RATE-OF-RETURN REGULATION AND EFFICIENCY: THE ECONOMICS OF THE NATURAL GAS PIPELINE AND DISTRIBUTION INDUSTRY

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EMF SR1

April 1984

Energy Modeling Forum
Stanford University
Stanford, CA 94305

ACKNOWLEDGMENTS

The author wishes to acknowledge the many helpful suggestions provided by James L. Sweeney, John P. Weyant and Donald A. Dunn.

The research contained herein is sponsored by the U.S. Department of Energy, Grant DE-FG03-791R-10297. The author is, however, solely responsible for the contents of this paper.

The activities of the Energy Modeling Forum are funded by gifts from corporate affiliates and by grants from or contracts with the U.S. Department of Energy and the Electric Power Research Institute. The corporate affiliates are:

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rules-of-thumb for setting the capital productivity requirement which might be used by the price-minimizing or welfare-maximizing regulator.

The author then shows how wellhead price controls, whether on all or only some supply sources, could lead to increases in institutional costs. Although the study is primarily theoretical, it concludes with a brief discussion of the empirical evidence for the principal testable results.

ABSTRACT

This study considers how rate-of-return regulation with wellhead price controls might affect the operating efficiency of gas utilities. It thus differs from much of the previous literature on rate-of-return regulation, which has focused on how regulation might cause an improper mixture of capital and other inputs--the Averch-Johnson effect.

The author argues that firm managers have some ability to pursue objectives of their own, which differ from simple profit-maximization. The managers may be thought of as maximizing a utility function of profits and the institutional costs arising from their pursuit of these other objectives, subject to a budget constraint. Regulatory policies affect the shape of the budget constraint, and therefore the combination of profit and institutional costs chosen by the firm. Unlike Averch-Johnson models, firm behavior in this model changes smoothly as the allowed rate of return is reduced below the cost of capital, a result in accord with the observed behavior of gas utilities. However, as the allowed rate of return is reduced, institutional costs tend to increase.

The author also argues that regulators have a second little-noted instrument of control: their power to deny approval for new capital investments. If the regulators prohibit unproductive capital investment, this could induce the utility to purchase gas beyond the point where its marginal revenue product equals marginal cost, a result also in accord with the observed behavior of gas utilities. If the firm were assumed to be a strict profit-maximizer, there would be simple

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CHAPTER 1

INTRODUCTION

This study is an examination of how natural gas pipeline and distribution company efficiency may be affected by three policy instruments at the disposal of regulators under a system of rate-of-return regulation with wellhead price controls. The three policy instruments to be considered are the regulator's powers to set the allowed rate-of-return, to approve or deny new capital investments, and to set the wellhead price. The study considers the likely effect of these instruments on both the efficiency of the firm's capital investment strategies, and the efficiency with which the firm conducts its operations. The objective is to give regulatory decisionmakers an improved awareness of the consequences of their actions, so as to allow more enlightened regulatory policies. Although the study focuses on the natural gas pipeline and distribution industries, many of the results are generally applicable to any rate-of-return regulated firm.

This chapter introduces the background and motivation for the study. The first section introduces some of the terminology that will be used in discussing the gas industry and gives a brief overview of the structure of the industry. The second section is a similar brief overview of rate-of-return regulation. The third section discusses wellhead price controls, and points out the lack of studies addressing their effects on gas pipeline and distribution company behavior. The fourth section discusses two concerns economists have raised about rate-of-return regulation: uneconomic capital investments, as in the Averch-Johnson model, and excessive operating costs. The lack of literature rigorously addressing the latter concern is pointed out. The fifth section discusses two properties of the Averch-Johnson model not in accord with observed behavior of gas pipeline and distribution companies, which deserve further examination. The sixth section discusses three criticisms of the assumptions of the Averch-Johnson model that have been raised in the literature, and that also deserve further examination. The final section presents a brief overview of the study.

1.1 Structure of the Natural Gas Industry

As background to the study, it is appropriate to begin with a brief idealized description of the structure of the natural gas industry in the United States, which may be thought of as three industries in vertical sequence. Gas producers explore for, develop, and operate gas fields; in many cases they are also oil producers. The producers generally sell gas to gas pipeline companies, at prices referred to as wellhead prices, under long-term contracts. Ceilings on these wellhead prices for many categories of gas are set by the Natural Gas Policy Act of 1978,[1] and administered by the Federal Energy Regulatory Commission (FERC).

The gas pipeline companies transport the gas to the consuming areas, and there resell it to local distribution companies at prices known as citygate prices. FERC also regulates the citygate prices of

[1] Natural Gas Policy Act of 1978, U.S. Code, supp. 5, title 15, secs. 3311-3432 (1982).

interstate pipelines, but under a different law--the Natural Gas Act of 1938[2]. Like most public utilities, prices are regulated so as to allow the pipeline company to recover its operating costs plus a "fair" return on its investment, a system known to economists as rate-of-return regulation.

The local distribution companies, in turn, resell the gas to final consumers. Distribution companies, too, are generally subject to rate-of-return regulation; however, the regulation is usually administered by state agencies, such as a state public utility commission.[3]

Although gas pipeline companies and local distribution companies serve very different markets and have very different cost characteristics, both are subject to rate-of-return regulation, and should respond in much the same way to regulatory incentives. Hence, this study will focus on the behavior of generic gas firms, which might be either pipelines or local distribution companies.

1.2 Rate-of-Return Regulation

To implement rate-of-return regulation, the regulatory commission first establishes a book value of the firm's assets, known as the rate

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[2] Natural Gas Act, U.S. Code, title 15, sec. 717 (1976).

[3] A more thorough description of the structure of the natural gas industry may be found in Arlon R. Tussing and Connie C. Barlow, An Introduction to the Gas Industry with Special Reference to the Proposed Alaska Highway Gas Pipeline, report prepared for the State of Alaska Legislative Affairs Agency (Fairbanks: University of Alaska Institute for Social and Economic Research, 1978). See also Richard J. Pierce, Jr., Natural Gas Regulation Handbook (New York: Executive Enterprises Publications, 1980) and Arlon R. Tussing and Connie C. Barlow, The Natural Gas Industry: Evolution, Structure, and Economics (Cambridge, MA: Ballinger Publishing Company, 1984).

base. It also establishes an allowed rate of return. The allowed rate of return times the rate base is used by the regulators as a ceiling on the firm's earnings. If earnings are below this level, a rate increase may be justified; if earnings are above this level, a rate reduction may be in order.

The regulatory ceiling on earnings is not, however, continuously binding. Tariffs are usually set based on historical cost--typically, the firm is required to set a tariff that would have yielded it the allowed rate of return in some prior "test year", perhaps with some adjustments for expected changes in costs. This delay between the time a firm's revenues or costs change and the time tariffs are modified to reflect these changes means the firm may actually earn more or less than the allowed rate of return. The delay is known to economists as regulatory lag. Furthermore, since there are costs involved in modifying tariffs, neither the company nor the regulators will usually seek to alter the tariffs unless the firm's earnings fall outside some range of acceptability about the allowed rate of return.[4]

1.3 Wellhead Price Controls

Regulation of wellhead natural gas prices has been a controversial issue; however, the FERC and its predecessor, the Federal Power Commission, have been regulating wellhead prices paid by interstate pipelines in one form or another since the Supreme Court's Phillips

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[4] A good source of background on rate-of-return regulation is Alfred E. Kahn, The Economics of Regulation; Principles and Institutions, 2 vols. (New York: John Wiley and Sons, 1970).

Decision[5] of 1954. A policy on wellhead price controls was fixed by statute for the first time with the passage of the Natural Gas Policy Act of 1978 (or N.G.P.A.), an artful compromise between proponents and opponents of price controls.[6] The N.G.P.A. sets a variety of different ceiling prices for gas supplies, based upon such factors as the age of the well, well depth, onshore versus offshore location, distance of well from other wells, type of producing formation, rate of production, and provisions of the existing gas sales contract (interstate versus intrastate, price, contract date, whether it is a "rollover contract"). Wellhead prices for certain types of "high cost" gas were deregulated immediately by the N.G.P.A., with most gas supplies developed since passage of the act ("new" gas), as well as supplies dedicated to intrastate commerce prior to the act, to be deregulated in 1985. "Old" gas, dedicated to interstate commerce prior to the act, will remain price-controlled indefinitely.[7] This policy of regulating the price of some gas supplies but not others has come to be known as partial wellhead price controls.

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[5] Phillips Petroleum Company vs. Wisconsin, 347 U.S. 672 (1954).

[6] For an analysis of the history and politics of wellhead price controls, see Pietro S. Nivola, "Energy Policy and the Congress: The Politics of the Natural Gas Policy Act of 1978," Public Policy 28 (Fall, 1980): 491-543 and M. Elizabeth Sanders, The Regulation of Natural Gas: Policy and Politics 1938-1978 (Philadelphia: Temple University Press, 1981).

[7] For a summary of the provisions of the N.G.P.A., see Pierce, pp. 36-76.

It would be a mistake to try to attribute too much rationality to the ceiling prices established by the N.G.P.A. There does appear, however, to be a pattern of higher prices for gas supplies which are more costly to produce, or over which the producer has more discretion about whether to produce. In other words, the N.G.P.A. establishes what an economist would call a discriminatory pricing policy by setting higher prices for gas with higher supply elasticities.

Wellhead price controls, combined with rate-of-return regulation, lead gas firms to practice average-cost pricing, buying various categories of gas at different prices and reselling the resulting mix at more or less a single price, equal to the average cost of the gas purchased, plus a markup to cover the firm's own costs. It is not unusual to find pipelines selling gas for less than the cost of the most expensive gas they purchase.[8]

There is an extensive literature dealing with the subject of wellhead price controls; however, most of it is concerned with the effects of wellhead price controls on gas producers or consumers, rather than pipeline or distribution companies.[9] Several studies have

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[8] See Section 6.1 below

[9] A good bibliography of academic studies through the mid-1970's is contained in Paul W. MacAvoy and Robert S. Pindyck, The Economics of the Natural Gas Shortage (1960-1980) (Amsterdam: North-Holland Publishing, 1975), pp. 251-256. Two recent studies with opposite conclusions are Glen C. Loury, "An Analysis of the Efficiency and Inflationary Impact of the Decontrol of Natural Gas Prices (Washington, DC: Natural Gas Supply Association, April, 1981), and Energy Action Educational Foundation, The Decontrol of Natural Gas Prices: A Price Americans Can't Afford (Washington, DC: Energy Action Educational Foundation, February, 1981).

empirically examined gas pipeline purchasing behavior in an attempt to determine if the wellhead gas market is competitive enough to function efficiently without wellhead price controls.[10]

1.4 Rate-of-Return Regulation and Efficiency

Rate-of-return regulation has raised two concerns among economists. The first has been raised informally by a number of economists,[11] yet has received little attention in the literature. It is that the cost-plus nature of rate-of-return regulation may not give the firm a strong enough incentive to minimize expenses, resulting in unnecessarily high operating costs. Despite the economists' misgivings, traditional models of firm behavior suggests that regulated firms would, under normal conditions, minimize operating costs.[12]

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[10]The most thorough of these studies is Paul W. MacAvoy, Price Formation in Natural Gas Fields: A study in Competition, Monopsony and Regulation (New Haven: Yale University Press, 1962). Other studies are Edward J. Neuner, The Natural Gas Industry: Monopoly and Competition in Field Markets (Norman: University of Oklahoma Press, 1960) and Leslie Cookenboo, Jr., Competition in the Field Market for Natural Gas, The Rice Institute Pamphlet, 44 (January, 1958). A more recent study is U.S. Federal Trade Commission, Economic Structure and Behavior in the Natural Gas Production Industry, by Joseph P. Mulholland, staff report to the Bureau of Economics (Washington, DC: Government Printing Office, February, 1979).

[11]See, for example, Kahn, v.2, p.48; William J. Baumol and Alvin K. Klevorick, "Input Choices and Rate of Return Regulation: An Overview of the Discussion", Bell Journal of Economics and Management Science 1 (Autumn, 1970): 188-189; Stephen Breyer, Regulation and Its Reform (Cambridge: Harvard University Press, 1982), p. 47; or F.M. Scherer, Industrial Market Structure and Economic Performance, Second edition (Chicago: Rand McNally, 1980), p. 483.

[12]See Elizabeth E. Bailey, Economic Theory of Regulatory Constraint (Lexington, MA: D.C. Heath and Company, Lexington Books, 1973), p. 42.

The second concern is that the firm may attempt to increase its profits by making uneconomic capital investments so as to enlarge its rate base. The classic model of this problem is that of Averch and Johnson.[13] The problem may be visualized graphically if one plots the firms's potential profit as a function of capital investment, as in Figure 1. Note that here I am talking about profits as an economist would define them, that is, earnings in excess of the opportunity cost of capital, not as an accountant would define them.

An unregulated firm would maximize its profits by setting capital

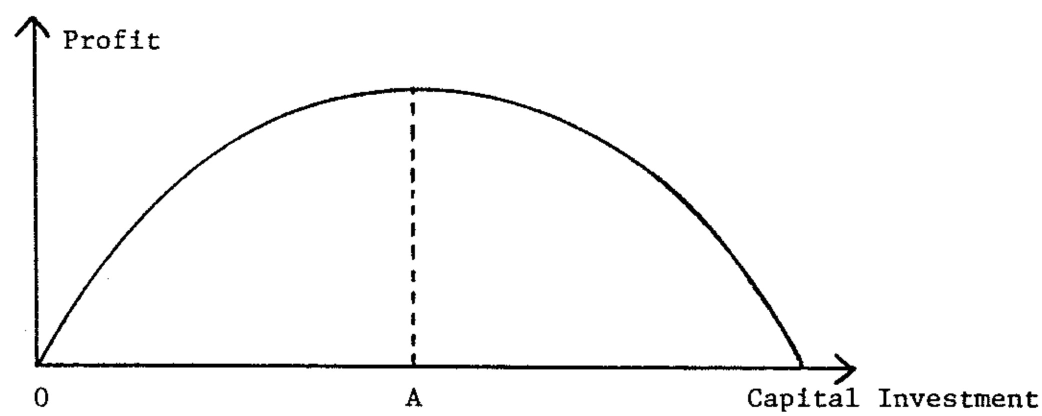

Figure 1
Potential Profit vs. Capital Investment

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[13]Harvey Averch and Leland Johnson, "Behavior of the Firm Under Regulatory Constraint", American Economic Review 52 (December, 1962): 1052-1069. A similar result was suggested independently by Stanislaw H. Wellisz, "Regulation of Natural Gas Pipeline Companies: An Economic Analysis", Journal of Political Economy 71 (February, 1963): 30-43.

equal to A in the diagram. However, under rate-of-return regulation with an allowed rate of return greater than the opportunity cost of capital, there is a ceiling on the firm's profits proportional to its capital investment. This is the straight line C shown in Figure 2. If regulation is effective in restricting the firm's profits, the firm's profits at point A will be above the ceiling, thus forcing it to expand its capital investment. This expansion has two effects. First, since the firm is expanding capital investment beyond the profit-maximizing point, potential profits are reduced. Second, by enlarging its rate base, the ceiling on profits is increased. At some point, which I have labeled B in Figure 2, potential profits will equal the profit ceiling, and the firm will be at its new profit-maximizing profit, subject to the

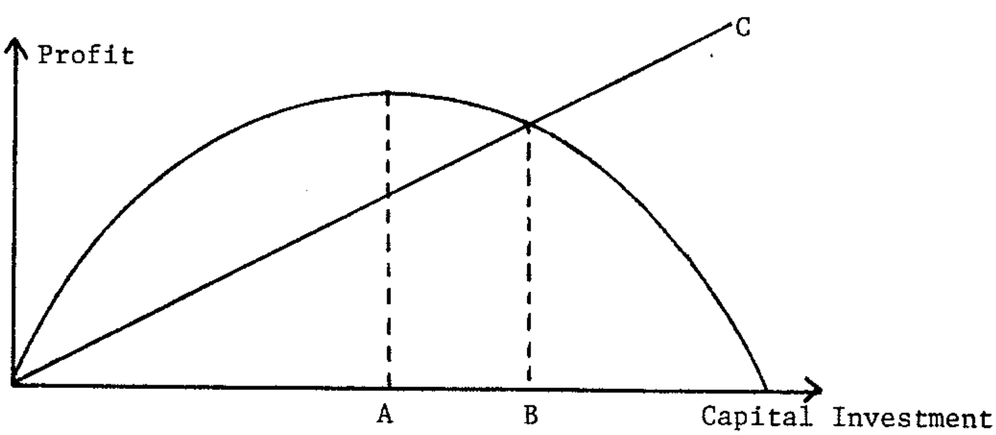

Figure 2
The Averch-Johnson Model

rate-of-return constraint.[14]

Now all of this is, to some extent, well and good. An objective of regulation is, presumably, to make the firm produce more than the profit-maximizing output, so as to lower prices to consumers; an expansion of capital investment will usually accomplish this objective. The problem here is that while capital investment is being expanded beyond the point where its marginal revenue product equals its marginal cost (the profit-maximizing point), the use of other inputs is not.[15] This results in a mixture of inputs which is overly capital intensive, and, as a result, excessively costly.

1.5 Two Anomalous Results of the Averch-Johnson Model

The Averch-Johnson model can be criticized both for the unrealism of its assumptions, and for two properties of the model which do not accord with the observed behavior of gas firms or other rate-of-return regulated firms. As the latter two anomalies provide a unifying theme for this study, I shall discuss them first.

The first anomaly is that the Averch-Johnson model predicts that as the firm's allowed rate of return is lowered toward the opportunity cost of capital, the firm will expand its capital investment. However, when the firm's allowed rate of return actually reaches the cost of capital,

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[14]A more thorough discussion of the Averch-Johnson Model and the ensuing literature may be found in Bailey. The graphical treatment of the Averch-Johnson model presented here is adapted from that book, which is, in turn, adapted from the work of E.E. Zajac, "A Geometrical Treatment of Averch-Johnson's Behavior of the Firm Model", American Economic Review 60 (March, 1970): 117-125.

[15]For a discussion of this result see Roger Sherman, "The Rate of Return Regulated Firm is Schizophrenic", Applied Economics 4 (March, 1972): 23-31.

the results become indeterminant--the firm can invest anywhere within a wide range, since no matter what it does it makes zero economic profit. If the allowed rate of return is set below the cost of capital, the firm should simply close up and go out of business, since no matter what it does it will lose money.[16]

To see this graphically, consider Figure 3, where again the hump-shaped curve represents the firm's potential profit as a function of capital investment. As the allowed rate of return is lowered, the

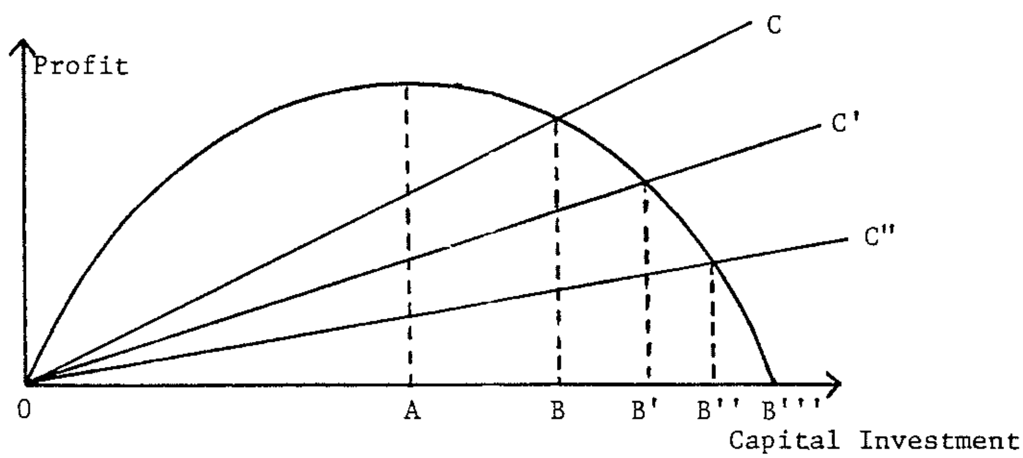

Figure 3
Effect of a Reduction in the Allowed Rate of Return

constraint curve C shifts downward to C' and on to C"; the optimum level of capital investment of the firm shifts upward from B to B' and on to B". When the allowed rate of return equals the cost of capital, the constraint curve lies on the capital investment axis. Since the firm's

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[16]This anomaly is discussed and derived mathematically in Baumol and Klevorick, pp. 162-190.

profits are now everywhere constrained to zero, the firm is free to set its capital investment anywhere between 0 and B". The implication for the regulator seeking to act in the best interests of consumers is to set the allowed rate of return above the cost of capital, but as close to it as possible.

This anomaly is of more than academic interest, since regulators generally seek to give the firm a "fair" allowed rate of return, which is usually interpreted as an allowed rate of return equal to the investor's opportunity cost of capital. In fact, in recent years, regulators have not raised allowed rates of return as quickly as market interest rates have risen, with the result that in many cases allowed rates of return have been below the opportunity cost of capital. In 1981, for example, the investor-owned gas utility industry had operating income equal to only 11.6% of net plant[17]; this compares to an average 14.8% rate on six-month commercial paper during the year.[18] Despite this, gas firms are not going out of business, nor is there, to the best of my knowledge, any discontinuity in their behavior when allowed rates of return drop below the opportunity cost of capital.

Furthermore, there is a certain absurdity in the implications of this model. How close can a regulator set the allowed rate of return to the opportunity cost of capital before the firm regards the allowed rate

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[17]American Gas Association, Gas Facts, 1981 (Arlington, VA: American Gas Association, 1982), p. 178.

[18]U.S. Bureau of Economic Analysis, "Current Business Statistics", Survey of Current Business 63 (January, 1983): S-14.

of return as, for all practical purposes, equal to the opportunity cost of capital, and everything goes haywire? Is it 1%? .01%? .0001%? The opportunity cost of capital is, of course, not a very precisely defined number anyway. Clearly, there is something lacking in this theory, but what?

This chapter has already alluded to the second anomaly. In the Averch-Johnson model, the firm continues to purchase all non-capital inputs only up to the point where their marginal revenue product equals their marginal cost. Therefore, the firm should never purchase gas at a price higher than its resale price. Yet, as already noted, one frequently observes pipeline companies doing just that.

1.6 The Assumptions of the Averch-Johnson Model

One can also criticize the unrealism of the assumptions upon which the Averch-Johnson model is based. Three criticisms are probably the most important.[19] First, as already noted, the rate-of-return constraint on profits is not, in practice, continuously binding; rather it is the tariffs which are binding. Once the tariff has been fixed, the firm has an incentive to minimize costs, so as to improve its profits. This cost minimizing incentive runs counter to the Averch-Johnson incentive to make uneconomic capital investments. Second, regulators are not as naive and/or passive as the Averch-Johnson model assumes. They can, and sometimes do, disallow the firm from passing on to consumers the

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[19] See Paul L. Joskow and Roger C. Noll, "Regulation in Theory and Practice: an Overview", in Studies in Public Regulation, ed. Gary Fromm (Cambridge: M.I.T. Press, 1981), p. 10-14.

cost of imprudent expenditures, or may simply forbid the firm from making certain types of expenditures altogether. Hence, it should be possible to regulate away at least the worst of the Averch-Johnson tendency toward overinvestment. Third, the Averch-Johnson model is often interpreted as assuming that the size of the firm's capital stock can be adjusted at will in response to changes in the allowed rate of return. Actual public utility capital has too long a life and too low a salvage value to be used in this fashion. In fact, risk-aversion may drive firms to reduce their investment, so as to decrease their exposure in the potentially hostile environment of the future.

All three of these criticisms of the assumptions of the Averch-Johnson model are valid. However, I do not believe they necessarily detract from the usefulness of the model, or models based on similar assumptions, as vehicles for gaining insight into the possible consequences of regulatory policies. The first criticism, dealing with the non-continuously binding nature of the rate-of-return constraint, points up the fact that the firm faces a tradeoff between the short-run losses an uneconomic investment entails, and the long-run benefits to be gained after the next rate adjustment. One could modify the Averch-Johnson model to reflect this tradeoff explicitly, as Bailey and Coleman have already done.[20] They show that as the regulatory lag is lengthened, the firm tends to move toward cost-minimization. In effect, the Averch-Johnson model can be viewed as a limiting case, where the regulatory lag

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[20]Elizabeth E. Bailey and Roger D. Coleman, "The Effect of Lagged Regulation in an Averch-Johnson Model", Bell Journal of Economics and Management Science 2 (Spring, 1971): 278-292.

approaches zero. Actual firms will be somewhere on the continuum between the kind of behavior predicted by an Averch-Johnson type assumption of no regulatory lag, and the (not very interesting) cost-minimizing behavior predicted by a model with very long lags. As long as one keeps this caveat in mind, it should be possible to learn a good deal from a model which does not explicitly take lags into account.

The second criticism, dealing with the assumption that regulators do not have control over firm expenditures, does not detract from the value of a model such as Averch and Johnson's for exploring how firms might behave if regulators lack the ability or vigilance to control the firm's expenditures. Nevertheless, the regulator's ability to approve or deny new capital investments is an important policy instrument, which has not been examined in the literature. I shall return to this issue and modify the assumption in Chapter 3.

The third criticism, dealing with the assumption of easy adjustment of the firm's capital stock, is really a criticism of one interpretation of the model. The Averch-Johnson model is a static model. One may invoke the assumption of an easily adjustable capital stock in applying the model to a dynamic situation. However, I believe a better approach is to assume that utility capital does have long life and low salvage value. One must then recognize that the allowed rate of return and cost of capital which the firm will use in its decisionmaking are not necessarily the current values of these variables, but anticipated values over the life of the investment. Since these anticipations usually change slowly, the Averch-Johnson model would not usually lead one to expect the firm to make rapid shifts in capital investment. I shall discuss the application of the model to dynamic situations further in Section 2.4.

1.7 Overview of the Study

Chapter 2 introduces a model based on the assumption that firm managers have objectives other than profit-maximization, which take on crucial importance when the firm's profit opportunities are severely curtailed under rate-of-return regulation. The new model behaves in a smooth fashion as the allowed rate of return is reduced below the cost of capital. More importantly, however, the model can be used to predict how the firm's operating costs might respond to changes in the allowed rate of return. Chapter 3 introduces another model in which regulators have the power to require that new capital investments be productive. It is shown how a capital productivity requirement could induce utilities to purchase gas with a marginal cost higher than its marginal revenue product. Rules of thumb for how price-minimizing or welfare-maximizing regulators might use this power are discussed.

Chapter 4 introduces a model with both management objectives other than profit and a capital productivity requirement. The potential impact of the capital productivity requirement on the firm's operating costs is discussed. Chapter 5 uses this model to explore how a gas utility's operating costs might be affected by wellhead price controls. Chapter 6 discusses the available empirical evidence for the model's predictions. Chapter 7 summarizes the results of the study, and gives suggestions for further research.

CHAPTER 2

GAS FIRM BEHAVIOR WHEN PROFIT-MAXIMIZATION IS NOT THE ONLY OBJECTIVE

It is the author's contention that to investigate the effects of rate-of-return regulation on operating costs, one must look beyond the traditional profit-maximizing model of the firm. The new model should take into account the other objectives of the individuals who make up the firm and the environment of imperfect information in which the firm operates. This chapter proposes a model of firm behavior under rate-of-return regulation based on the assumption that, instead of maximizing profits alone, firm managers maximize utility. This utility is a function of both profit and the institutional costs resulting from any other objectives the managers may be pursuing.

This new model can be used to explain why a firm might not minimize its operating costs, as a profit-maximizing firm generally would. It can also be used to explain firm behavior when rate-of-return regulation forces the firm to have zero or even negative profits. A major theme of the chapter is that a lowering of the allowed rate of return toward the cost of capital would cause the utility-maximizing manager to gradually shift away from profit-maximization toward the maximization of institutional costs.

The first section of this chapter discusses assumptions about the nature of the firm and the nature of institutional costs which could underlie this model, and presents a brief overview of the relevant literature. The second section discusses how the managers would go about maximizing utility; it thus presents the basic model. It is shown how firm behavior is predictable under this model at values of the allowed rate of return equal to or below the cost of capital, thus explaining the first anomaly discussed in the preceding chapter. The third section uses the model to explore how the managers might respond to a lowering of the allowed rate of return. It is shown how this is likely to lead to increasing institutional costs in the firm's operations. The fourth section discusses the need to carefully define what one means by the allowed rate of return and cost of capital in applying this model to the real world, where both may fluctuate over time. The fifth section discusses the principal practical result of the model: that in setting the allowed rate of return regulators may face a tradeoff between profits and institutional costs.

2.1 Objectives Other Than Profit-Maximization

This section examines why it is appropriate to model firm managers as maximizers of utility, where utility is a function of profit and the institutional costs resulting from any other objectives the managers may be pursuing. The first subsection examines the nature of these institutional costs. It is argued that they could take the form of expenses which benefit management directly, or of excessive factor payments, or of suboptimal use of inputs. The second subsection examines the underlying assumptions about the nature of the firm which could result in such a model. It is proposed that the firm be viewed as a set of contracts among individuals, formulated and enforced in an environment where information is costly.

2.1.1 Institutional Costs

In traditional neoclassical theory, firms are black boxes which automatically select inputs and outputs so as to maximize profit. Many economists have questioned this assumption, generally after observing that firms are composed of human beings with diverse objectives and limited capabilities. The earlier critical literature generally aimed at proposing models to be used as alternatives to the neoclassical model of the firm. Three of these models are especially interesting because they suggest other categories of costs which might arise, beyond those which would be incurred by a traditional neoclassical firm.

The three models to be examined in this subsection are the "expense preference" theories of Williamson, the "organizational slack" theories of Cyert and March, and the "X-inefficiency" theories of Leibenstein. The expenses which are the focus of these models are, respectively, those which benefit management directly, excessive factor payments, or expenses due to suboptimal use of inputs. Although each of the three authors has used somewhat pejorative language, connoting "waste", to describe these expenses, I believe they are more properly viewed as a cost of doing business arising from the fact that economic activity must be conducted by human beings organized under certain economic institutions. Hence, I will describe such expenses as "institutional costs." Institutional costs may be contrasted with "physical costs," which would occur even in the idealized neoclassical model. Just as one can often reduce physical costs by improving physical technology, one can often reduce institutional costs by improving institutional technology.

My purpose here is not, however, to construct a new theory of institutional costs. It is, rather, to use the concept in addressing the effects of one important type of institutional technology: rate-of-return regulation. The three models to be discussed in this subsection are presented as illustrations of the concept of institutional costs.

The first model is the "expense preference" model of Oliver Williamson. He argues that firm managements maximize a utility function which is increasing in profit and certain categories of expenses of direct benefit to management. The latter might include management salaries, excessive staff, or "perks", such as plush offices, generous travel and entertainment allowances, and company-sponsored recreation programs. As a result, the firm would incur costs for these expense categories in excess of that which could be justified on the basis of profit-maximization alone.[1]

The second model is the "organizational slack" model of Cyert and March. They argue that all organizations are coalitions of individuals

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[1] Oliver E. Williamson, The Economics of Discretionary Behavior; Managerial Objectives in a Theory of the Firm (Englewood Cliffs, NJ: Prentice-Hall, 1964).

with various objectives. The firm's goals are set by a bargaining process, with side payments being made to individuals whose objectives cannot otherwise be satisfied. Since this market for side payments is not well defined, there may be a tendency for these payments to grow to levels in excess of the amounts required to keep individuals in the coalition, leading to a problem they refer to as "organizational slack." During hard times, a more vigorous search for excessive payments is made by the bargaining participants, and organizational slack is reduced.[2] The emphasis in organizational slack theory is thus on excessive factor payments.

A third model is the X-inefficiency model of Harvey Leibenstein. His approach is rather a elaborate one, which starts with the premise that only individuals, not firms, can have objectives or utility. Individuals within an organization exhibit a constant tendency to shift their efforts, so as to contribute to their own utilities, rather than to profit-maximization ("effort entropy"). The result, in a single output firm, is that actual output is less than the maximum possible with the given inputs, a phenomena Leibenstein calls "X-inefficiency." He argues that all firms exhibit some degree of X-inefficiency, which can be held in check only through the vigorous efforts of the management. An environment where the pressure on management to perform is low, such as a monopoly or a firm operating on a cost-plus contract, leads to high X-inefficiency. A tight competitive environment leads to low

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[2]Richard M. Cyert and James G. March, A Behavioral Theory of the Firm (Englewood Cliffs, NJ: Prentice-Hall, 1963).

X-inefficiency.[3] In any case, the emphasis in X-inefficiency theory is on suboptimal use of inputs.

In the author's opinion, the difference between "organizational slack" and "X-inefficiency" theories is not a sharp one. Both theories explicitly reject the notion of utility maximization. Cyert and March see firm behavior as "adaptively rational." This means that there is search for new modes of behavior ("standard operating procedures") only when performance does not meet expectations. Leibenstein proposes a somewhat similar concept of "inert areas." This describes a set of behavior patterns ("effort positions") any one of which an individual would find acceptable, and not seek to change, unless confronted by new opportunities or constraints.

Although these two models provide interesting descriptions of firm behavior and of the sources of institutional costs, they do not provide a very useful construct for obtaining testable hypothesis or for doing analysis of concrete problems.[4] This is primarily due to their rejection of maximizing behavior without offering a useable alternative.

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[3] Leibenstein has written a number of papers and several books about his theories. These are reviewed in Harvey Leibenstein, "A Branch of Economics Is Missing: Micro-Micro Theory", Journal of Economic Literature, 18 (June, 1979): 477-502. The original paper is "Allocative Efficiency vs. X-Efficiency", American Economic Review, 56 (June, 1966): 392-415. The most thorough exposition of X-efficiency theory is contained in his book, Beyond Economic Man; A New Foundation for Microeconomics (Cambridge: Harvard University Press, 1976).

[4] For an exposition of this critique of X-Efficiency theory see George J. Stigler, "The Xistence of X-Efficiency", American Economic Review, 66 (March, 1976): 213-216. Leibenstein responds in "X-inefficiency Xists--Reply to an Xorcist", American Economic Review, 68 (March, 1978): 203-211.

Williamson's expense preference model does not have these problems. It, however, deals with only one aspect of institutional costs, and does not really explain how managers get the discretionary power to pursue objectives other than profit which he ascribes to them. In the next subsection, I therefore examine some other approaches to the analysis of institutional costs.

2.1.2 The Nature of the Firm

In recent years, a new stream of literature has developed reexamining the theory of the firm. Rather than providing an alternative to the neoclassical approach, this literature tends to integrate a new model of the firm into the neoclassical framework by modifying a few assumptions. In this approach, firms are viewed as a set of contractual relationships among individuals, with what I call institutional costs becoming a focus of attention. Consistent with this approach, I will argue that, for our purposes, a gas firm can be viewed as a contract between utility-maximizing managers and a regulator who can impose certain constraints. Management utility will be a function of profit and institutional costs.

Although no new comprehensive theory of the firm has yet developed, two clear trends in this new literature may be noted. The first has been to end the dichotomy between consumer theory and the theory of the firm by extending the utility-maximization hypothesis to the individuals who make up the firm. The second has been to recognize that there are very important informational constraints on the ability to negotiate and enforce contracts. The imperfect contracting between the individuals who make up the firm explains the existence of institutional costs.[5]

This literature has been developing by viewing the firm both from a "macro" and "micro" perspective. The macro literature has tried to explain firm organization as a system of minimizing institutional costs. A key paper was that of Alchian and Demsetz, who saw firms as emerging so as to reduce the amount of shirking which takes place in joint or team production. The members of the team can make themselves better off by contracting with a manager who can monitor, and appropriately reward, each member's contributions.[6] Williamson has extended this type of analysis to other types of opportunistic behavior by individuals in order to give explanations for the emergence of a variety of economic institutions. He also considers the problems posed by the individuals' limited knowledge and computational capabilities ("bounded rationality").[7]

In contrast to this macro view is what has become known as the "agency literature", which focuses on the construction of contracts between a principal and an agent, given limitations on information. Ideally, the principal would want to reward the agent for actions taken

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[5]This literature is reviewed in Louis De Alessi, "Property Rights, Transaction Costs, and X-Efficiency: An Essay in Economic Theory", American Economic Review, 73 (March, 1983): 64-81.

[6]Armen A. Alchian and Harold Demsetz, "Production, Information Costs, and Economic Organization", American Economic Review, 62 (December, 1972): 777-795.

[7]Oliver E. Williamson, Markets and Hierarchies; Analysis and Anti-Trust Implications, (New York: Free Press, 1975).

in his interests. However, the world may often be such that the principal cannot costlessly determine whether the agent has taken actions in his interest. So the principal must base his reward on some type of imperfect information, such as the outcome of the actions, which may reflect other types of risks as well. The agency literature can be viewed as exploring the tradeoffs faced by the principal in determining how much of this risk to impose on the agent. As the amount of risk which the agent bears increases, the agent has an increasing incentive to take actions in the principal's interest, but will also generally demand compensation with an increasingly large expected value. Generally speaking, it will not be optimal to construct a contract where the agent bears the entire risk, and therefore acts as the principal would have acted.[8] Applying this type of model to relationships between owners and managers, or supervisors and subordinates, one could readily see how institutional costs might arise.

There have been several papers which have combined the macro and micro approaches. Most of this work has been normative in nature-attempting to derive optimal institutional structures for handling

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[8]Examples of this literature are: Michael Spence and Richard Zeckhauser, "Insurance, Information, and Individual Action", American Economic Review, 61 (May, 1971): 380-387; Stephen A. Ross, "The Economic Theory of Agency: The Principal's Problem", American Economic Review, 63 (May 1973): 134-139; Steven Shavell, "Risk Sharing and Incentives in the Principal Agent Relantionship", Bell Journal of Economics, 10 (spring, 1979): 55-73; and Bengt Holmstrom, "Moral Hazard and Observability", Bell Journal of Economics, 10 (Spring, 1979): 74-91.

specific types of problems.[9] There are two descriptive papers worth noting here, however. The first is a paper by Fama, which points out that if the managerial labor market is efficient and rational, as capital markets are generally assumed to be, managers would have to ultimately bear the costs or benefits of their actions in future compensation, thus eliminating the incentive problem.[10] This author would argue, however, that managers acquire a large amount of firm-specific knowledge, inhibiting competition for managers between firms. Furthermore, top managers frequently have effective control over the firm,[11] inhibiting competition for managers within the firm. Hence, the managerial labor market is less than efficient and rational. Nevertheless, Fama's paper points up the importance of these assumptions about the managerial labor market in explaining firm behavior.

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[9] Examples of this literature are: Joseph E. Stiglitz, "Incentives and Risk Sharing in Sharecropping", Review of Economic Studies, 41 (April, 1974): 219-255; Joseph E. Stiglitz, "Incentives, Risk, and Information: Notes Towards a Theory of Hierarchy", Bell Journal of Economics, 6 (Autumn, 1975): 552-579; James A. Mirrlees, "The Optimal Structure of Incentives and Authority Within an Organization", Bell Journal of Economics, 7 (Spring, 1976): 105-131; Stanley Baiman and Joel S. Demski, "Economically Optimal Performance Evaluation and Control Systems", 18 supplement (1980) pp. 184-220; and John Christensen, "Communication in Agencies", Bell Journal of Economics, 12 (Autumn, 1981): 661-674.

[10] Eugene F. Fama, "Agency Problems and the Theory of the Firm", Journal of Political Economy, 88 (April, 1980): 288-307.

[11] Three classic studies emphasizing the control of top managers over large corporations are Adolf A. Berle and Gardiner Means, The Modern Corporation and Private Property (New York: MacMillan, 1932); Robert Aaron Gordon, Business Leadership in the Large Corporation (Washington, DC: The Brookings Institution, 1945); and Myles L. Mace, Directors: Myth and Reality (Boston: Harvard University Graduate School of Business Administration, 1971).

A second important descriptive paper is that of Jensen and Meckling. They argue that a particular contractual structure may be modeled by assuming a utility-maximizing manager faced with a budget constraint. The manager's utility function is an increasing function of both the present value of the firm and the present value of the "non-pecuinary benefits consumed" by the manager. Their discussion reveals the latter to be almost synonymous with what I call institutional cost. By showing how the budget constraint is altered, Jensen and Meckling then use their model to explore how the firm might be affected by such things as outside equity financing, monitoring and bonding of the management, and the issuance of debt.[12]

My study focuses on the contract between the managers and the regulator. A paper by Baron and Myerson has shown how the agency approach can be applied to specify an optimal regulatory contract.[13] My study, however, will be limited to an examination of rate-of-return regulation. As such, a descriptive approach, like that of Jensen and Meckling, appears to be more appropriate. Like them, I shall model the gas firm by assuming utility-maximizing managers faced with a budget constraint. Utility will be an increasing function of the firm's profit and institutional costs. The utility function and the budget constraint already reflect all of the firm's other contracts, including that

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[12]Michael C. Jensen and William H. Meckling, "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure", Journal of Financial Economics, 3 (October, 1976): 305-360.

