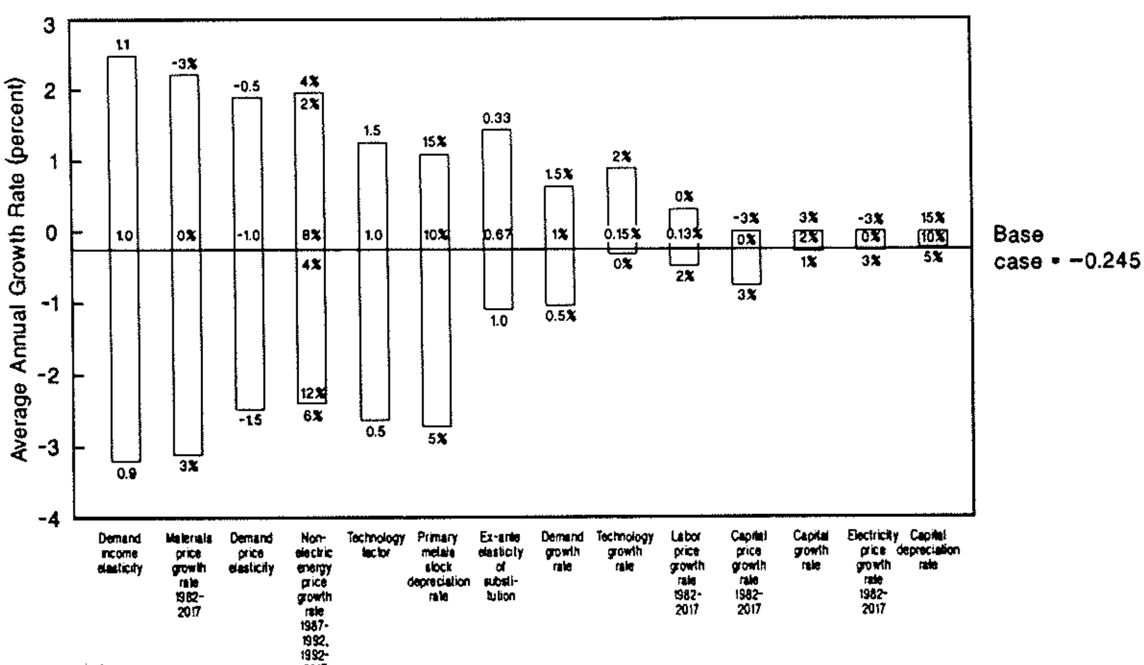

Fig. 8b. Sensitivity analysis for non-electric energy demand growth rate: 1982–2017.

expressed as the sum of the electricity use in each vintage, i, operating during that period, E_i, divided by the sum of the outputs produced by each vintage during the period, Y_i.

That is

$$I_t = \frac{E_1 + E_2 + \cdots + E_t}{Y_1 + Y_2 + \cdots + Y_t},$$

which can be written as

$$I_t = \frac{E_1}{\sum Y_i} \frac{Y_1}{Y_1} \frac{K_1}{K_1} \frac{\sum K_i}{\sum K_i} + \frac{E_2}{\sum Y_i} \frac{Y_2}{Y_2} \frac{K_2}{K_2} \frac{\sum K_i}{\sum K_i} + \cdots$$

or

$$I_t = \frac{E_1}{Y_1} \frac{Y_1}{\sum Y_i} \frac{K_1}{K_1} \frac{\sum K_i}{\sum K_i} + \frac{E_2}{Y_2} \frac{Y_2}{\sum Y_i} \frac{K_2}{K_2} \frac{\sum K_i}{\sum K_i} + \cdots,$$

which upon rearrangement yields

$$I_t = \frac{E_1}{Y_1} \times \frac{Y_1/K_1}{\sum Y_i/\sum K_i} \times \frac{K_1}{\sum K_i} + \frac{E_2}{Y_2} \times \frac{Y_2/K_2}{\sum Y_i/\sum K_i} \times \frac{K_2}{\sum K_i} + \cdots$$

$$\begin{matrix}\text{Average}\\ \text{intensity}\end{matrix} = \begin{bmatrix}\text{vintage}\\ \text{intensity}\\ \text{level}\end{bmatrix} \begin{bmatrix}\text{vintage}\\ \text{relative}\\ \text{capital}\\ \text{utilization}\end{bmatrix} \begin{bmatrix}\text{vintage}\\ \text{capital}\\ \text{proportion}\end{bmatrix} + \cdots.$$