[13]David P. Baron and Roger B. Myerson, "Regulating A Monopolist With Unknown Costs", Econometrica, 50 (July, 1982): 911-930.

between the managers and the owners. Hence, the utility function and budget constraint incorporate a certain structure of incentive payments to managers and monitoring of their actions by outsiders. I shall assume this structure to be independent of actions taken by the regulator.

Utility is an increasing function of profit either because management compensation is some function of profits, or because managers believe higher profits result in increased job security, since profits are what the owners monitor. Utility is an increasing function of institutional costs for one of two reasons. First, these costs may represent expenditures which contribute directly to management utility. Second, they may be costs which would require effort to reduce through bargaining or searching for lower factor costs and better utilization of inputs. The utility of institutional costs may be reduced by the owners' monitoring efforts, which may sometimes permit them to penalize the managers for at least some types of institutional costs.

The regulators, for their part, are assumed to be incapable of directly controlling institutional costs. The assumption is justified, since under a rate-of-return regulation system, regulators really are in a weak position. About all the regulators can do when they happen to notice an inappropriate cost is to disallow the firm from passing that particular cost through to consumers. Even this requires concrete legal evidence, and lengthly administrative processes, which will not usually be practical except in flagrant cases.[14] Without the owners' ability to hire and fire managers, or make incentive payments to them, regulators have little means to even attempt to control institutional costs directly.

2.2 Firm Behavior Under Rate-of-Return Regulation

Firm managers whose utility is an increasing function of profit and institutional costs will go about maximizing that utility just as would the consumer in neoclassical theory. Specifically, the managers will choose to operate at the point of tangency between the highest attainable isoutility curve and the budget constraint they face. The regulators can influence this solution by taking actions which alter the shape or location of the budget constraint. By analyzing how a regulatory action affects the managers' budget constraint, one can use this utility-maximization framework to predict the firm's response to a regulatory action. The first subsection below discusses the manager's budget constraint, and how it would be affected by a rate-of-return limitation on profits. The second subsection discusses the managers' utility function and the utility-maximizing solution. In both subsections, these results are explained verbally and graphically. The section concludes with a mathematical formulation subsection, which explains the model in a mathematically formal fashion. As no new results are presented in this mathematical formulation subsection, or

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[14]See Martin T. Farris and Roy J. Sampson, Public Utilities; Regulation, Management, and Ownership (Boston: Houghton-Mifflin, 1973), pp. 94-99 and Kahn, v.2, p. 47.

any of the other mathematical formulation sections throughout this study, readers who are interested only in a general overview may skip these subsections.

It is important to understand that the institutional costs to be discussed in this study are assumed to be part of the firm's operating costs. In practice, institutional costs will also arise in the firm's capital expenditures. However, an investigation of the latter form of institutional costs must be left for future research.

2.2.1 The Budget Constraint

The managers of every firm must allocate their resources so as to achieve one of many feasible combinations of profit and institutional costs. If the firm managers are maximizing a utility function of profit and institutional costs, they will always choose to operate at a point which maximizes profit for any given level of institutional costs, and vice-versa. The set of such feasible points constitutes the budget constraint which the managers face, which may be plotted on a graph with profit on one axis and institutional costs on the other.

I shall assume that the firms's physical costs (that is, non-institutional costs) are independent of the level of institutional costs chosen by the managers.[15] Then, in an untaxed unregulated environment, if the managers desired to increase profit by a dollar from a point on

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[15] One could make alternative assumptions about the interaction of physical costs and institutional costs. Three different approaches are discussed in R. Rees, "A Reconsideration of the Expense Preference Theory of the Firm", Economica, 41 (August, 1974): 295-307. My approach corresponds to Rees' case (i).

the budget constraint, they would have to reduce institutional costs by a dollar. Hence, the budget constraint will be straight line with slope of minus one, as shown in Figure 4. Actually, such a situation hardly

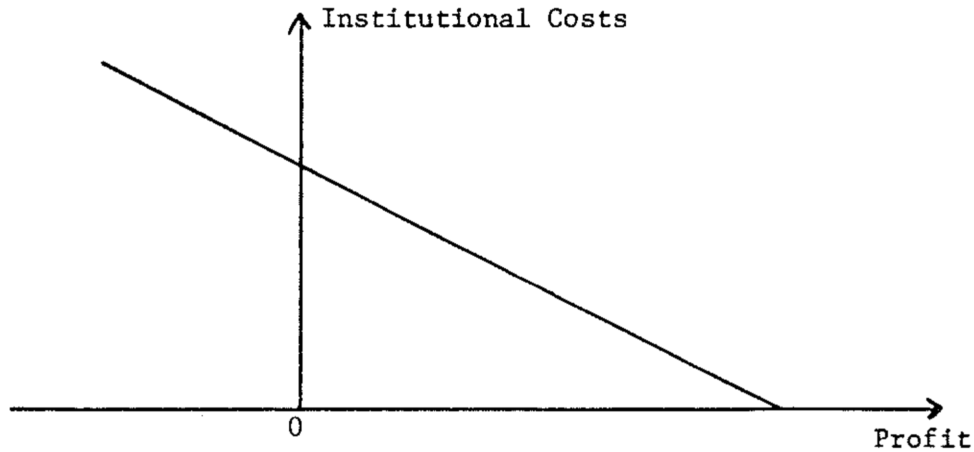

Figure 4
Budget Constraint of Unregulated Firm

ever arises in the real world. Most institutional costs are tax-deductible; with a roughly 50% corporate income tax, the slope of the budget constraint (institutional costs vs. after-tax profit) becomes closer to minus two.

It is the effect of rate-of-return regulation on the budget constraint that is of primary concern in this study, however. Without rate-of-return regulation, the managers would choose the same level of capital investment no matter where on the budget constraint they choose to operate. The firm can maximize profit, at any given level of institutional costs, just as would an ordinary profit-maximizing firm, by setting the value of the marginal product of capital equal to its marginal cost. Under rate-of-return regulation, however, the firm becomes subject to a profit ceiling based on the amount of capital investment it has, and these observations no longer apply.

Let π^* be what the regulatory ceiling on profit would be if the managers chose the amount of capital investment they would choose in an unregulated environment. As long as the managers choose a combination of profit and institutional costs such that profit is less than π^*, rate-of-return regulation has no effect. Hence, the budget constraint is unaffected by rate-of-return regulation at values of profit less than π^*.

If, however, the managers wish profits greater than π^* when the allowed rate of return is greater than the cost of capital, they must invest more than the amount of capital they would choose if the firm were unregulated. This will be costly, reducing the potential institutional costs at any given level of profit below what they would be if the firm were unregulated. Hence, the budget constraint will be lower and steeper than in an unregulated environment at values of profit greater than π^*. The new budget constraint will be as shown in Figure 5.

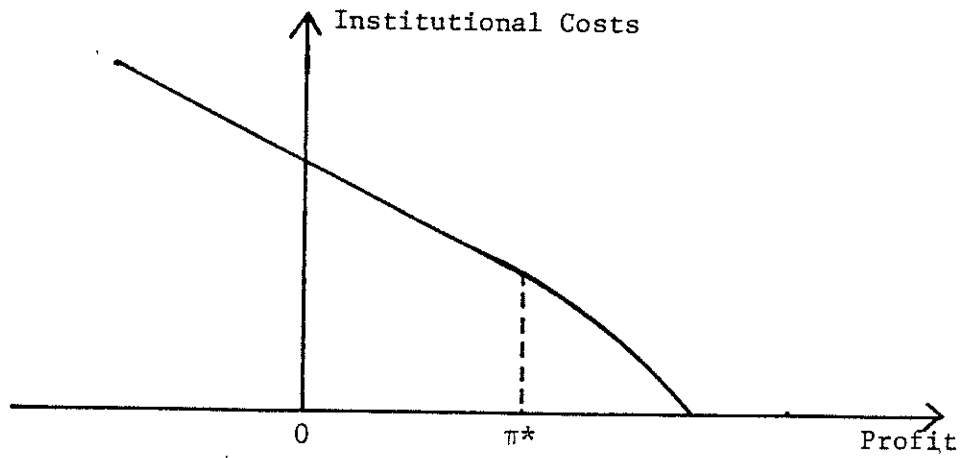

Figure 5
Budget Constraint of a Rate-of-Return Regulated Firm
With an Allowed Rate of Return Greater Than the Cost of Capital

If the allowed rate of return is reduced, π* will be reduced. Furthermore, the budget constraint will become lower and steeper, since the managers must acquire increasingly large amounts of capital to achieve a given profit ceiling. A resulting family of budget constraints is shown in Figure 6.

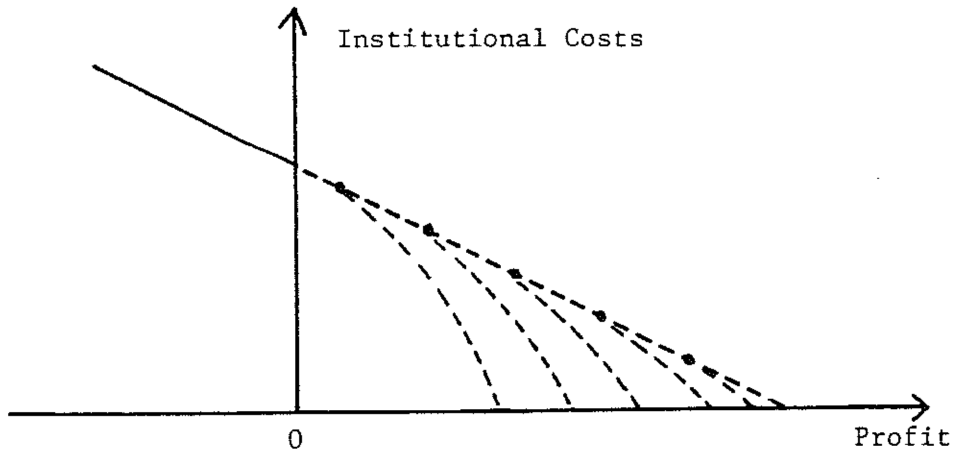

Figure 6
Effect of a Reduction in the Allowed Rate of Return
on the Budget Constraint

When the allowed rate of return exactly equals the cost of capital, π^* will equal zero and the budget constraint will be vertical at the institutional costs axis as shown in Figure 7. There is then no way the firm can have a positive profit. Note that, assuming the allowed rate of return is greater than or equal to the cost of capital, the budget

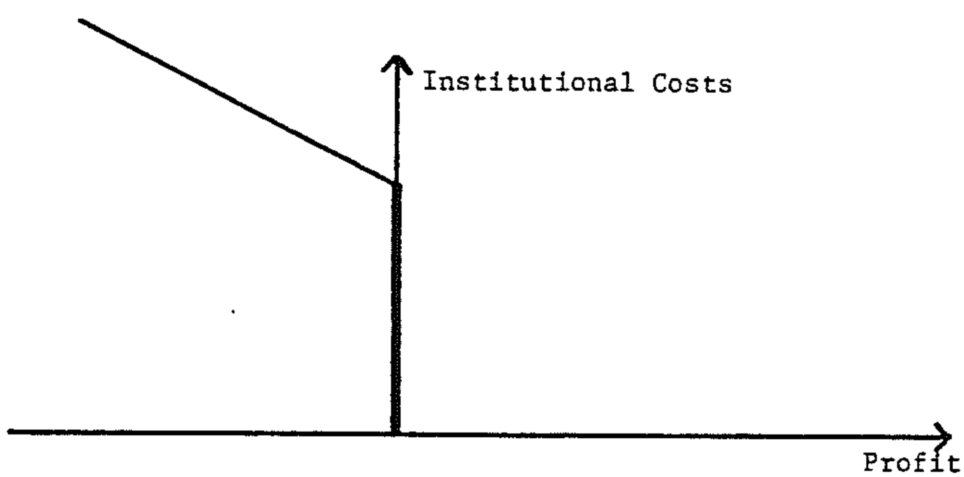

Figure 7
Budget Constraint of a Rate-of-Return Regulated
Firm with an Allowed Rate of Return Equal to the Cost of Capital

constraint always intersects the institutional costs axis at a value equal to the maximum amount of profit the firm could have earned in an untaxed, unregulated environment. This is because of the one-to-one tradeoff between profit and institutional costs in an untaxed unregulated environment.

I can similarly plot the budget constraint when the allowed rate of return is less than the cost of capital. In this case, π^*, will be negative, but the managers can reduce their losses by investing less than the amount of capital they would choose for an unregulated firm. The new locus always passes through the origin, since, while positive profits are impossible, the managers can reduce their losses to zero by reducing capital investment to zero and going out of business. A budget constraint for an allowed rate of return less than the cost of capital might appear as in Figure 8.

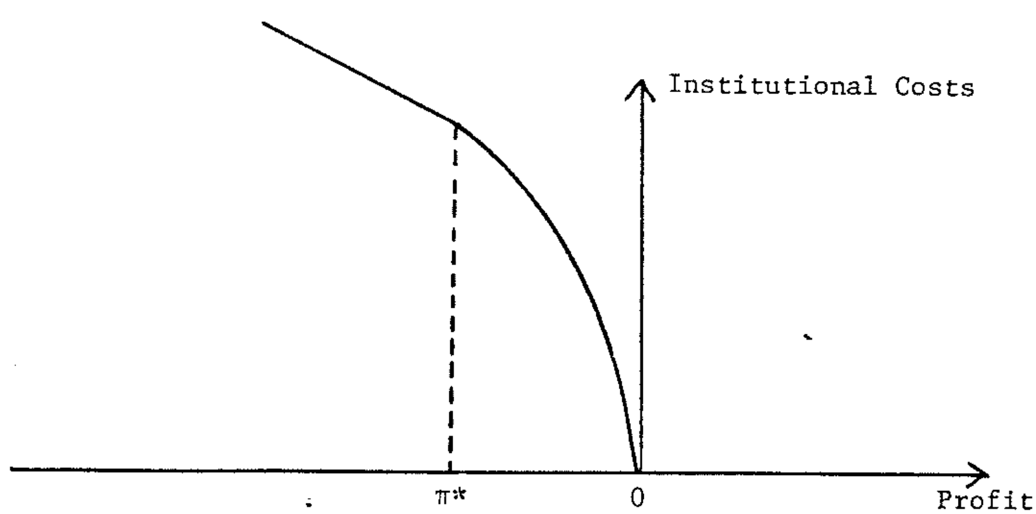

Figure 8
Budget Constraint of a Rate-of-Return Regulated Firm
With an Allowed Rate of Return Less Than the Cost of Capital

A profit-maximizing firm with an allowed rate of return less than the cost of capital must, of course, go out of business. However, the utility-maximizing managers assumed in this study may prefer to keep the firm in business even when this is not in the best interests of the firm's stockholders. If the regulators set the allowed rate of return high enough that the firm could cover its operating costs and fixed charges, such a firm would not go bankrupt.

2.2.2 Utility Maximization

Given a budget constraint, one can determine the combination of profit and institutional costs which the managers will choose by plotting isoutility curves (points of equal utility) as shown in Figure 9. Utility increases with increasing amounts of profit or institutional costs, hence the curves represent increasing values of utility as one

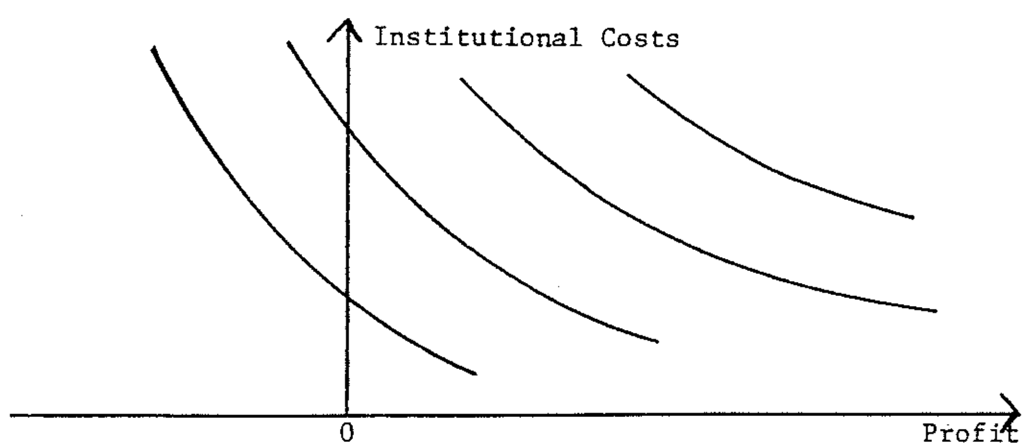

Figure 9
Isoutility Curves

moves up and to the right. The managers will seek to operate on the highest possible isoutility curve consistent with the budget constraint which they face. This will be where an isoutility curve is just tangent to the budget constraint. Figure 10 shows one such solution. In this figure, the managers will choose to operate at a point to the right of π^*, where the rate-of-return constraint is binding. This is usual case

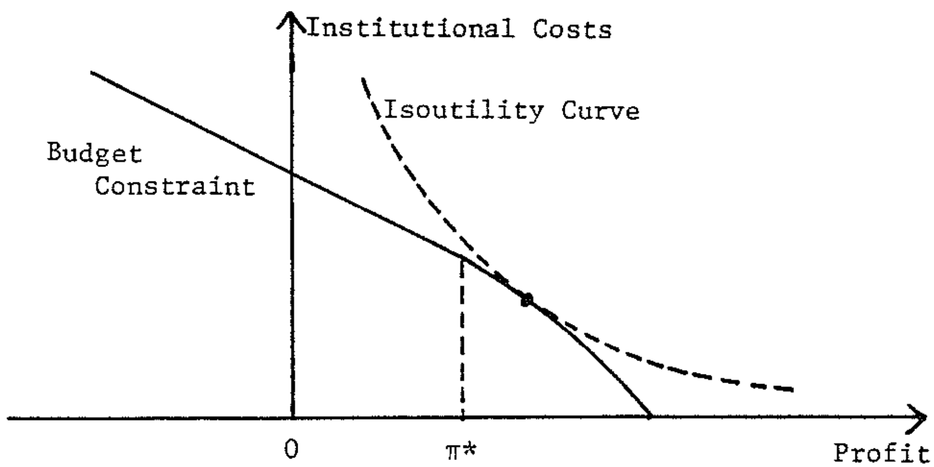

Figure 10
Utility-Maximizing Solution
with the Rate-of-Return Constraint Binding

for rate-of-return regulated firms; however, there is no necessary reason it has to be this way. Figure 11 shows how a different utility

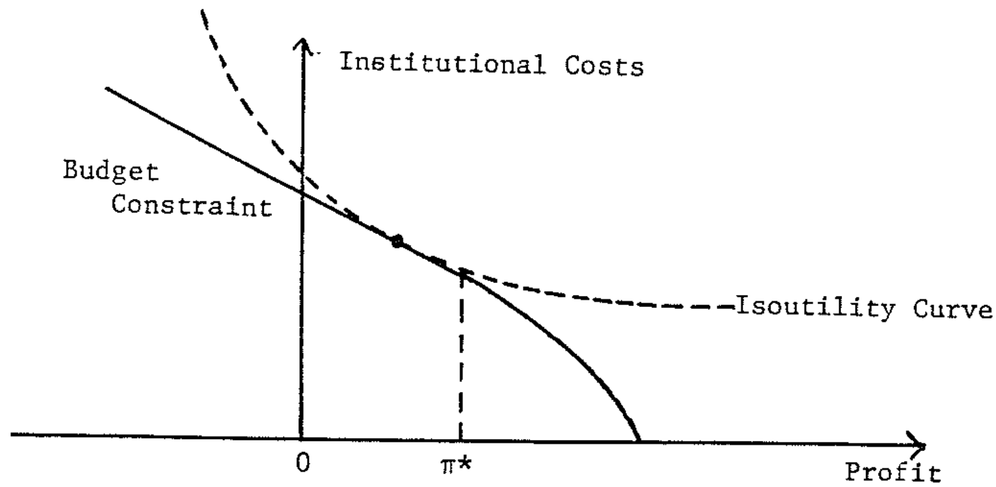

Figure 11
Utility-Maximizing Solution with
Rate-of-Return Constraint Nonbinding

function might result in the managers choosing a value of profit less than π^*, where the rate-of-return constraint is not binding.

These two figures were drawn assuming an allowed rate of return greater than the cost of capital. One could easily draw similar figures for an allowed rate of return less than the cost of capital, or for an allowed rate equal to the cost of capital with the managers choosing to operate at a point to the left of the institutional costs axis. The one situation where the tangency condition would not apply is if the allowed rate of return equals the cost of capital and the managers choose to operate at the zero-profit point. In this case, the kink in the budget constraint at the point where profit equals zero will touch the isoutility curve as shown in Figure 12.

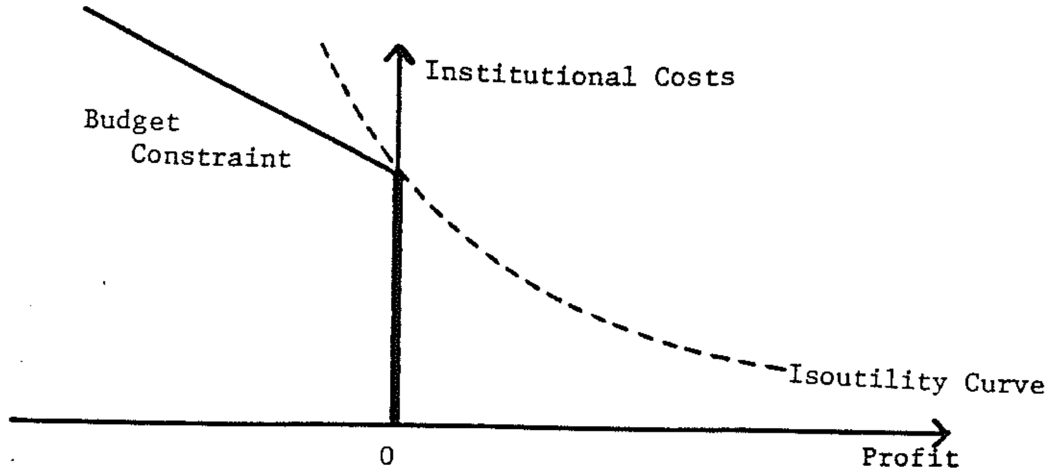

Figure 12
Utility-Maximizing Solution With an Allowed Rate of Return Equal to the Cost of Capital and the Rate-of-Return Constraint Binding

2.2.M. Mathematical Formulation

This subsection presents a mathematical formulation of the model discussed above. I will show how the budget constraint is derived, and show that it has the shape described above, as well as the claimed responses to changes in the allowed rate of return. I will also show how the utility-maximizing solution for the managers is at the point of tangency of the budget constraint in π - x space and an isoutility curve. The case of an unregulated firm will be considered first, then the firm subject to a rate-of-return constraint.

The notation to be used in this subsection is as follows:

π = real economic profit;

x = institutional costs;

k = quantity of capital invested;

g = quantity of gas sold;

$z(k,g)$ = minimum operating cost required to sell quantity of gas g with capital investment k;

$p(g)$ = consumer price of gas $(p'(g) < 0)$;

$f(g)$ = wellhead price of gas $(f'(g) > 0)$;

c = constant representing value of any rents accruing to firm from partial wellhead price controls (see below);

s = allowed rate of return;

r = cost of capital;

$U(\pi,x)$ = the manager's utility function (concave).

I assume that g is always larger than the quantity of gas available to the firm from price-controlled sources, allowing me to represent the rents accruing to the firm from partial wellhead price controls as a constant c. The firm's total gas purchase costs are therefore $f(g)g - c$.

My notation scheme differs from the standard textbook approach to modeling firm behavior in two minor respects. First, z( ) serves the role of "labor" in the standard textbook approach. My notation recognizes that there are many non-capital inputs to a firm, which I have lumped into a single input. I measure these other inputs by their monetary value rather than their physical quantity; this allows me to simplify the model by not having to specify the price of these inputs. Second, I make z( ) a function of capital investment k and output g, with output an independent variable. The traditional approach would be to have output dependent upon capital investment and z according to a production function, with z being an independent variable. My approach generally produces simpler, more intuitive results.

I also define the firm's net revenue function to be

$$R(k,g) = \big(p(g) - f(g)\big)g + c - z(k,g) - rk. \qquad (2.2.M/1)$$

This function represents the total resources which the managers have to divide between profits and institutional costs. I assume throughout this study that R(k,g) is a strictly concave function. This means that the function has the distinctive upside-down bowl shape, as depicted in Figure 13 below. The following theorem may help to convince the reader of the plausibility of this assumption.

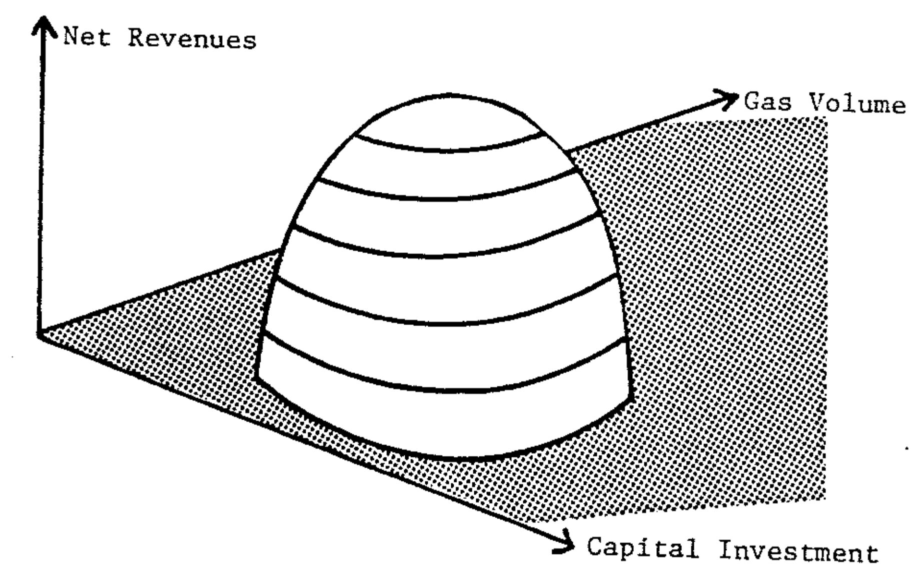

Figure 13
The Net Revenue Function

<u>Theorem 2.2.M.A</u>: If

1) the gas firm's underlying production function for gas transportation $g = h(z,k)$ is strictly concave with $h_1(z,k) > 0$ and $h_2(z,k) > 0$; and

2) if the difference between marginal revenue from the sale of gas, and the marginal cost of gas to the firm moves in negative direction as gas sales increase, that is if

$$\frac{\partial}{\partial g}\left[p'(g)g + p(g) - f'(g)g - f(g)\right] < 0,$$

then the firm's net revenue function, as defined by (2.2.M/1), is strictly concave.

Proof: To demonstrate the strict concavity of $R(k,g)$ it is sufficient to show that $R_{11}(k,g) < 0$ and $R_{11}(k,g)R_{22}(k,g) - \left(R_{12}(k,g)\right)^2 > 0$. I will begin by obtaining expressions for $z_{11}(k,g)$, $z_{12}(k,g)$, and $z_{22}(k,g)$ in terms of the partials of the production function $g = h(z,k)$. I shall then use these expressions to obtain the desired properties of the partials of $R(k,g)$.

One can obtain expressions for $z_1(k,g)$ and $z_2(k,g)$ in terms of the partials of the production function as follows. First, totally differentiating $g = h(z,k)$ yields

$$dg = h_1(z,k)dz + h_2(z,k)dk.$$

One can write this expression in terms of k and g alone by substituting $z = z(k,g)$:

$$dg = h_1\big(z(k,g),k\big)dz + h_2\big(z(k,g),k\big)dk.$$

This implies

$$z_1(k,g) = \frac{\partial z}{\partial k} = \frac{-h_2(z(k,g),k)}{h_1(z(k,g),k)} \qquad (2.2.M/2)$$

and

$$z_2(k,g) = \frac{\partial z}{\partial g} = \frac{1}{h_1(z(k,g),k)}. \qquad (2.2.M/3)$$

These relationships may be used to determine the second partials of $z(k,g)$:

$$z_{11}(k,g) = \frac{-[h_{21}(z(k,g),k)z_1(k,g) + h_{22}(z(k,g),k)]}{h_1(z(k,g),k)}$$

$$+ \frac{h_2(z(k,g),k)\ [h_{11}(z(k,g),k)z_1(k,g) + h_{12}(z(k,g),k)]}{[h_1(z(k,g),k)]^2} ;$$

$$z_{12}(k,g) = \frac{-h_{11}(z(k,g),k)z_1(k,g) - h_{12}(z(k,g),k)}{[h_1(z(k,g),k]^2} ;$$

$$z_{22}(k,g) = \frac{-h_{11}(z(k,g),k)z_2(k,g)}{[h_1(z(k,g),k)]^2} .$$

Substituting for $z_1(k,g)$ and $z_2(k,g)$ using (2.2.M/2) and (2.2.M/3), and dropping the arguments for ease of notation:

$$z_{11}(k,g) = \frac{-h_{11}h_2^{\ 2} + 2h_{12}h_1h_2 - h_{22}h_1^{\ 2}}{h_1^{\ 3}} ; \qquad (2.2.M/4)$$

$$z_{12}(k,g) = \frac{h_{11}h_2 - h_{12}h_1}{h_1^{\ 3}} ; \qquad (2.2.M/5)$$

$$z_{22}(k,g) = \frac{-h_{11}}{h_1^3} . \qquad (2.2.M/6)$$

Since the numerators of (2.2.M/4) and (2.2.M/6) are positive by the strict (quasi-) concavity of $h(z,k)$, while the denominator is assumed positive, both $z_{11}(k,g)$ and $z_{22}(k,g)$ are positive.

To show that $R_{11}(k,g) < 0$, one merely notes that, by taking the partial of (2.2.M/1),

$$R_{11}(k,g) = -z_{11}(k,g), \qquad (2.2.M/7)$$

which is negative since $z_{11}(k,g)$ is positive. One can similarly take the partials of (2.2.M/1) to obtain

$$R_{12}(k,g) = -z_{12}(k,g) \qquad (2.2.M/8)$$

and

$$R_{22}(k,g) = \frac{\partial}{\partial g}\left[p'(g)g + p(g) - f'(g)g - f(g)\right] - z_{22}(k,g). \qquad (2.2.M/9)$$

Using the above expressions allows one to write

$$R_{11}(k,g)R_{22}(k,g) - \left(R_{11}(k,g)\right)^2$$

$$= -z_{11}(k,g)\frac{\partial}{\partial g}\left[p'(g)g + p(g) - f'(g)g - f(g)\right]$$

$$+ z_{11}(k,g)z_{22}(k,g) - \left(z_{12}(k,g)\right)^2. \qquad (2.2.M/10)$$

The first term in the above expression is positive since I have shown that $z_{11}(k,g) > 0$ and assumed that

$$\frac{\partial}{\partial g}\left[p'(g)g + p(g) - f'(g)g - f(g)\right] < 0.$$

The other two terms may be written using (2.2.M/4) - (2.2.M/6) as

$$z_{11}(k,g)z_{22}(k,g) - \left(z_{12}(k,g)\right)^2$$

$$= - \left(\frac{-h_{11}h_2^{\;2} + 2h_{12}h_1h_2 - h_{22}h_1^{\;2}}{h_1^{\;3}}\right)\frac{h_{11}}{h_1^{\;3}}$$

$$- \left(\frac{h_{11}h_2 - h_{12}h_1}{h_1^{\;3}}\right)^2$$

$$= \frac{h_{22}h_{11} - h_{12}^{\;2}}{h_1^{\;4}}.$$

This expression is positive by the strict concavity of $h(z,k)$. So (2.2.M/10) is positive. I have thus shown that, under the given assumptions, $R(k,g)$ is a strictly concave function.

Q.E.D.

2.2.M.1 The Unregulated Firm

In this mathematical subsection, I will refer to the budget constraint which I have graphed and discussed earlier as "the budget constraint in π - x space", to distinguish it from the firm's overall budget constraint, which is a function of k and g as well as π. The budget constraint in π - x space is defined to be the largest value of x which the firm could achieve at each value of π.

For an unregulated firm, the budget constraint in π - x space may be obtained by maximizing

$$x = \left(p(g) - f(g)\right) + c - z(k,g) - rk - \pi$$

over g and k, where both must be non-negative. The first-order conditions for this maximization requires that the derivatives of this function with respect to g and k equal zero:

$$\frac{dx}{dg} = p'(g) + p(g) - f'(g) - f(g) - z_2(k,g) = 0; \qquad (2.2.M.1/1)$$

$$\frac{dx}{dk} = -z_1(k,g) - r = 0. \qquad (2.2.M.1/2)$$

Lemma 2.2.A: Equations (2.2.M.1/1) and (2.2.M.1/2) yield unique optimum values for g and k which are independent of π.

Proof: Note that the objective function may be written as $x = R(k,g) - \pi$. Since $R(k,g)$ is strictly concave, the objective function itself is a strictly concave function of k and g as well. Hence, the first order conditions yield unique optimum values. Since neither condition is a function of π, the solution must be independent of π.

Q.E.D.

By this Lemma, one may write the budget constraint in π - x space for the unregulated firm as

$$x = \left(p(g^*) - f(g^*)\right)g^* + c - z(k^*,g^*) - rk^* - \pi, \qquad (2.2.M.1/3)$$

where g* and k* are the unique values of g and k satisfying (2.2.M.1/1) and (2.2.M.1/2). The following theorem about the shape of the budget constraint in π - x space follows easily.

Theorem 2.2.M.1.A: The slope of the budget constraint in π - x space is minus one.

Proof: Since g* and k* are independent of π, the derivative of (2.2.M.1/3) with respect to π yields

$$\frac{dx}{d\pi} = -1.$$

Q.E.D.

A goal of this subsection is to show that the solution chosen by the managers lies at the point of tangency between the budget constraint in π - x space and an isoutility curve. To demonstrate this, I start by deriving the first-order conditions for the solution to the managers' problem. The managers seek to maximize utility

$$U(\pi,x),$$

subject to the budget constraint

$$\left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi - x \geq 0.$$

Maximization is over π, x, g, and k; where x, g, and k, but not π, must be non-negative. The Lagrangian will be

$$L = U(\pi,x) + \lambda\left[\left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi - x\right].$$

The first-order conditions[16] require that:

$$\frac{\partial L}{\partial \pi} = U_1(\pi,x) - \lambda = 0; \qquad (2.2.M.1/4)$$

$$\frac{\partial L}{\partial x} = U_2(\pi,x) - \lambda \leq 0; \qquad (2.2.M.1/5)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial g} = \lambda[p'(g)+p(g)-f'(g)g-f(g)-z_2(k,g)] \leq 0; \qquad (2.2.M.1/6)$$

$$\frac{\partial L}{\partial g} g = 0; \qquad g > 0;$$

$$\frac{\partial L}{\partial k} = \lambda[-z_1(k,g) - r] \leq 0; \qquad (2.2.M.1/7)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = (p(g) - f(g))g + c - z(k,g) - rk - \pi - x \geq 0; \qquad (2.2.M.1/8)$$

$$\frac{\partial L}{\partial \lambda} \lambda = 0; \qquad \lambda \geq 0.$$

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[16]Throughout this study, the first-order conditions are presented in the format used by William J. Baumol in his _Economic Theory and Operations Analysis_, 4th ed. (Englewood Cliffs, NJ: Prentice-Hall, 1977), Chap. 8. The format is technically correct and easy to remember. However, it can leave the reader with the misleading impression that the Lagrange multipliers (the λ's) are mathematically analogous to the other variables. In fact, the condition that $\partial L/\partial\lambda \geq 0$ is simply a way of restating the constraint, while the condition that $[\partial L/\partial\lambda]\lambda = 0$ restates the requirement that either the constraint be an equality or the Lagrange multiplier equal zero (or both).

I shall assume that g, k, and x are all greater than zero at the solution, and that π and x have positive marginal utilities, that is, $U_1(\pi,x) > 0$ and $U_2(\pi,x) > 0$. These assumptions are sufficient to insure that conditions (2.2.M.1/5) - 2.2.M.1/8) are equalities with $\lambda > 0$. I now prove the desired theorem.

Theorem 2.2.M.1.B: The solution chosen by the managers lies at the point of tangency between the budget constraint in π - x space and an isoutility curve in π - x space.

Proof: Note that equations (2.2.M.1/6) and (2.2.M.1/7) are equivalent to (2.2.M.1/1) and (2.2.M.1/2). By Lemma 2.2.A, this means that they must yield unique values of g and k, which are the same as those on the budget constraint in π - x space. Substituting these values into (2.2.M.1/8) yields an equation equivalent to (2.2.M.1/3), implying that the solution chosen by the managers must lie on the budget constraint in π - x space.

To show tangency, it remains to demonstrate that the slope of the isoutility curve equals the slope of the budget constraint in π - x space at the solution. An isoutility curve is defined by $U(\pi,x)$ = constant. Total differentiation yields a slope

$$\frac{dx}{d\pi} = - \frac{U_1(\pi,x)}{U_2(\pi,x)}.$$

But solving (2.2.M.1/4) and (2.2.M.1/5) yields

$$-\frac{U_1(\pi,x)}{U_2(\pi,x)} = -1,$$

indicating that the slope of the isoutility curve is minus one. By Theorem 2.2.M.1.A, the slope of the budget constraint in π - x space is minus one, so the two slopes are the same.

Q.E.D.

2.2.M.2. The Regulated Firm

The budget constraint in π - x space for the regulated firm is defined the same way as it is for the unregulated firm, only the maximization is performed subject to the rate-of-return constraint. The budget constraint in π - x space may be obtained by maximizing

$$x = \big(p(g) - f(g)\big) + c - z(k,g) - rk - \pi$$

subject to the rate-of-return constraint

$$(s-r)k - \pi \geq 0.$$

Maximization is over g and k, where both must be non-negative. The Lagrangian will be

$$L = \big(p(g) - f(g)\big) + c - z(k,g) - rk - \pi + \lambda[(s-r)k - \pi].$$

The first-order conditions require that:

$$\frac{\partial L}{\partial g} = p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) \leq 0; \quad (2.2.M.2/1)$$

$$\frac{\partial L}{\partial g} g = 0; \qquad g \geq 0;$$

$$\frac{\partial L}{\partial k} = -z_1(k,g) - r + \lambda(s-r) \leq 0; \quad (2.2.M.2/2)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = (s-r)k - \pi \geq 0; \quad (2.2.M.2/3)$$

$$\frac{\partial L}{\partial \lambda} \lambda = 0; \qquad \lambda \geq 0.$$

I shall assume that g and k are greater than zero at the solution, which implies that (2.2.M.2/1) and (2.2.M.2/2) must be equalities.

For ease of notation, define the budget constraint in π - x space to be

$$x = B(\pi,s).$$

It will be useful throughout the remainder of this chapter to know some of the partials of $B(\pi,s)$ given that the rate-of-return constraint is an equality with λ greater than zero. The following series of lemmas give these partials.

Lemma 2.2.B: If the rate-of-return constraint is an equality, and $s \neq r$, then

$$\frac{dk}{d\pi} = \frac{1}{s-r} \,. \tag{2.2.M.2/4}$$

This is, of course, positive if $s > r$ and negative if $s < r$.

Proof: If the rate-of-return constraint (2.2.M.2/3) is an equality, total differentiation yields the desired result immediately.

Q.E.D.

Lemma 2.2.C: If the rate of return constraint is an equality, and $s \neq r$, then

$$\frac{d^2k}{d\pi^2} = 0.$$

Proof: Taking the derivative of (2.2.M.2/4) with respect to k yields the desired result immediately.

Q.E.D.

Lemma 2.2.D: If the rate-of-return constraint is an equality, and $s \neq r$, then

$$\frac{dk}{ds} = \frac{-k}{s-r}.$$

This is, of course, negative if $s > r$ and positive if $s < r$.

Proof: If the rate-of-return constraint (2.2.M.2/3) is an equality, total differentiation yields the desired result immediately.

Q.E.D.

Lemma 2.2.E: If the rate-of-return constraint is an equality, and $s \neq r$, then

$$\frac{d^2k}{d\pi ds} = \frac{-1}{(s-r)^2}.$$

This is, of course, always negative.

Proof: Proof taking the derivative of (2.2.M.2/4) with respect to s yields the desired result immediately.

Q.E.D.

Lemma 2.2.F: If the rate-of-return constraint is an equality, then

$$\frac{d}{d\pi}\left(-z_1(k,g)\right) < 0$$

if $s > r$ and

$$\frac{d}{d\pi}\left(-z_1(k,g)\right) > 0$$

if $s < r$ at points on the budget constraint in π - x space.

Proof: Taking the derivative,

$$\frac{d}{d\pi}\left(-z_1(k,g)\right) = \left(-z_{11}(k,g) - z_{12}(k,g)\frac{dg}{dk}\right)\frac{dk}{d\pi} .$$

Since $dk/d\pi > 0$ if $s > r$ and $dk/d\pi < 0$ if $s < r$ by Lemma 2.2.B, a demonstration that

$$-z_{11}(k,g) - z_{12}(k,g)\frac{dg}{dk} < 0$$

will be sufficient to prove the theorem.

By differentiating (2.2.M.2/1),

$$\frac{dg}{dk} = \frac{z_{12}(k,g)}{\frac{\partial}{\partial g}[p'(g)g + p(g) - f'(g)g - f(g)] - z_{22}(k,g)} .$$

Hence,

$$-z_{11}(k,g) - z_{12}(k,g)\frac{dg}{dk} =$$

$$-z_{11}(k,g) - \frac{(z_{12}(k,g))^2}{\frac{\partial}{\partial g}[p'(g)g + p(g) - f'(g)g - f(g)] - z_{22}(k,g)}.$$

Substituting again, using (2.2.M.7) - (2.2.M/9), yields

$$-z_{11}(k,g) - z_{12}(k,g)\frac{dg}{dk} = R_{11}(k,g) - \frac{(-R_{12}(k,g))^2}{R_{22}(k,g)}. \quad (2.2.M.2/5)$$

But, by the strict concavity of R(k,g),

$$R_{11}(k,g)R_{22}(k,g) - (R_{12}(k,g))^2 > 0.$$

Dividing through by $R_{22}(k,g)$, which is negative by the strict concavity of R(k,g), yields

$$R_{11}(k,g) - \frac{(-R_{12}(k,g))^2}{R_{22}(k,g)} < 0.$$

So, by (2.2.M.2/5),

$$- z_{11}(k,g) - z_{12}(k,g)\frac{dg}{dk} < 0$$

as claimed.

Q.E.D.

Lemma 2.2.G: If the rate-of-return constraint is an equality with $\lambda > 0$, and if $s \neq r$, then

$$B_1(\pi,s) < -1.$$

Proof: By definition,

$$B(\pi,s) = \big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi,$$

so,

$$B_1(\pi,s) = \left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right]\frac{dg}{d\pi}$$

$$- \left[z_1(k,g) + r\right]\frac{dk}{d\pi} - 1.$$

However, by (2.2.M.2/1), the first expression in brackets must always equal zero, so

$$B_1(\pi,s) = \left[-z_1(k,g) - r\right]\frac{dk}{d\pi} - 1. \qquad (2.2.M.2/6)$$

By (2.2.M.2/2), $-z_1(k,g) - r < 0$ if $s > r$ and $-z_1(k,g) - r > 0$ if $s < r$. By Lemma 2.2.B, $dk/d\pi > 0$ if $s > r$ and $dk/d\pi < 0$ if $s < r$. Hence, the first term in (2.2.M.2/6) is negative and $B_1(\pi,s) < -1$ as claimed.

Q.E.D.

Lemma 2.2.H: If the rate-of-return constraint is an equality with $\lambda > 0$, and if $s \neq r$, then

$$B_2(\pi,s) > 0.$$

Proof: By definition,

$$B(\pi,s) = \left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi,$$

so

$$B_2(\pi,s) = \left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right]\frac{dg}{ds} - \left[z_1(k,g) + r\right]\frac{dk}{ds}.$$

However, by (2.2.M.2/1), the first expression in brackets must always equal zero, so

$$B_2(\pi,s) = \left[-z_1(k,g) - r\right]\frac{dk}{ds}. \qquad (2.2.M.2/7)$$

By (2.2.M.2/2), $-z_1(k,g) - r < 0$ if $s > r$ and $-z_1(k,g) - r > 0$ if $s < r$. By Lemma 2.2.D, $dk/ds < 0$ if $s > r$ and $dk/ds > 0$ if $s < r$. Hence, $B_2(\pi,s) > 0$ as claimed.

Q.E.D.

Lemma 2.2.I: If the rate of return constraint is an equality with $\lambda > 0$ and $s \neq r$, then

$$B_{11}(\pi,s) < 0.$$

Proof: Taking the derivative of (2.2.M.2/6) with respect to π yields

$$B_{11}(\pi,s) = \frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{d\pi}) + [-z_1(k,g) - r]\frac{d^2k}{d\pi^2} .$$

By Lemma 2.2.C, $d^2k/d\pi^2 = 0$, so this expression simplifies to

$$B_{11}(\pi,s) = \frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{d\pi}). \qquad (2.2.M.2/8)$$

Since r is exogenous, Lemma 2.2.F implies that

$$\frac{d}{d\pi}[-z_1(k,g) - r] < 0$$

if $s > r$ and

$$\frac{d}{d\pi}[-z_1(k,g) - r] > 0$$

if $s < r$. Lemma 2.2.B implies that $dk/d\pi > 0$ if $s > r$ and $dk/d\pi < 0$ if $s < r$. Hence $B_{11}(\pi,s) < 0$ as claimed.

Q.E.D.

Lemma 2.2.J: If the rate-of-return constraint is an equality with $\lambda > 0$, then

$$B_{12}(\pi, s) > 0$$

if $s > r$.

Proof: Taking the derivative of (2.2.M.2/7) with respect to s yields

$$B_{12}(\pi, s) = \frac{d}{d\pi}\left[-z_1(k,g) - r\right]\left(\frac{dk}{ds}\right) + \left[-z_1(k,g) - r\right]\frac{d^2k}{d\pi ds}. \tag{2.2.M.2/9}$$

Since r is exogenous, Lemma 2.2.F implies that

$$\frac{d}{d\pi}\left[-z_1(k,g) - r\right] < 0$$

if $s > r$. By Lemma 2.2.D, $dk/ds < 0$ if $s > r$. Hence, the first term of (2.2.M.2/9) is positive if $s > r$. By (2.2.M.2/2), $-z_1(k,g) - r < 0$ if $s > r$; by Lemma 2.2.E, $d^2k/d\pi ds < 0$ if $s \neq r$. Hence, the second term of (2.2.M.2/9) is positive if $s > r$ as well. So $B_{12}(\pi, s) > 0$ if $s > r$, as claimed.

Q.E.D.

Define π^* to be the largest profit the firm could make under the rate-of-return constraint given the capital investment on the budget constraint in π - x space in the absence of a rate-of-return constraint. The following lemma provides an explicit expression for π^*.

Lemma 2.2.K: $\pi^* = (s-r)k^*$.

Proof: In Lemma 2.2.A, I showed that if there were no rate-of-return constraint there would be unique values for g and k on the budget constraint in π - x space, which I called g^* and k^*. Given capital investment k^*, the largest profit the firm could earn under the rate-of-return constraint may be determined from (2.2.M.2/3):

$$\pi^* = (s-r)k^*. \tag{2.2.M.2/10}$$

Q.E.D.

I am now in a position to explore the shape of the budget constraint in π - x space, and how it responds to changes in s. I shall first consider the budget constraint in π - x space over the range $\pi \leq \pi^*$.

<u>Theorem 2.2.M.2.A</u>: For values of $\pi \leq \pi^*$, the budget constraint in π - x space of the rate-of-return regulated firm is the same as that for the unregulated firm.

Proof: In Lemma 2.2.A, I showed that if there were no rate-of-return constraint, there would be unique values for g and k on the budget constraint in π - x space, which I called g^* and k^*. If $\pi < \pi^*$, then g^* and k^* are also feasible solutions for the firm under the rate-of-return constraint. To see this, note that if $\pi \leq \pi^*$ then

$$(s-r)k^* - \pi \geq (s-r)k^* - \pi^*.$$

Since the right side of this inequality is equal to zero by (2.2.M.2/10), g^* and k^* satisfy the rate-of-return constraint (2.2.M.2/3). It follows that g^* and k^* must be the unique optimum values of g and k for the rate-of-return regulated firm with $\pi \leq \pi^*$ as well.

One may then write the budget constraint in π - x space of the rate-of-return regulated firm with $\pi \leq \pi^*$ as:

$$x = \left(p(g^*) - f(g^*)\right)g^* + c - z(k^*,g^*) - rk^* - \pi.$$

But this is the same as the budget constraint in π - x space of the unregulated firm (2.2.M.1/3). So the two are the same.

Q.E.D.

The following Lemma will allow me to apply Lemmas 2.2.G - 2.2.J to the analysis of the budget constraint over the range $\pi > \pi^*$.

Lemma 2.2.L: If $\pi > \pi^*$ then at any solution to (2.2.M.2/1) - (2.2.M.2/3) the rate-of-return constraint (2.2.M.2/3) must be an equality and λ must be greater than zero.

Proof: Assume there were a solution to (2.2.M.2/1) - (2.2.M.2/3) with $\pi > \pi^*$, and $\lambda = 0$. Then (2.2.M.2/1) and (2.2.M.2/2) become equivalent to (2.2.M.1/1) and (2.2.M.1/2) respectively. By Lemma 2.2.A, the latter equations yield $g = g^*$ and $k = k^*$ as unique solutions. Now if $k = k^*$, (2.2.M.2/3) requires that $(s-r)k^* - \pi \geq 0$. But if $\pi > \pi^*$ then

$$(s-r)k^* - \pi < (s-r)k^* - \pi^*.$$

Since the right side of this inequality is equal to zero by (2.2.M.2/10), I have a contradiction, showing that $\lambda > 0$ if $\pi > \pi^*$. If $\lambda > 0$, then (2.2.M.2/3) must be an equality, since $(\partial L/\partial\lambda)\lambda = 0$.

Q.E.D.

The following theorems describe the shape of the budget constraint in π - x space over the range $\pi > \pi^*$, and its response to changes in s.

<u>Theorem 2.2.M.2.B</u>: If s = r, then the budget constraint in π - x space is undefined for values of $\pi > \pi^*$.

Proof: If s = r, then by (2.2.M.2/10), $\pi^* = 0$. But if s = r, (2.2.M.2/3) requires $\pi \leq 0$. So the budget constraint in π - x space is undefined for values of $\pi > \pi^*$.

Q.E.D.

<u>Theorem 2.2.M.2.C</u>: If $\pi > \pi^*$, the slope of the budget constraint in π - x space is less than minus one.

Proof: If $\pi > \pi^*$ on the budget constraint in π - x space, then, by Lemma 2.2.L, the rate-of-return constraint is an equality with $\lambda > 0$, and, by Theorem 2.2.M.2.B, $s \neq r$. So by Lemma 2.2.G,

$$\frac{dx}{d\pi} = B_1(\pi,s) < -1,$$

indicating that the slope of the budget constraint in π - x space is less than minus one.

Q.E.D.

Theorem 2.2.M.2.D: If $\pi > \pi^*$, the slope of the budget constraint in π - x space becomes increasingly negative as π increases.

Proof: If $\pi > \pi^*$ on the budget constraint in π - x space, then by Lemma 2.2.L, the rate-of-return constraint is an equality with $\lambda > 0$, and, by Theorem 2.2.M.2.B, $s \neq r$. By Theorem 2.2.M.2.C, if $\pi > \pi^*$ the slope of the budget constraint in π - x space is negative. Since Lemma 2.2.I requires that

$$\frac{d^2x}{d\pi^2} = B_{11}(\pi,s) < 0,$$

this slope must become increasingly negative as π increases.

Q.E.D.

Theorem 2.2.M.2.E: If $\pi > \pi^*$, then an increase in s causes the budget constraint in π - x space to shift upward.

Proof: If $\pi > \pi^*$ on the budget constraint in π - x space, then by Lemma 2.2.L, the rate-of-return constraint is an equality with $\lambda > 0$, and, by Theorem 2.2.M.2.B, $s \neq r$. So by Lemma 2.2.H,

$$\frac{dx}{ds} = B_2(\pi,s) > 0,$$

indicating that an increase in s causes the budget constraint in π - x space to shift upward.

Q.E.D.

Theorem 2.2.M.2.F: If $\pi > \pi^*$ and $s > r$, then an increase in s causes the budget constraint in π - x space to become less negatively sloped.

Proof: If $\pi > \pi^*$ on the budget constraint in π - x space, then by Lemma 2.2.L, the rate of return constraint is an equality with $\lambda > 0$. By Theorem 2.2.M.2.C, if $\pi > \pi^*$, the slope of the budget constraint in π - x space is negative. Since Lemma 2.2.J requires that if $s > r$

$$\frac{d^2x}{d\pi ds} = B_{12}(\pi,s) > 0,$$

this slope must become less negative as s increases.

Q.E.D.

Theorem 2.2.M.2.G: An increase in s causes the value of π^* to increase.

Proof: Totally differentiating (2.2.M.2/10) yields

$$\frac{d\pi^*}{ds} = k^* > 0,$$

indicating that an increase in s causes the value of π^* to increase.

Q.E.D.

For the rate-of-return regulated firm, I also wish to show that the solution chosen by the managers lies at the point of tangency between the budget constraint in π - x space and an isoutility curve. To demonstrate this, I must again derive the first-order conditions for the solution to the managers' problem.

The managers seek to maximize utility

$$U(\pi,x),$$

subject to the budget constraint

$$(p(g) - f(g))g + c - z(k,g) - rk - \pi - x \geq 0,$$

and the rate of return constraint

$$(s-r)k - \pi \geq 0.$$

Maximization is over π, x, g, and k, where x, g, and k, but not π, must be non-negative. The Lagrangian will be

$$L = U(\pi,x) + \lambda_1[(p(g) - f(g))g + c - z(k,g) - rk - \pi - x]$$
$$+ \lambda_2[(s-r)k - \pi].$$

The first order conditions require that:

$$\frac{\partial L}{\partial \pi} = U_1(\pi,x) - \lambda_1 - \lambda_2 = 0; \qquad (2.2.M.2/11)$$

$$\frac{\partial L}{\partial x} = U_2(\pi,x) - \lambda_1 \leq 0; \qquad (2.2.M.2/12)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial g} = \lambda_1[p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)] \leq 0; \qquad (2.2.M.2/13)$$

$$\frac{\partial L}{\partial g} g = 0; \qquad g \geq 0;$$

$$\frac{\partial L}{\partial k} = \lambda_1[-z_1(k,g) - r] + \lambda_2[s-r] \leq 0; \qquad (2.2.M.2/14)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda_1} = \big(p(g)-f(g)\big)g + c - z(k,g) - rk - \pi - x \geq 0; \quad (2.2.M.2/15)$$

$$\frac{\partial L}{\partial \lambda_1}\lambda_1 = 0; \qquad \lambda_1 \geq 0$$

$$\frac{\partial L}{\partial \lambda_2} = (s-r)k - \pi \geq 0; \quad (2.2.M.2/16)$$

$$\frac{\partial L}{\partial \lambda_2}\lambda_2 = 0; \qquad \lambda_2 \geq 0.$$

I shall, again, assume that g, k, and x are all greater than zero at the solution, and that π and x have positive marginal utilities, that is $U_1(\pi,x) > 0$ and $U_2(\pi,x) > 0$. These assumptions are sufficient to insure that conditions (2.2.M.2/12) - (2.2.M.2/15) are equalities with $\lambda_1 > 0$. I now prove the desired theorem.

Theorem 2.2.M.2.H: If $s \neq r$, the solution chosen by the managers of the rate-of-return regulated firm lies at the point of tangency between the budget constraint in π - x space and an isoutility curve in π - x space. If s = r, the solution either lies on the budget constraint in π - x space at $\pi = 0$ or lies at the point of tangency.

Proof: I first show that the solution chosen by the managers must lie on the budget constraint in π - x space. Suppose the contrary. Since the budget constraint in π - x space is defined to be the largest value of x which the managers could achieve at each value of π, this would imply that the chosen value of x was less than the managers could have achieved at the chosen value of π. Since I assume x to have positive marginal utility, the managers could improve on this solution by moving to larger value of x. But this is a contradiction, since the initial solution is optimal.

To show tangency, it remains to demonstrate that the slope of the isoutility curve equals the slope of the budget constraint in π - x space at the solution. An isoutility curve is defined by $U(\pi,x)$ = constant. Total differentiation yields a slope

$$\frac{dx}{d\pi} = -\frac{U_1(\pi,x)}{U_2(\pi,x)} .$$

Hence, one may obtain an explicit expression for the slope of the isoutility curve at the solution by solving (2.2.M.2/11) - (2.2.M.2/16) for $-U_1(\pi,x)/U_2(\pi,x)$.

The budget constraint is defined to be

$$x = \big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi.$$

Hence, the slope of the budget constraint in π - x space will be

$$\frac{dx}{d\pi} = [p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)]\frac{dg}{d\pi} - [z_1(k,g)+r]\frac{dk}{d\pi} - 1.$$

Note, however, that (2.2.M.2/1) requires that the first term in brackets equal zero at points on the budget constraint in π - x space. So the slope simplifies to

$$\frac{dx}{d\pi} = -[z_1(k,g) + r]\frac{dk}{d\pi} - 1, \qquad (2.2.M.2/17)$$

here $dk/d\pi$ is determined by conditions (2.2.M.2/1) - (2.2.M.2/3).

I shall consider two cases, depending upon the value of λ_2 at the solution.

i) Assume that the solution to (2.2.M.2/11) - (2.2.M.2/16) is such that $\lambda_2 = 0$. Then (2.2.M.2/11) and (2.2.M.2/12) may be solved to eliminate λ_1, yielding

$$-\frac{U_1(\pi,x)}{U_2(\pi,x)} = -1.$$

This indicates that the slope of an isoutility curve must equal minus one at this solution. Now if $\lambda_2 = 0$ in (2.2.M.2/11) - (2.2.M.2/16), then $\lambda = 0$ at this same solution in (2.2.M.2/1) - (2.2.M.2/3). But if $\lambda = 0$, then (2.2.M.2/2) requires that

$$-[z_1(k,g) + r] = 0.$$

So (2.2.M.2/17) simplifies to

$$\frac{dx}{d\pi} = -1,$$

indicating that the slope of the budget constraint in $\pi - x$ space is minus one. So the two slopes are the same if $\lambda_2 = 0$.

ii) Assume the solution to (2.2.M.2/11) - (2.2.M.2/16) is such that $\lambda_2 > 0$. If $s \neq r$, one may solve (2.2.M.2/11), (2.2.M.2/12), and (2.2.M.2/14) to eliminate λ_1 and λ_2, yielding

$$\frac{-U_1(\pi,x)}{U_2(\pi,x)} = \frac{-[z_1(k,g) + r]}{s - r} - 1$$

as the slope of the isoutility curve at the solution.

Now if $\lambda_2 > 0$ in (2.2.M.2/11) - (2.2.M.2/16), then $\lambda > 0$ at this same solution in (2.2.M.2/1) - (2.2.M.2/3). But if $\lambda > 0$, then the rate-of-return constraint (2.2.M.2/3) must be an equality, so, by Lemma 2.2.B, if $s \neq r$, $dk/d\pi = 1/s-r$. Substituting in (2.2.M.2/17) yields

$$\frac{dx}{d\pi} = \frac{-[z_1(k,g) + r]}{s - r} - 1$$

as the slope of the budget constraint in π - x space at the solution. Hence, if $s \neq r$, the two slopes are the same if $\lambda_2 > 0$. If $s = r$, (2.2.M.2/16) requires $\pi = 0$.

So I have shown that in both cases, if $s \neq r$, (2.2.M.2/11) - (2.2.M.2/16) require that the slope of an isoutility curve equal the slope of the budget constraint in π - x space. Hence, if $s \neq r$, the solution chosen by the managers must lie at a point of tangency between the budget constraint in π - x space and an isoutility curve. Similarly, even if $s = r$ in case i, the solution is at a point of tangency. However, if $s = r$ in case ii, then $\pi = 0$.

Q.E.D.

2.3. Effect of Changing the Allowed Rate of Return

I am now ready to use this model to explore a principal question about rate-of-return regulation: what happens as the firm's allowed rate of return is reduced? In this section, I show graphically how, when the allowed rate of return exceeds the cost of capital, a reduction in the allowed rate of return tends to cause a reduction in profits and an increase in institutional costs. I then examine the effect of small reductions in the allowed rate-of-return, which may be broken down into a substitution effect and an income effect. A mathematical subsection follows, which repeats the analysis in a more formal fashion.

There are two published papers which have explored the effects of a reduction in the allowed rate of return on the profits and institutional costs of a firm with utility-maximizing managers. The two papers reach different conclusions regarding the impacts of a reduction in the allowed rate of return on institutional costs. Unfortunately, both papers are very sketchy, providing neither intuitive insight into the logic behind their results, nor considering the case of allowed rates of return at or below the cost of capital.

Crew and Kleindorfer[17] modeled a firm under rate-of-return regulation whose management utility is an increasing function of profits and staff. The firm was assumed to have a Cobb-Douglas production function of labor and capital, an exponential consumer price function of output and advertising, a linear management utility function of profits and staff, and a minimum level of staff dependent on output and advertising (which was, in fact, always binding). It should be noted that staff and

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[17]Michael A. Crew and Paul R. Kleindorfer, "Managerial Discretion and Public Utility Regulation", Southern Economic Journal, 45 (January, 1979): 696-709.

labor are two completely distinct inputs. They showed, using numerical simulations with two different assumed demand elasticities, that profits declined as the allowed rate of return was reduced, while output, advertising, and staff increased.

Crew and Kleindorfer's work has been somewhat generalized by Arzac and Edwards.[18] They assumed a rate-of-return regulated firm whose management utility is an increasing function of both profit and unproductive management perks. The firm was assumed to have a general production function of capital and labor. Using comparative statics techniques, they found that as the allowed rate of return was lowered by a small amount, profits declined, however, the direction of the change in perks could not, in general, be predicted.

A similar dichotomy of results appears in my model, depending upon whether one is considering the overall tendencies of a reduction in the allowed rate of return, or the effects of a small reduction in the allowed rate of return. The overall tendencies may be demonstrated by assuming the rate-of-return constraint is always binding, and plotting a set of tangency points as in Figure 14. It can be seen that as the

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[18] Enrique R. Arzac and Franklin R. Edwards, "Efficiency in Regulated and Unregulated Firms; An Iconoclastic View of the Averch-Johnson Thesis" in Problems in Public Utility Regulation, ed. Michael A. Crew (Lexington, MA: D.C. Heath and Company, Lexington Books, 1979) pp. 41-54.

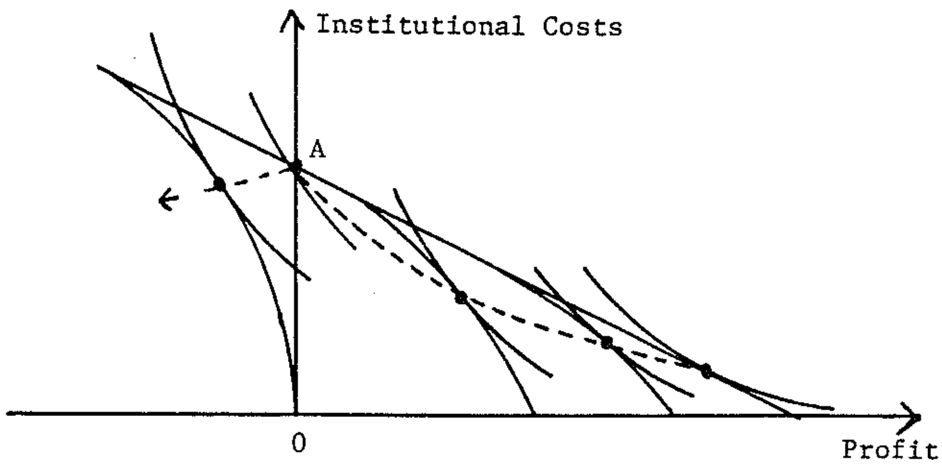

Figure 14
Effect of a Reduction in the Allowed Rate of Return
on the Utility-Maximizing Solution

allowed rate of return is reduced profits tend to decline, and institutional costs tend to increase, as long as the allowed rate of return exceeds the cost of capital. At the point where the allowed rate of return equals the cost of capital, institutional costs equal the maximum profits the firm could have earned in an untaxed unregulated environment, while profits equal zero.

To understand the impact of an small reduction in the allowed rate of return on profits and institutional costs, one must consider separately the two effects that a reduction in the allowed rate of return has on each of these variables. There is, first, a substitution effect, due to the changing slope of the budget constraint altering the profit/ institutional costs tradeoff. Second, there is an income effect, due to the leftward shifting of the budget constraint lowering the managers achievable level of utility.

Figure 15 illustrates the two effects. Budget constraint A is initially tangent to isoutility curve U_1 at point e. If the allowed rate of return is lowered, the slope of the budget constraint becomes steeper and shifts to the left. If only the slope of the budget constraint were changed, without changing the isoutility curve to which it was tangent, one would get a new budget constraint A', and a new point

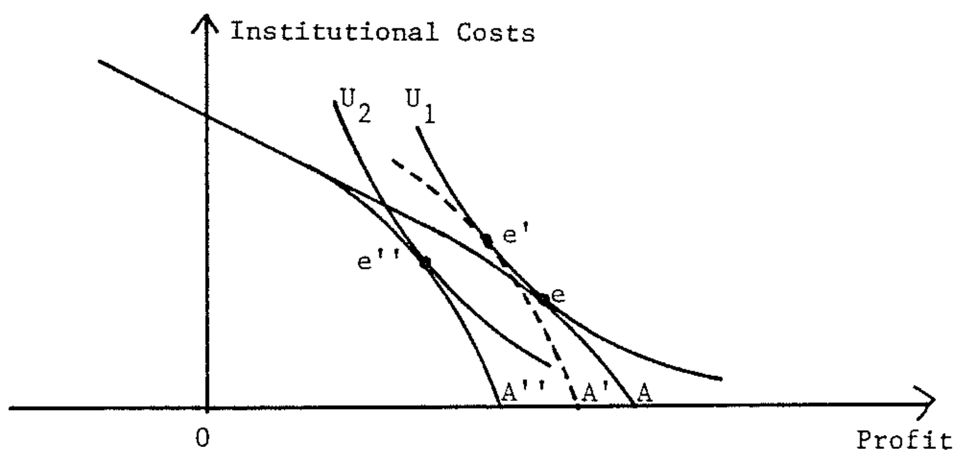

Figure 15
Substitution and Income Effects of a Reduction in the Allowed Rate of Return Given an Allowed Rate of Return Greater than the Cost of Capital.

of tangency e'. The difference between e and e' is the substitution effect. Now shifting the isoutility curve to the left, with no change in slope, to its actual new position further shifts the point of tangency with the isoutility curve to its new equilibrium e". The difference between e' and e" is the income effect.

It can be seen from Figure 15 that the substitution effect causes institutional costs to increase and profit to decrease, while the income effect causes both institutional costs and profit to decrease. The two effects work in the same direction for profit, implying that profit must decline as the rate of return is reduced. However, they work in opposite directions for institutional costs, implying that it cannot be said what will happen to institutional costs after a small change in the allowed rate of return.

Figure 15 is drawn assuming the allowed rate of return is greater than the cost of capital. If the allowed rate of return is less than the cost of capital, the direction of the substitution effect cannot be determined, since one does not know whether the slope of the budget constraint becomes more or less steep as the allowed rate of return is reduced. This substitution effect is shown by the difference between e and e' in Figure 16. In the Figure, the substitution effect causes institutional costs to decrease and profit to increase as the allowed

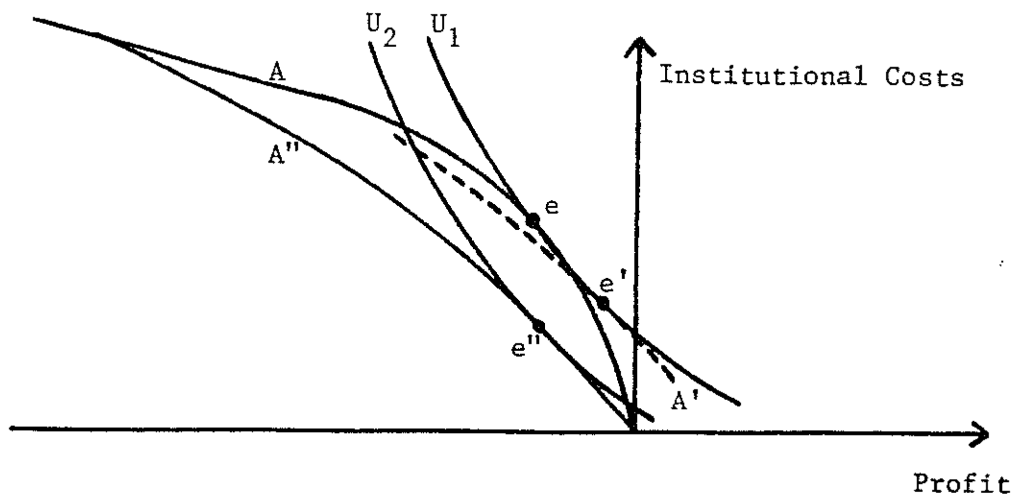

Figure 16
Substitution and Income Effects of a Reduction in the Allowed Rate of Return Given an Allowed Rate of Return Less than The Cost of Capital

rate of return is reduced, but I could have drawn it so the substitution effect was the other way around. The income effect, shown by the difference between e' and e", continues to cause both profit and institutional costs to decrease as the allowed rate of return is reduced.

Table 1 summarizes these results for a small reduction in the allowed rate of return. The income effects are based on the assumption that the utility functions are such that profit and institutional costs are "normal goods," the amount of which would increase if the firm's budget constraint were to shift rightward.

TABLE 1

EFFECTS OF REDUCTION IN THE ALLOWED RATE OF RETURN

Variable	Relationship of Allowed Rate of Return to Cost of Capital	Substitution Effect	Income Effect	Total Effect
Profit	greater than	-	-	-
	less than	?	-	?
Institutional Costs	greater than	+	-	?
	less than	?	-	?

How can the indeterminant effect of a small reduction in the allowed rate of return on institutional costs be reconciled with the overall tendency for institutional costs to increase as the allowed rate of return is reduced toward the cost of capital? To understand the latter tendency, one should look at the behavior of the firm when the allowed rate of return equals the cost of capital, with the rate-of-return constraint assumed binding. Since the firm is prohibited from earning a positive profit in this situation, the managers would become institutional cost maximizers. They will choose to set prices and buy inputs as would an unregulated monopolist, with the benefits being taken as institutional costs instead of profit. This corresponds to point A in Figure 14, where the budget constraint has become vertical at the institutional costs axis. Hence, for allowed rates of return greater than or equal to the cost of capital, institutional costs would be maximized at an allowed rate of return equal to the cost of capital.

The path of institutional costs as the allowed rate of return is lowered may, however, vary greatly with the shape of the managers' isoutility curve. For example, if the isoutility curves are nearly vertical, indicating a strong management preference for profits over institutional costs, institutional costs will remain low until the allowed rate of return almost reaches the cost of capital, at which point they quickly move to the institutional costs-maximizing solution, as shown in Figure 17. In the limit, if the manager's isoutility curves were vertical, indicating strict profit maximization, their behavior would be like the Averch-Johnson model described in Chapter 1. Institutional costs would be zero until the allowed rate of return equalled the cost of capital, at which point the results become indeterminant. It is certainly conceivable that for some utility functions, institutional costs could drop with a reduction in the allowed rate of return over some range, despite the generally increasing tendency.

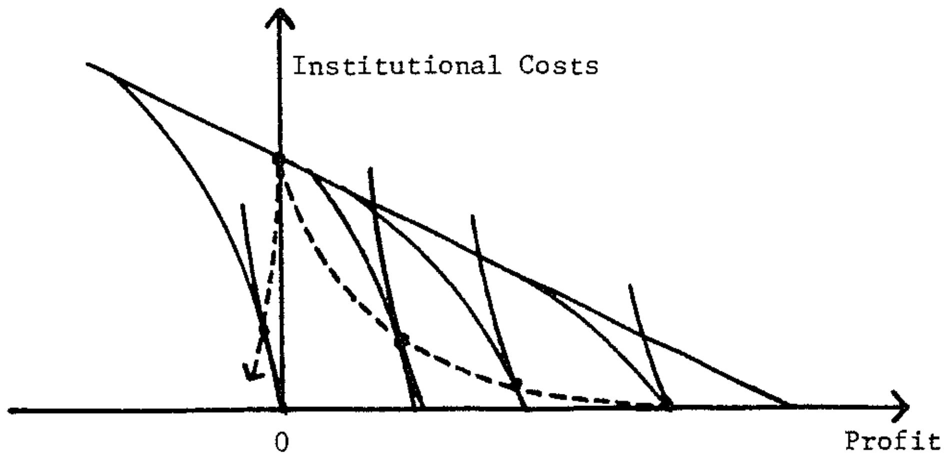

Figure 17
Effect of a Reduction in the Allowed Rate of Return
Given Strong Manager Preference for Profits

An interesting special case of this model arises if one assumes a utility function of the form $\pi + U(x)$, where π is profit and $U(x)$ is some increasing function of institutional costs x. In this case, the substitution effect of a reduction in the allowed rate of return can be shown to outweigh the income effect, meaning that institutional costs increase even for small reductions in the allowed rate of return. Crew and Kleindorfer used a utility function of this form, explaining their results.

2.3.M Mathematical Formulation

The main thrust of this subsection is to formally demonstrate the comparative statics results shown in Table 1. I will also consider the special case of a utility function of the form $U(\pi,x) = \pi + U(x)$. Throughout this subsection, I shall use the same model, assumptions, and notation as were used in Section 2.2.M. I delay consideration of the case of s = r until section 2.5.M.

The managers' problem may be restated in terms of π and x alone as maximize utility

$$U(\pi,x),$$

subject to the budget constraint

$$B(\pi,s) - x \leq 0.$$

The Lagrangian for this problem will be:

$$L = U(\pi,x) + \lambda[B(\pi,s) - x] .$$

The first-order conditions require that:

$$\frac{dL}{d\pi} = U_1(\pi,x) + \lambda B_1(\pi,s) = 0; \qquad (2.3.M/1)$$

$$\frac{dL}{dx} = U_2(\pi,x) - \lambda \leq 0; \qquad (2.3.M/2)$$

$$\frac{dL}{dx}\, x = 0; \qquad x \geq 0;$$

$$\frac{dL}{d\lambda} = B(\pi,s) - x \leq 0; \qquad (2.3.M/3)$$

$$\frac{dL}{d\lambda}\, \lambda = 0; \qquad \lambda \geq 0.$$

If one assumes $U_2(\pi,x) > 0$ at the solution, that is the managers are not satiated in institutional costs, then λ must be greater than zero. If one also assumes $x > 0$ at the solution, then all three conditions above become equalities.

The second-order conditions require that the determinant

$$D = \begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix}$$

be greater than zero. Note that I have dropped the parameters of the partials of U( ) and B( ) for compactness of notation.

To find the effect of a small change in s, ds, conditions (2.3.M/1) - (2.3.M/3) may be totally differentiated. This yields a system of simultaneous equations in $d\pi$, dx, $d\lambda$ and ds. Dropping the parameters of U( ) and B( ), the system may be expressed in matrix notation as

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{21} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{12} \\ 0 \\ -B_2 \end{bmatrix} ds. \qquad (2.3.M/4)$$

Since I am interested in the behavior of the firm subject to a binding rate-of-return constraint, I shall assume that the rate-of-return constraint (2.2.M.2/3) must be an equality, and that the λ in conditions (2.2.M.2/1) - (2.2.M.2/3) must be greater than zero at the solution chosen by the managers. These assumptions allow me to apply Lemmas 2.2.B - 2.2.J. I am now in a position to demonstrate the comparative statics results.

Theorem 2.3.M.A: The substitution effect of an increase in the allowed rate of return on profit $(d\pi/ds)_s$ is positive if $s > r$.

Proof: To derive the substitution effect, one wants to add some compensation, call it y, to the budget constraint (2.3.M/3) so as to hold the solution to the same isoutility curve when s changes. If this were done, (2.3.M/4) could be rewritten as

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{21} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{12} \\ 0 \\ -B_2 - \frac{dy}{ds} \end{bmatrix} ds. \qquad (2.3.M/5)$$

If one knew dy/ds, this system could be solved for the substitution effect $(d\pi/ds)_s$. But it is possible to solve for dy/ds. For by the third equation of this system

$$B_1 d\pi - dx = \left(-B_2 - \frac{dy}{ds}\right)ds.$$

Now by (2.3.M/1) and (2.3.M/2)

$$B_1 = -\frac{U_1}{U_2},$$

so, substituting back,

$$-\frac{U_1}{U_2}\,d\pi - dx = \left(-B_2 - \frac{dy}{ds}\right)ds. \qquad (2.3.M/6)$$

If the solution is to be held to the same isoutility curve $U(\pi,x)$ = constant, total differentiation may be used to show that

$$\frac{dx}{d\pi} = -\frac{U_1}{U_2},$$

which implies that the left side of (2.3.M/6) is equal to zero. Thus, it must be that

$$\frac{dy}{ds} = B_2$$

if the solution is to be held to a given isoutility curve.

Substituting into (2.3.M/5) yields the system

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{21} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{12} \\ 0 \\ 0 \end{bmatrix} ds, \tag{2.3.M/7}$$

which may be solved for the substitution effect $(d\pi/ds)_s$. Applying Cramer's rule[19],

$$\left(\frac{d\pi}{ds}\right)_s = \frac{\begin{vmatrix} -\lambda B_{12} & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ 0 & -1 & 0 \end{vmatrix}}{D},$$

or

$$\left(\frac{d\pi}{ds}\right)_s = \frac{\lambda B_{12}}{D}. \tag{2.3.M/8}$$

Since I assume $\lambda = U_2(\pi,x) > 0$, while, by Lemma 2.2.J, $B_{12} > 0$ if $s > r$, I have shown that $(d\pi/ds)_s > 0$ if $s > r$.

Q.E.D.

Theorem 2.3.M.B: The substitution effect of an increase in the allowed rate of return on institutional cost $(dx/ds)_s$ is negative if $s > r$.

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[19] See James M. Henderson and Richard E. Quandt, Macroeconomic Theory; A Mathematical Approach, 2nd. ed. (New York: McGraw-Hill, 1971), pp. 386-388.

Proof: One can solve system (2.3.M/7) for the substitution effect $(dx/ds)_s$. Applying Cramer's rule,

$$\left(\frac{dx}{ds}\right)_s = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{12} & B_1 \\ U_{21} & 0 & -1 \\ B_1 & 0 & 0 \end{vmatrix}}{D},$$

or

$$\left(\frac{dx}{ds}\right)_s = \frac{\lambda B_{12} B_1}{D}. \tag{2.3.M/9}$$

But, I can substitute from (2.3.M/8) to write

$$\left(\frac{dx}{ds}\right)_s = \left(\frac{d\pi}{ds}\right)_s B_1.$$

By Lemma 2.2.G, $B_1 < 0$, so $(dx/ds)_s$ always has the opposite sign of $(d\pi/ds)_s$. Since by Theorem 2.3.M.A, $(d\pi/ds)_s > 0$ if $s > r$, I have shown that $(dx/ds)_s < 0$ if $s > r$.

Q.E.D.

Define profit to be a "normal good" if its level would increase with a relaxation of the budget constraint. One could relax the budget constraint by adding a positive y to the left side of (2.3.M/3). Hence, if a small increase in y produces an increase in profit π, that is if $d\pi/dy > 0$, then profit is a normal good.

Theorem 2.3.M.C: If profit is a normal good, then the income effect $(d\pi/ds)_y$ of an increase in the allowed rate of return on profit is positive.