Thus, each vintage's contribution to the average intensity of all vintages can be thought of as the product of three terms: (1) the intensity of electricity use in that vintage; (2) the capacity utilization (output per unit of capital stock) of that vintage relative to the average utilization rate of all the available capacity, and (3) the fraction of the total capacity available represented by capacity of that vintage.

Fig. 9 shows the results of our first simulation experiment in which electricity prices decrease at three percent per year from 1958 to 2017. Other factor prices are held fixed at their 1958 levels and GNP grows at three percent per year from 1958 to 2017.

Fig. 9(a) shows the intensity of electricity use in each vintage. More electricity is used in each successive vintage as electricity prices decline. That it is easier to adjust electricity use in newly installed vintages than in ones already in place is evident from the much more dramatic increase in the new vintage intensities compared to the increase in intensity of electricity use in each individual vintage shown in the figure. Fig. 9(b) shows the vintage relative capacity utilization – output divided by capital available – of each vintage relative to the capacity utilization of all vintages in service. In the example, since electricity prices change smoothly, the relative capacity

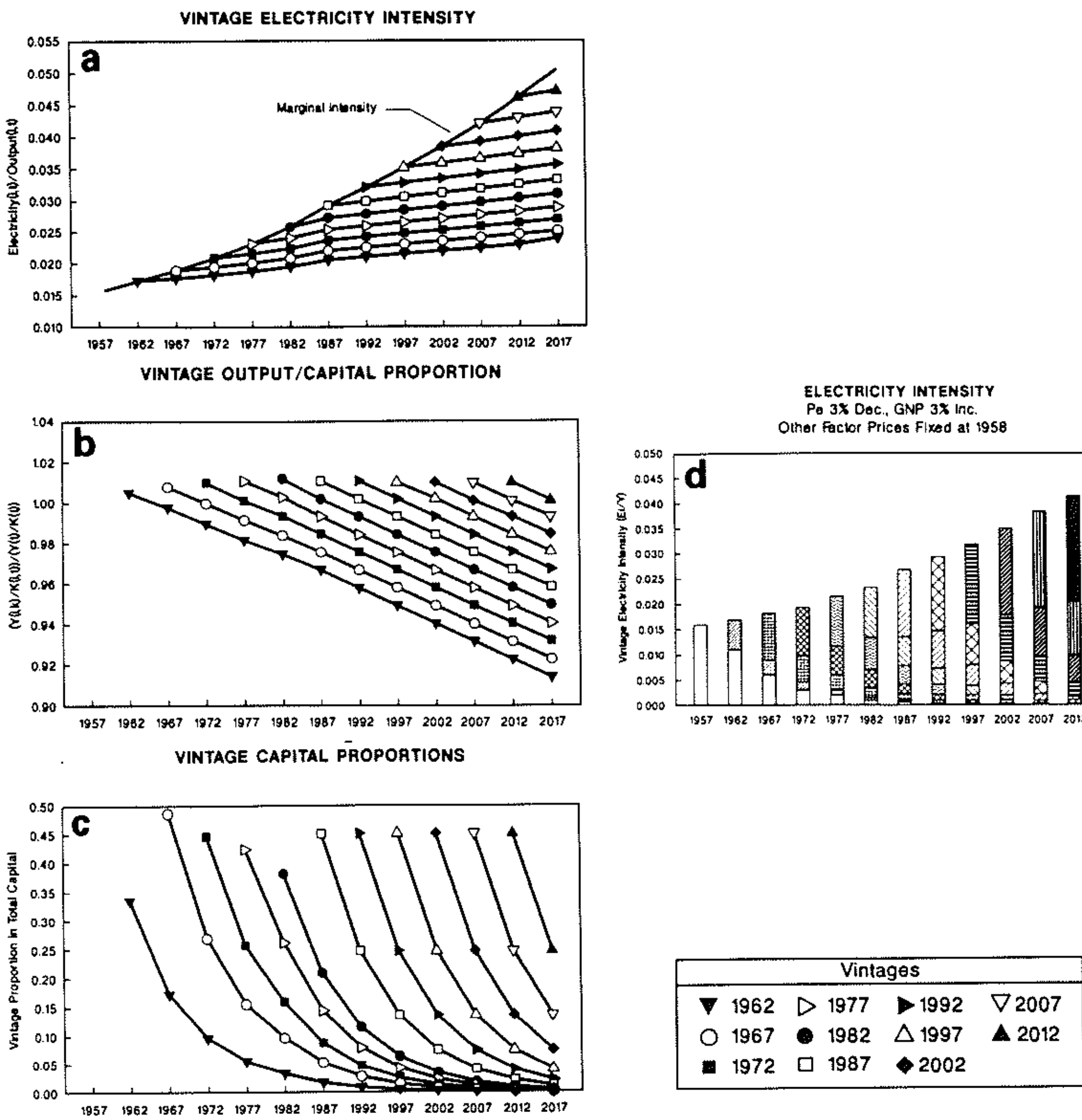

Fig. 9. Results for example no. 1.

utilization of each vintage declines slowly over time as electricity prices move steadily away from the level that prevailed when the equipment in it was first installed and the vintage becomes less suited to its current environment. Because electricity has a small factor share, the capacity utilization effect is small; declining by only ten percent over a 60 year period. Finally, fig. 9(c) shows the vintage capital proportions, the capital available from each vintage as a proportion of the total capital equipment available. These proportions decline over time as the capital in each vintage depreciates. Fig. 9(d) shows the average intensity of electricity use – defined as the average electricity use per unit of output from all vintages, as well as the contribution of each active vintage to the average overall intensity.

In our second experiment, electricity prices are assumed to decline at three percent per year through 1972, then increase at three percent per year from