Proof: Since profit is a normal good, $d\pi/dy > 0$. To find an expression for $d\pi/dy$, I totally differentiate (2.3.M/1) - (2.3.M/3) to obtain

$$\begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{21} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix} \begin{vmatrix} d\pi \\ dx \\ d\lambda \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -1 \end{vmatrix} dy. \tag{2.3.M/10}$$

Applying Cramer's rule,

$$\frac{d\pi}{dy} = \frac{\begin{vmatrix} 0 & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ -1 & -1 & 0 \end{vmatrix}}{D},$$

or:

$$\frac{d\pi}{dy} = \frac{U_{12} + B_1 U_{22}}{D}. \tag{2.3.M/11}$$

To find the income effect $(d\pi/ds)_y$, one first finds the total effect $d\pi/ds$, and subtracts the substitution effect $(d\pi/ds)_s$. The total effect $d\pi/ds$ may be found by applying Cramer's rule to system (2.3.M/4)

$$\frac{d\pi}{ds} = \frac{\begin{vmatrix} -\lambda B_{12} & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ -B_2 & -1 & 0 \end{vmatrix}}{D}$$

$$= \frac{\lambda B_{12} + B_2 \left[U_{12} + B_1 U_{22}\right]}{D}.$$

Subtracting the substitution effect (2.3.M/8) yields the income effect

$$\left(\frac{d\pi}{ds}\right)_y = \frac{B_2\left[U_{12} + B_1 U_{22}\right]}{D},$$

or, substituting (2.3.M/11),

$$\left(\frac{d\pi}{ds}\right)_y = \left(\frac{d\pi}{dy}\right)B_2.$$

Since $B_2 > 0$ by Lemma 2.2.H and $d\pi/dy > 0$ if profit is a normal good $(d\pi/ds)_y$ must be positive.

Q.E.D.

As in the case of profit, define institutional costs to be a "normal good" if their level would increase with a relaxation of the budget constraint.

<u>Theorem 2.3.M.D</u>: If institutional costs are a normal good, then the income effect $(dx/ds)_y$ of an increase in the allowed rate of return on institutional costs is positive.

Proof: Since institutional costs are a normal good, the dx/dy obtained by solving (2.3.M/10) is positive. Applying Cramer's rule,

$$\frac{dx}{dy} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & 0 & B_1 \\ U_{21} & 0 & -1 \\ B_1 & -1 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{dy} = \frac{-(U_{11} + \lambda B_{11}) - U_{12}B_1}{D}. \qquad (2.3.M/12)$$

To find the income effect $(dx/ds)_y$, one finds the total effect dx/ds, and subtracts the substitution effect $(d\pi/ds)_s$. The total effect dx/ds may be found by applying Cramer's rule to system (2.3.M/4),

$$\frac{dx}{ds} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{12} & B_1 \\ U_{21} & 0 & -1 \\ B_1 & -B_3 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{ds} = \frac{\lambda B_{12}B_1 + B_2[-(U_{11} + \lambda B_{11}) - U_{12}B_1]}{D}. \qquad (2.3.M/13)$$

Subtracting the substitution effect (2.3.M/9) yields the income effect

$$\left(\frac{dx}{ds}\right)_y = \frac{B_2[-(U_{11} + \lambda B_{11}) - U_{12}B_1 \;]}{D}$$

or, substituting (2.3.M/12),

$$\left(\frac{dx}{ds}\right)_y = \left(\frac{dx}{dy}\right)B_2.$$

Since $B_2 > 0$ by Lemma 2.2.H and $dx/dy > 0$ if institutional costs are a normal good, $(dx/ds)_y$ must be positive.

Q.E.D.

Thus, I have demonstrated the results shown in Table 1. One special utility function will provide stronger results, as stated in the following theorem.

<u>Theorem 2.3.M.E</u>: If the manager's utility function is of the form $U(\pi,x) = \pi + U(x)$, then the overall effect of an increase in the allowed rate of return on institutional costs is negative if $s > r$.

Proof: If the utility function has the form $U(\pi,x) = \pi + U(x)$, then $U_{11} = U_{12} = 0$, and the overall effect dx/ds given in (2.3.M/13) simplifies to

$$\frac{dx}{ds} = \frac{\lambda[B_{12}B_1 - B_2B_{11}]}{D}.$$

Substituting for B_1, B_2, B_{11}, and B_{12} using (2.2.M.2/6) - (2.2.M.2/9) yields

$$\lambda\{\frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{ds}) + [-z_1(k,g) - r]\frac{d^2k}{d\pi ds}\}$$

$$X\{[-z_1(k,g) - r]\frac{dk}{d\pi} - 1\}$$

$$\frac{dx}{ds} = \frac{- \lambda\{[-z_1(k,g) - r]\frac{dk}{ds}\} \{\frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{d\pi})\}}{D}$$

$$= \frac{-\lambda\frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{ds}) + \lambda[-z_1(k,g) - r](\frac{d^2k}{d\pi ds})\{[-z_1(k,g) - r]\frac{dk}{d\pi} - 1\}}{D}.$$

Now, substituting again, using (2.2.M.2/6), yields

$$\frac{dx}{ds} = \frac{-\lambda\frac{d}{d\pi}[-z_1(k,g) - r](\frac{dk}{ds}) + \lambda[-z_1(k,g) - r](\frac{d^2k}{d\pi ds})B_1}{D}.$$

Since r is exogenous,

$$\frac{d}{d\pi}[-z_1(k,g) - r] < 0$$

if $s > r$ by Lemma 2.2.F; $dk/ds < 0$ if $s > r$ by Lemma 2.2.D; and $\lambda = U_2(\pi,x) > 0$ by assumption; the left term in the numerator is negative. Since $-z_1(k,g) - r < 0$ if $s > r$ by (2.2.M.2/2), $d_2k/d\pi ds < 0$ by Lemma 2.2.E, and $B_1 < 0$ by Lemma 2.2.G, the right term in the numerator is negative as well. Hence, the overall effect dx/ds is negative.

Q.E.D.

2.4 Dynamic Considerations

The model presented in the previous sections, like most models of rate-of-return regulation, abstracts from reality by assuming a static world, with a single cost of capital and a single allowed rate of return, which never vary. Actual capital comes in a variety of forms, with the cost of each fluctuating constantly. Regulators respond to changes in the cost of capital by periodically adjusting the firm's allowed rate of return. Investment decisions are usually made incrementally, given the anticipated environment the firm will face over the life of the investment.

A fully dynamic model of regulated firm behavior would be extraordinarily complex, and will not be attempted here. There are, however, three points to be made in this section; an understanding of these points will help the reader to relate my model and its results to the dynamic reality. In the first subsection, I point out that the allowed rates of return and costs of capital used by managers and investors in their decisionmaking are really prospective values over the life of a proposed investment. In the second subsection, I argue that firm operation with an allowed rate of return below the cost of capital is not inconsistent with the operation of capital markets. In the third subsection, I point out the important distinction between marginal and average rates of return, which is easily missed in applied discussions of rate-of-return regulation.

2.4.1 Prospective vs. Current Values

As long as the firm is at least partly equity financed, both the cost of capital[20] and allowed rate of return on the firm's investment will fluctuate over time. Therefore, in their decision-making on new investments, firm managers need to consider the prospective values of both variables over the life of the investment. The current allowed rate of return and the current cost of capital should be but one data point that managers take into account in forming their expectations about the future. The managers will undoubtedly want to consider the regulatory commission's performance over its entire history. They will probably also try to assess the kind of pressures the regulators may be operating under in the foreseeable future. Since so many factors go into the formation of these long-term expectations, they will probably change slowly over time.

Investors, too, will base their decisions on prospective, rather than current, allowed rates of return and costs of capital. Managers and investors need not hold the same expectations, however, the two will probably tend to track each other closely, since each will have access to much the same information.

There are many ways one could alter the models in this study to make them more "realistic" in their handling of expectations. One could think of managers or investors as having probability distributions on

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[20]For a discussion of the concept of the cost of capital and how it can be empirically estimated, see Stewart C. Myers, "The Application of Finance Theory to Public Utility Rate Cases", Bell Journal of Economics and Management Science, 3 (Spring, 1972): 58-97.

the allowed rate of return and cost of capital in each period, or one could have explicit models of the formation of expectations. However, I believe one can relate the models to the real world without losing too many insights by simply thinking of the allowed rates of return and costs of capital in the model as single prospective values.

2.4.2. Financing New Investments

In Section 2.2., I showed how a (prospective) allowed rate of return below the (prospective) cost of capital would lead managers to maintain a lower level of investment than would the unregulated firm, but, in many cases, to still maintain a positive level of investment. One may be tempted to argue that this is inconsistent with the workings of capital markets, which will not provide funds for investments with a negative net present value. In this subsection, I will indicate how such investments are indeed possible, given the dynamic nature of the investment process.

It is true that a new firm with an allowed rate of return below the cost of capital would be unable to raise funds in the capital market. Suppose, however, that investors initially anticipated an allowed rate of return above the cost of capital, thereby permitting the firm to raise funds to start-up. If, at some later date, the regulators defy the investors expectations, and lower the firm's allowed rate of return or, equivalently, fail to increase it when the cost of capital increases, there would be no reason the firm could not continue operations, even with a prospective allowed rate of return below the prospective cost of capital. Given the low salvage value of much gas firm capital, continued operations would generally also be in the best interests of the firm's stockholders.

More interesting than simply the firm's continued operation at an allowed rate of return below the cost of capital is the continued ability of the managers to raise new capital, should they choose to do so, simply by selling additional equity. Since this new equity must be priced such that the net present value of the anticipated returns is zero at the cost of capital, the present value of any anticipated real economic profit or loss generated by the new investment must accrue to the value of the old equity. What will happen is that when the capital markets first anticipate the new investment, the market value of the firm will increase or decrease by the anticipated present value of the resulting economic profit or loss. This will maintain the firm's stock price such that the net present value of the anticipated returns is zero. The new stock may then be sold at this price as well.

The managers could, therefore, raise new capital when the firm has an allowed rate of return below the cost of capital. It is certainly not in the interests of the firm's stockholders for them to do so, since increased investment means increased real economic losses, resulting in a decline in the firm's market value. But this whole model is based on the argument that the managers have some power to take actions not in the owners' interests. So there is nothing here which is inconsistent, either internally or with the theory of capital markets.

2.4.3 Average vs. Marginal Values

Like most other prices, allowed rates of return and costs of capital can have marginal values different from their average values. As elsewhere in economics, the proper comparison is usually between marginal values. For example, to determine how much a given investment increases the firm's profit ceiling, one would compare the *marginal* allowed rate of return on the investment to the *marginal* cost of capital.

One example of how a difference between average and marginal allowed rates of return could develop is if the regulators threaten to punish the firm with a lower (average) allowed rate of return if it fails to make some investment, or reward it with a higher allowed rate of return if it does. In this case, the marginal allowed rate of return on that particular investment would certainly exceed the firm's average allowed rate of return.

The most common discrepancies between average and marginal values are, however, due to the existence of debt capital. Debt has the unique property that its cost to the firm is fixed at the time the securities are issued. Hence, at any given time, the firm's average cost of capital including the cost of outstanding debt, will probably differ from the marginal cost of new capital. Furthermore, the issuance of new capital changes the firm's average cost of capital. Since regulators generally set the firm's average allowed rate of return based on the firm's average cost of capital,[21] there is a linkage between the issuance of new capital and the firm's allowed rate of return. This implies that the average allowed rate of return also differs from the marginal allowed rate of return.

One must, therefore, be wary of the kind of argument I made in Section 1.5, where I quoted figures to show that gas utility rates of return in 1981 were below the cost of capital. The gas utility rate-of-return figure quoted is an average, while the cost-of-capital figure is a marginal. Since some gas utilities probably could have persuaded, or did persuade, regulators to grant them higher allowed rates of return to cover the high cost of any newly issued capital, marginal allowed rates of return would have been higher than the average I quoted. Marginal allowed rates of return, like most other marginals used in economics, are not always easily estimated in practice, but can be estimated given certain assumptions about how the regulators will adjust the (average) allowed rate of return in response to a new investment.

The extension of my models to the case where marginal allowed rates of return and cost of capital could differ from average appears to be straightforward. One could simply reformulate the model with average allowed rate of return and cost of capital as functions of capital investment, rather than constants. This would, however, cause a great

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[21]for a discussion of this point, see Gordon R. Corey, "The Averch-Johnson Proposition; A Critical Analysis", Bell Journal of Economics and Management Science, 2 (Spring, 1971): 358-373.

increase in analytical complexity without a corresponding increase in significant insights. For most purposes, one can relate the models to the real world by simply thinking of the allowed rates of return and costs of capital in the model as marginal values.

2.5 Practical Implications

This chapter has proposed a new model of firm behavior under rate-of-return regulation based on the assumption that firm managers maximize a utility function of profits and institutional costs, instead of profits alone. The model was then used to examine the impacts of one policy instrument available to regulators under rate-of-return regulation with wellhead price controls: the setting of the allowed rate of return. The model has two key advantages over a strict profit-maximizing model, such as that of Averch and Johnson. First, its results accord better with the observed behavior of regulated firms, since the model predicts their behavior will change smoothly as the allowed rate of return is lowered to, or below, the cost of capital. Second, the model provides an approach for examining how rate-of-return regulation affects the firm's incentives to minimize operating costs.

The main practical implication of this chapter is that setting an appropriate rate of return is likely to involve a tradeoff--as profits are reduced, institutional costs tend to increase. Intuitively, if a firm's opportunities for profit are restricted, while opportunities for meeting other objectives are not, managers will place greater emphasis on meeting the other objectives[22]. A lowering of the allowed rate of return toward the cost of capital restricts the firm's profit opportunities, resulting in a gradual shift away from profit-maximization toward the maximization of other objectives. The managers' pursuit of these other objectives will be costly, taking the form of increased institutional costs.

Consumer prices as a function of the allowed rate of return will reflect this tradeoff, probably reaching a minimum somewhere between the rate of return the firm would earn as an unregulated monopolist and the cost of capital. To see this, recall that I have argued that at an allowed rate of return equal to the cost of capital, the firm purchases inputs and produces output just as would an unregulated monopolist. This is because, with no opportunity to earn a profit, the managers become institutional cost maximizers, behaving just like unregulated monopolists, but taking the benefits as institutional costs instead of profits. At allowed rates of return above the cost of capital, but below what the firm would earn as an unregulated monopolist, the managers will choose to have more capital investment than would an unregulated monopolist, so as to increase their profit ceiling. At allowed rates of return below the cost of capital, the managers will choose to have less capital investment than would an unregulated monopolist, so as

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[22]This intuitive argument is elaborated in Armen A. Alchian and Reuben A. Kessel, "Competition, Monopoly and the Pursuit of Money", in Aspects of Labor Economics, A Conference of the Universities-National Bureau Committee for Economic Research (Princeton: Princeton University Press, 1962), pp. 157-183.

to cut their losses, but will still choose a positive level of investment if the disutility of the firm's losses to the managers is outweighed by other sources of utility the firm can provide.

If one makes the very reasonable assumption for gas utilities that marginal operating costs are a declining function of capital investment, this would imply that the managers will choose to sell more gas, and hence have lower prices, than an unregulated monopolist when the allowed rate of return is greater than the cost of capital, but below what the firm would earn if it were an unregulated monopolist. Similarly they will sell less gas, and hence have higher prices, than an unregulated monopolist when the allowed rate of return is below the cost of capital. One could therefore visualize the relationship between the allowed rate of return and consumer price as being something like that shown in Figure 18, where r is the cost of capital, s* is the rate of

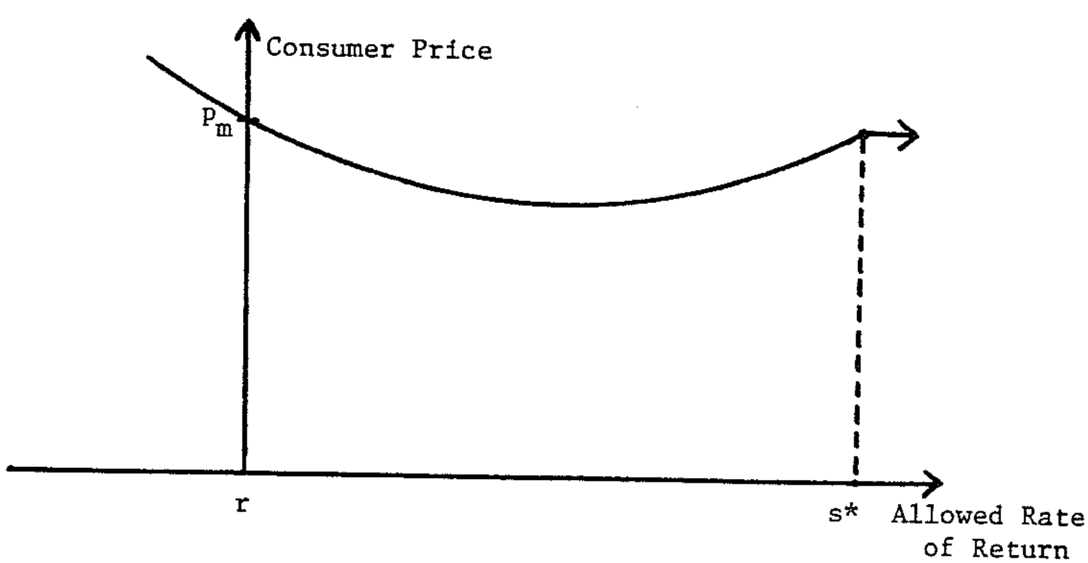

Figure 18
Allowed Rate-of-Return vs. Consumer Price

return the firm would earn as an unregulated monopolist, and p_m the unregulated monopoly price.

The results of this model are somewhat extreme if applied directly to the real world. In particular, the conclusion that the firm with an allowed rate of return equal to the cost of capital would price like an unregulated monopolist should not taken literally. In the real world, the tendencies toward high institutional costs would be mitigated, but not eliminated, by the short-run incentives to cost-minimize resulting from regulatory lag. The model does, however, provide a rigorous basis for one widely held regulatory rule-of-thumb--that regulators should set the allowed rate of return above the cost of capital to encourage the firm to operate efficiently.[23]

The model discussed in this chapter is still not in accord with the observed behavior of gas firms in that it still predicts that each non-capital input will be purchased only up to the point where its marginal cost equals its marginal revenue product. It cannot therefore explain why gas firms are commonly observed purchasing gas at prices which exceed resale value less marginal transmission costs. To explain this phenomenon, one must introduce the second instrument available to regulators--the power to approve or deny new capital investments. This is the subject of the next chapter.

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[23]For a discussion of this rule-of-thumb, see Averch and Johnson, pp. 1061-1062.

2.5.M Mathematical Formulation

In this subsection, I will show why the relationship between allowed rate of return and price is as shown in Figure 18. I first show that if $s = r$, the firm has the same output and price as an unregulated monopolist. The cases of $s > r$ and $s < r$ will then be considered.

Theorem 2.5.M.A: If $s = r$, the managers will always choose $g = g^*$ and $k = k^*$, where g^* and k^* is the solution which would be chosen by the unregulated monopolist. Since price is a function of g, the price charged by this firm $p(g^*)$ is the same as that charged by the unregulated monopolist.

Proof: If $s = r$, then (2.2.M.2/1) and (2.2.M.2/2) become identical to (2.2.M.1/1) and (2.2.M.1/2), which determine g and k for the unregulated firm. By Lemma 2.2.A, the latter equations yield unique optimum values, which I have called g^* and k^*.

Q.E.D.

It remains to consider the case of $r < s < s^*$, where s^* is the return earned by the unregulated monopolist, and the case of $s < r$. The results for these cases require the additional assumption that $z_{12}(k,g) < 0$, meaning, that marginal operating costs are a declining function of k. The following theorem helps to justify this assumption.

Theorem 2.5.M.B: If the underlying production function $g = h(z,k)$ is homothetic with $h_1(z,k) > 0$ and $h_2(z,k) > 0$, then $z_{12}(k,g) < 0$.

Proof: By total differentiation of $g = h(z,k)$, the slope of an isoproduct curve will be

$$\frac{\partial z}{\partial k} = \frac{-h_2(z,k)}{h_1(z,k)} .$$

Since $\partial z/\partial k$ is at the ratio of the first partials of a homothetic function, it will be homogeneous of degree zero.[24] Since one could, alternatively, write $\partial z/\partial k$ in terms of z and k as

$$\frac{\partial z}{\partial k} = z_1\big(k,h(z,k)\big),$$

$z_1\big(k,h(z,k)\big)$ is homogeneous of degree zero.

Then by Euler's theorem,[25]

$$\big[z_{11}\big(k,h(z,k)\big) + z_{12}\big(k,h(z,k)\big)h_2(z,k)\big]k$$

$$+ \big[z_{12}\big(k,h(z,k)\big)h_1(z,k)\big]z = 0,$$

or

$$z_{11}\big(k,h(z,k)\big)k + z_{12}\big(k,h(z,k)\big)\big[h_1(z,k)z + h_2(z,k)k\big] = 0.$$

I assume the marginal productivities of z and k, $h_1(z,k)$ and $h_2(z,k)$ to be positive. By (2.2.M/7), $z_{11}\big(k,h(z,k)\big) = -R_{11}\big(k,h(z,k)\big)$, which is positive by the strict concavity of $R(k,g)$. So $z_{12}\big(k,h(z,k)\big)$ must be negative.

Q.E.D.

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[24] For a proof of this result, see Baumol, p. 285, footnote 10.

[25] See ibid, p. 283.

The following lemma will be useful in proving later results.

Lemma 2.5.A: If $z_{12}(k,g) < 0$, then g and k always move in the same direction in response to a change in s.

Proof: The relationship between g and k must satisfy

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = 0$$

by (2.2.M.2/1), since I assume $g > 0$ at the solution. Differentiating yields

$$\frac{dg}{dk} = \frac{z_{12}(k,g)}{\frac{d}{dg}[p'(g)g + p(g) - f'(g)g - f(g)] - z_{22}(k,g)} .$$

Substituting, using (2.2.M/9) yields

$$\frac{dg}{dk} = \frac{z_{12}(k,g)}{R_{22}(k,g)} .$$

Since $z_{12}(k,g) < 0$ by assumption, while $R_{22}(k,g) < 0$ by the strict concavity of R(k,g), it follows that $dg/dk > 0$. This implies that g and k must always move in the same direction in response to a change in s.

Q.E.D.

The following theorem is the desired substantive result.

Theorem 2.5.M.C: If $z_{12}(k,g) > 0$, $s > r$, and $\pi > \pi^*$ at the solution, then $g > g^*$ and the consumer price, p(g), is less than the price the firm would charge as an unregulated monopolist $p(g^*)$. Similarly, if

$s < r$ and $\pi > \pi^*$ at the solution, then $g < g^*$ and $p(g)$ is greater than $p(g^*)$.

Proof: The rate-of-return constraint (2.2.M.2/3) requires that

$$\pi \leq (s-r)k.$$

By (2.2.M.2/10)

$$\pi^* = (s-r)k^*.$$

Using these two expressions and the assumption that $\pi > \pi^*$,

$$(s-r)k \geq \pi > \pi^* = (s-r)k^*,$$

or

$$(s-r)k > (s-r)k^*.$$

This implies $k > k^*$ if $s > r$ and $k < k^*$ if $s < r$.

By Theorem 2.5.M.A, if $s = r$ then $g = g^*$ and $k = k^*$. Since Lemma 2.5.A requires g and k to move in the same direction in response to a change in s, it must be that $g > g^*$ if $s > r$ and $g < g^*$ if $s < r$. Since $p'(g) < 0$, the results concerning price follow immediately.

Q.E.D.

Note that I have not shown that price as a function of allowed rate of return necessarily has the concave shape shown in Figure 17.

CHAPTER 3

CAPITAL INVESTMENT RESTRICTIONS UNDER PROFIT-MAXIMIZATION

This chapter introduces a second instrument generally used by gas firm regulators: restrictions on capital investment. The introduction will be made in the context of strict profit-maximizing managers for two reasons. The first is pedagogical: the effects of capital investment restriction on utility-maximizing managers is much the same as it is on profit-maximizing managers, but can be understood more easily in the context of profit-maximization. The second is that the profit-maximizing model helps to explain the appeal of capital investment restrictions to regulators, who may tend to think in terms of the traditional profit-maximizing theories. The case for using this instrument emerges stronger in the profit-maximizing case than in the utility-maximizing case. Furthermore, there are simple rules-of-thumb for using capital investment restrictions to accomplish regulatory objectives in the profit-maximizing case; these rules of thumb do not apply in the utility-maximizing case.

The first section discusses the nature and purposes of capital-investment restrictions. It will also be shown how capital-investment restrictions can explain why gas firms might sell gas with a marginal cost exceeding its marginal revenue product. The second section derives simple rules-of-thumb for capital-investment restriction that would be applied by price-minimizing or welfare-maximizing regulators. The third section derives similar rules-of-thumb for the special case of investments in new supply projects, where the benefits from the perspective of society may differ from the benefits from the perspective of the firm. The fourth section derives rules-of-thumb for the special case of investments in new consumer hookups, where an even more difficult issue of evaluating benefits arises. The final section offers a few reflections on the practical implications of the chapter. It will be recalled that firm behavior cannot be explained under a profit-maximizing model if the allowed rate of return is at or below the cost of capital, hence an allowed rate of return above the cost of capital will be assumed throughout this chapter.

3.1 Effect of Capital Investment Restrictions

Gas firms must not only obtain regulatory approval of their tariffs, but must also generally obtain approval of new capital investments. The regulators can, and do, deny approval for new projects not deemed to be in the public interest. This section begins with a brief examination of capital investment regulation at FERC. It then examines graphically how capital investment restrictions can alter the gas firm's choice of inputs, explaining the second anomaly of the Averch-Johnson model discussed in Chapter 1. This is the observed willingness of gas firms to sell gas with a marginal cost exceeding its marginal revenue product. A mathematical subsection follows which demonstrates this result more formally.

In Chapter 1, it was explained how, under the Averch-Johnson model, the firm might tend to use an overly capital-intensive mix of inputs. Regulatory policymakers have not been oblivious to this potential incentive for inefficiency inherent in rate-of-return regulation. They have generally responded by requiring gas firms to obtain prior approval of new facilities, with approval being granted only if the project is judged to be economically justified.

In the case of a FERC regulated pipeline, prior authorization by FERC, in the form of a Certificate of Public Convenience and Necessity, is generally required before the pipeline may undertake construction or expansion of a facility.[1] FERC has administratively defined the word "facility" to exclude replacement of deteriorated or obsolete existing facilities with substantially similar replacements; the addition of certain auxiliary installations to existing systems, such as valves; drips; yard and station piping; cathodic protection equipment; residual refining equipment; water pumping, treatment and cooling equipment; electrical and communication equipment and buildings; and taps on existing pipelines to enable the pipeline to take delivery of gas from a producer. New gas compressors are not excluded.[2]

For those projects requiring FERC approval, the applicants are required to submit a variety of data, including a complete engineering plan for the project; market data, including a detailed breakdown of

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[1]This requirement is contained in the Natural Gas Act. See U.S. Code, title 15, sec. 717 (c) (1976).

[2]Code of Federal Regulations, title 18, part 2.55 (1983).

historical and projected demands, names and descriptions of major customers, market surveys, information on past curtailments, and copies of sales and transportation agreements; detailed estimates of the capital costs of the facilities; financing plans; and projected impacts of the project on pipeline revenues and expenses.[3] The clear thrust here is to give the Commission enough data to make an economic evaluation of the project. Abbreviated applications are permitted for certain specifically defined smaller projects "provided it contains all information and supporting data necessary to explain fully the proposed project, its economic justification, its effect upon applicants present and future operations and upon the public proposed to be served..."[4] For major new projects the Commission staff will generally do an economic evaluation of the project, as well as feasibility studies of alternatives.[5]

The fact that regulators regulate not only tariffs, but capital investment as well, has received little attention in the economic literature on rate-of-return regulation. Joskow and Noll, in their critique of the Averch-Johnson model, argue that regulators do have some power to control costs associated with serious production inefficiencies.[6]

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[3] Code of Federal Regulations, title 18, part 157.14 (1983).

[4] Code of Federal Regulations, title 18, part 157.7(a) (1983).

[5] Interview with Kenneth A. Williams, Director of Pipeline and Producer Regulation, U.S. Federal Energy Regulatory Commission, September 9, 1982.

[6] Joskow and Noll, p. 12.

Breyer has made a similar argument.[7] However, to the best of the author's knowledge, there have been no published attempts to incorporate a regulatory restriction on capital investment into a model of firm behavior under rate-of-return regulation.

I will now show how, by introducing a restriction on capital investment, regulators can not only limit unproductive capital investment, but can also induce the firm to sell gas with a marginal cost exceeding its marginal revenue product. I assume the regulatory restriction on capital investment takes the form of a required marginal internal rate of return on all capital which the firm invests. This means the managers must demonstrate that each dollar of capital they invest yields a stream of benefits or cost reductions with an equivalent present value, where these benefits or cost reductions are discounted at the required marginal internal rate of return. I shall henceforth refer to this required marginal internal rate of return on capital as the "capital productivity requirement."

In the static environment I am considering, a capital productivity requirement for a cost-saving investment is equivalent to requiring the managers to demonstrate that each dollar invested yields cost savings at a rate equal to the capital productivity requirement. For example, if the capital productivity requirement is set at 15% per year, the managers would have to demonstrate that each dollar they invest saves them at least 15 cents per year in operating expenses.

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[7] Stephen Breyer, Regulation and Its Reform (Cambridge: Harvard University Press, 1982), pp. 49-50.

The capital productivity requirement obviously limits unproductive capital investment. The reason why it induces the firm to buy more gas than could be justified under a marginal cost equals marginal revenue product rule can also be made clear intuitively. The managers still wish to increase the firm's capital investment, despite the capital productivity requirement, so as to increase the firm's profit ceiling. Additional gas purchases permit the firm to justify additional capital investment at any given required capital productivity. Hence, there is an extra benefit to the managers from purchasing additional gas, which would not be present without the capital productivity requirement.

A graphical presentation should make this argument clearer. Figure 19 is a three-dimensional representation of a rate-of-return regulated firm without a capital productivity requirement. The central hump shows

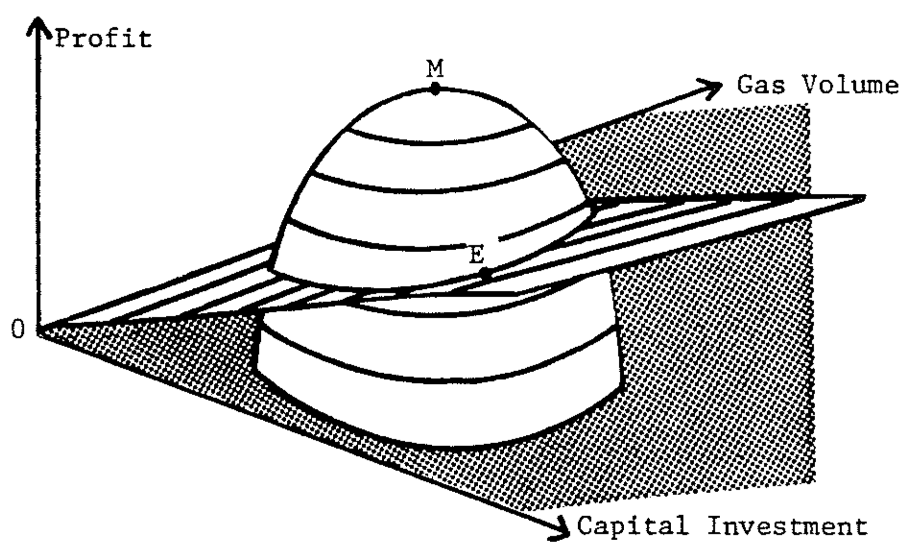

Figure 19
The Profit-Maximizing Firm Under a Rate-of Return Constraint

the firm's potential profit for each combination of capital investment and gas volume in an unregulated environment. Without regulation, the managers would choose to operate at the highest point on the hump, point M. Under a rate-of-return constraint, the firm must operate below the constraint plane, so the managers choose the highest profit point on the constraint plane, point E.

At point E, the firm satisfies the marginal cost of gas equals marginal revenue product rule which maximizes profit at any given level of capital investment. This can be seen clearly in Figure 20, which is a slice through point E in Figure 19 parallel to the profit and gas sales axis. Figure 20 thus represents the plane of all points having the same capital investment as point E. It can be seen that gas sales at point E are set to maximize the firm's profit at this level of capital investment.

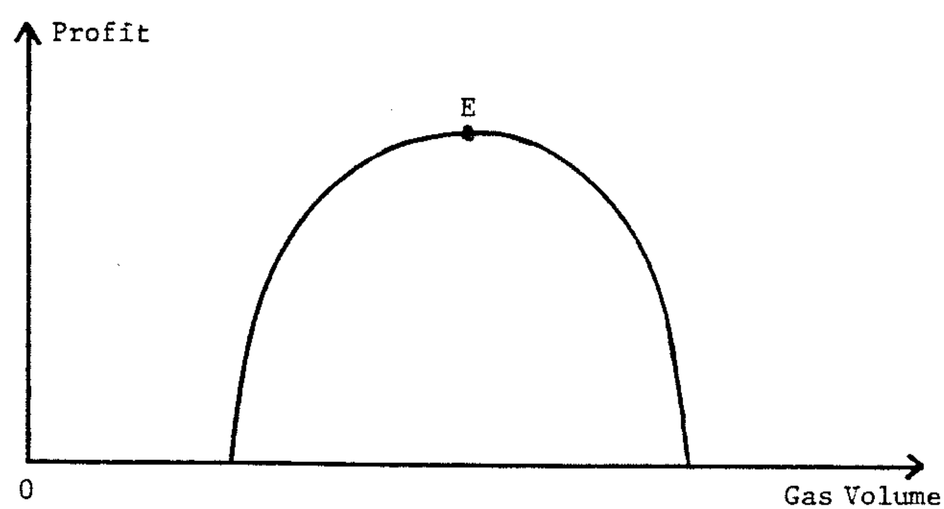

Figure 20
Profit-Maximizing Gas Sales without a Capital Productivity Requirement

The effects of adding a capital productivity requirement will be easier to understand if Figure 19 is first redrawn as seen from above, as in Figure 21. The latter is a contour diagram of all feasible points in Figure 19. The dotted curves in the figure represent isoprofit contours--points of equal profit. The solid curve is the intersection of the potential profit hump and the constraint plane; points inside the solid curve are infeasible by the rate-of-return constraint.

A capital productivity requirement, in effect, sets a ceiling on capital investment for any given volume of gas. This ceiling may be plotted on the same axis used in Figure 21. Presumably, this ceiling passes through the origin and rises with increasing gas volume. Hence,

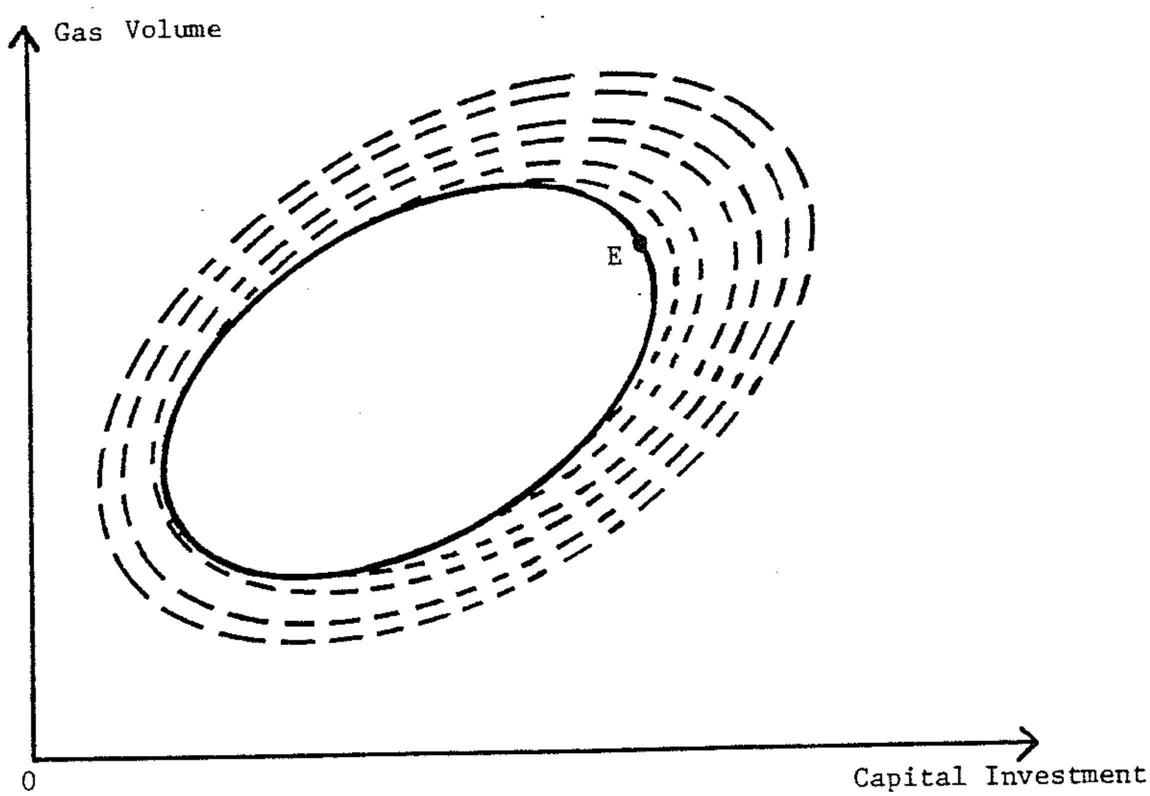

Figure 21
The Profit-Maximizing Firm Under Rate-of-Return Constraint--Top View

the ceiling would look as shown in Figure 22. Superimposing the capital productivity requirement in Figure 22 on the model without a capital productivity requirement shown in Figure 21 results in Figure 23. Since the firm must operate on or above the capital productivity requirement, the old optimum, point E, is now infeasible. The new optimum will be point F, which involves less capital investment, and hence lower profit, but larger gas sales.

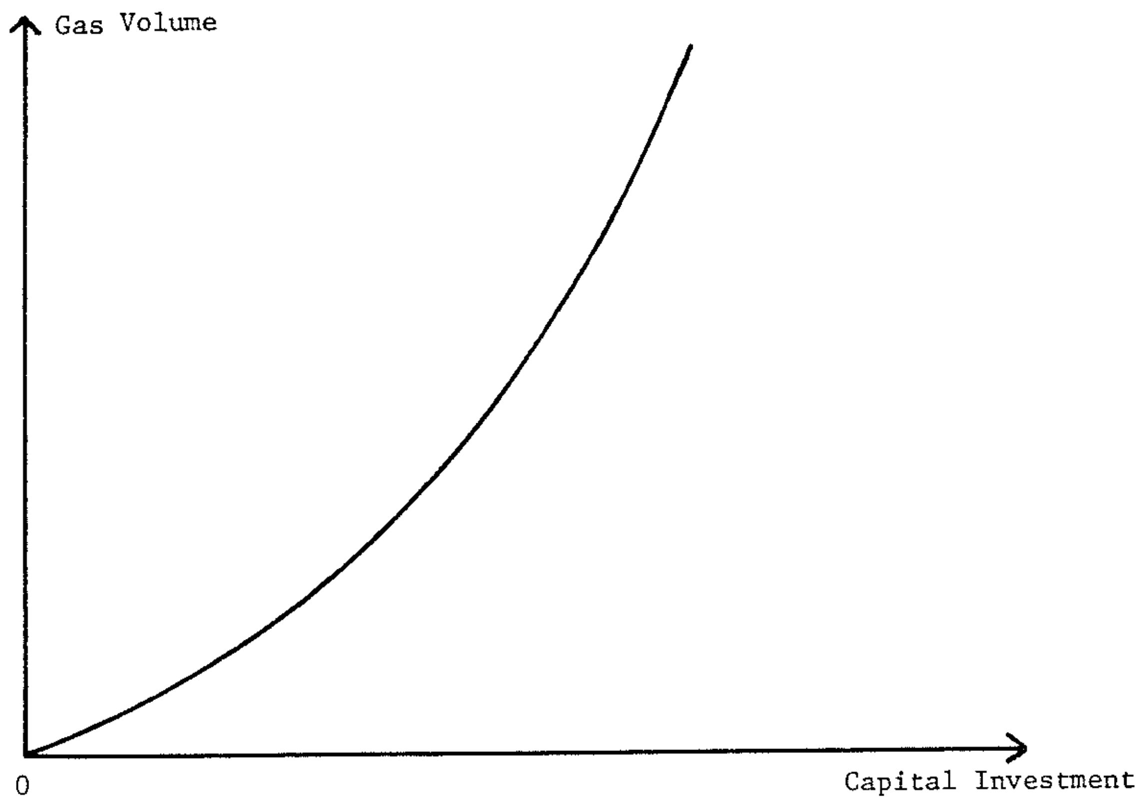

Figure 22
The Capital Productivity Requirement

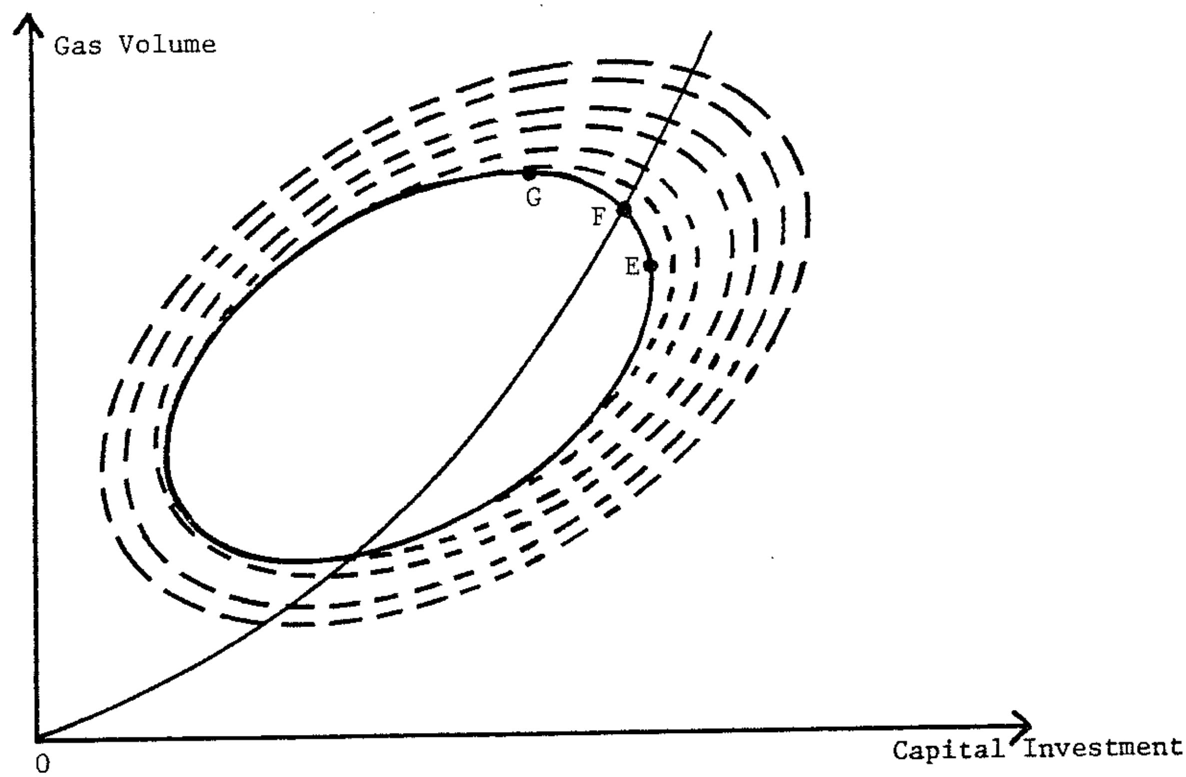

Figure 23
The Profit-Maximizing Firm Under Rate-of-Return Constraint
and a Capital Productivity Requirement

To see that under a capital productivity requirement the firm is purchasing gas beyond the point where its marginal cost equals its marginal revenue product, one may draw a diagram similar to Figure 20, but this time slicing through point F, as shown in Figure 24. This figure thus represents the plane of all points having the same capital investment as point F. It can be seen that gas sales at point F are set higher than the profit-maximizing point for this level of capital investment, where the marginal cost of gas would have been equal to its marginal revenue product.

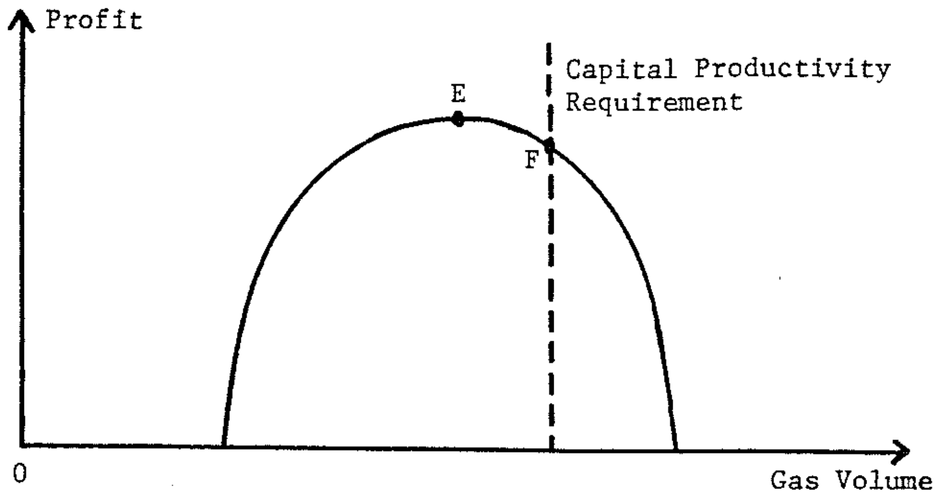

Figure 24
Profit-Maximizing Gas Sales with a Capital Productivity Requirement

3.1.M. Mathematical Formulation

In this section, I will demonstrate mathematically how a capital productivity requirement might lead the firm's managers to choose to purchase gas with a marginal cost greater than its marginal revenue product. I shall use the notation and assumptions that were introduced in Chapter 2. However, I introduce the variable β for the required capital productivity. Given the capital productivity requirement, the managers' problem is to maximize profit

$$(p(g) - f(g))g + c - z(k,g) - rk - x,$$

subject to the rate-of-return constraint

$$(p(g) - f(g))g + c - z(k,g) - sk - x \leq 0,$$

and the capital productivity requirement

$$- z_1(k,g) \geq \beta.$$

Maximization is over g, k, and x, where all three must be non-negative. Although institutional costs always turn out to be zero under the strict profit-maximization considered in this chapter, the variable x is included here because its presence makes it easier to prove the desired results.[8] The Lagrangian will be

$$L = (1-\lambda_1)\left[\left(p(g)-f(g)\right)g+c-z(k,g)-x\right]-(r-\lambda_1 s)k-\lambda_2\left(z_1(k,g)+\beta\right).$$

The first-order conditions require that:

$$\frac{\partial L}{\partial g} = (1-\lambda_1)[p(g)-p'(g)g-f(g)-f'(g)g-z_2(k,g)]-\lambda_2 z_{12}(k,g) \leq 0;$$

$$\frac{\partial L}{\partial g}\, g = 0; \qquad g > 0; \qquad (3.1.M/1)$$

$$\frac{\partial L}{\partial k} = -(1 - \lambda_1)z_1(k,g) - (r - \lambda_1 s) - \lambda_2 z_{11}(k,g) \leq 0; \qquad (3.1.M/2)$$

$$\frac{\partial L}{\partial k}\, k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial x} = -(1 - \lambda_1) \leq 0; \qquad (3.1.M/3)$$

$$\frac{\partial L}{\partial x}\, x = 0; \qquad x \leq 0;$$

$$\frac{\partial L}{\partial \lambda_1} = -\left[\left(p(g) - f(g)\right)g + c - z(k,g) - x\right] + sk \geq 0; \qquad (3.1.M/4)$$

$$\frac{\partial L}{\partial \lambda_1}\lambda_1 = 0; \qquad \lambda_1 \geq 0;$$

$$\frac{\partial L}{\partial \lambda_2} = -z_1(k,g) - \beta \geq 0; \qquad (3.1.M/5)$$

$$\frac{\partial L}{\partial \lambda_2}\lambda_2 = 0; \qquad \lambda_2 > 0.$$

---

[8]Bailey, p. 25, uses this same technique.

I assume $g > 0$ and $k > 0$ at the solution, hence (3.1.M/1) and (3.1.M/2) will be equalities.

Lemma 3.1.A: It is always true that $\lambda_1 < 1$ and $x = 0$.

Proof: $\lambda_1 \leq 1$ by (3.1.M/3). But if $\lambda_1 = 1$ then $\lambda_2 = 0$ by (3.1.M/1), since $z_{12}(k,g) < 0$ by assumption. However, $\lambda_1 = 1$ and $\lambda_2 = 0$ contradicts (3.1.M/2), since $s > r$ by assumption. So $\lambda_1 < 1$, which implies by (3.1.M/3) that $x = 0$.

Q.E.D.

I am now in a position to prove the desired result.

Theorem 3.1.M.A: If $\lambda_2 \neq 0$, then the marginal cost of gas exceeds its marginal revenue product.

Proof: (3.1.M/1) may be rewritten as

$$p(g) + p'(g)g - z_2(k,g) - f(g) - f'(g)g = \frac{\lambda_2 z_{12}(k,g)}{1 - \lambda_1}$$

The term on the left is the marginal revenue product of gas $p(g) + p'(g) - z_2(k,g)$ minus its marginal cost $f(g) + f'(g)g$. The term on the right is always negative, since $\lambda_2 > 0$ and $z_{12}(k,g) < 0$ by assumption (see Section 2.5.M), while $\lambda_1 < 1$ by Lemma 3.1.A. So I have shown that the marginal cost of gas exceeds its marginal revenue product.

Q.E.D.

3.2 Rules-of-Thumb for Restrictions on Capital Investment

It turns out that there are some simple rules-of-thumb which regulators might use to accomplish their objectives, given profit-maximizing behavior by firm managers. They involve setting an appropriate capital productivity requirement. The first subsection below considers the regulator seeking to minimize consumer prices. The second subsection considers the regulator seeking to maximize social welfare.

3.2.1 Price Minimization

In this subsection, I will show graphically why a regulator seeking to minimize consumer prices would set the capital productivity requirement equal to the allowed rate of return. Intuitively, the regulator seeks to assure that consumers get a marginal return from the firm's capital investment equal to the marginal return the consumers are required to pay on that capital investment. A mathematical subsection follows, which repeats the argument more formally.

The regulator can vary the inputs the managers choose in Figure 23 by changing the capital productivity requirement. An increase in the capital productivity requirement increases the amount of gas required to justify any given level of capital investment, and hence swings the required capital productivity curve upward and to the left. Note that if the firm is to earn its allowed rate of return it must operate at a point on the solid curve, where the rate of return constraint intersects the profit hump.

The regulator seeking to minimize consumer prices should therefore set the capital productivity constraint to pass through point G, the point on the solid curve with the largest gas volume. To see that capital productivity must equal the allowed rate of return at point G, Figure 23 may be sliced horizontally through point G to produce Figure 25. The potential profit hump in this figure touches the rate-of-return constraint at only one point--point G--and hence the two must be tangent

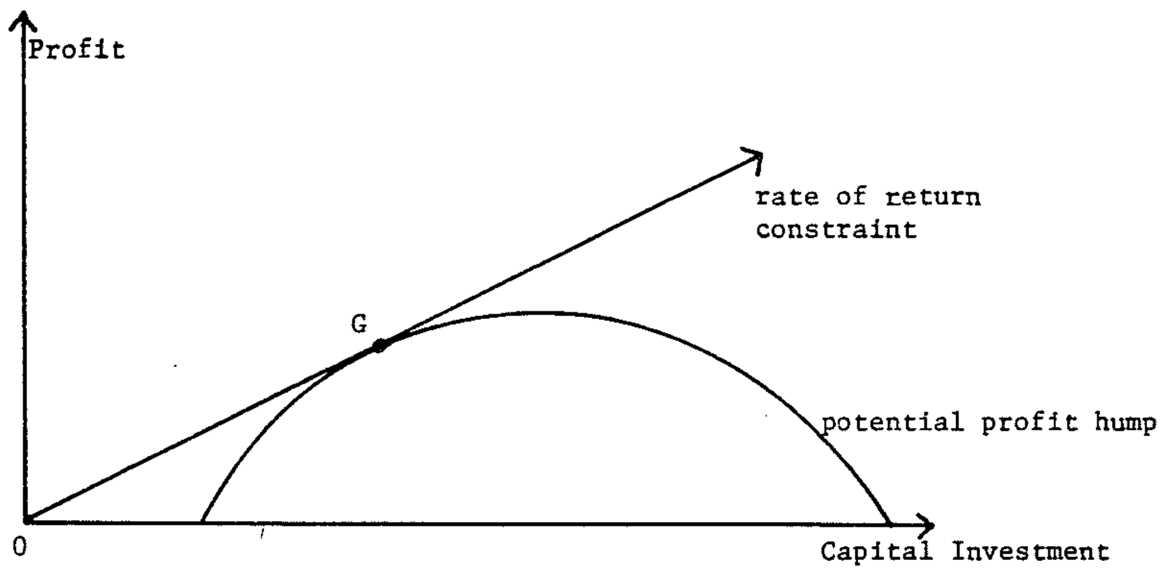

Figure 25
Equality at Capital Productivity and Allowed Rate-of-Return at the Point of Maximum Gas Volume

at point G. But if the potential profit hump and the rate-of-return constraint are tangent at point G, they have the same slope there. The slope of the rate-of-return constraint is the allowed rate of return, while the slope of the potential profit hump is the capital productivity. So a price-minimizing regulator would require a capital productivity equal to the allowed rate of return.

3.2.1.M Mathematical Formulation

In this subsection, I show mathematically why a price-minimizing regulator would seek to assure that the firm has a capital productivity equal to the allowed rate of return. I shall assume that the regulators are free to impose any kind of constraint they choose on the firm subject only to permitting the firm to earn its allowed rate of return.

The regulator's problem is to minimize consumer price p(g), subject to permitting the firm to earn its allowed rate of return,

$$\big(p(g) - f(g)\big)g + c - z(k,g) - sk \geq 0.$$

Maximization is over g and k, where both must be non-negative. The Lagrangian will be

$$L = g + \lambda\big[\big(p(g) - f(g)\big)g + c - z(k,g) - sk\big].$$

The first-order conditions require that:

$$\frac{\partial L}{\partial g} = 1 + \lambda\big[p(g)+p'(g)g-f(g)-f'(g)g-z_2(k,g)\big] \leq 0; \qquad (3.2.1.M/1)$$

$$\frac{\partial L}{\partial g}\, g = 0; \qquad g \geq 0;$$

$$\frac{\partial L}{\partial k} = \lambda\big[-z_1(k,g) - s\big] \leq 0; \qquad (3.2.1.M/2)$$

$$\frac{\partial L}{\partial k}\, k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = \big(p(g) - f(g)\big)g + c - z(k,g) - sk \geq 0; \qquad (3.2.1.M/3)$$

$$\frac{\partial L}{\partial \lambda}\, \lambda = 0; \qquad \lambda \geq 0.$$

I assume $g > 0$ and $k > 0$ at the solution, hence (3.2.1.M/1) and (3.2.1.M/2) will be equalities with $\lambda \neq 0$.

Theorem 3.2.1.A: The price-minimizing regulator should seek to assure that the firm has a capital productivity $-z_1(k,g)$ equal to s.

Proof: Since $\lambda \neq 0$, (3.2.1.M/2) requires $-z_1(k,g) = s$.

Q.E.D.

If the regulators impose upon the firm a requirement that capital productivity be at least equal to s, the price-minimizing solution will be among the firm's feasible solutions, since it satisfies both the rate-of-return constraint and the capital productivity requirement. I cannot rule out the possibility that some type of additional constraint may be necessary to induce the firm to operate at this price-minimizing solution. However, since there are already two constraints to determine only two variables, I would not normally expect additional constraints to be necessary.

An analogous argument applies to each of the remaining sections of this chapter. I cannot prove that simply constraining the firm to the stated rules-of-thumb is sufficient to induce the firm to operate at the solution desired by the regulators. However, in each case, the number of constraints equals the number of variables. The regulator should check that the desired solution is being selected by the firm.

3.2.2 Welfare Maximization

In this subsection, I consider the problem faced by the regulator with the broader objective of trying to maximize the welfare of society as a whole. I will show that the regulator should require a capital productivity less than the allowed rate of return, but greater than the cost of capital if the firm's average cost exceeds its marginal social cost; a capital productivity equal to the allowed rate of return if the firm's average cost equals its marginal social cost; and a capital productivity below the cost of capital if the firm's average cost is less than its marginal social cost.

Graphically, the welfare-maximizing regulator should choose a capital productivity requirement which gives a solution somewhere on arc G-E in Figure 23. Any other solution obtainable by adjusting the capital productivity requirement which permits the firm to obtain its allowed rate of return, specifically those along the extension of arc G-E to the left of G, will have less profit for the firm, but an equal amount of gas sales compared to some point on G-E. Hence, one can always improve the firm's profits by moving from a solution to the left of G to a solution to the right of G, without making anyone else worse off. This would imply the regulator should set the capital productivity requirement at a level less than or equal to the allowed rate of return, but greater than or equal to what the managers would choose for themselves if there were no requirement; the capital productivity the managers would choose for themselves must be lower than the cost of capital, since the rate-of-return constraint leads the managers to expand investment beyond the point where capital productivity equals the cost of capital. Unfortunately, this is about as far as one can go with a graphical analysis of welfare-maximization.

The standard rule for maximizing the total welfare of society requires that the prices of all inputs equal the social value of their marginal products, while the prices of all outputs equal their marginal social costs. A "first-best" welfare-maximizing solution therefore requires a capital productivity equal to the cost of capital. This solution also requires the firm to charge a consumer price for gas equal to its marginal social cost. However, under rate-of-return regulation, the firm must charge a gas price equal to its delivered average cost, including the allowed rate of return on capital. It would be a coincidence for this delivered average cost of gas to equal its marginal social cost when capital productivity equals the cost of capital.

If capital productivity is set equal to the cost of capital when the firm's average cost is below marginal social cost, the firm must charge consumers less than marginal social cost to avoid earning excessive profits. Under these circumstances, there is no way the standard welfare-maximizing rules can be followed. However, a "second-best" welfare-maximizing solution can be obtained by setting the capital productivity requirement a bit lower than the cost of capital. This would result in increased capital investment, lowering the firm's marginal social cost and raising the firm's average cost. In a similar manner, if the firm's average cost exceeds marginal social cost, the regulator should set the capital productivity requirement a bit greater than the cost of capital, but always below the allowed rate of return.

Note that from the perspective of society, gas at the wellhead should be valued at its price, not its marginal cost to the gas firm. Hence, the marginal social cost of delivered gas will equal the wellhead price of gas plus the firm's marginal operating costs.

One may be tempted to assume that gas firms would always have marginal social cost below average cost, since gas firms are generally regarded as being characterized by increasing returns to scale. Such may not be the case, however, for a gas firm under partial wellhead price controls. Under these circumstances, the firm will have available a limited amount of cheap price-controlled gas, but must purchase remaining supplies at a higher market wellhead price. Hence, the wellhead price of gas will exceed the average wellhead cost of gas, offsetting any economies of scale in gas transmission.

3.2.2.M Mathematical Formulation

In this subsection, I will demonstrate mathematically that if the firm's marginal social cost exceeds its average social cost, the welfare-maximizing regulator would seek to insure that the firm has a capital productivity below the cost of capital; if the firm's marginal social cost equals its average cost, the welfare-maximizing regulator would seek to insure that the firm has a capital productivity equal to the cost of capital; if the firm's marginal social cost is below its average cost, the welfare-maximizing regulator would seek to insure that the firm has a capital productivity above the cost of capital, but below the allowed rate of return. I again assume the regulators are free to impose any kind of constraint they choose on the firm, subject only to permitting the firm to earn its allowed rate of return.

I shall also assume that the regulators optimize each firm independently, ignoring the impacts that regulation of one firm may have on the wellhead gas market faced by other firms. Technically, the regulator should be optimizing for all firms simultaneously,[9] to do so, however, would be prohibitively complicated. The author's intuition is that the impacts of one firm's departure from pareto-optimality on the others is probably small enough to be of little practical consequence. However, this is an empirical issue.

The problem faced by the welfare-maximizing regulator is to maximize the sum of consumer, producer, and gas firm surplus,

$$\int_0^g \big(p(g) - f(g)\big)dg - z(k,g) - rk - x,$$

subject to the firm earning its allowed rate of return,

$$p(g)g - f(g)g + c - z(k,g) - sk - x = 0.$$

This constraint could be binding from above or below, depending upon the circumstances. Maximization is over g, k, and x, where all three must be non-negative. The variable x is, again, included in the model as a mathematical convenience. The Lagrangian will be:

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[9]For a critique of "piecemeal welfare economics" see R.G. Lipsey and Kelvin Lancaster, "The General Theory of the Second Best," Review of Economic Studies, 24 (1956-1957), pp. 11-32.

$$L = \int_0^g (p(g) - f(g))dg - z(k,g) - rk - x$$
$$+ \lambda[p(g)g - f(g)g + c - z(k,g) - sk - x].$$

The first order conditions require that:

$$\frac{\partial L}{\partial g} = (1 + \lambda)[p(g) - f(g) - z_2(k,g)] + \lambda[p'(g)g - f'(g)g] \leq 0;$$
$$\frac{\partial L}{\partial g} g = 0; \qquad g > 0; \qquad (3.2.2.M/1)$$

$$\frac{\partial L}{\partial k} = -(1 + \lambda)z_1(k,g) - (r + \lambda s) \leq 0; \qquad (3.2.2.M/2)$$
$$\frac{\partial L}{\partial k} k = 0; \qquad k > 0;$$

$$\frac{\partial L}{\partial x} = -(1 + \lambda) \leq 0; \qquad (3.2.2.M/3)$$
$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = p(g)g - f(g)g + c - z(k,g) - sk - x = 0; \qquad (3.2.2.M/4)$$

I assume $g > 0$ and $k > 0$ at the solution, hence (3.2.2.M/1) and (3.2.2.M/2) will be equalities.

Lemma 3.2.A: It is always true that $\lambda > -1$ and $x = 0$.

Proof: $\lambda \geq -1$ by (3.2.2.M/3). But $\lambda = -1$ contradicts (3.2.2.M/2), since $s > r$ by assumption. So $\lambda > -1$, which implies by (3.2.2.M/3) that $x = 0$.

Q.E.D.

Lemma 3.2.B:

$$(1 + \lambda)\left[f(g) + \left(-c + z(k,g) + sk\right)/g - f(g) - z_2(k,g)\right] + \lambda\left[p'(g)g - f'(g)g\right] = 0. \qquad (3.2.2.M/5)$$

Proof: Solving (3.2.2.M/4) for p(g) and substituting into (3.2.2.M/1) yields (3.2.2.M/5).

Q.E.D.

I am now in a position to state the substantive theorems of this subsection.

Theorem 3.2.2.M.A: If the firm's marginal social costs $f(g) + z_2(k,g)$ equal average costs $f(g) + (-c + z(k,g) + sk)/g$, then the welfare-maximizing regulator would seek to insure that capital productivity $-z_1(k,g)$ is equal to r.

Proof: If $f(g) + z_2(k,g) = f(g) + (-c + z(k,g) + sk)/g$, then the first term in brackets in (3.2.2.M/5) equals zero. Since $p'(g) < 0$ and $f'(g) > 0$ by assumption, the second term in brackets must be negative, so $\lambda = 0$. But if $\lambda = 0$, (3.2.2.M/2) requires $-z_1(k,g) = r$.

Q.E.D.

Theorem 3.2.2.M.B: If the firm's marginal social costs $f(g) + z_2(k,g)$ exceed average costs $f(g) + \left(z(k,g) + sk\right)/g$, then the welfare-maximizing regulator would seek to insure that capital productivity $-z_1(k,g)$ is less than r.

Proof: If $f(g) + z_2(k,g) > f(g) + \left(-c + z(k,g) + sk\right)/g$, then the first term in brackets in (3.2.2.M/5) is negative. Since $p'(g) < 0$ and $f'(g) > 0$ by assumption, the second term in brackets must be negative as well, so $-1 < \lambda < 0$. Now by (3.2.2.M/2),

$$-z_1(k,g) = \frac{r + \lambda s}{1 + \lambda}.$$

If $-1 < \lambda < 0$, then

$$\frac{r + \lambda s}{1 + \lambda} < \frac{r + \lambda r}{1 + \lambda} = r.$$

So $-z_1(k,g) < r$.

Q.E.D.

Theorem 3.2.2.M.C: If the firm's marginal social costs $f(g) + z_2(k,g)$ are less than average costs $f(g) + \left(-c + z(k,g) + sk\right)/g$, then the welfare-maximizing regulator would seek to insure that capital productivity $-z_1(k,g)$ is greater than r, but less than s.

Proof: If $f(g) + z_2(k,g) < f(g) + \left(-c + z(k,g) + sk\right)/g$, then the first term in brackets in (3.2.2.M/5) is positive. Since $p'(g) < 0$ and $f'(g) > 0$ by assumption, the second term in brackets must be negative, so $\lambda > 0$ or $\lambda < -1$. But $\lambda > -1$ by Lemma 3.2.A, so $\lambda > 0$. Now by (3.2.2.M/2),

$$-z_1(k,g) = \frac{r + \lambda s}{1 + \lambda}.$$

If $\lambda > 0$, then

$$s = \frac{s + \lambda s}{1 + \lambda} > \frac{r + \lambda s}{1 + \lambda} > \frac{r + \lambda r}{1 + \lambda} = r.$$

So $s > -z_1(k,g) > r$.

Q.E.D.

3.3. Regulatory Policies on New Supply Projects

Investments in new supply projects yield savings in the form of a reduction in gas purchase costs. In attempting to apply the rules-of-thumb proposed in the last section to the analysis of new supply projects, the regulator faces the additional problem of how to value this reduction in gas purchase costs. Is the savings to be viewed from the perspective of the firm or the perspective of society? Gas from alternative sources should be valued from the firm's perspective at marginal purchase cost to the firm, but from society's perspective at its market price.

This section will extend the analysis of the preceding section to treat the special problems presented by new supply projects. Examples of such projects might include gathering lines into a new gas field, facilities for importing liquefied natural gas, or a synthetic gas plant. Modified rules-of-thumb are derived, which regulators might apply given profit-maximizing behavior by gas firms.

In the first subsection, I consider the case of the regulator seeking to minimize price to consumers. Since any savings in the regulated firm's costs are passed through to consumers, the regulator should value the cost savings from a new supply project from the perspective of the firm. The resulting required capital productivity should equal the allowed rate of return, as in the previous section. In the second subsection, I consider the case of the regulator seeking to maximize social welfare. In this case, the regulator should value the cost savings from the project as they affect welfare from the perspective of society, but as they affect the firm's rate-of-return constraint from the perspective of the firm. The capital productivity rule-of-thumb derived in the preceding section fails, and must be replaced with a new type of rule-of-thumb.

3.3.1 Price Minimization

The capital productivity rule derived in the previous section for the price-minimizing regulator is easily extended to investments in new supply projects. It should, however, be added that the savings from the new supply project should be valued from the perspective of the firm, rather than the perspective of society as a whole. This is because any savings to the firm will be passed through to consumers.

One can determine if a supply project meets a given capital productivity requirement by asking whether each dollar invested yields a savings in gas purchase costs with an equivalent present value, where these savings are discounted at the required capital productivity. In the static environment I am considering, this is equivalent to requiring that each dollar invested yield savings in gas purchase costs at a rate equal to the required capital productivity. This savings in gas purchase costs should be measured by comparison to the cheapest alternative source of supply available to the firm without additional investment. Since the perspective of the firm is being taken, the savings should be calculated based upon the marginal purchase costs to the firm, rather than the prices, of gas from the two sources. The capital productivity requirement should again equal the allowed rate of return.

In the case of gas supply projects there is, however, a simpler equivalent way of stating this rule-of-thumb. This is to fix a ceiling on the marginal cost of gas to the firm from sources requiring additional investment, where marginal cost includes capital cost evaluated at the allowed rate of return. This marginal cost ceiling would be set equal to the marginal cost of gas available to the firm without additional capital investment.

3.3.1.M Mathematical Formulation

In this subsection, I shall demonstrate that the price-minimizing regulator would seek to assure equal marginal costs of gas from all non-price controlled sources, where marginal cost includes capital cost evaluated at the allowed rate of return. Gas from effectively price-controlled sources would be purchased up to the limit available to the firm, at which point its marginal cost would be less than the marginal cost of gas from non-price controlled sources. These conditions will be shown equivalent to assuring a capital productivity on sources requiring capital investment equal to, or greater than, the cost of capital.

I shall assume there are n potential sources of supply $i = 1, \ldots, n$, available. Let

g_i = gas purchased from source i;

$f_i(g_i)$ = wellhead price of gas purchased from source i;

$k_{2i}(g_i)$ = capital investment required to purchase quantity g_i of gas from source i;

k_1 = investment in other plant;

$z(k_1, \sum_i g_i)$ = operating cost;

$p(\sum_i g_i)$ = consumer price;

s = allowed rate of return.

Consumer price minimization is equivalent to output maximization, hence the regulator's problem is to maximize

$$\sum_i g_i$$

subject to permitting the firm to earn its allowed rate of return,

$$p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 \geq 0;$$

and, possibly, to wellhead price ceilings $f_i(g_i) \leq F_i$, which are equivalent to constraints on supply,

$$g_i \leq G_i.$$

I assume sources i = m,...,n, are subject to the wellhead price ceiling. Maximization is over g_i for i = 1,..., n, and k, where all these variables must be non-negative. The Lagrangian will be

$$L = \sum_i g_i + \lambda_1 \left[p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i) g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 \right]$$

$$+ \sum_i \lambda_{2i}(G_i - g_i) \; .$$

The first order conditions require that:

$$\frac{dL}{dg_j} = 1 + \lambda_1 \left[p'(\sum_i g_i)(\sum_i g_i) + p(\sum_i g_i) - f_j'(g_j) g_j - f_j(g_j) - z_2(k_1, \sum_i g_i) - sk_{2j}'(g_j) \right] \leq 0; \qquad (3.3.1.M/1)$$

$$\frac{dL}{dg_j} g_j = 0; \qquad g_j \geq 0; \qquad j = 1,\ldots,m-1;$$

$$\frac{\partial L}{dg_j} = 1 + \lambda_1 \left[p'(\sum_i g_i)(\sum_i g_i) + p(\sum_i g_i) - f_j'(g_j) g_j - f_j(g_j) - z_2(k_1, \sum_i g_i) - sk_{2j}'(g_j) \right] - \lambda_{2j} \leq 0; \qquad (3.3.1.M/2)$$

$$\frac{\partial L}{\partial g_j} g_j = 0; \qquad g_j \geq 0; \qquad j = m,\ldots,n;$$

$$\frac{\partial L}{\partial k_1} = \lambda_1 [-z_1(k_1, \sum_i g_i) - s] \leq 0; \qquad (3.3.1.M/3)$$

$$\frac{\partial L}{\partial k_1} k_1 = 0; \qquad k_1 \geq 0;$$

$$\frac{dL}{d\lambda_1} = p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 \geq 0;$$

$$\frac{dL}{d\lambda_1} \lambda_1 = 0; \qquad \lambda_1 \geq 0; \qquad (3.3.1.M/4)$$

$$\frac{dL}{d\lambda_{2j}} = G_j - g_j \geq 0; \qquad (3.3.1.M/5)$$

$$\frac{dL}{d\lambda_{2i}} \lambda_{2j} = 0; \qquad \lambda_{2j} \geq 0 \qquad j = m, \ldots, n.$$

I assume that $g_j > 0$ for $j = 1, \ldots, n$ and $k_1 > 0$ at the solution, implying that (3.3.1.M/1) - (3.3.1.M/3) are equalities with $\lambda_1 \neq 0$.

Theorem 3.3.1.M.A: Given two sources a and b which are not subject to a wellhead price ceiling, the price-minimizing regulator would seek to assure that

$$f_a'(g_a)g_a + f_a(g_a) + sk_{2a}'(g_a) = f_b'(g_b)g_b + f_b(g_b) + sk_{2b}'(g_b). \qquad (3.3.1.M/6)$$

In words, the marginal cost of gas from the two sources are equal, where this marginal cost includes capital cost evaluated at the allowed rate of return s.

Proof: Since sources a and b are not subject to the wellhead price ceiling, it must be that $a < n - 1$ and $b < n - 1$. Subtracting (3.3.1.M/1) with $i = b$ from (3.3.1.M/1) with $i = a$ and dividing through by λ_1 yields the desired result.

Q.E.D.

<u>Corollary</u>: If the firm could buy gas from source a without further capital investment, then (3.3.1.M/6) is equivalent to setting the productivity of capital invested in source b equal to s.

Proof: Solve (3.3.1.M/6) with $k'_{2a}(g_a) = 0$ for s yielding

$$\frac{f'_a(g_a)g_a + f_a(g_a) - f'_b(g_b)g_b - f_b(g_b)}{k'_{2b}(g_b)} = s.$$

The term on the left is the marginal reduction in gas purchase costs to the firm with an increase in g_b and corresponding decrease in g_a, divided by the marginal increase in capital investment with the increase in g_b. Hence, the term on the left is the productivity of capital invested in source b, which is being set equal to s.

Q.E.D.

<u>Theorem 3.3.1.M.B</u>: Given a source a which is not subject to a wellhead price ceiling and a source b subject to a wellhead price ceiling, the price-minimizing regulator would seek to assure that either:

1) $g_b < G_b$ and

$$f'_a(g_a)g + f_a(g_a) + sk'_{2a}(g_a) = f'_b(g_b)g_b + f_b(g_b) + sk'_{2b}(g_b); \tag{3.3.1.M/7}$$

or 2) $g_b = G_b$ and

$$f'_a(g_a)g + f_a(g_a) + sk'_{2a}(g_a) \geq f'_b(g_b)g_b + f_b(g_b) + sk'_{2b}(g_b). \tag{3.3.1.M/8}$$

In words, either the marginal cost of gas from the two sources are equal, or the firm buys all gas available from source b, at which point its marginal cost is less than or equal to the marginal cost of gas from source a. Marginal cost includes capital cost evaluated at the allowed rate of return s.

Proof: Since source a is not subject to the wellhead price ceiling, it must be that $a < n - 1$; since source b is subject to the wellhead price ceiling, it must be that $b \geq n$. By (3.3.1.M/5), $g_b \leq G_b$. So there are two cases to consider.

1) Assume $g_b < G_b$ at the solution. Then $\lambda_{2b} = 0$ by (3.3.1.M/5). Subtracting (3.3.1.M/2) with $i = b$ from (3.3.1.M/1) with $i = a$ and dividing through by λ_1 yields (3.3.1.M/7).

2) Assume $g_b = G_b$ at the solution. Subtracting (3.3.1.M/2) with $i = b$ from (3.3.1.M/1) with $i = a$ and dividing through by λ_1 yields

$$f'_a(g_a)g + f_a(g_a) + sk'_{2a}(g_a) = f'_b(g_b)g_b + f_b(g_b) + sk'_{2b}(g_b) + \frac{\lambda_{2b}}{\lambda_1},$$

which is equivalent to (3.3.1.M/8).

Q.E.D.

Corollary: If the firm could buy gas from source a without further capital investment, then (3.3.1.M/8) is equivalent to setting the productivity of capital invested in source b greater than or equal to s.

Proof: Solve (3.3.1.M/8) with $k_{2a}(g_a) = 0$ for s yielding

$$\frac{f_a'(g_a)g_a + f_a(g_a) - f_b'(g_b)g_b - f_b(g_b)}{k_{2b}'(g_b)} \geq s.$$

As in the previous corollary, the term on the left is the productivity of capital invested in source b.

Q.E.D.

3.3.2 Welfare Maximization

The capital productivity rule derived in the Section 3.2.2 for the welfare-maximizing regulator is less easily extended to investments in new supply projects. As before, a first-best welfare maximum would require that the consumer price of gas equal its marginal social cost, while capital productivity should equal the cost of capital. One need only note that in calculating capital productivity the savings in gas purchase costs from the new supply project is calculated from the perspective of society rather than the perspective of the firm, based on differences in price rather than marginal cost. Unfortunately, if a regulator were to follow these first-best principles, it would be a coincidence for the firm to earn its allowed rate of return.

A second-best welfare maximization will require the regulator to permit some departure from these principles. The regulator's objective is to permit those departures which minimize welfare loss while holding the firm to its allowed rate of return. Since the savings in gas purchase costs from the project should be viewed from the perspective of society as they affect welfare, but from the perspective of the firm as they affect the rate of return constraint, the second-best welfare-maximizing rule-of-thumb is no longer conveniently stated as a capital productivity requirement. The new rule-of-thumb involves the ratio of the effect on welfare of an additional unit of gas to the effect on the rate-of-return constraint of an additional unit of gas.

A measure of how much an additional unit of gas from a given source of supply contributes to welfare is the consumer price of gas (that is, its value to consumers) minus the wellhead price of gas (its resource cost to society), minus the marginal operating cost to the gas firm, minus the cost of the capital invested in the supply project to produce the gas. Call this measure the "marginal gain in welfare" of gas from a source. Note that under the first-best welfare maximization, the marginal gain in welfare of gas from each source will be zero; no improvement of the outcome is possible.

It is also possible to construct a measure of how much an additional unit of gas from a given source lowers the firm's earnings relative to its allowed earnings. This will be the marginal purchase cost of the gas to the firm, plus the marginal operating costs to the firm, plus allowed earnings on the capital invested in the supply project, minus marginal revenues from the sale of the gas. Call this measure the "marginal drain on earnings" of gas from a source.

The second-best rule-of-thumb for the welfare-maximizing regulator is to require that the ratio of marginal gain in welfare to marginal drain on earnings for each supply source be greater than some minimum value, where the source produces a positive marginal drain on earnings. The managers could then expand supplies from each non-price-controlled supply source until the ratio equalled this ceiling, and expand supplies from price-controlled sources until either the ratio equalled this ceiling or until all available supplies had been contracted for.

To see why welfare-maximization requires equal ratios from all non-price-controlled sources, suppose there is some solution giving the firm its allowed rate of return at which these ratios for different sources were not equal. Then one could obtain another solution which also gives the firm its allowed rate of return, but has higher social welfare, by expanding supplies from sources with large ratios and reducing supplies from sources with small ratios. Although the managers may not be able to expand supplies from price-controlled sources until the ratio reaches the desired level, welfare would be increased by expanding supplies from price-controlled sources with large ratios, or negative drain on earnings, as much as possible.

3.2.2.M Mathematical Formulation

In this subsection, I will show that a welfare-maximizing regulator would seek to assure equal ratios of marginal gain in welfare to marginal drain on earnings from all non-price-controlled sources. Effectively price controlled gas would be purchased up to the limit available, at which point either its marginal drain on earnings will be negative, or its ratio will be above the desired ratio for non-price controlled sources. As in Section 3.1.2.M, I assume the regulators maximize welfare for each firm independently.

As in Section 3.2.1.M, I assume the firm has n potential sources of gas supply available, $i = 1, \ldots, n$. I will use the same notation in this section as I used there. The regulator's problem is to maximize welfare

$$\int_0^{\sum_i g_i} p(g)dg - \sum_i \int_0^{g_i} f_i(g_i)dg_i - z(k_1, \sum_i g_i) - r\sum_i k_{2i}(g_i) - rk_1 - x,$$

subject to the constraint that the firm earn its allowed rate of return,

$$p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 - x = 0,$$

and perhaps subject to a wellhead price ceiling, $f_i(g_i) \leq F_i$, which is equivalent to a ceiling on gas supply,

$$g_i \leq G_i.$$

I assume sources $i = m, \ldots, n$ are subject to the wellhead price ceiling. Maximization is over g_i for $i = 1, \ldots, n$ as well as k and x, where all these variables must be non-negative. The variable x is again included in the model as a mathematical convenience.

The Lagrangian will be

$$L = \int_0^{\sum_i g_i} p(g)dg - \sum_i \int_0^{g_i} f_i(g_i)dg_i - z(k_1, \sum_i g_i) - r\sum_i k_{2i}(g_i) - rk_1 - x)$$

$$+ \lambda_1 [p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 - x]$$

$$+ \sum_i \lambda_{2i}(G_i - g_i).$$

For ease of notation, I make the following additional definitions:

$$MGW_j = \frac{d}{dg_j}[\int_0^{\sum_i g_i} p(g)dg - \sum_i \int_0^{g_i} f(g_i)dg_i - z(k_1, \sum_i g_i) - r\sum_i k_{2i}(g_i) - rk_1 - x]$$

= marginal gain in welfare from sale of gas from source j;

$$MDE_j = -\frac{d}{dg_j}[p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k_1, \sum_i g_i) - s\sum_i k_{2i}(g_i) - sk_1 - x]$$

= marginal drain on earnings from sale of gas from source j.