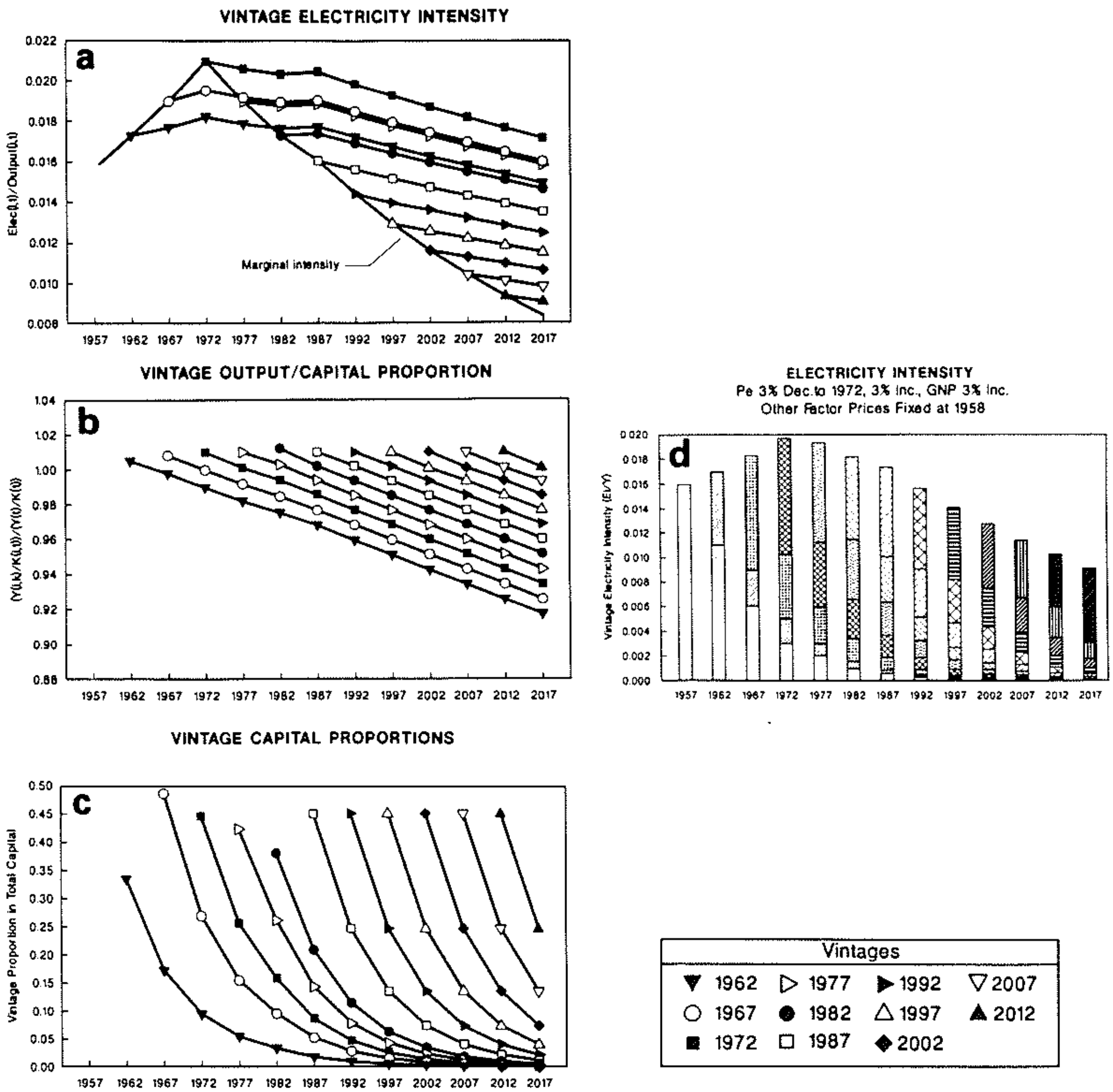

Fig. 10. Results for example no. 2.

1973 to 2017. Non-electric factor input prices are again fixed at their 1958 levels, and GNP again is assumed to grow at three percent per year throughout the entire 1958 to 2017 time horizon.

Results from this experiment are shown in fig. 10 (which is organized according to the same format employed in fig. 9). In this experiment, fig. 10(a) shows that the vintage intensity level in new equipment rises until 1972 and then declines. Again, there is far more flexibility in the energy intensity of new equipment than there is in existing equipment. The results for vintage relative capacity utilization shown in fig. 10(b) is very similar to that shown in fig. 9(b) for the first experiment. Fig. 10(d) shows that in this experiment, the average electricity intensity rises through 1972 and then declines.

In a third experiment, electricity prices are again assumed to decline at three percent per year through 1973, then increase at three percent per year

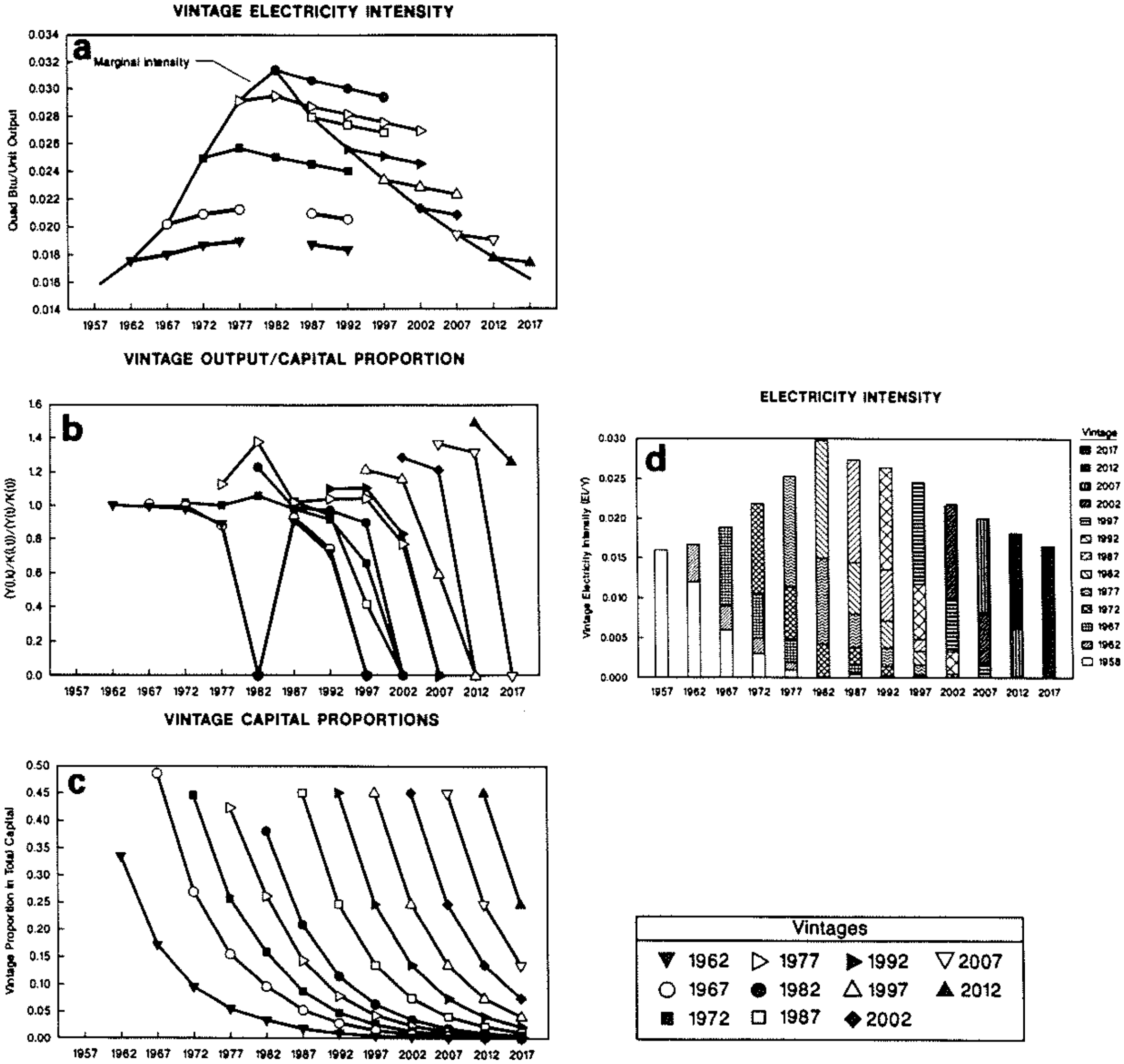

Fig. 11. Results for example no. 3.

from 1982 to 2017. Non-electric factor input prices, however, are assumed to follow their actual trajectories through 1982 and a reasonable set of projections from 1973 to 2017. GNP is again assumed to grow at three percent per year throughout the entire 1958 to 2017 time horizon. These assumptions prescribe a cleaner transition from a low energy price regime to a high energy price regime than actually occurred in 1973–74.

Results from this experiment are shown in fig. 11. The pattern of vintage electricity intensities for this example [shown in fig. 11(a)] rises and falls just like in fig. 10, but the vintage intensity peaks in 1982 instead of 1972 on account of increases in the prices of the other factors from 1972 to 1982.

Comparison of the capacity utilization results for this example [shown in fig. 11(b)] with those shown in fig. 10(b) shows the importance of other factor prices in determining electricity demand and the advantage of a vintaged multi-factor approach to industrial energy demand modeling. Because of significant changes in the prices of other factors, output from vintages installed under previous price regimes can be drastically reduced. In fact, in this experiment we see that in 1982, output from all vintages installed in and before 1967 are taken out of production entirely. Also in 1982, all vintages installed from 1972 onwards are operated intensively as factor prices change towards their design specifications. By 1987, however, other prices have risen to the extent that it becomes desirable to produce some output with the pre-1972 vintages. As factor prices continue to change, the capacity utilization falls to zero for all vintages installed in 1982 and previously. Fig. 10(c) shows how the vintage capital proportions change over time. The contribution of these complex relationships to electricity intensity are shown in fig. 10(d) which displays the contribution of each vintage to the average intensity of all vintages operating in each year.