The first-order conditions require that:

$$\frac{\partial L}{\partial g_j} = MGW_j - \lambda_1 MDE_j \leq 0; \qquad (3.3.2.M/1)$$

$$\frac{\partial L}{\partial g_j} g_j = 0; \qquad g_j \geq 0; \qquad j = 1, \ldots, m - 1;$$

$$\frac{\partial L}{\partial g_j} = MGW_j - \lambda_i MDE_j - \lambda_{2j} \leq 0; \qquad (3.3.2.M/2)$$

$$\frac{\partial L}{\partial g_j} g_j = 0; \qquad g_j \geq 0; \qquad j = m, \ldots, n;$$

$$\frac{\partial L}{\partial k} = -(1 + \lambda_1)z_1(k_1,\sum_i g_i) - (r + \lambda_1 s) \leq 0; \qquad (3.3.2.M/3)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial x} = -(1 + \lambda_1) \leq 0; \qquad (3.3.2.M/4)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial \lambda_1} = p(\sum_i g_i)(\sum_i g_i) - \sum_i f_i(g_i)g_i - z(k,\sum_i g_i)$$

$$- s\sum_i k_{2i}(g_i) - sk_1 - x = 0; \qquad (3.3.2.M/5)$$

$$\frac{\partial L}{\partial \lambda_{2j}} = G_j - g_j \geq 0; \qquad (3.3.2.M/6)$$

$$\frac{\partial L}{\partial \lambda_{2j}} \lambda_{2j} = 0; \qquad \lambda_{2j} \geq 0. \qquad j = 1,\ldots, n.$$

I assume $g_j > 0$ for $j = 1,\ldots, n$ and $k > 0$ at the solution, implying that (3.3.2.M/1) - (3.3.2.M/3) are equalities.

Lemma 3.3.A: It is always true that $\lambda_1 > -1$ and $x = 0$.

Proof: $\lambda_1 \geq -1$ by (3.3.2.M/4). But $\lambda_1 = -1$ contradicts (3.3.2.M/3), since $s > r$ by assumption. So $\lambda_1 > -1$, which implies by (3.3.2.M/4) that $x = 0$.

Q.E.D.

Lemma 3.3.B: It is always true that $MGW_j > -MDE_j$.

Proof: By definition,

$$MGW_j + MDE_j = -p'(\sum_i g_i)(\sum_i g_i) + f_j'(g_j)g_j + (s-r)k_{2j}'(g_j).$$

Since $p'(\ ) < 0$, $f_j'(g_j) > 0$, $s > r$, and $k_{2j}'(g_j) > 0$ by assumption, this shows that

$$MGW_j + MDE_j > 0,$$

from which the desired result follows immediately.

Q.E.D.

Theorem 3.3.2.M.A: The welfare-maximizing regulator would seek to assure that

$$MDE_j > 0$$

and $$\frac{MGW_j}{MDE_j} = \lambda_1 \qquad (3.3.2.M/7)$$

for all non-price-controlled sources $j = i, \ldots, m - 1$.

Proof: Assume $MDE_j = 0$. Then $MGW_j = 0$ also by (3.3.2.M/1). But $MDE_j = MGW_j = 0$ contradicts Lemma 3.3.B. So $MDE_j \neq 0$. Solving (3.3.2.M/1) for λ_1 yields (3.3.2.M/7). Since $\lambda_1 > -1$ by Lemma 3.3.A, this implies that

$$\frac{MGW_j}{MDE_j} > -1.$$

Assume $MDE_j < 0$. Then it must be that $MGW_j < -MDE_j$, contradicting Lemma 3.3.B. So $MDE_j > 0$.

Q.E.D.

Theorem 3.3.2.M.B: For all price-controlled sources $j = m, \ldots, n$, the welfare-maximizing regulator would seek to assure that either

1) $g_j < G_j$, $MDE_j > 0$,

and $$\frac{MGW_j}{MDE_j} = \lambda_1 \qquad (3.3.2.M/8)$$

or 2) $g_j = G_j$ and either

a) $MDE_j \leq 0$

or b) $MDE_j > 0$ and

$$\frac{MGW_j}{MDE_j} \geq \lambda_1.$$

Proof: By (3.3.2.M/6), $g_j \leq G_j$. So there are two cases to consider.

1) Assume $g_j < G_j$ at the solution. Then $\lambda_{2j} = 0$ by (3.3.2.M/6). Assume $MDE_j = 0$. Then $MGW_j = 0$ also by (3.3.2.M/2). But $MDE_j = MGW_j = 0$ contradicts Lemma 3.3.B. So $MDE_j \neq 0$. Solving (3.3.2.M/2) for λ_1 yields (3.3.2.M/8). Since $\lambda_1 > -1$ by Lemma 3.3.A, this implies that

$$\frac{MGW_j}{MDE_j} > -1.$$

Assume $MDE_j < 0$. Then it must be that $MGW_j < -MDE_j$, contradicting Lemma 3.3.B. So $MDE_j > 0$.

2) Assume $g_j = G_j$ at the solution. There are then two cases to consider.

a) If $MDE_j \leq 0$, then (3.3.2.M/2) can always be satisfied for any value of λ_1 by the appropriate choice of λ_{2j}. For $\lambda_1 > -1$ by Lemma

3.3.A, hence

$$- \lambda_1 MDE_j \geq MDE_j.$$

This implies that

$$MGW_j - \lambda_1 MDE_j \geq MGW_j + MDE_j.$$

The expression on the right is positive by Lemma 3.3.B, hence so is the expression on the left. Thus (3.3.2.M/2) can be satisfied by an appropriate non-negative λ_{2j}.

b) If $MDE_j > 0$ then (3.3.2.M/2) implies

$$\frac{MGW_j}{MDE_j} \geq \lambda_1.$$

Q.E.D.

The welfare-maximizing rules for capital investments derived in Section 3.2.2 can also be expressed as a ratio rule. Solving (3.3.2.M/3) for λ_1 yields

$$\lambda_1 = \frac{-z_1(k,\sum_i g_i) - r}{s + z_1(k,\sum_i g_i)}.$$

The numerator is the capital productivity minus cost of capital, or marginal welfare of capital investment, while the denominator is the allowed rate of return minus capital productivity, or marginal drain on earnings of capital investment.

3.4 Regulatory Policies on New Consumer Hookups

Investments in new consumer hookups may yield benefits to the gas firm in the form of additional revenues, plus additional benefits to gas producers and consumers. As with investments in new supply projects, the question arises as to what perspective should be taken in evaluating these benefits. This section extends the analysis of the preceding sections to treat the special problems presented by investments in new consumer hookups. Since the question of whether to hookup a new class of consumers is somewhat inseparable from the question of how different classes of consumers should be charged, consumer pricing will be discussed in this section as well. Modified rules-of-thumb are derived, which regulators might apply, given profit-maximizing behavior by gas firms.

In the first subsection, I consider the case of the regulator seeking to maximize consumer surplus. Here the regulator should view the benefits of the project as they affect consumer surplus from the perspective of consumers, but as they affect the firm's rate-of-return constraint from the perspective of the firm. The rule-of-thumb in this case is therefore expressed as a ratio rule, similar to the one for welfare-maximizing investment in new supply projects derived in the previous section. In the second subsection, I consider the case of the regulator seeking to maximize social welfare. The optimum policy again takes the form of a ratio rule.

3.4.1 Consumer Surplus Maximization

Even in the case of regulators seeking to act in the interest of consumers, the evaluation of new hookups necessarily involves some difficult interpersonal comparisons. Should the regulators seek to benefit primarily consumers already hooked up to the system, potential consumers who could be hooked up to the system, or some combination of both? Economic theory cannot provide an answer to this question. I shall assume that the regulators seek to maximize consumer surplus, meaning that they choose to maximize total consumer benefits without regard for the equity with which these benefits are distributed.

For the rate-of-return regulated firm, the marginal effect of a cost-saving investment on consumer surplus exactly equals its marginal effect on the firm's allowable costs. Hence, in the preceding two sections, one could maximize consumer surplus or, equivalently, minimize consumer price, by minimizing the firm's allowable costs. For investments in new hookups, there is always some increase in consumer surplus which cannot be recovered as revenues by the firm. There is, therefore, no analogous rule for setting the consumer surplus-maximizing number of new hookups. The consumer surplus-maximizing rule turns out to be a ratio rule, very similar to the one I derived for the welfare-maximizing investment in new supply projects in Section 3.3.2.

An additional consumer hookup increases consumer surplus by an amount equal to the area under the consumer's demand curve minus total payments by the consumer. Call this amount the "marginal consumer surplus" of the additional hookup. Since the area under the consumers demand curve is always greater than or equal to the payments from the new consumer (otherwise, the consumer would not choose to hook-up), marginal consumer surplus is non-negative.

It is also possible to construct a measure of how much each new hookup lowers the firm's earnings relative to its allowed earnings. This measure will be the marginal purchase cost of gas supplied to the new consumer plus the marginal operating costs to the firm of supplying the gas plus allowed earnings on the capital invested in the new hookup minus revenues from the new consumer. Call this the "marginal drain on earnings" from the new hookup. Note that this is exactly the same concept as the "marginal drain on earnings" from the sale of gas discussed in Section 3.3.2.

If the marginal drain on earnings for a new hookup is positive, the rule which should be followed by the consumer surplus-maximizing regulator is to set a minimum level on the ratio between the marginal consumer surplus and the marginal drain on earnings. That is, the managers should be required to show that each new hookup produces at least a given number of dollars of consumers surplus per dollar of additional charges which will have to be levied on other consumers. If the marginal drain on earnings from a new hookup are negative, the new hookup should be automatically permitted. Not only does such a hookup produce consumer surplus for the newly hooked-up consumers, but it also reduces the charges which the firm must levy on other consumers, thereby increasing their consumer surpluses as well.

A similar rule applies to the pricing of gas. The sale of an additional unit of gas increases consumer surplus by an amount equal to the value of the gas to the consumer--its price--minus the additional payment for the gas--its marginal revenue to the gas firm. This will be the "marginal consumer surplus" from the sale of an additional unit of gas. It will always be positive. The marginal drain on earnings from the sale of gas will be as defined in Section 3.3.2. It should by now come as no surprise that the consumer surplus-maximizing rule for gas pricing is that each consumer should have the same ratio of marginal consumer surplus to the marginal drain on earnings as every other consumer, and that this ratio should be the same as the minimum of the ratio set for new hookups. Intuitively, every decision the managers make, whether to sell additional gas or hookup new consumers, should produce some given increase in consumer surplus per dollar of additional charges which will have to be levied on other consumers.

3.4.1.M Mathematical Formulation

In this subsection, I shall derive the ratio rules which the consumer surplus-maximizing regulator would seek to assure are met by all new consumer hookups and the gas firm's consumer prices. I assume there are n classes of potential consumers $i = 1,\ldots, n$, with all consumers in a given class having identical demand curves. Let:

g_i = gas sold to each consumer in class i;

$p_i(g_i)$ = consumer price to consumer class i;

q_i = number of consumers in class i hooked-up to the system;

$k_{2i}(q_i)$ = investment required to hook-up q_i consumers of class i;

Q_i = total population of class i customers;

k_1 = investment in other plant;

$z(k_1,\sum_i g_i q_i)$ = operating cost;

$f(\sum_i g_i q_i)$ = wellhead price of gas;

c = constant representing value of any rents accruing to firm from partial wellhead price controls;

s = allowed rate of return.

The consumer surplus-maximizing regulator's problem is then to maximize consumer surplus

$$\sum_i [\int_0^{g_i} p_i(g)dg - p_i(g_i)g_i]q_i,$$

subject to permitting the firm to earn its allowed rate of return,

$$\sum_i p_i(g_i)g_i q_i - z(k_1,\sum_i g_i q_i) - f(\sum_i g_i q_i)(\sum_i g_i q_i)$$
$$+ c - s\left(k_1 + \sum_i k_{2i}(q_i)\right) \geq 0,$$

and the limit on the number of consumers in each class,

$$q_i \leq Q_i; \quad i = 1, \ldots, n.$$

Maximization is over g_i and q_i for $i = 1, \ldots, n$, where all these variables must be non-negative.

For ease of notation, I make the following additional definitions:

$$MCSH_j = \int_0^{g_j} p_j(g)dg - p_j(g_j)g_j$$

= marginal consumer surplus from new hookup to consumer of class j;

$$MCSG_j = -p'_j(g_j)g_j$$

= marginal consumer surplus from sale of gas to consumer of class j;

$$MDEH_j = \frac{d}{dq_j}\left[\sum_i p_i(g_i)g_iq_i - z(k_1,\sum_i g_iq_i) - f(\sum_i g_iq_i)(\Sigma g_iq_i) - s\left(k_1+\sum_i k_{2i}(q_i)\right)\right]$$

= marginal drain on earnings of new hookup to consumer of class j;

$$MDEG_j = -\frac{d}{dg_j}\left[\sum_i p_i(g_i)g_iq_i - z(k_1,\sum_i g_iq_i) - f(\sum_i g_iq_i)(\Sigma g_iq_i) - s\left(k_1+\sum_i k_{2i}(q_i)\right)\right]$$

= marginal drain on earnings from sale of gas to consumer of class j.

Lemma 3.4.A: $MCSH_j > 0$.

Proof: Since $p'_j(g_j) < 0$ by assumption, $p_j(g) > p_j(g_j)$ for $g < g_j$.

Hence,

$$\int_0^{g_j} p_j(g)dg > p_j(g_j)g_j$$

Q.E.D.

Lemma 3.4.B: $MCSG_j > 0$.

Proof: Since $p'_j(g_j) < 0$, $-p'_j(g_j) > 0$.

Q.E.D.

The Lagrangian will be

$$L = \sum_i \Big[\int_0^{g_i} p_i(g)dg - p_i(g_i)g_i\Big]q_i + \lambda_1\Big[\sum_i p_i(g_i)g_iq_i - z(k_1,\sum_i g_iq_i)$$

$$- f(\sum_i g_iq_i)(\sum_i g_iq_i) + c - s\big(k_1 + \sum_i k_{2i}(q_i)\big)\Big] + \sum_i \lambda_{2i}(Q_i - q_i).$$

The first-order conditions require that:

$$\frac{dL}{dq_j} = MCSH_j - \lambda_1 MDEH_j - \lambda_{2j} \leq 0; \qquad (3.4.1.M/1)$$

$$\frac{dL}{dq_j}q_j = 0_j; \qquad q_j \geq 0; \qquad j = 1,\ldots, n;$$

$$\frac{dL}{dg_j} = MCSG_j - \lambda_1 MDEG_j \leq 0; \qquad (3.4.1.M/2)$$

$$\frac{dL}{dg_j}g_j = 0; \qquad g_j \geq 0; \qquad j = 1,\ldots, n;$$

$$\frac{dL}{dk_1} = \lambda_1\big[-z_1(k_1,\sum_i g_iq_i) - s\big] \leq 0; \qquad (3.4.1.M/3)$$

$$\frac{dL}{dk_1}k_1 = 0; \qquad k_1 \geq 0; \qquad j = 1,\ldots, n;$$

$$\frac{dL}{d\lambda_1} = \sum_i p_i(g_i)g_iq_i - z(k_1,\sum_i g_iq_i) - f(\sum_i g_iq_i)(\sum_i g_iq_i) + c$$

$$- s\big(k_1 + \sum_i k_{2i}(q_i)\big) \leq 0; \qquad (3.4.1.M/4)$$

$$\frac{dL}{d\lambda_1}\lambda_1 = 0; \qquad \lambda_1 \geq 0;$$

$$\frac{dL}{d\lambda_{2j}} = Q_j - q_j \geq 0; \qquad (3.4.1.M/5)$$

$$\frac{dL}{d\lambda_{2j}}\lambda_{2j} = 0; \qquad \lambda_{2j} \geq 0. \qquad j = 1,\ldots, n.$$

I assume $q_j > 0$ and $g_j > 0$ for $j = 1, \ldots, n$ and $k_1 > 0$ at the solution, hence (3.4.1.M/1) - (3.4.1.M/3) are equalities.

Theorem 3.4.1.M.A: The consumer surplus-maximizing regulator would seek to insure that either:

1) $q_j < Q_j$, $MDEH_j > 0$

and $\frac{MCSH_j}{MDEH_j} = \lambda_1$, (3.4.1.M/6)

or 2) $q_j = Q_j$ and either

a) $MDEH_j \leq 0$

or b) $MDEH_j$ and $\frac{MCSH_j}{MDEH_j} \geq \lambda_1$.

Proof: By (3.4.1.M/5), $q_j \leq Q_j$. So there are two cases to consider.

1) Assume $q_j < Q_j$ at the solution. Then $\lambda_{2j} = 0$ by (3.4.1.M/5). Assume $MDEH_j \leq 0$. Since $\lambda_1 \geq 0$ by (3.4.1.M/4), this would imply that $MCSH_j \leq 0$ by (3.4.1.M/1), which contradicts Lemma 3.4.A. Hence, $MDEH_j > 0$. Dividing (3.4.1.M/1) through by $MDEH_j$ yields (3.4.1.M/6).

2) Assume $q_j = Q_j$ at the solution. There are then two cases to consider

a) If $MDEH_j \leq 0$ then (3.4.1.M/1) can always be satisfied for any value of λ_1 by appropriate choice of λ_{2j}. For $\lambda_1 \geq 0$ by (3.4.1.M/4), while $MCSH_j > 0$ by Lemma 3.4.A. Hence, if $MDEH_j \leq 0$, then

$$MCSH_j - \lambda_1 MDEH_j > 0.$$

So (3.4.1.M/1) can be satisfied by the appropriate non-negative λ_{2j}.

b) If $MDEH_j > 0$ then (3.4.1.M/1) implies

$$\frac{MCSH_j}{MDEH_j} \geq \lambda_1 .$$

Q.E.D.

Theorem 3.4.1.M.B: The consumer surplus-maximizing regulator will seek to assure that the gas firm's pricing policies satisfy $MDEG_j > 0$ and

$$\lambda_1 = \frac{MCSG_j}{MDEG_j} \tag{3.4.1.M/7}$$

for all consumers.

Proof: Assume $MDEG_j \leq 0$. Since $\lambda_1 \geq 0$ by (3.4.1.M/4), this would imply that $MCSG_j \leq 0$ by (3.4.1.M/2). But this contradicts Lemma 3.4.B. Hence $MDEG_j > 0$. Dividing (3.4.1.M/2) through by $MDEG_j$ yields (3.4.1.M/7).

Q.E.D.

Note that both the minimum ratio for new hookups specified by Theorem 3.4.1.M.A and the ratio for gas prices specified by Theorem 3.4.1.M.B are equal to λ_1.

3.4.2 Welfare Maximization

The first-best rules for welfare maximization would require a consumer price of gas equal to its marginal social cost. It would also require the installation of new hookups until the marginal gain in welfare from new hookups is zero. Unfortunately, it would be a coincidence if a gas firm which followed these rules were to earn its allowed rate of return. Here again, the ratio rule provides a second best solution.

In the case of new hookups, the solution requires that the ratio of the marginal gain in welfare from each new hookup to the marginal drain on earnings from the new hookup be greater than some minimum value if the marginal drain on earnings from the hookup is positive. If the marginal drain on earnings from the new hookup is negative, the hookup should always be permitted. In setting gas sales, the ratio of the marginal gain welfare from the sale of gas to the marginal drain on earnings from the sale of gas would be required to be the same for each consumer, and the same as the minimum acceptable ratio for new hookups. Intuitively, if any of these ratios were to differ, one could improve welfare, while holding the gas firm at its allowed rate of return, by increasing hookups and/or gas sales where the ratios are high and reducing hookups and/or gas sales where the ratios are low.

3.4.2.M Mathematical Formulation

In this subsection, I shall derive the ratio rules which the welfare-maximizing regulator would seek to assure are met by all new consumer hookups and the gas firm's consumer prices. As in Section 3.2.2.M, I assume the regulators maximize for each firm independently. As in Section 3.4.1.M, I assume there are n classes of potential consumers $i = 1, \ldots, n$, with all consumers in a given class having identical demand curves. I use the notation which was introduced in that section.

The welfare-maximizing regulator's problem is to maximize welfare

$$\sum_i q_i \int_0^{g_i} p_i(g)dg - \int_0^{\sum_i g_i q_i} f(g)dg - z(k_1, \sum_i g_i q_i) - r\left(k_1 + \sum_i k_{2i}(q_i)\right) - x,$$

subject to the firm earning its allowed rate of return,

$$\sum_i p_i(g_i) g_i q_i - z(k_1, \sum_i g_i q_i) - f(\sum_i g_i q_i)(\sum_i g_i q_i) + c$$

$$- s\left(k_1 + \sum_i k_{2i}(q_i)\right) - x = 0,$$

and the limit on the number of consumers in each class,

$$q_i \leq Q_i; \qquad i = 1, \ldots, n.$$

Maximization is over q_i and g_i for $i = 1,\ldots,n$, and x, where all these variables must be non-negative. The variable x is again included in the model as a mathematical convenience.

The Lagrangian will be

$$L = \sum_i q_i \int_0^{g_i} p_i(g)dg - \int_0^{\sum_i g_i q_i} f(g)dg - z(k_1, \sum_i g_i q_i) - r\left(k_1 + \sum_i k_{2i}(q_i)\right) - x$$

$$+ \lambda_1\left[\sum_i p_i(g_i) g_i q_i - z(k_1, \sum_i g_i q_i) - f(\sum_i g_i q_i)(\sum_i g_i q_i) + c\right.$$

$$\left. - s\left(k_1 + \sum_i k_{2i}(q_i)\right) - x\right] + \sum_i \lambda_{2i}(Q_i - q_i).$$

For ease of notation, I make the following additional definitions:

$$MGWH_j = \frac{d}{dq_j}\left[\sum_i q_i \int_0^{g_i} p_i(g)dg - \int_0^{\sum_i g_i q_i} f(g)dg - z(k_1, \sum_i g_i q_i) - r\left(k_1 + \sum_i k_{2i}(q_i)\right) - x\right]$$

= marginal gain in welfare from new hookup to consumer of class j;

$$MGWG_j = \frac{d}{dg_j}\left[\sum_i q_i \int_0^{g_i} p_i(g)dg - \int_0^{\sum_i g_i q_i} f(g)dg - z(k_1, \sum_i g_i q_i) - r\left(k_1 + \sum_i k_{2i}(q_i)\right) - x\right]$$

= marginal gain in welfare from sale of gas to consumer of class j.

Recall that $MDEH_j$ and $MDEG_j$ were defined in Section 3.4.1.M.

The first-order conditions require that:

$$\frac{\partial L}{\partial q_j} = MGWH_j - \lambda_1 MDEH_j - \lambda_{2j} \leq 0; \qquad (3.4.2.M/1)$$

$$\frac{\partial L}{\partial q_j} q_j = 0; \qquad q_j \geq 0; \qquad j = 1, \ldots, n;$$

$$\frac{\partial L}{\partial g_j} = MGWG_j - \lambda_1 MDEG_j \leq 0; \qquad (3.4.2.M/2)$$

$$\frac{\partial L}{\partial g_j} g_j = 0; \qquad g_j \geq 0; \qquad j = 1, \ldots, n;$$

$$\frac{\partial L}{\partial k_1} = -(1 + \lambda_1) z_1(k_1, \sum_i g_i q_i) - (r + \lambda_1 s) \leq 0; \qquad (3.4.2.M/3)$$

$$\frac{\partial L}{\partial k} k_1 = 0; \qquad k_1 \geq 0;$$

$$\frac{\partial L}{\partial x} = -(1 + \lambda_1) \leq 0; \qquad (3.4.2.M/4)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_i p_i(g_i)g_i q_i - z(k_1, \sum_i g_i q_i) - f(\sum_i g_i q_i)(\sum_i g_i q_i) + c$$

$$- s\left(k_1 + \sum_i k_{2i}(q_i)\right) - x = 0; \qquad (3.4.2.M/5)$$

$$\frac{\partial L}{\partial \lambda_{2j}} = Q_j - q_j \geq 0; \qquad (3.4.2.M/6)$$

$$\frac{\partial L}{\partial \lambda_{2j}} \lambda_{2j} = 0; \qquad \lambda_{2j} \geq 0; \qquad j = 1, \ldots, n.$$

I assume $q_j > 0$ and $g_j > 0$ for $j = 1, \ldots, n$ and $k_1 > 0$ at the solution, hence (3.4.2.M/1) - (3.4.2.M/3) will be equalities.

Lemma 3.4.C: It is always true that $\lambda_1 > -1$ and $x = 0$.

Proof: $\lambda_1 \geq -1$ by (3.4.2.M/4). But $\lambda_1 = -1$ contradicts (3.4.2.M/3), since $s > r$ by assumption. So $\lambda_1 > -1$, which implies by (3.4.2.M/4) that $x = 0$.

Q.E.D.

Lemma 3.4.D: It is always true that $MGWH_j > - MDEH_j$.

Proof: By definition,

$$MGWH_j + MDEH_j = \int_0^{g_j} p_j(g)dg - p_j(g_j)g_j + f'(\sum_i g_i q_i)(\sum_i g_i q_i)g_j + (s-r)k'_{2j}(q_j).$$

Since $p'_j(\ ) < 0$, $f'(\ ) > 0$, and $k'_{2j}(q_j) > 0$ by assumption, the desired result follows immediately.

Q.E.D.

Lemma 3.4.E: It is always true that $MGWG_j > MGEG_j$.

Proof: By definition,

$$MGWG_j + MDEG_j = -p_j'(g_j)g_jq_j + f'(\sum_i g_iq_i)q_j(\sum_i g_iq_i).$$

Q.E.D.

Since $p_i'(\ ) < 0$ and $f'(\ ) > 0$ by assumption, the desired result follows immediately.

Q.E.D.

<u>Theorem 3.4.2.M.A</u>: The welfare-maximizing regulator would seek to insure that either:

1) $q_j < Q_j$, $MDEH_j > 0$ and $\frac{MGWH_j}{MDEH_j} = \lambda_1$ (3.4.2.M/7)

or 2) $q_j = Q_j$ and either

a) $MDEH_j \leq 0$

or b) $MDEH_j > 0$ and $\frac{MGWH_j}{MDEH_j} \geq \lambda_1$.

Proof: By (3.4.2.M/6), $q_j \leq Q_j$. So there are two cases to consider.

1) Assume $q_j < Q_j$ at the solution. Then $\lambda_{2j} = 0$ by (3.4.2.M/6). Assume $MDEH_j = 0$. Then $MGWH_j = 0$ also by (3.4.2.M/1). But $MDEH_j = MGWH_j = 0$ contradicts Lemma 3.4.D. So $MDEH_j \neq 0$. Solving (3.4.2.M/1) for λ_1 yields (3.4.2.M/7). Since $\lambda_1 > -1$ by Lemma 3.4.C, this implies that

$$\frac{MGWH_j}{MDEH_j} > -1.$$

Assume $MDEH_j < 0$. Then it must be that $MGWH_j < -MDEH_j$, contradicting Lemma 3.4.D. So $MDEH_j > 0$.

2) Assume $q_j = Q_j$ at the solution. There are then two cases to consider:

a) If $MDEH_j \leq 0$ then (3.4.2.M/1) can always be satisfied for any value of λ_1 by the appropriate choice of λ_{2j}. For $\lambda_1 \geq -1$ by Lemma 3.4.C, hence

$$-\lambda_1 MDEH_j \geq MDEH_j.$$

This implies that

$$MGWH_j - \lambda_1 MDEH_j \geq MGWH_j + MDEH_j.$$

The expression the right is positive by Lemma 3.4.D, hence so is the expression on the left. Thus (3.4.2.M/1) can be satisfied by an appropriate non-negative λ_{2j}.

b) If $MDEH_j > 0$ then (3.4.2.M/1) implies

$$\frac{MGWH_j}{MDEH_j} \geq \lambda_1.$$

Q.E.D.

<u>Theorem 3.4.2.M.B</u>: The welfare-maximizing regulator will seek to assure that the gas firm's pricing policies satisfy

$$MDEG_j > 0$$

$$\text{and} \quad \frac{MGWG_j}{MDEG_j} = \lambda_1 \qquad (3.4.2.M/8)$$

for all consumers.

Proof: Assume $MDEG_j = 0$. Then $MGWG_j = 0$ also by (3.4.2.M/2). But $MDEG_j = MGWG_j = 0$ contradicts Lemma 3.4.E. So $MDEG_j \neq 0$. Solving (3.4.2.M/2) for λ_1 yields (3.4.2.M/8). Since $\lambda_1 > -1$ by Lemma 3.4.C, this implies that

$$\frac{MGWG_j}{MDEG_j} > -1.$$

Assume $MDEG_j < 0$. Then it must be that $MGWG_j < -MDEG_j$, contradicting Lemma 3.4.E. So $MDEG_j > 0$.

Q.E.D.

Note that both the minimum ratio for new hookups specified by Theorem 3.4.2.M.A and the ratio for gas prices specified by Theorem 3.4.2.M.B are equal to λ_1.

3.5 Practical Implications

This chapter has discussed a second instrument available to regulators under rate-of-return regulation: restrictions on capital investment. The application of this instrument was explored in the context of a strict profit-maximizing firm. It was shown how restrictions on capital investment, in the form of a required capital productivity, can induce the firm to purchase gas beyond the point where its marginal cost equals its marginal revenue product.

Restrictions on capital investment may be an especially appealing policy to regulators who perceive firms as strict profit-maximizers, since there are simple rules-of-thumb which a consumer surplus-maximizing or welfare-maximizing regulator might follow. For most cost-saving investments, the consumer surplus-maximizing regulator would require a capital productivity (marginal internal rate-of-return) equal to the allowed rate of return. The welfare-maximizing regulator would require a capital productivity less than the allowed rate of return but greater than the cost of capital if the firm's average cost exceeds its marginal social cost, or a capital productivity below the cost of capital if the firm's average cost is less than its marginal social cost.

Investments in new supply projects and new hookups pose special problems, since the effect of these investments on consumer surplus or welfare may differ from their impact on the firm's rate-of-return constraint. In the case of a supply project, the consumer surplus-maximizing regulator would again require a capital productivity, measured from the perspective of the firm, equal to the allowed rate of return. The welfare-maximizing regulator would require a minimum ratio of the marginal gain in welfare to the marginal drain on earnings for each supply project, if the project would create a drain on earnings. In the case of new consumer hookups, the consumer surplus-maximizing regulator would require a minimum ratio of marginal consumer surplus to marginal drain on earnings from each hookup, if the hookup would create a drain on earnings. The welfare-maximizing regulator would require a minimum ratio of marginal gain in welfare to marginal drain on earnings from each hookup, if the hookup would create a drain on earnings.

Of course, the model I have proposed is somewhat idealized. Informational problems would limit the regulators' ability to implement these policies. First, the regulators do not have the resources to evaluate the productivity of each and every dollar the firm may wish to invest, so they will tend to look at the average productivity of the entire project all at once. Second, the regulators must actually base their decisions on forecasts of the impacts of capital investment, since actual impacts are never known until a project is completed, if then; this adds risk and uncertainty to the decisionmaking. Third, the regulators must generally base their forecasts partly on evidence provided by the managers itself; this evidence may be manipulated by the managers to their own advantage. Finally, for investments in new hookups, there are practical difficulties in estimating consumer surplus which would prevent exact implementation of the proposed rules-of-thumb. However, this model does capture the basic appeal of capital investment restrictions to regulators, and provides some insight into the effects of this little-acknowledged regulatory instrument.

CHAPTER 4

GAS FIRM BEHAVIOR WHEN PROFIT MAXIMIZATION IS NOT THE ONLY OBJECTIVE AND CAPITAL INVESTMENT IS RESTRICTED

This chapter introduces a model of gas firm behavior which combines the assumption of utility-maximizing managers examined in Chapter 2 with the restrictions on capital investment examined in Chapter 3. If firm managements do, indeed, have objectives other than profit-maximization, then restrictions on capital investment could affect the emphasis the firm places on profits compared to other objectives. This could potentially alter the results of the two previous chapters. Although, in fact, this chapter does not demonstrate any dramatically new results, it does allow one to identify which of the results of these previous chapters can be extended to this new model. The chapter also lays the groundwork for the examination of the effects of wellhead price controls, which will be made in the following chapter.

The first section examines how the capital productivity requirement would affect the budget constraint faced by the managers of a rate-of-return regulated firm. The second section uses these results to examine how changes in the capital productivity requirement would affect the firm's profits and institutional costs. The third section examines how changes in the allowed rate of return would affect the firm's profit and institutional costs when the firm is subject to a capital productivity requirement. The final section offers a few reflections on the practical implications of the chapter.

4.1 Another Look at the Firm's Budget Constraint

With or without a constraint on capital productivity, the combination of profit and institutional costs which maximizes a manager's utility function will be found at the point of tangency between the firm's budget constraint and an isoutility curve. The impact of the capital productivity requirement will be through its impact on the budget constraint. This section therefore examines how the capital productivity requirement affects the budget constraint.

First, consider the case of an allowed rate of return greater than the cost of capital. Recall that without a capital productivity requirement, the budget constraint is of the general form shown in Figure 26. π^* represents the ceiling on the firm's profit assuming the managers invest the amount of capital they would choose without regulation. Standard theory of the firm requires that, in the absence of regulation, the managers would choose an amount of capital investment such that capital productivity exactly equaled the cost of capital. At all values of profit less than π^* the profit ceiling is not binding, so the managers behave as they would without rate-of-return regulation. Hence, capital productivity exactly equals the cost of capital. To

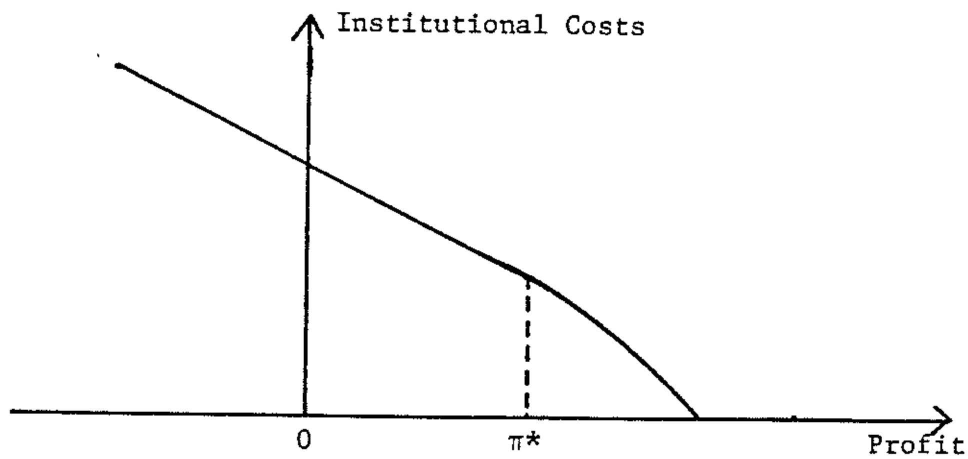

Figure 26
Budget Constraint of a Rate-of-Return Regulated Firm
with an Allowed Rate of Return Greater than the Cost of Capital

attain values of profit greater than π^*, the managers must invest more capital than they would in the absence of regulation, so as to increase the profit ceiling. Hence, as one moves to values of profit greater than π^*, capital productivity must decline.

It follows that, if the regulator were to set a capital productivity requirement less than the cost of capital, there would be no impact at values of profit less than π^*, or even at values of profit greater than π^* but less than some π^{**}. At π^{**} the productivity of capital would have dropped low enough that the capital productivity requirement becomes binding. The capital productivity requirement would make it more expensive for the managers to expand profits beyond π^{**} than it would be under the rate-of-return constraint alone. This is because they must not only acquire capital at a cost which exceeds the value of its marginal product, but must also acquire gas at a cost which exceeds the value of its marginal product, so as to justify the capital. As a result, the budget constraint will be lower at values of profit greater than π^{**} than it would be without the capital productivity requirement. The new budget constraint is shown in Figure 27.

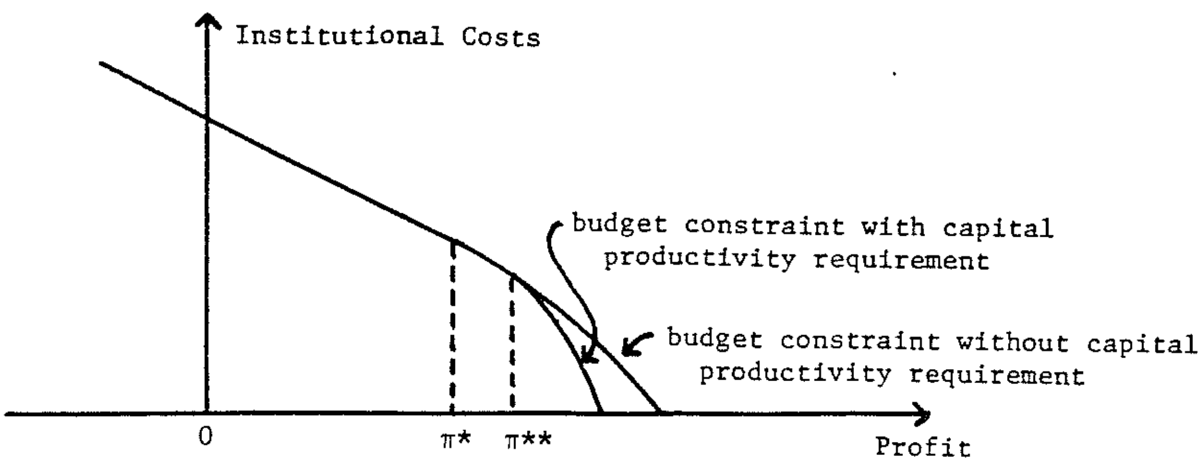

Figure 27
Effect of Capital Productivity Requirement Less Than the Cost of Capital Given an Allowed Rate of Return Greater than the Cost of Capital

If one were to increase the capital productivity requirement, π^{**} would shift leftward, while the budget constraint would drop lower to the right of π^{**}. One would expect the constraint to become steeper to the right of π^{**} as well, as increasingly large increases in gas throughput become necessary to justify a given increase in capital investment, and hence a given increase in profit. This latter result, however, I have only been able to prove for a homogeneous production function with non-decreasing returns to scale. The difficulty stems from the fact that increasing the capital productivity requirement means increasing the gas throughput at any given level of capital investment. Therefore, if the firm had a decreasing returns to scale production function, it is conceivable that this larger volume of gas throughput could produce an offsetting tendency for smaller increases in gas throughput to justify a given increase in capital investment. The non-decreasing returns to scale production function assumption is, however, not terribly restrictive, since increasing returns to scale provides the rationale for regulating gas firms to begin with.

If the regulators were to set the capital productivity requirement greater than the cost of capital, the budget constraint would be everywhere affected, since capital productivity would be everywhere less than or equal to the cost of capital without the capital productivity requirement. In order to satisfy the requirement, the firm must use a more costly mix of capital and other inputs at any given level of profit. As a result, the constraint shifts downward everywhere. Figure 28 illustrates the general shape of the new budget constraint.

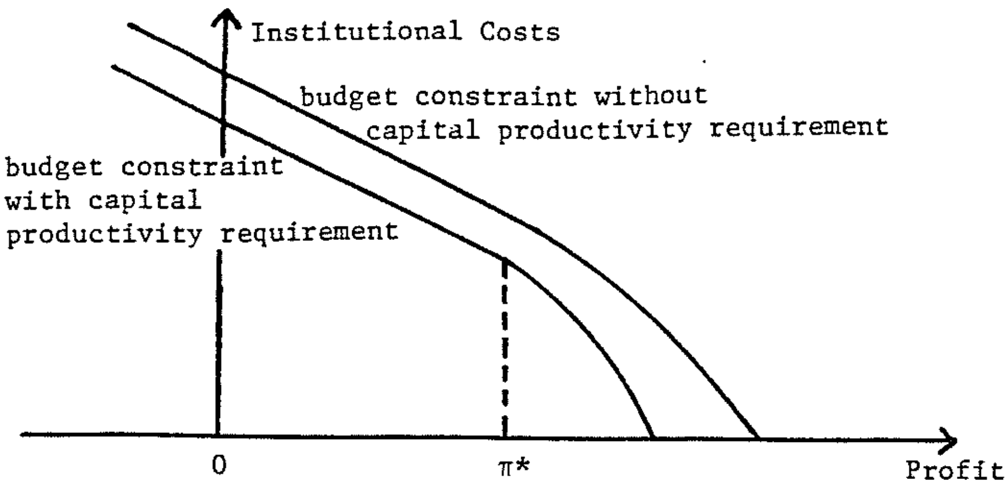

Figure 28
Effect of a Capital Productivity Requirement Greater Than the Cost of Capital Given an Allowed Rate of Return Greater Than the Cost of Capital

If one were to increase the capital productivity requirement, the budget constraint would continue to drop lower. At values of profit greater than π^*, where the rate-of-return constraint is binding, one would also expect the budget constraint to become steeper, as increasingly large increases in gas throughput become necessary to justify a given increase in capital investment, and hence a given increase in profit. But, again, I can only prove this for a homogeneous production function with increasing returns to scale. At values of profit less than π^* the slope of the budget constraint is unaffected by an increase in the capital productivity requirement. This is because the capital productivity requirement would not affect the profit/institutional costs tradeoff the managers would face in the absence of a rate-of-return constraint. Since the capital productivity requirement does alter the amount of capital the firm would choose to invest in the absence of a rate-of-return constraint, which in turn determines π^*, π^* itself may be affected by an increase in the capital productivity requirement. The direction of the change cannot, in general, be determined.

The situation when the allowed rate-of-return is less than the cost of capital should also be considered. Recall that in this case π^* is negative, since the firm is required to lose money if it invests the amount of capital it would invest without rate-of-return regulation. The firm can reduce its losses below π^* only by reducing its capital investment below the amount it would invest if unregulated, until losses are reduced to zero when capital investment reaches zero. Figure 29 illustrates the budget constraint when the allowed rate-of-return is less than the cost of capital and there is no capital productivity

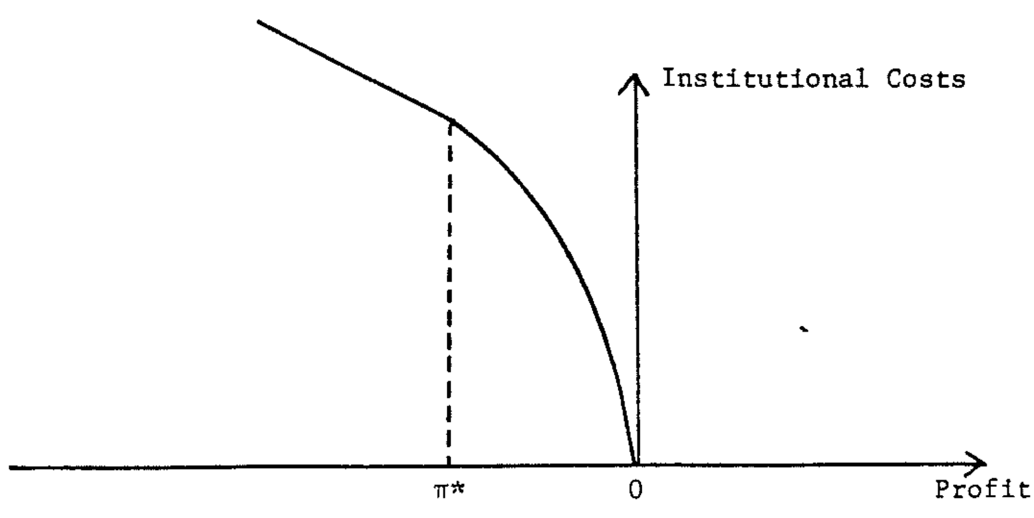

Figure 29
Budget Constraint of a Rate-of-Return Regulated Firm
with an Allowed Rate of Return Less Than the Cost of Capital

requirement. At values of profit less than π^*, capital productivity equals the cost of capital, as before. At values of profit greater than π^*, capital investment is being reduced, and hence capital productivity is increasing.

Now suppose the regulators set a capital productivity requirement less than the cost of capital. This requirement would automatically be met by the firm, since the firm would always choose a capital productivity greater than or equal to the cost of capital. Hence, the requirement will have no impact on the firm's budget constraint.

Suppose, then, that the regulators set the capital productivity requirement greater than the cost of capital. The budget constraint will certainly be affected at values of profit less than π^*, where capital productivity would otherwise equal the cost of capital. The budget constraint will also be affected at values of profit greater than π^*, but less than some π^{**}. At π^{**} capital productivity would have risen high enough to meet the requirement.

At values of profit less than π^*, the firm must use a more costly mix of capital and other inputs under the capital productivity requirement, hence the budget constraint shifts downward. At values of profit greater than π^* but less than π^{**}, the locus must also shift downward. However, the amount of the shift diminishes as one moves toward π^{**}, as the difference between the capital productivity the firm would choose without the requirement and the required capital productivity diminishes. At values of profit greater than π^{**}, capital productivity would have been at least as large as required anyway, and the requirement has no effect. The new budget constraint is illustrated in Figure 30.

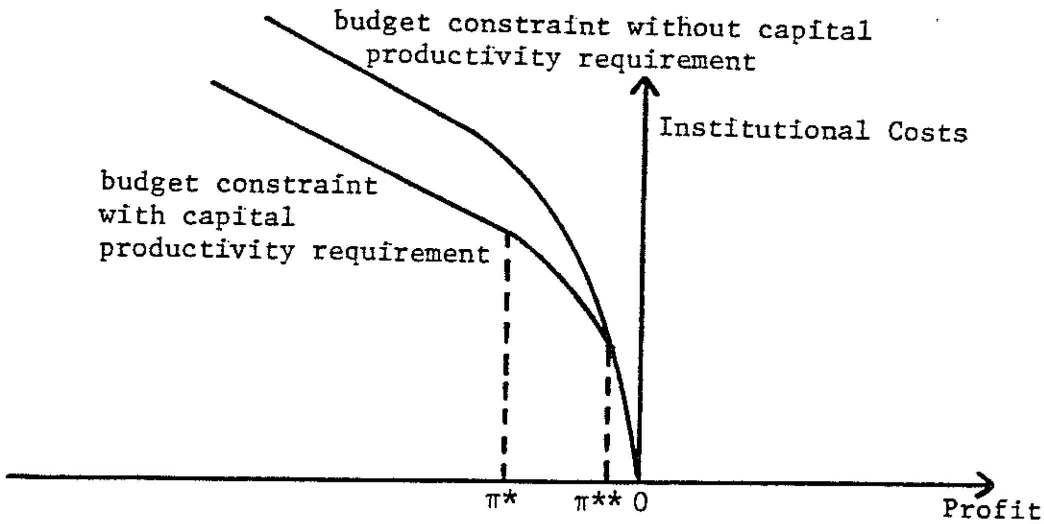

Figure 30
Effect of Capital Productivity Requirement Greater Than the Cost of Capital Given an Allowed Rate of Return Less Than the Cost of Capital

If one were to increase the capital productivity requirement, π^{**} would shift rightward, while the budget constraint would drop lower at values of profit less than π^{**}. For values of profit less than π^*, the slope of the constraint would again be unaffected. For values of profit between π^* and π^{**}, the slope of the constraint would probably become less negative, as increasingly large reductions in gas throughput become possible with a given reduction in capital investment, and hence a given decrease in losses. However, I can only prove this for a homogeneous production function with increasing returns to scale.

Finally, consider the case of an allowed rate-of-return equal to the cost of capital. Recall that, in the absence of a capital productivity requirement, the budget constraint would be vertical at the institutional cost axis. This is because it is impossible for the firm to earn a positive profit in this case, but since the rate-of-return constraint is not binding at values of profit less than or equal to zero, the managers could operate as they would in the absence of regulation over that interval. The budget constraint is therefore as shown in Figure 31.

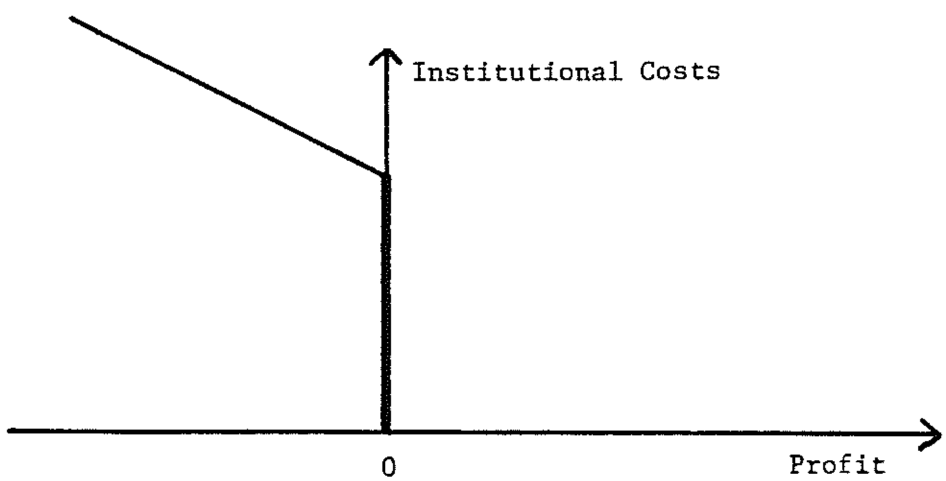

Figure 31
Budget Constraint of a Rate-of-Return Regulated Firm
With an Allowed Rate of Return Equal to the Cost of Capital

Since, in the absence of regulation, the managers would set capital productivity equal to the cost of capital, a capital productivity requirement less than or equal to the cost of capital would have no impact on this budget constraint. A capital productivity requirement greater than the cost of capital would produce a downward shift in the budget constraint, as the managers are forced to use a more costly mix of capital and other inputs. The slope of the budget constraint would be unaffected, since the capital productivity requirement would not affect the profit/institutional costs tradeoff in the absence of the rate-of-return constraint. Figure 32 illustrates how a capital productivity requirement greater than the cost of capital would shift the budget constraint.

Table 2 summarizes the results of this section.

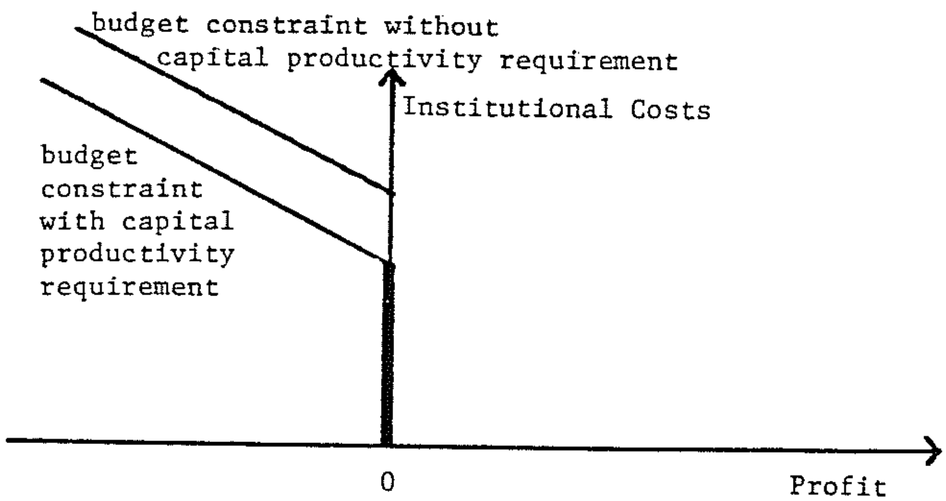

Figure 32
Effect of a Capital Productivity Requirement Greater Than the Cost of Capital Given an Allowed Rate of Return Equal to the Cost of Capital

TABLE 2

EFFECT OF AN INCREASE IN THE REQUIRED CAPITAL PRODUCTIVITY
ON THE FIRM'S BUDGET CONSTRAINT

Relationship of Allowed Rate of Return to Cost of Capital	Relationship of Required Capital Productivity to Cost of Capital	Effect on Position of Budget Constraint	Effect on Slope of Budget Constraint*
greater than	less than or equal to	shifts downward if $\pi > \pi^{**}$; no effect if $\pi \leq \pi^{**}$	more negative if $\pi > \pi^{**}$; no effect if $\pi \leq \pi^{**}$
greater than	greater than	shifts downward everywhere	more negative if $\pi > \pi^{*}$; no effect if $\pi \leq \pi^{*}$
less than	less than or equal to	no effect	no effect
less than	greater than	shifts downward if $\pi < \pi^{**}$; no effect if $\pi \geq \pi^{**}$	less negative if $\pi^{*} < \pi < \pi^{**}$; no effect if $\pi \leq \pi^{*}$ or $\pi \geq \pi^{**}$
equal to	less than or equal to	no effect	no effect
equal to	greater than	shifts downward everywhere	no effect

*Assumes homogeneous production function with non-decreasing returns to scale.

4.1.M Mathematical Formulation

This subsection presents a mathematical formulation of the model discussed above. I will show how the budget constraint is derived, and that it has the shape described above, as well as the responses to increases in the required capital productivity shown in Table 2. I will show how a capital productivity requirement could lead utility-maximizing managers to purchase gas with a marginal cost greater than its marginal revenue product. I will also show how the utility-maximizing solution for the managers, subject to a capital productivity requirement, continues to be at the point of tangency of the budget constraint in π - x space and an isoutility curve. This subsection uses the basic notation and assumptions which were introduced in Section 2.2.M, with the addition of the variable β for required capital productivity.

The budget constraint in π - x space is defined in this subsection as the largest value of x which the firm could achieve at each value of π, where the firm is subject to the rate-of-return constraint and the capital productivity requirement. The budget constraint in π - x space may be obtained by maximizing

$$\left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi$$

subject to the rate-of-return constraint

$$(s-r)k - \pi \geq 0,$$

and the capital productivity requirement

$$-z_1(k,g) - \beta \geq 0.$$

Maximization is over g and k, where both must be non-negative. The Lagrangian will be

$$L = \big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi$$

$$+ \lambda_2\big((s-r)k - \pi\big) - \lambda_3\big(z_1(k,g) + \beta\big).$$

The first-order conditions require that:

$$\frac{\partial L}{\partial g} = p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)$$

$$- \lambda_3 z_{12}(k,g) \leq 0;$$

$$\frac{\partial L}{\partial g} g = 0; \qquad g \geq 0;$$

$$\frac{\partial L}{\partial k} = -z_1(k,g) - r + \lambda_2(s-r) - \lambda_3 z_{11}(k,g) \leq 0; \qquad (4.1.M/2)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda_2} = (s-r)k - \pi \geq 0; \qquad (4.1.M/3)$$

$$\frac{\partial L}{\partial \lambda_2}\lambda_2 = 0; \qquad \lambda_2 \geq 0;$$

$$\frac{\partial L}{\partial \lambda_3} = -z_1(k,g) - \beta \geq 0; \qquad (4.1.M/4)$$

$$\frac{\partial L}{\partial \lambda_3}\lambda_3 = 0; \qquad \lambda_3 \geq 0.$$

I shall assume that g and k are greater than zero at the solution, which implies that (4.1.M/1) and (4.1.M/2) must be equalities.

The following theorem is a result introduced in Chapter 3, which easily transfers to the firm with utility-maximizing managers. It shows how a capital productivity requirement might lead the firm managers to choose to purchase gas with a marginal cost greater than its marginal revenue product.

Theorem 4.1.M.A: At all points on the budget constraint in π - x space for which $\lambda_3 \neq 0$, the marginal cost of gas exceeds its marginal revenue product.

Proof: By (4.1.M/1),

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = \lambda_3 z_{12}(k,g).$$

The term on the left is the marginal revenue product of gas $p'(g)g + p(g) - z_2(k,g)$ minus its marginal cost $f'(g)g + f(g)$. The term on the right is always negative, since $\lambda_3 > 0$ by assumption and $z_{12}(k,g) < 0$ by assumption (see Section 2.5.M). So the marginal cost of gas must exceed its marginal revenue product.

Q.E.D.

The remaining theorems of this section are more easily proven given the set of lemmas which follow. The first two lemmas deal with capital productivity on the budget constraint in π - x space when the capital productivity requirement is non-binding. They will be useful later in exploring how a capital productivity requirement might alter the budget constraint in π - x space.

Lemma 4.1.A: If the capital productivity requirement is non-binding, then the capital productivity $-z_1(k,g)$ at any point on the budget constraint in π - x space with $\pi \leq \pi^*$ will equal r.

Proof: By Lemma 2.2.A, there are unique values for g and k on the budget constraint in π - x space of the unregulated firm, which I have called g^* and k^*. If the capital productivity requirement is assumed non-binding, and if $\pi < \pi^*$, then g^* and k^* are also feasible solutions for the firm under a rate-of-return constraint. To see this, note that if $\pi \leq \pi^*$ then

$$(s-r)k^* - \pi \geq (s-r)k^* - \pi^*.$$

Since the right side of this inequality is equal to zero by Lemma 2.2.K, g^* and k^* satisfy the rate-of-return constraint (4.1.M/3). Hence, g^* and k^* must be the unique values of g and k on the budget constraint in π - x space of the rate-of-return regulated firm as well. But by (2.2.M.1/2), $-z_1(k^*,g^*) = r$, as claimed.

Q.E.D.

Lemma 4.1.B: If the capital productivity requirement is non-binding, then capital productivity $-z_1(k,g)$ at points on the budget constraint in π - x space with $\pi > \pi^*$ declines monotonically with increases in π if $s > r$ and increases monotonically with increases in π if $s < r$.

Proof: If the capital productivity requirement is non-binding, then (4.1.M/4) is satisfied by assumption and λ_3 must equal zero. In this case (4.1.M/1) - (4.1.M/3) become identical to (2.2.M.2/1) - (2.2.M.2/3), and the lemmas of Section 2.2.M.2 apply. By Lemma 2.2.L, if $\pi > \pi^*$ the rate-of-return constraint must be an equality. So by Lemma 2.2.F,

$$\frac{d}{d\pi}\left(-z_1(k,g)\right) < 0$$

if $s > r$ and

$$\frac{d}{d\pi}\left(-z_1(k,g)\right) > 0$$

if $s < r$ at points on the budget constraint in π - x space with $\pi > \pi^*$. This shows that capital productivity on the budget constraint in π - x space with $\pi > \pi^*$ declines monotonically with increases in π if $s > r$ and increases monotonically with increases in π if $s < r$, as claimed.

Q.E.D.

For notational convenience, define the budget constraint in π - x space, subject to a capital productivity requirement, to be

$$x = B(\pi, s, \beta).$$

It will be useful throughout the remainder of this chapter to know some of the partials of $B(\pi,s,\beta)$, given that the rate-of-return constraint and the capital productivity requirement are equalities with λ_2 and λ_3 greater than zero. The following series of lemmas gives the needed partials. Note first, however, that the rate-of-return constraint (4.1.M/3) is identical to the rate-of-return constraint (2.2.M.2/3).

Hence, Lemmas 2.2.B - 2.2.E, which are derived using only this constraint, also apply to a firm subject to a capital productivity requirement.

Lemma 4.1.C: If the capital productivity requirement is an equality, then

$$\frac{dg}{dk} = \frac{-z_{11}(k,g)}{z_{12}(k,g)} .$$

This is always positive.

Proof: If the capital productivity requirement is an equality, then (4.1.M/4) is an equality. Totally differentiating yields

$$\frac{dg}{dk} = \frac{-z_{11}(k,g)}{z_{12}(k,g)} .$$

By (2.2.M/7), $-z_{11}(k,g) = R_{11}(k,g)$, which is negative by the strict concavity of R(k,g). I have assumed that $z_{12}(k,g) < 0$ (see Section 2.5.M). Hence,

$$\frac{dg}{dk} > 0.$$

Q.E.D.

Lemma 4.1.D: If the rate-of-return constraint and the capital productivity requirement are equalities and s is not equal to r, then

$$\frac{dg}{d\pi} = \frac{-z_{11}(k,g)}{z_{12}(k,g)}\left(\frac{1}{s-r}\right) .$$

This will be positive if $s > r$ and negative if $s < r$.

Proof: By the chain rule,

$$\frac{dg}{d\pi} = \frac{dg}{dk}\frac{dk}{d\pi}.$$

Using Lemmas 4.1.C and 2.2.B to substitute for dg/dk and dk/dπ yields

$$\frac{dg}{d\pi} = \frac{-z_{11}(k,g)}{z_{12}(k,g)}\left(\frac{1}{s-r}\right) .$$

Since, by Lemma 4.1.C,

$$\frac{dg}{dk} = \frac{-z_{11}(k,g)}{z_{12}(k,g)} > 0,$$

it must be that $dg/d\pi > 0$ if $s > r$ and $dg/d\pi < 0$ if $s < r$.

Q.E.D.

Lemma 4.1.E: If the rate-of-return constraint is an equality and s is not equal to r, then

$$\frac{dk}{d\beta} = 0 .$$

Proof: If the rate-of-return constraint is an equality and s is not equal to r, (4.1.M/3) is an equality, determining k as a function of exogenous variables s, r, and π alone. Hence, it must be that

$$\frac{dk}{d\beta} = 0.$$

Q.E.D.

Lemma 4.1.F: If the rate-of-return constraint and capital productivity requirement are equalities with s not equal to r, then

$$\frac{dg}{d\beta} = \frac{-1}{z_{12}(k,g)} > 0.$$

Proof: If the capital productivity requirement is an equality, then (4.1.M/4) is an equality. Differentiating yields

$$-z_{11}(k,g)\,\frac{dk}{d\beta} - z_{12}(k,g)\,\frac{dg}{d\beta} = 1.$$

By the preceding lemma, $dk/d\beta = 0$ if the capital productivity requirement is an equality with s not equal to r. So

$$\frac{dg}{d\beta} = \frac{-1}{z_{12}(k,g)}\ .$$

Since I assume that $z_{12}(k,g) < 0$ (see Section 2.5.M), this expression is positive.

Q.E.D.

Lemma 4.1.G: If the rate-of-return constraint is an equality and s is not equal to r, then

$$\frac{d^2k}{d\pi d\beta} = 0.$$

Proof: By Lemma 2.2.B, if the rate-of-return constraint is an equality then

$$\frac{dk}{d\pi} = \frac{1}{s-r}.$$

Taking the derivative with respect to β yields the desired result.

Q.E.D.

<u>Lemma 4.1.H</u>: If the underlying production function $g = h(z,k)$ is homothetic, and the capital productivity requirement is an equality then

$$\frac{dz}{dk} = \frac{z}{k} .$$

Proof: One can express the capital productivity requirement $-z_1(k,g) = \beta$ in terms of the underlying production function $g = h(z,k)$. To do so, I differentiate the production function with g held fixed to obtain

$$\beta = \frac{\partial z}{\partial k} = \frac{-h_2(z,k)}{h_1(z,k)} .$$

If $h(z,k)$ is homothetic then the term on the right, call it $v(z,k)$ is homogeneous of degree zero. Totally differentiating $v(z,k) = \beta$ yields an expression for dz/dk,

$$\frac{dz}{dk} = \frac{-v_2(z,k)}{v_1(z,k)} .$$

But, by Euler's Theorem, since $v(z,k)$ is homogeneous of degree zero,

$$v_1(z,k)z + v_2(z,k)k = 0,$$

or

$$\frac{z}{k} = \frac{-v_2(z,k)}{v_1(z,k)} \text{ .}$$

Hence,

$$\frac{dz}{dk} = \frac{z}{k} \text{ .}$$

Q.E.D.

Lemma 4.1.I: Assume that the gas firm's underlying production function $g = h(z,k)$ is homogeneous with non-decreasing returns to scale and that inputs have positive marginal products. Assume also that the rate-of-return constraint and the capital productivity requirements are equalities. Then

$$\frac{d^2 g}{d\pi d\beta} > 0$$

if $s > r$ and

$$\frac{d^2 g}{d\pi d\beta} < 0$$

if $s < r$.

Proof: By the chain rule and Lemma 2.2.B,

$$\frac{d^2 g}{d\pi d\beta} = \frac{d}{d\beta}\left(\frac{dg}{d\pi}\right) = \frac{d}{d\beta}\left(\frac{dg}{dk}\,\frac{dk}{d\pi}\right) = \frac{d}{d\beta}\left(\frac{dg}{dk}\right)\,\frac{1}{s-r} \text{ .}$$

If dg/dk is a function of z and k, then I can write,

$$\frac{d}{d\beta}\left(\frac{dg}{dk}\right)\,\frac{1}{s-r} = \frac{\partial}{\partial z}\left(\frac{dg}{dk}\right)\,\frac{dz}{d\beta}\,\frac{1}{s-r} + \frac{\partial}{\partial k}\,\left(\frac{dg}{dk}\right)\frac{dk}{d\beta}\,\frac{1}{s-r}.$$

But, by Lemma 4.1.E, $dk/d\beta = 0$, so this simplifies to,

$$\frac{d^2g}{d\pi d\beta} = \frac{\partial}{\partial z}\left(\frac{dg}{dk}\right) \frac{dz}{d\beta} \frac{1}{s-r} . \qquad (4.1.M/5)$$

One can obtain an expression for dg/dk as a function of z and k by differentiating the production function $g = h(z,k)$,

$$\frac{dg}{dk} = h_2(z,k) + h_1(z,k) \frac{dz}{dk} .$$

Since the underlying production function is homothetic and the capital productivity requirement is an equality I can substitute for dz/dk using Lemma 4.1.H,

$$\frac{dg}{dk} = h_2(z,k) + h_1(z,k) \frac{z}{k} .$$

Taking the partial of dg/dk with respect to z,

$$\frac{\partial}{\partial z}\left(\frac{dg}{dk}\right) = h_{12}(z,k) + h_{11}(z,k) \frac{z}{k} + h_1(z,k) \frac{1}{k} .$$

If $h(z,k)$ is homogeneous of degree ≥ 1, that is, with non-decreasing returns to scale, then $h_1(z,k)$ is homogeneous of degree ≥ 0. By Euler's Theorem,

$$h_{11}(z,k)z + h_{12}(z,k)k \geq 0.$$

Since inputs are assumed to have positive marginal products, $h_1(z,k) > 0$. Hence,

$$\frac{\partial}{\partial z}\left(\frac{dg}{dk}\right) > 0.$$

Now,

$$\frac{dz}{d\beta} = z_1(k,g)\,\frac{dk}{d\beta} + z_2(k,g)\,\frac{dg}{d\beta}\,.$$

By Lemma 4.1.E, $dk/d\beta = 0$; by (2.2.M/3), $z_2(k,g) > 0$ since I assume $h_1(z,k) > 0$; by Lemma 4.1.F, $dg/d\beta > 0$. Hence $dz/d\beta > 0$.

So I have shown that $d^2g/d\pi d\beta$, as expressed in (4.1.M/5) is positive if $s > r$ and negative if $s < r$.

Q.E.D.

Lemma 4.1.J: If the rate-of-return constraint and capital productivity requirement are equalities with λ_2 and λ_3 greater than zero and s is not equal to r, then

$$B_1(\pi,s,\beta) < -1.$$

Proof: By definition,

$$B(\pi,s,\beta) = \big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi,$$

so

$$B_1(\pi,s,\beta) = [p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{}$$

$$-[z_1(k,g) + r]\frac{dk}{d\pi} - 1.$$

Substituting for $dg/d\pi$ and $dk/d\pi$ using Lemmas 4.1.D and 2.2.B yields

$$B_1(\pi,s,\beta) = -[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)] \frac{z_{11}(k,g)}{z_{12}(k,g)} \left(\frac{1}{s-r}\right)$$

$$- [z_1(k,g) + r]\frac{1}{s-r} - 1.$$

But if g and k lie on the budget constraint in π - x space (4.1.M/1) and (4.1.M/2) may be solved simultaneously to eliminate λ_3 yielding

$$-[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)] \frac{z_{11}(k,g)}{z_{12}(k,g)} \left(\frac{1}{s-r}\right)$$

$$- [z_1(k,g) + r] \frac{1}{s-r} = - \lambda_2,$$

so

$$B_1(\pi,s,\beta) = -\lambda_2 - 1.$$

Since I assume $\lambda_2 > 0$, I have shown $B_1(\pi,s,\beta) < -1$.

Q.E.D.

Lemma 4.1.K: If both the rate-of-return constraint and the capital productivity requirement are equalities with λ_2 and λ_3 greater than zero and s is not equal to r then

$$B_3(\pi,s,\beta) < 0.$$

Proof: By definition,

$$B(\pi,s,\beta) = \big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi,$$

so

$$B_3(\pi,s,\beta) = [p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{d\beta}$$

$$- [z_1(k,g) + r]\frac{dk}{d\beta} \; . \qquad (4.1.M/6)$$

By Lemma 4.1.E, $dk/d\beta = 0$, while, by Lemma 4.1.F, $dg/d\beta > 0$. By (4.1.M/1),

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = \lambda_3 z_{12}(k,g),$$

which is negative since I assume $\lambda_3 > 0$ and $z_{12}(k,g) < 0$ (see Section 2.5.M). Hence, $B_3(\pi,s,\beta) < 0$.

Q.E.D.

Lemma 4.1.L: Assume that the underlying production function $g = h(z,k)$ is homogeneous with non-decreasing returns to scale, and that inputs have positive marginal products. Assume also that the return constraint and capital productivity requirement are equalities with λ_2 and λ_3 greater than zero. Then

$$B_{13}(\pi,s,\beta) < 0$$

if $s > r$ and

$$B_{13}(\pi,s,\beta) > 0$$

if $s < r$.

Proof: From (4.1.M/6),

$$B_3(\pi,s,\beta) = [p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{d\beta}$$

$$- [z_1(k,g) + r]\frac{dk}{d\beta} ,$$

so

$$B_{13}(\pi,s,\beta) = \frac{\partial}{\partial g}[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{d\pi}\frac{dg}{d\beta}$$

$$- z_{12}(k,g)\frac{dk}{d\pi}\frac{dg}{d\beta}$$

$$+ [p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{d^2g}{d\pi d\beta}$$

$$- z_{12}(k,g)\frac{dg}{d\pi}\frac{dk}{d\beta} - z_{11}(k,g)\frac{dk}{d\pi}\frac{dk}{d\beta} - [z_1(k,g)+r]\frac{d^2k}{d\beta d\pi}. \quad (4.1.M/7)$$

Consider the first two terms together. Using Lemmas 4.1.D, 4.1.F, and 2.2.B, these may be written as

$$\frac{\partial}{\partial g}[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{1}{s-r}\right)\left(\frac{1}{z_{12}(k,g)}\right)$$

$$+ z_{12}(k,g)\left(\frac{1}{s-r}\right)\left(\frac{1}{z_{12}(k,g)}\right)$$

$$= \frac{\partial}{\partial g}[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{z_{11}(k,g)}{(z_{12}(k,g))^2}\left(\frac{1}{s-r}\right) + \frac{1}{s-r}.$$

Substituting, using (2.2.M/7) - (2.2.M/9), yields

$$\left[\frac{-R_{22}(k,g)\ R_{11}(k,g)}{(R_{12}(k,g))^2} + 1\right]\frac{1}{s-r}.$$

Since

$$R_{11}(k,g)\, R_{22}(k,g) - \left(R_{12}(k,g)\right)^2 > 0$$

by the strict concavity of the revenue function, the term in brackets will be negative. This implies the first two terms of (4.1.M/7) are negative if $s > r$ and positive if $s < r$.

Consider, then, the third term. By (4.1.M/1),

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = \lambda_3 z_{12}(k,g),$$

which is negative since $\lambda_3 > 0$ by assumption and $z_{12}(k,g) < 0$ by assumption (see Section 2.5.M). By Lemma 4.1.I,

$$\frac{d^2g}{d\pi d\beta} > 0$$

if $s > r$ and

$$\frac{d^2g}{d\pi d\beta} < 0$$

if $s < r$. Hence, the third term of (4.1.M/7) is also negative if $s > r$ and positive if $s < r$.

The last three terms of (4.1.M/7) equal zero, since, by Lemma 4.1.E, $dk/d\beta = 0$ and, by Lemma 4.1.G,

$$\frac{d^2k}{d\pi d\beta} = 0.$$

So $B_{13}(\pi,s,\beta) < 0$ if $s > r$ and $B_{13}(\pi,s,\beta) > 0$ if $s > r$.

Q.E.D.

The final lemma in this series provides a useful property of π^*.

Lemma 4.1.M: If $\beta \leq r$, then the value of π^* is the same as it would be in the absence of a capital productivity requirement.

Proof: π^* was defined in Section 2.2.M.2 to be the largest profit the firm could make under the rate-of-return constraint given the capital investment on the budget constraint in π - x space in the absence of the rate-of-return constraint. In Lemma 2.2.A, I showed that if there were no rate-of-return constraint or capital productivity requirement, there would be unique values for g and k on the budget constraint in π - x space, which I called g^* and k^*. By (2.2.M.1/2), capital productivity at this solution $-z_1(k^*,g^*)$ equals r. Hence, this solution remains feasible under a capital productivity requirement of $\beta < r$. It follows that g^* and k^* must be the unique values of g and k under this capital productivity requirement as well. Given capital investment k^*, the largest profit the firm could earn under the rate-of-return constraint may be determined from (4.1.M/3):

$$\pi^* = (s-r)k^*.$$

This value for π^* is the same as that specified by Lemma 2.2.K for the firm in the absence of a capital productivity requirement.

Q.E.D.

4.1.M.1 The Case of $s > r$ and $\beta < r$

Having laid out the necessary lemmas, I will now use them to explore how a capital productivity requirement affects the budget constraint in π - x space. In this subsection, I consider the case of $s > r$ and $\beta \leq r$. Note that, by Lemma 4.1.M, π^* is unaffected by the capital productivity requirement in this case.

Theorem 4.1.M.A: Assume $s > r$, $\beta \leq r$, and that the capital productivity requirement affects the budget constraint in π - x space. Then there exists a $\pi^{**} \geq \pi^*$ such that the budget constraint in π - x space is unaffected by the capital productivity requirement at all values of $\pi \leq \pi^{**}$.

Proof: By Lemma 4.1.A, if the capital productivity requirement is not binding, then capital productivity $-z_1(k,g)$ at any point on the budget constraint in π - x space with $\pi \leq \pi^*$ will equal r. Hence, the introduction of a capital productivity requirement of $\beta \leq r$ will have no impact on the budget constraint in π - x space if $\pi \leq \pi^*$. By Lemma 4.1.B, if the capital productivity requirement is not binding, then $-z_1(k,g)$ at points on the budget constraint in π - x space with $\pi > \pi^*$ declines monotonically with increases in π if $s > r$. Clearly, $-z_1(k,g)$ must drop below β at some point if the capital productivity requirement is to have any impact on the budget constraint in π - x space. Let π^{**} be the value of π at which $-z_1(k,g) = \beta$, with π^{**} defined to equal π^* if $\beta = r$. Since $-z_1(k,g) \geq \beta$ at all values of $\pi \leq \pi^{**}$ in the absence of a capital productivity requirement, the requirement will not affect the budget constraint in π - x space at values of $\pi \leq \pi^{**}$.

Q.E.D.

<u>Theorem 4.1.M.1.B</u>: If $s > r$ and $\beta < r$, then an increase in β reduces the value of π^{**}.

Proof: By Lemma 4.1.A, if the capital productivity requirement is not binding, then the capital productivity $-z_1(k,g)$ on the budget constraint in π - x space at $\pi = \pi^*$ equals r. By Lemma 4.1.B, if $s > r$ this $-z_1(k,g)$ declines monotonically with increases in π over the interval $\pi > \pi^*$. Hence, if $\beta < r$, an increase in β reduces the value of π at which $-z_1(k,g) = \beta$, which is π^{**}.

Q.E.D.

I now make the additional assumption that if $s > r$, $\beta \leq r$ and $\pi > \pi^{**}$, then λ_2 and λ_3 are greater than zero at points on the budget constraint in π - x space. As the following theorem demonstrates, both the rate of return constraint and the capital productivity requirement affect the values of g or k on the budget constraint in π - x space if $\pi > \pi^{**}$. One would, therefore, normally expect the associated Lagrange multipliers λ_2 and λ_3 to be positive. I have, however, been unable to rule out the possibility of a pathological solution. So I must assume that λ_2 and λ_3 are indeed positive.

<u>Theorem 4.1.M.1.C</u>: If $s > r$, $\beta \leq r$, and $\pi > \pi^{**}$, and one were to drop either the rate-of-return constraint or the capital productivity

requirement, the value of g or k at points on the budget constraint in π - x space would change.

Proof: Consider, first, dropping the rate-of-return constraint, leaving only the capital productivity requirement. The new solution in this case may be found by first noting that if there were neither a rate-of-return constraint nor a capital productivity requirement, then, by Lemma 2.2.A there are unique values for g and k on the budget constraint in π - x space, which I have called g* and k*. By (2.2.M.1/2), $-z_1(k^*,g^*) = r$, so the addition of a capital productivity requirement of $\beta \leq r$ does not affect the feasibility of this solution. Hence, g* and k* must be the optimum values for the firm subject to this capital productivity requirement as well. But this solution is not feasible under the rate-of-return constraint. For if $\pi > \pi^{**} \geq \pi^*$, then,

$$(s-r)k^* - \pi < (s-r)k^* - \pi^*.$$

Since the right side of this inequality equals zero by Lemma 2.2.K, this is a contradiction of the rate-of-return constraint, (4.1.M/3). So the value of k must change if one were to drop the rate-of-return constraint.

Now consider dropping the capital productivity requirement, leaving only the rate-of-return constraint. By definition of π^{**}, capital productivity, $-z_1(k,g)$ equals β at $\pi = \pi^{**}$ on the budget constraint in π - x space in the absence of a capital productivity requirement. By Lemma 4.1.B, $-z_1(k,g)$ would therefore be less than β at values of

$\pi > \pi^{**}$ in the absence of the capital productivity requirement. Since this solution is infeasible if there is a requirement that $-z_1(k,g)$ be greater than or equal to β, the values of g or k must change if one were to drop the capital productivity requirement.

Q.E.D.

Having offered some justification for my assumption that λ_2 and λ_3 are greater than zero if $\pi > \pi^{**}$, the following two theorems demonstrate how the budget constraint in π - x space would be affected by a change in β if $\pi > \pi^{**}$.

Theorem 4.1.M.D: If $s > r$, $\beta \leq r$ and $\pi > \pi^{**}$ with λ_2 and λ_3 greater than zero, then an increase in β shifts the budget constraint in π - x space downward.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. So by Lemma 4.1.K,

$$B_3(\pi,s,\beta) < 0$$

if $s > r$. This shows that an increase in β shifts the budget constraint in π - x space downward.

Q.E.D.

Theorem 4.1.M.1.E: Assume that $s > r$, $\beta < r$, and $\pi > \pi^{**}$ with λ_2 and λ_3 greater than zero. Assume also that the underlying production function is homogenous with non-decreasing returns to scale and that inputs have positive marginal products. Then an increase in β makes the budget constraint in π - x space more negatively sloped.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 4.1.J,

$$B_1(\pi, s, \beta) < -1,$$

indicating that the slope of the budget constraint in π - x space is negative. By Lemma 4.1.L, if the underlying production function is homogeneous with non-decreasing returns to scale and inputs have positive maginal products, then

$$B_{13}(\pi, s, \beta) < 0,$$

if $s > r$. This indicates that the budget constraint in π - x space becomes more negatively sloped as β increases.

Q.E.D.

4.1.M.2 The Case of $s > r$ and $\beta > r$

The following subsection explores how a capital productivity requirement affects the budget constraint in π - x space when $s > r$ and $\beta > r$.

Lemma 4.1.N: The values of g and k on the firm's budget constraint in π - x space in the absence of a rate-of-return constraint must be independent of π.

Proof: In the absence of a rate-of-return constraint, the budget constraint in π - x space may be obtained by maximizing

$$\left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi$$

subject to the capital productivity requirement

$$-z_1(k,g) - \beta \geq 0.$$

Maximization is over g and k. Since π does not appear in the capital productivity requirement and appears only as an additive constant in the objective function, the optimum values of g and k must be independent of π.

Q.E.D.

I assume g' and k' to be the unique values of g and k on the firm's budget constraint in π - x space in the absence of a rate of return constraint. I call this an assumption, rather than a definition, since I have been unable to rule out the possibility of multiple optima.

Recall that π^* was defined to be the largest profit the firm could make under the rate-of-return constraint given the capital investment on the budget constraint in π - x space in the absence of the rate-of-return constraint.

<u>Lemma 4.1.0</u>: $\pi^* = (s-r)k'$.

Proof: I assume g' and k' to be the unique values of g and k on the firm's budget constraint in π - x space. Given capital investment k', the largest profit the firm could make under the rate-of-return constraint may be determined from (4.1.M/3)

$$\pi^* - (s-r)k'. \qquad (4.1.M.2/1)$$

Q.E.D.

<u>Lemma 4.1.P</u>: If $\pi \leq \pi^*$, then the values of g and k on the budget constraint in π - x space are the same as they would be in the absence of a rate-of-return constraint. That is $g = g'$ and $k = k'$.

Proof: If $\pi \leq \pi^*$, then by (4.1.M.2/1)

$$\pi \leq (s-r)k'.$$

This indicates that g' and k' are feasible values of g and k under the rate-of-return constraint. Since g' and k' are assumed to be the unique values of g and k on the firm's budget constraint in π - x space in the absence of a rate-of-return constraint, they must also be the unique values under the rate-of-return constraint.

Q.E.D.

<u>Lemma 4.1.Q</u>: $\lambda_2 = 0$ at points on the budget constraint in π - x space with $\pi \leq \pi^*$.