6. Conclusions

These preliminary tests of our simple industrial energy demand projection methodology have been instructive. Consideration of all factors of production, the competition between electric and non-electric energy inputs, and vintaging of the capital stock of energy using equipment all appear to have been essential in our understanding past energy demand trends and in projecting future ones. In particular, consideration of the factors of production other than energy appear to result in a model for industrial electricity demand that is superior to that reported on in Peck and Weyant (1985).

The correspondence between model projections and actual energy demands by the primary metals industry from 1958 to 1982 was not perfect. Part of the problem may be the reliability of the historical data on the inputs to – and outputs from – this industry. As mentioned in section 4, there is also, however, a somewhat unique technological trend in this particular industry that makes matching history with a simple aggregate model like that proposed here a difficult challenge. Conventional steel making technologies utilize fossil fuel intensive processes like a blast furnace in conjunction with either an open hearth or direct reduction process to produce virgin steel ingots. However, it is also possible to use newer electric-arc furnaces to convert scrap steel directly to ingots. The favorable economics of this technology coupled with the availability of sufficient scrap has led to a dramatic increase in steel production by this method over the past 10 to 15 years. This trend has resulted in a marked shift from fossil fuel to electricity inputs to the primary metals industry that may be difficult to discern from aggregate fuel use data. Indeed, this technological trend may well explain the tendency of our model to under-predict electricity demand, particularly during the post-embargo era. Better results – particularly for electricity demand – can be expected if factor specific rates of technological change are incorporated into the model. Whether the method developed here can be applied to other industries, to the whole manufacturing sector, or at the utility service region level is an open question. The requisite data and parameter estimates must be obtained before the structure of the model can be tested. Based on our experience with the model for the primary metals industry, it is likely that significant insights about energy demands by other industries could be gained through the use of this model as well.

Our experience here brings with it several prescriptions for electric utility planners. First, great uncertainty surrounds any forecast of future electricity demand. Even a perfect demand forecasting model relies crucially on factor price and economic growth rate assumptions that are highly uncertain and to which any electricity demand projection is likely to be quite sensitive. In addition, no model is guaranteed to produce a perfect forecast, even if reliable inputs could be obtained. The data reliability, aggregation, and estimation problems involved in the construction of any model are likely to increase rather than decrease the reliability of any electricity demand forecast. Although uncertainty abounds in any electricity demand analysis, our second conclusion is that a simple model can often help in identifying and quantifying the major causes of that uncertainty. Insights gained from exercising such a model can help planners strike the appropriate balance between an analysis that assumes nothing is known and one that assumes that everything is known with perfect certainty.

Appendix A

Table A.1
Historical data.

Year	Capital (1972$ × 10^6)	Labor (hundred)	Electric energy (trillion BTUs)	Non-electric energy (trillion BTUs)	Materials (millions 1972$)	Output (millions 1972$)	Shipments (millions 1972$)
1958	29,466	8,491	332	2,713	16,118	37,067	36,836
1959	29,329	9,097	406	2,892	18,825	41,868	42,129
1960	29,902	9,205	429	3,099	19,320	42,211	41,856
1961	29,870	8,557	427	2,940	19,243	42,109	41,588
1962	29,687	8,784	457	3,056	20,223	44,488	44,514
1963	29,868	8,881	492	3,163	20,989	47,132	46,960
1964	30,528	9,385	539	3,541	24,050	51,996	51,906
1965	31,626	9,882	583	3,720	27,043	57,084	57,040
1966	33,206	10,245	618	3,792	29,144	61,185	60,585
1967	35,044	10,008	672	3,595	26,792	56,388	55,901
1968	36,650	9,946	685	3,695	28,390	58,491	58,743
1969	37,717	10,243	769	3,964	30,816	61,147	60,976
1970	38,472	9,772	740	3,818	29,190	55,264	54,915
1971	38,488	8,973	723	3,545	27,472	53,170	53,319
1972	38,370	9,228	778	3,723	30,657	58,942	58,430
1973	38,279	9,954	929	4,036	35,524	68,228	68,865
1974	39,142	10,010	1,020	3,907	40,143	69,662	69,670
1975	39,744	8,557	858	3,306	33,345	53,956	53,067
1976	40,086	8,748	915	3,423	35,716	58,068	57,769
1977	40,405	8,853	987	3,317	37,278	59,462	59,536
1978	40,514	9,208	1,032	3,515	38,859	63,280	63,116
1979	40,535	9,519	1,074	3,507	39,364	64,074	64,077
1980	40,449	8,543	1,007	2,866	33,059	56,346	56,765
1981	40,440	8,213	1,015	2,837	32,440		57,083
1982	39,598	6,384	728	1,867	23,637		42,531

Appendix B

Assuming parameters α, A_K, L_L, A_M, A_E, A_N and ρ and prices P_K, P_L, P_M, P_E and P_N are known, the ex-ante production function is given by

$$Y=\alpha(A_K K^{-\rho}+A_L L^{-\rho}+A_M M^{-\rho}+A_E E^{-\rho}+A_N N^{-\rho})^{-1/\rho},$$

where Y = output, K = capital, L = labor, M = materials, E = electricity and N = non-electric energy.

Note that α is assumed to change over time at the rate g_α.

Given the parameter, r, the ex-post production function will be derived:

$$Y=\beta(B_K K^{-r}+B_L L^{-r}+B_M M^{-r}+B_E E^{-r}+B_N N^{-r})^{-1/r}.$$

Using the known parameters and prices, and the fact that $B_K+B_L+B_M+$

$B_E + B_N = 1$, B_K, B_L, B_M, B_E and B_N may be determined by equating the slopes of the ex-ante and ex-post isoquants and setting them equal to the slopes of the isocost curves, giving

$$B_K = \left[1 + \frac{P_L}{P_K}\left(\frac{P_L}{P_K}\frac{A_K}{A_L}\right)^{-(1+r)/(1+\rho)} + \frac{P_M}{P_K}\left(\frac{P_M}{P_K}\frac{A_K}{A_M}\right)^{-(1+r)/(1+\rho)} + \frac{P_E}{P_K}\left(\frac{P_E}{P_K}\frac{A_K}{A_E}\right)^{-(1+r)/(1+\rho)} + \frac{P_N}{P_K}\left(\frac{P_N}{P_K}\frac{A_K}{A_N}\right)^{-(1+r)/(1+\rho)} \right]^{-1},$$

$$B_L = B_K \frac{P_L}{P_K}\left(\frac{P_L}{P_K}\frac{A_K}{A_L}\right)^{-(1+r)/(1+\rho)},$$

$$B_N = B_K \frac{P_N}{P_K}\left(\frac{P_N}{P_K}\frac{A_K}{A_N}\right)^{-(1+r)/(1+\rho)},$$

$$B_E = B_K \frac{P_E}{P_K}\left(\frac{P_E}{P_K}\frac{A_K}{A_E}\right)^{-(1+r)/(1+\rho)},$$

$$B_M = B_K \frac{P_M}{P_K}\left(\frac{P_M}{P_K}\frac{A_K}{A_M}\right)^{-(1+r)/(1+\rho)}.$$

The remaining parameter, β, is determined by first solving for K^0 (the capital required for unit output from the ex-ante production function) in terms of A_K, A_L, A_M, A_E, A_N, α, ρ, and factor prices:

$$K^0 = \frac{1}{\alpha}\left[A_K + A_L\left(\frac{P_L}{P_K}\frac{A_K}{A_L}\right)^{\rho/(1+\rho)} + A_M\left(\frac{P_M}{P_K}\frac{A_K}{A_M}\right)^{\rho/(1+\rho)} + A_E\left(\frac{P_E}{P_K}\frac{A_K}{A_E}\right)^{\rho/(1+\rho)} + A_N\left(\frac{P_N}{P_K}\frac{A_K}{A_N}\right)^{\rho/(1+\rho)} \right]^{1/\rho}.$$

The remaining input factors, L, N, E and M, are found for unit output by the ex-post production function in terms of the parameters B_K, B_L, B_M, B_N, B_E and r, and the prices:

$$L^0 = K^0\left(\frac{P_L}{P_K}\frac{B_K}{B_L}\right)^{-1/(1+r)},$$

$$N^0 = K^0\left(\frac{P_N}{P_K}\frac{B_K}{B_N}\right)^{-1/(1+r)},$$

$$E^0 = K^0\left(\frac{P_E}{P_K}\frac{B_K}{B_E}\right)^{-1/(1+r)},$$

$$M^0 = K^0\left(\frac{P_M}{P_K}\frac{B_K}{B_M}\right)^{-1/(1+r)},$$

Finally, β is determined:

$$\beta = (B_K(K^0)^{-r} + B_L(L^0)^{-r} + B_M(M^0)^{-r} + B_E(E^0)^{-r} + B_N(N^0)^{-r})^{1/r}.$$

A variable cost function for the ex-post production function is derived and then differentiated to find the marginal cost function. Inverting the marginal cost function results in a short-run supply curve with output, Y, as a function of short-run marginal cost:

$$Y = \beta K\left[\frac{1-(\Gamma/m\beta)^{r/(1+r)}}{B_K}\right]^{1/r},$$

where m = marginal cost and

$$\Gamma = \sum_{i=2}^{5} P_i\left[\sum_{j=2}^{5} B_j\left(\frac{B_i}{B_j}\frac{P_j}{P_i}\right)^{r/(1+r)}\right]^{1/r}.$$

An industry supply curve is formed by summing the supply curves for each vintage of capital. The model is based on exogenous capital stock governed by the functions. Capital Investment,

$$K(t) = K_0 \mathrm{e}^{gt}, \quad \text{and}$$

Capital Depreciation,

$$K(\text{vintage } i, \text{ year } (t+n)) = K(\text{vintage } i, \text{ year } (t))\,\mathrm{e}^{-dn}.$$

An accelerator version demand function is used, based on total stock demand (S) is given by

$$S = D_0 p^j I^v,$$

where p is the price, I is the demand growth factor and j and v are price and income elasticities, respectively.

Demand is determined by the growth in stock, the derivative of this equation:

$$\frac{\mathrm{d}s}{\mathrm{d}t}=S_t\left[j\frac{1}{P}\frac{\mathrm{d}p}{\mathrm{d}t}+v\frac{1}{I}\frac{\mathrm{d}I}{\mathrm{d}t}\right]+\varDelta S_{t-1},$$

where $\varDelta$ is the depreciation rate of the stock. Ignoring the price factor, $\mathrm{d}p/\mathrm{d}t$, and calculating on a yearly basis gives

$$\text{Demand}(t+1)=D_0\left[p_{t-1}^j I_{t-1}^v\cdot\varDelta+p_t^j I_t^v v\left(1-\frac{I_{t-1}}{I_t}\right)\right].$$

The market clearing price, p, is found by equating supply and demand. The output from each vintage of capital is calculated from the supply curve, using p as the marginal cost.

Finally, given the outputs, Y, capital, K, parameters and prices, the optimal inputs are determined:

$$L=\left[\frac{\dfrac{Y^{-r}}{\beta}-B_K K^{-r}}{B_L+B_M\left(\dfrac{P_L}{P_M}\dfrac{B_M}{B_L}\right)^{-r/(1+r)}+B_E\left(\dfrac{P_L}{P_E}\dfrac{B_E}{B_L}\right)^{-r/(1+r)}+B_N\left(\dfrac{P_L}{P_N}\dfrac{B_N}{B_L}\right)^{-r/(1+r)}}\right]^{-1/r},$$

$$E=L\left(\frac{P_L}{P_E}\frac{B_E}{B_L}\right)^{1/(1+r)},$$

$$N=L\left(\frac{P_L}{P_N}\frac{B_N}{B_L}\right)^{1/(1+r)},$$

$$M=L\left(\frac{P_L}{P_M}\frac{B_M}{B_L}\right)^{1/(1+r)}.$$

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