Proof: By Lemma 4.1.P, if $\pi \leq \pi^*$ then $g = g'$ and $k = k'$ on the budget constraint in π - x space. If $\pi < \pi^*$, then by (4.1.M.2/1)

$$\pi < (s-r)k'.$$

By (4.1.M/3) this implies $\lambda_2 = 0$ over this range. But if the solution with $\lambda_2 = 0$ satisfies (4.1.M/1) - (4.1.M/4) with $\pi < \pi^*$, it must also satisfy these equations with $\pi = \pi^*$. This is because only (4.1.M/3) is a function of π, and it is satisfied by $k = k'$ and $\pi = \pi^*$ by (4.1.M.2/1).

Q.E.D.

<u>Lemma 4.1.R</u>: If $\beta \geq r$, the capital productivity requirement (4.1.M/4) is an equality at all points on the budget constraint in π - x space with $\pi \leq \pi^*$.

Proof: Assume (4.1.M/4) were an inequality at some point on the budget constraint in π - x space with $\pi \leq \pi^*$, implying that $\lambda_3 = 0$ at that point. By Lemma 4.1.Q, $\lambda_2 = 0$ as well, hence $-z_1(k,g) = r$ by (4.1.M/2). But if (4.1.M/4) is an inequality, then $-z_1(k,g) > \beta$, which is a contradiction since $\beta \geq r$. So (4.1.M/4) must be an equality at all points on the budget constraint in π - x space.

Q.E.D.

<u>Theorem 4.1.M.2.A</u>: If $s > r$, $\beta > r$ and $\pi \leq \pi^*$, then an increase in β shifts budget constraint in π - x space downward.

Proof: Recall that the budget constraint in π - x space is defined to be

$$x = [p(g) - f(g)]g + c - z(k,g) - rk - \pi,$$

so I must show that

$$\frac{dx}{d\beta} = \left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right] \frac{dg}{d\beta}$$

$$- [z_1(k,g) + r] \frac{dk}{d\beta} < 0. \qquad (4.1.M.2/2)$$

Differentiating (4.1.M/4), which is an equality by Lemma 4.1.R., yields

$$-z_{11}(k,g) \frac{dk}{d\beta} - z_{12}(k,g) \frac{dg}{d\beta} = 1. \qquad (4.1.M.1/3)$$

Now, by Lemma 4.1.Q, λ_2 must equal zero since $\pi \leq \pi^*$. Hence, (4.1.M/1) - (4.1.M/2) may be solved to eliminate λ_3 yielding

$$z_{12}(k,g) = \frac{[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]z_{11}(k,g)}{-z_1(k,g) - r} .$$

Substituting for $z_{12}(k,g)$ in (4.1.M.1/3)

$$-z_{11}(k,g) \frac{dk}{d\beta}$$

$$- \frac{[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]z_{11}(k,g)}{-z_1(k,g) - r} \frac{dg}{d\beta} = 1,$$

or

$$[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{d\beta} - [z_1(k,g) + r]\frac{dk}{d\beta}$$

$$= \frac{z_1(k,g) + r}{z_{11}(k,g)} .$$

The term on the left is $dx/d\beta$ by (4.1.M.2/2). The numerator on the right is negative since $-z_1(k,g) = \beta > r$ by (4.1.M/4). The denominator equals $-R_{11}(k,g)$ by (2.2.M/7), which is positive by the strict concavity of $R(k,g)$. Hence, $dx/d\beta < 0$, and an increase in β shifts the budget constraint downward.

Q.E.D.

Theorem 4.1.M.2.B: If $s > r$, $\beta > r$, and $\pi \leq \pi^*$, then the slope of the budget constraint in π - x space is -1 regardless of the value of β.

Proof: Lemma 4.1.P and the defintion of the budget constraint in π - x space require that if $\pi \leq \pi^*$ then

$$x = \left(p(g') - f(g')\right)g' + c - z(k',g') - rk' - \pi.$$

Taking the derivative with respect to π yields

$$\frac{dx}{d\pi} = -1,$$

showing that the slope of the budget constraint in π - x space is -1 regardless of the value of β.

Q.E.D.

For the case of $s > r$ and $\beta > r$, I will make the assumption that if $\pi > \pi^*$ then λ_2 and λ_3 are greater than zero at points on the budget constraint in π - x space. As the following theorem demonstrates, both the rate-of-return constraint and the capital productivity requirement affect the values of g or k on the budget constraint in π - x space if

$\pi > \pi^*$. One would, therefore, normally expect the associated Lagrange multipliers λ_2 and λ_3 to be positive. I have, however, been unable to rule out the possibility of a pathological solution. So I must assume λ_2 and λ_3 are indeed positive.

<u>Theorem 4.1.M.2.C</u>: If $s > r$ and $\beta > r$, and $\pi > \pi^*$, and one were to drop either the rate-of-return constraint or the capital productivity requirement, the values of g or k at points on the budget constraint in $\pi - x$ space would change.

Proof: Consider, first, dropping the rate-of-return constraint, leaving only the capital productivity requirement. Recall that π^* was defined to be the largest profit the firm could make under the rate-of-return constraint given the capital investment on the budget constraint in $\pi - x$ space in the absence of the rate-of-return constraint. Since $\pi > \pi^*$, the capital investment on the budget constraint in $\pi - x$ space after dropping the rate-of-return constraint would, therefore, have been insufficient under the rate-of-return constraint. So the values of g or k must change when the rate-of-return constraint is dropped.

Now consider dropping the capital productivity requirement, leaving only the rate-of-return constraint. By Lemmas 4.1.A and 4.1.B, capital productivity would be less than or equal to r everywhere on the budget constraint in $\pi - x$ space in the absence of a capital productivity requirement. Hence, the values of g and k after dropping the capital productivity requirement would have been infeasible under the capital productivity requirement. So the values of g or k must change when the capital productivity requirement is dropped.

Q.E.D.

Having offered some justification for the assumption that λ_2 and λ_3 are greater than zero if $\pi > \pi^*$, the following two theorems demonstrate how the budget constraint in $\pi - x$ space would be affected by a change in β if $\pi > \pi^*$.

Theorem 4.1.M.2.D: If $s > r$, $\beta > r$, and $\pi > \pi^*$ with λ_2 and λ_3 greater than zero, then an increase in β shifts the budget constraint in $\pi - x$ space downward.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 4.1.K,

$$B_3(\pi, s, \beta) < 0$$

if $s > r$. This shows that an increase in β shifts the budget constraint in $\pi - x$ space downward.

Q.E.D.

Theorem 4.1.M.2.E: Assume that $s > r$, $\beta > r$ and $\pi > \pi^*$ with λ_2 and λ_3 greater than zero. Assume also that the underlying production function is homogeneous with non-decreasing returns to scale and that inputs have positive marginal products. Then an increase in β makes the budget constraint in $\pi - x$ space more negatively sloped.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 4.1.J,

$$B_1(\pi,s,\beta) < -1,$$

indicating that the slope of the budget constraint in π - x space is negative. By Lemma 4.1.L, if the underlying production function is homogeneous with non-decreasing returns to scale and inputs have positive marginal products, then

$$B_{13}(\pi,s,\beta) < 0,$$

if $s > r$, indicating that the budget constraint in π - x space becomes more negative as β increases.

Q.E.D.

4.1.M.3 The Case of $s < r$ and $\beta < r$.

This subsection is a short one, since I only need to prove one theorem. Note that, by Lemma 4.1.M, π^* is unaffected by the capital productivity requirement in this case.

Theorem 4.1.M.3.A: If $s < r$, then a capital productivity requirement of $\beta \leq r$ has no impact on the budget constraint in π - x space.

Proof: Lemmas 4.1.A and 4.1.B taken together imply that, in the absence of a capital productivity requirement, capital productivity will be greater than or equal to r at all points on the budget constraint in π - x space if $s < r$. Since this solution remains feasible under a capital productivity requirement of $\beta \leq r$, it must be that a capital productivity requirement of $\beta \leq r$ has no impact on the budget constraint in π - x space.

Q.E.D.

4.1.M.4 The Case of $s < r$ and $\beta > r$

The following subsection explores how a capital productivity requirement affects the budget constraint in π - x space when $s < r$ and $\beta > r$. In this case, π^* may be determined as in Section 4.1.M.2.

Theorem 4.1.M.4.A: Assume $s < r$ and $\beta > r$. Then there exists a $\pi^{**} > \pi^*$ such that the budget constraint in π - x space is unaffected by the capital productivity requirement at values of $\pi \geq \pi^{**}$.

Proof: Note first that if $s < r$, then the budget constraint is undefined at values of $\pi \geq 0$. To see this, note that a positive π with $s < r$ would contradict (4.1.M/3).

By Lemma 4.1.A, if the capital productivity requirement is not binding, then capital productivity $-z_1(k,g)$ equals r at $\pi \leq \pi^*$ on the budget constraint in π - x space. By Lemma 4.1.B, if $s < r$ and $\pi > \pi^*$ this capital productivity increases monotonically with increases in π. Define π^{**} to be the point at which $-z_1(k,g)$ equals β, or, if $-z_1(k,g)$ is everywhere less than β on the budget constraint in π - x space, define π^{**} to equal zero. Capital productivity, $-z_1(k,g)$, must then be greater than or equal to β at all values of π greater than or equal to π^{**} for which the budget constraint in π - x space is defined. Thus, a capital productivity requirement would not affect the budget constraint in π - x space at values of $\pi \geq \pi^{**}$.

Q.E.D.

Theorem 4.1.M.4.B: If $s < r$ and $\beta > r$, then an increase in β increases the value of π^{**} if $\pi^{**} \neq 0$.

Proof: By Lemma 4.1.B, if the capital productivity requirement is not binding, then capital productivity $-z_1(k,g)$, at points on the budget constraint in π - x space with $\pi > \pi^*$ increases monotonically with increases in π if $s < r$. Hence, an increase in β increases the value of π at which $-z_1(k,g) = \beta$, which is π^{**} if $\pi^{**} \neq 0$.

Q.E.D.

I will now make the additional assumption that if $s < r$, $\beta > r$, and $\pi^* < \pi < \pi^{**}$, then $\lambda_2 > 0$ and $\lambda_3 > 0$ at points on the budget constraint in π - x space. As the following theorem demonstrates, both the rate-of-return constraint and the capital productivity requirement affect the values of g or k chosen by the firm if $\pi^* > \pi > \pi^{**}$. One would, therefore, normally expect the associated Lagrange multipliers λ_2 and λ_3 to be positive. I have, however, been unable to rule out the possibility of a pathological solution. So I must assume that λ_2 and λ_3 are indeed positive.

Theorem 4.1.M.4.C: If $s < r$, $\beta > r$ and $\pi^* > \pi > \pi^{**}$, and one were to drop either the rate-of-return constraint or the capital productivity requirement, the values of g or k at points on the budget constraint in π - x space would change.

Proof: Consider, first, dropping the rate-of-return constraint, leaving only the capital productivity requirement. Recall that π^* was defined to be the largest profit the firm could achieve under the rate-of-return constraint given the capital investment on the budget constraint in π - x space in the absence of a rate-of-return constraint. Since $\pi > \pi^*$, the capital investment on the budget constraint in π - x space after dropping the rate-of-return constraint would, therefore, have been insufficient under the rate-of-return constraint. So the values of g or k must change when the rate-of-return constraint is dropped.

Now consider dropping the capital productivity requirement, leaving only the rate-of-return constraint. Recall that π^{**} was defined to be the point at which $-z_1(k,g)$ equals β, or if $-z_1(k,g)$ is everywhere less than β on the budget constraint in π - x space, then π^{**} was defined to equal zero. In either case, Lemma 4.1.B requires that, in the absence of a capital productivity requirement, $-z_1(k,g) < \beta$ for $\pi^* > \pi > \pi^{**}$. Hence, the values of g and k after dropping the capital productivity requirement would have been infeasible under the capital productivity requirement. So the solution must change when the capital productivity requirement is dropped.

Q.E.D.

Having offered some justification for the assumption that λ_2 and λ_3 are greater than zero if $\pi^* < \pi < \pi^{**}$, the following two theorems demonstrate how the budget constraint in π - x space would be affected by a change in β over this interval.

Theorem 4.1.M.4.D: If $s < r$, $\beta > r$, and $\pi^* > \pi > \pi^{**}$ with λ_2 and λ_3 greater than zero, then an increase in β shifts the budget constraint in π - x space downward.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3), and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 4.1.K,

$$B_3(\pi,s,\beta) < 0$$

if $s < r$. This shows that an increase in β shifts the budget constraint in π - x space downward.

Q.E.D.

Theorem 4.1.M.4.E: Assume that $s < r$, $\beta > r$ and $\pi^* > \pi > \pi^{**}$ with λ_2 and λ_3 greater than zero. Assume also that the underlying production function is homogeneous with non-decreasing returns to scale and that inputs have positive marginal products. Then an increase in β makes the budget constraint in π - x space less negatively sloped.

Proof: If λ_2 and λ_3 are greater than zero, then both the rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 4.1.J,

$$B_1(\pi,s,\beta) < -1,$$

indicating that the slope of the budget constraint in π - x space is negative. By Lemma 4.1.L, if the underlying production function is homogeneous with non-decreasing returns to scale and inputs have positive marginal products, then

$$B_{13}(\pi,s,\beta) > 0,$$

if $s < r$. This indicates that the budget constraint in π - x space becomes less negative as β increases.

Q.E.D.

Next, I consider the behavior of the budget constraint in π - x space for values of $\pi \leq \pi^*$.

Theorem 4.1.M.4.F: If $s < r$, $\beta > r$ and $\pi \leq \pi^*$, then an increase in β shifts the budget constraint in π - x space downward.

Proof. The proof of this theorem is identical to the proof of Theorem 4.1.M.2.A.

Q.E.D.

Theorem 4.1.M.4.G: If $s < r$, $\beta > r$ and $\pi < \pi^*$, then the slope of the budget constraint in π - x space is -1 regardless of the value of β.

Proof: The proof of this theorem is identical to the proof of theorem 4.1.M.2.B.

Q.E.D.

4.1.M.5 The Case of s = r.

This subsection concludes my discussion of the budget constraint in π - x space under a capital productivity requirement with an examination of the case of $s = r$.

Theorem 4.1.M.5.A: If $s = r$ then π^* always equals zero, regardless of any capital productivity requirement.

Proof: By Lemma 4.1.0,

$$\pi^* = (s-r)k',$$

where k' is the level of capital investment on the budget constraint in π - x space in the absence of a rate-of-return constraint. It can be seen that if $s = r$, π^* must equal zero regardless of the value of k'. Therefore, π^* always equals zero, regardless of any capital productivity requirement.

Q.E.D.

Theorem 4.1.M.5.B: If $s = r$, the budget constraint in π - x space is undefined for values of $\pi > \pi^* = 0$.

Proof: Any π on the budget constraint in π - x space must satisfy (4.1.M/3)

$$\pi \leq (s-r)k.$$

It can be seen that if $s = r$, no positive π can satisfy this condition.

Q.E.D.

Theorem 4.1.M.5.C: If $s = r$, then as long as $\beta \leq r$ a capital productivity requirement has no impact on the budget constraint in π - x space.

Proof: By Lemma 4.1.A, if the capital productivity requirement is not binding then capital productivity $-z_1(k,g)$, at any point on the budget constraint in π - x space with $\pi \leq \pi^*$ will equal r. By Theorem 4.1.M.5.B, the budget constraint in π - x space is only defined for $\pi \leq \pi^*$, so capital productivity at any point on the budget constraint in π - x space will equal r in the absence of a capital productivity requirement. A capital productivity requirement of $\beta \leq r$ does not affect the feasibility of this solution, so the capital productivity requirement of $\beta \leq r$ has no impact on the budget constraint in π - x space.

Q.E.D.

Theorem 4.1.M.5.D: If s = r and $\beta > r$, then an increase in β shifts the budget constraint in π - x space downward.

Proof: By Theorem 4.1.M.5.B, the budget constraint in π - x space is only defined over values of $\pi \leq \pi^*$ if s = r. The rest of the proof is then identical to the proof of Theorem 4.1.M.2.A.

Q.E.D.

Theorem 4.1.M.5.E: If s = r and $\beta > r$, then the slope of the budget constraint in π - x space is -1 regardless of the value of β.

Proof: By Theorem 4.1.M.5.B, the budget constraint in π - x space is only defined over values of $\pi \leq \pi^*$ if s = r. The rest of the proof is then identical to Theorem 4.1.M.2.B.

Q.E.D.

4.1.M.6 The Tangency Condition

Having examined how the budget constraint in π - x space is affected by the capital productivity requirement, it remains to show that the solution chosen by the firm managers continues to lie at the tangency of this budget constraint in π - x space and an isoutility curve. To demonstrate this, I must first derive the first-order conditions for the solution to the managers' problem.

The firm managers' problem is to maximize utility

$$U(\pi,x),$$

subject to the budget constraint

$$\big(p(g) - f(g)\big)g - z(k,g) - rk - \pi - x \geq 0,$$

the rate of return constraint

$$(s-r)k - \pi \geq 0,$$

and the capital productivity requirement

$$-z_1(k,g) - \beta \geq 0,$$

where x, g and k, but not π, must be non-negative. The Lagrangian will be

$$L = U(\pi,x) + \lambda_1^*\left[\big(p(g) - f(g)\big)g + c - z(k,g) - rk - \pi - x\right]$$
$$+ \lambda_2^*\big((s-r)k - \pi\big) - \lambda_3^*\big(z_1(k,g) + \beta\big).$$

The first-order conditions require that:

$$\frac{\partial L}{\partial \pi} = U_1(\pi,x) - \lambda_1^* - \lambda_2^* = 0; \qquad (4.1.M.6/1)$$

$$\frac{\partial L}{\partial x} = U_2(\pi,x) - \lambda_1^* \leq 0; \qquad (4.1.M.6/2)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial g} = \lambda_1^*[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]$$

$$-\lambda_3^* z_{12}(k,g) \leq 0; \qquad (4.1.M.6/3)$$

$$\frac{\partial L}{\partial g} g = 0; \qquad g \geq 0;$$

$$\frac{\partial L}{\partial k} = -\lambda_1^*\left(z_1(k,g)+r\right) + \lambda_2^*\left(s-r\right) - \lambda_3^* z_{11}(k,g) \leq 0; \qquad (4.1.M.6/4)$$

$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda_1^*} = \left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi - x \geq 0; \qquad (4.1.M.6/5)$$

$$\frac{\partial L}{\partial \lambda_1^*}\lambda_1^* = 0; \qquad \lambda_1^* \geq 0;$$

$$\frac{\partial L}{\partial \lambda_2^*} = (s-r)k - \pi \geq 0; \qquad (4.1.M.6/6)$$

$$\frac{\partial L}{\partial \lambda_2^*}\lambda_2^* = 0; \qquad \lambda_2^* \geq 0;$$

$$\frac{\partial L}{\partial \lambda_3^*} = -z_1(k,g) - \beta \geq 0; \qquad (4.1.M.6/7)$$

$$\frac{\partial L}{\partial \lambda_3^*}\lambda_3^* = 0; \qquad \lambda_3^* \geq 0.$$

I shall assume that g, k, and x are all greater than zero at the solution, and that π and x have positive marginal utilities, that is $U_1(\pi,x) > 0$ and $U_2(\pi,x) > 0$. These assumptions are sufficient to insure that conditions (4.1.M.6/2) - (4.1.M.6/5) are equalities with $\lambda_1^* > 0$. I now prove the desired theorem.

Theorem 4.1.M.6.A: If $s \neq r$, the solution chosen by the managers of the rate-of-return regulated firm subject to a capital productivity requirement lies at the point of tangency between the budget constraint in π - x space and an isoutility curve in π - x space. If s = r, the solution either lies on the budget constraint at $\pi = 0$ or lies at the point of tangency.

Proof: I first show that the solution chosen by the managers must lie on the budget constraint in π - x space. Suppose the contrary. Since the budget constraint in π - x space is defined to be the largest value of x which the firm could achieve at each value of π, this would imply that the chosen value of x was less than the firm could have achieved at the chosen value of π. Since I assume x to have positive marginal utility, the managers could improve on this solution by moving to a larger value of x. But this is a contradiction, since the initial solution is assumed optimal. So the solution must lie on the budget constraint in π - x space.

To show tangency, it remains to demonstrate that the slope of the isoutility curve equals the slope of the budget constraint in π - x space at the solution. An isoutility curve is defined by $U(\pi,x)$ = constant. Total differentiation yields a slope

$$\frac{dx}{d\pi} = -\frac{U_1(\pi,x)}{U_2(\pi,x)}.$$

Hence, one may obtain an explicit expression for the slope of the isoutility curve at the solution by solving (4.1.M.6/1) - (4.1.M.6/7) for $-U_1(\pi,x)/U_2(\pi,x)$.

The budget constraint is defined to be

$$x = \left(p(g) - f(g)\right)g + c - z(k,g) - rk - \pi.$$

Hence, the slope of the budget constraint in π - x space is

$$\frac{dx}{d\pi} = [p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{dg}{d\pi}$$

$$- [z_1(k,g) + r]\frac{dk}{d\pi} - 1, \qquad (4.1.M.6/8)$$

where $dg/d\pi$ and $dk/d\pi$ are determined from the conditions for g and k on the budget constraint in π - x space (4.1.M/1) - (4.1.M/4).

i) Assume the solution to (4.1.M.6/1) - (4.1.M.6/7) is such that $\lambda_2^* = \lambda_3^* = 0$. Then (4.1.M.6/1) and (4.1.M.6/2) may be solved to eliminate λ_1^* yielding

$$-\frac{U_1(\pi,x)}{U_2(\pi,x)} = -1.$$

This indicates that the slope of an isoutility curve must equal minus one at the solution. Since (4.1.M.6/3) requires that

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = 0$$

and (4.1.M.6/4) requires that

$$-[z_1(k,g) + r] = 0$$

at the solution, (4.1.M.6/8) simplifies to

$$\frac{dx}{d\pi} = -1,$$

indicating that the slope of the budget constraint in $\pi - x$ space is minus one. So the two slopes are the same if $\lambda_2^* = \lambda_3^* = 0$.

ii) Assume the solution to (4.1.M.6/1) - (4.1.M.6/7) is such that $\lambda_2^* > 0$ and $\lambda_3^* = 0$. If $s \neq r$, one may solve (4.1.M.6/1), (4.1.M.6/2), and (4.1.M.6/4) to eliminate λ_1^* and λ_2^*, yielding

$$-\frac{U_1(\pi,x)}{U_2(\pi,x)} = \frac{-[z_1(k,g) + r]}{s-r} - 1$$

as the slope of the isoutility curve at the solution.

If $\lambda_3^* = 0$, then (4.1.M.6/3) requires that

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = 0,$$

so (4.1.M.6/8) simplifies to

$$\frac{dx}{d\pi} = -[z_1(k,g) + r]\,\frac{dk}{d\pi} - 1. \qquad (4.1.M.6/9)$$

Now if $\lambda_2^* > 0$ and $\lambda_3^* = 0$ in (4.1.M.6/1) - (4.1.M.6/7) then $\lambda_2 > 0$ and $\lambda_3 = 0$ at the same solution in (4.1.M/1) - (4.1.M/4). But if $\lambda_2 > 0$, then the rate-of-return constraint (4.1.M/3) must be an equality, so by Lemma 2.2.B, if $s \neq r$,

$$\frac{dk}{d\pi} = \frac{1}{s - r} .$$

Substituting in (4.1.M.6/9) yields

$$\frac{dx}{d\pi} = \frac{-[z_1(k,g) + r]}{s-r} - 1$$

as the slope of the budget constraint in π - x space at the solution. So if $s \neq r$, the two slopes are the same if $\lambda_2^* > 0$ and $\lambda_3^* = 0$. If $s = r$, (4.1.M.6/6) requires $\pi = 0$.

iii) Assume the solution to (4.1.M.6/1) - (4.1.M.6/7) is such that $\lambda_2^* = 0$ and $\lambda_3^* > 0$. Then (4.1.M.6/1) and (4.1.M.6/2) may be solved to eliminate λ_1^* yielding

$$-\frac{U_1(\pi,x)}{U_2(\pi,x)} = -1.$$

This indicates that the slope of an isoutility curve must equal minus one at the solution.

If $\lambda_2^* = 0$ and $\lambda_3^* > 0$ in (4.1.M.6/1) - (4.1.M.6/7), then $\lambda_2 = 0$ and $\lambda_3 > 0$ at the same solution in (4.1.M/1) - (4.1.M/4). But if $\lambda_2 = 0$, (4.1.M/1) and (4.1.M/2) may be solved simultaneously to eliminate λ_3 yielding

$$\left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right] \frac{z_{11}(k,g)}{z_{12}(k,g)}$$

$$+ \left[z_1(k,g) + r\right] = 0.$$

If $\lambda_3 > 0$, (4.1.M/4) is an equality, so one may use Lemma 4.1.C to substitute for $z_{11}(k,g)/z_{12}(k,g)$

$$-\left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right] \frac{dg}{dk}$$

$$+ \left[z_1(k,g) + r\right] = 0.$$

Multiplying through by $-dk/d\pi$ yields

$$\left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right] \frac{dg}{d\pi}$$

$$- \left[z_1(k,g) + r\right] \frac{dk}{d\pi} = 0.$$

So (4.1.M.6/8) simplifies to

$$\frac{dx}{d\pi} = -1.$$

This indicates that the slope of the budget constraint in π - x space is minus one. So the two slopes are the same if $\lambda_2^* = 0$ and $\lambda_3^* > 0$.

iv) Assume the solution to (4.1.M.6/1) - (4.1.M.6/7) is such that $\lambda_2^* > 0$ and $\lambda_3^* > 0$. If $s \neq r$, one may solve (4.1.M.6/1) - (4.1.M.6/4) to eliminate λ_1^*, λ_2^*, and λ_3^* yielding

$$- \frac{U_1(\pi,x)}{U_2(\pi,x)} = -[p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)] \frac{z_{11}(k,g)}{(s-r)z_{12}(k,g)}$$

$$- \frac{[z_1(k,g) + r]}{s-r} - 1$$

as the slope of the isoutility curve at the solution.

But if $\lambda_2^* > 0$ and $\lambda_3^* > 0$ in (4.1.M.6/1) - (4.1.M.6/7), then $\lambda_2 > 0$ and $\lambda_3 > 0$ in (4.1.M/1) - (4.1.M/4), and rate-of-return constraint (4.1.M/3) and the capital productivity requirement (4.1.M/4) are equalities. Then by Lemma 2.2.B, $dk/d\pi = 1/(s-r)$ and, by Lemma 4.1.D,

$$\frac{dg}{d\pi} = \frac{-z_{11}(k,g)}{z_{12}(k,g)}\left(\frac{1}{s-r}\right)$$

if $s \neq r$. Substituting in (4.1.M.6/8) yields

$$\frac{dx}{d\pi} = -[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)]\frac{z_{11}(k,g)}{(s-r)z_{12}(k,g)}$$

$$- \frac{[-z_1(k,g) + r]}{s-r} - 1$$

as the slope of the budget constraint in $\pi - x$ space at the solution. So if $s \neq r$, the two slopes are the same if $\lambda_2^* > 0$ and $\lambda_3^* > 0$. If $s = r$ (4.1.M.6/6) requires that $\pi = 0$.

So I have shown that, in each of these four cases, if $s \neq r$ (4.1.M.6/1) - (4.1.M.6/7) require that the slope of an isoutility curve equal the slope of the budget constraint in π - x space. Hence, if $s \neq r$, the solution chosen by the managers must lie at a point of tangency between the budget constraint in π - x space and an isoutility curve. In cases ii and iv, if s = r then the solution lies on the budget constraint at $\pi = 0$; in the other two cases the solution is at the point of tangency even with s = r.

Q.E.D.

4.2 Effect of a Change in Required Capital Productivity

This section discusses how a change in the required capital productivity might affect the level of profit and institutional costs selected by the firm managers. Throughout the section, I will assume that both the rate-of-return constraint and capital productivity requirement are binding at the solution selected by the managers.

As with a change in the allowed rate of return, a change in the capital productivity requirement will have two effects on both profit and institutional costs. The changing slope of the budget constraint alters the tradeoff between profit and institutional costs, leading to a substitution effect. The shifting of the budget constraint lowers the managers' achievable level of utility, producing an income effect.

Figure 33 illustrates the two effects, assuming an allowed rate of

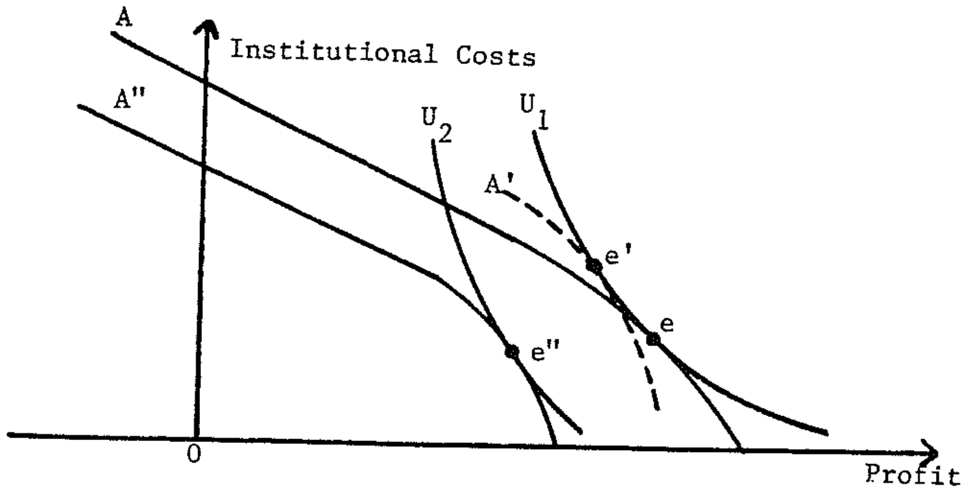

Figure 33
Substitution and Income Effects of an Increase in the Capital Productivity Requirement Given an Allowed Rate of Return Greater Than the Cost of Capital

return greater than the cost of capital. Budget constraint A is initially tangent to isoutility curve U_1 at point e. If the capital productivity requirement is increased, the slope of the budget constraint becomes steeper (assuming a homogeneous production function with non-decreasing returns to scale) and shifts to the left. If only the slope of the budget constraint were changed, without changing the isoutility curve to which it was tangent, one would get a new budget constraint A', and a new point of tangency e'. The difference between e and e' is the substitution effect.

Now shifting the isoutility curve to the left, with no change in slope, to its actual new position further shifts the point of tangency with the isoutility curve to its new equilibrium e". The difference between e' and e" is the income effect. It can be seen from Figure 33 that the substitution effect causes institutional costs to increase and profit to decrease, while the income effect causes institutional costs and profit to decrease. The two effects work in the same direction for profit, implying that profit must decline when the required capital productivity increases. However, they work in opposite directions for institutional costs, implying it cannot be said what will happen to institutional costs. Figure 33 is drawn assuming a required capital productivity greater than the cost of capital; however, the effects would be the same if the required capital productivity were less than the cost of capital.

Figure 34 illustrates the two effects assuming an allowed rate of return less than the cost of capital. If the required capital productivity is increased in this case, the budget constraint becomes less steep

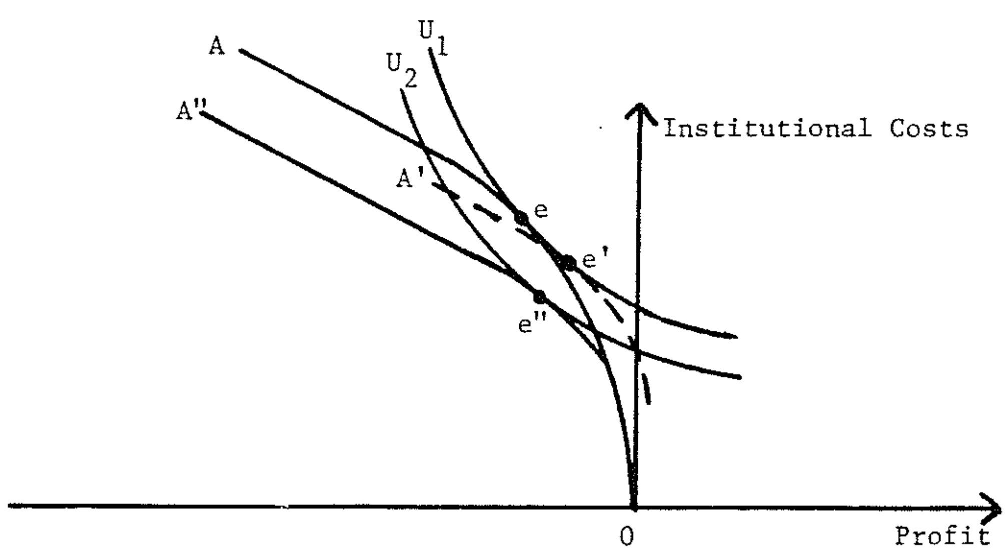

Figure 34
Substitution and Income Effects of an Increase in the Capital Productivity Requirement Given an Allowed Rate of Return Less Than the Cost of Capital

(assuming a homogenous production function with non-decreasing returns to scale) and shifts to the left. The substitution effect is again represented by the difference between e and e', while the income effect is represented by the difference between e' and e". It can be seen from Figure 34 that the substitution effect causes institutional costs to decrease and profit to increase, while the income effect causes both institutional costs and profit to decrease. The two effects work in the same direction for institutional costs, implying that institutional costs must decline as the required capital productivity is increased. However, they work in opposite directions for profit, implying it cannot be said what will happen to profit. Table 3 summarizes these results.

TABLE 3

EFFECTS OF AN INCREASE IN THE REQUIRED PRODUCTIVITY OF CAPITAL

Variable	Relationship of Allowed Rate of Return to Cost of Capital	Substitution Effect*	Income Effect	Total Effect
Profit	greater than	-	-	-
	less than	+	-	?
Institutional Costs	greater than	+	-	?
	less than	-	-	-

*Assuming a homogeneous production function with non-decreasing returns-to-scale.

4.2.M Mathematical Formulation

The main thrust of this subsection is to formally demonstrate the comparative statics results shown in Table 3. I shall use the same model, assumptions, and notation here as were used in Section 4.1.M.

If one assumes that both the rate-of-return constraint and the capital productivity requirement are binding at the solution, with $\lambda_2 > 0$ and $\lambda_3 > 0$, the managers' problem is maximize utility

$$U(\pi, x),$$

subject to the budget constraint

$$B(\pi, s, \beta) - x \geq 0.$$

Maximization is over π and x, where x, but π, must be non-negative. The Lagrangian will be

$$L = U(\pi, x) + \lambda[B(\pi, s, \beta) - x].$$

The first-order conditions require that:

$$\frac{\partial L}{\partial \pi} = U_1(\pi, x) + \lambda\left[B_1(\pi, s, \beta)\right] = 0; \qquad (4.2.M/1)$$

$$\frac{\partial L}{\partial x} = U_2(\pi, x) - \lambda \leq 0; \qquad (4.2.M/2)$$

$$\frac{\partial L}{\partial x} x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = B(\pi,s,\beta) - x \geq 0; \qquad (4.2.M/3)$$

$$\frac{\partial L}{\partial \lambda}\lambda = 0; \qquad \lambda \geq 0.$$

If one assumes $U_2(\pi,x) > 0$ at the solution, that is the managers are not satiated in institutional costs, then λ must be greater than zero. If one also assumes that $x > 0$ at the solution, then the two conditions (4.1.M/2) and (4.1.M/3) become equalities.

The second order conditions require that the determinant:

$$D = \begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix}$$

be greater than zero. Note that I have dropped the parameters of the partials of $U(\pi,x)$ and $B(\pi,s,\beta)$ for compactness of notation.

To find the effect of a small change in β, $d\beta$, conditions (4.1.M/1) and (4.1.M/3) may be totally differentiated. This yields a system of simultaneous equations in $d\pi$, dx, $d\lambda$, and $d\beta$. Dropping the parameters of $U(\pi,x)$ and $B(\pi,s,\beta)$, the system may be expressed in matrix notation as

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{13} \\ 0 \\ -B_3 \end{bmatrix} d\beta. \qquad (4.2.M/4)$$

I am now in a position to demonstrate the comparative statics results.

Theorem 4.2.M.A: Assume that the underlying production function is homogeneous with non-decreasing returns to scale and that inputs have positive marginal products. Then the substitution effect of an increase in the required capital productivity on profit $(d\pi/d\beta)_s$, is negative if $s > r$ and positive if $s < r$.

Proof: To derive the substitution effect, one wants to add some compensation, call it y, to the budget constraint (4.2.M/3) so as to hold the solution to the same isoutility curve when β changes. If this were done, (4.2.M/4) could be rewritten as

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{13} \\ 0 \\ -B_3 - \frac{dy}{d\beta} \end{bmatrix} d\beta \qquad (4.2.M/5)$$

If one knew $dy/d\beta$, this system could be solved for the substitution effect $(d\pi/d\beta)_s$. But it is possible to solve for $dy/d\beta$. For, by the third equation of this system,

$$B_1 d\pi - dx = (-B_3 - \frac{dy}{d\beta})d\beta.$$

Now, by (4.2.M/1) and (4.2.M/2),

$$-\frac{U_1}{U_2} = B_1,$$

so substituting back,

$$-\frac{U_1}{U_2}\,d\pi - dx = \left(-B_3 - \frac{dy}{d\beta}\right)d\beta. \tag{4.2.M/6}$$

If the solution is to be held to the same isoutility curve $U(\pi,x)$ = constant, total differentiation may be used to show that

$$\frac{dx}{d\beta} = -\frac{U_1}{U_2},$$

which implies that the left side of (4.2.M/6) is equal to zero. Thus, it must be that

$$\frac{dy}{d\beta} = -B_3$$

if the solution is to be held to a given isoutility curve.

Substituting into (4.1.M/5) yields the system

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{13} \\ 0 \\ 0 \end{bmatrix} d\beta \tag{4.2.M/7}$$

which may be solved for the substitution effect $(d\pi/d\beta)_s$. Applying Cramer's rule,

$$\left(\frac{d\pi}{d\beta}\right)_s = \frac{\begin{vmatrix} -\lambda B_{13} & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ 0 & -1 & 0 \end{vmatrix}}{D},$$

or

$$\left(\frac{d\pi}{d\beta}\right)_s = \frac{\lambda B_{13}}{D}. \qquad (4.2.M/8)$$

I assume $\lambda = U_2(\pi,x) > 0$. If the underlying production function is homogeneous with non-decreasing returns to scale and inputs have positive marginal products, then, by Lemma 4.1.L, $B_{13} < 0$ if $s > r$ and $B_{13} > 0$ if $s < r$. Hence I have shown that

$$\left(\frac{d\pi}{d\beta}\right)_s < 0$$

if $s > r$ and

$$\left(\frac{d\pi}{d\beta}\right)_s > 0$$

if $s < r$.

Q.E.D.

Theorem 4.2.M.B: Assume that the underlying production function is homogeneous with non-decreasing returns to scale and that inputs have positive marginal products. Then the substitution effect of an increase in required capital productivity on institutional costs, $(dx/d\beta)_s$ is positive if $s > r$ and negative if $s < r$.

Proof: One can solve system (4.2.M/7) for the substitution effect $(dx/d\beta)_s$. Applying Cramer's rule,

$$\left(\frac{dx}{d\beta}\right)_s = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{13} & B_1 \\ U_{12} & 0 & -1 \\ B_1 & 0 & 0 \end{vmatrix}}{D},$$

or

$$\left(\frac{dx}{d\beta}\right)_s = \frac{\lambda B_{13} B_1}{D}. \qquad (4.2.M/9)$$

But, I can substitute from (4.2.M/8) to write

$$\left(\frac{dx}{d\beta}\right)_s = \left(\frac{d\pi}{d\beta}\right)_s B_1.$$

By Lemma 4.1.J, $B_1 < -1$. Thus, $(dx/d\beta)_s$ always has the opposite sign of $(d\pi/d\beta)_s$. So if the underlying production function is homogeneous with non-decreasing returns to scale and inputs have positive marginal products,

$$\left(\frac{dx}{d\beta}\right)_s > 0$$

if s > r and

$$\left(\frac{dx}{d\beta}\right)_s < 0$$

if s < r.

Q.E.D.

Recall from Section 2.3.M that profit is defined to be a "normal good" if its level would increase with a relaxation of the budget constraint. One could relax the budget constraint by adding a positive y to the left side of (4.2.M/3). Hence, if a small increase in y produces an increase in profit π, that is if $d\pi/dy > 0$, then profit is a normal good.

Theorem 4.2.M.C: If profit is a normal good, then the income effect $(d\pi/d\beta)_y$ of an increase in the required capital productivity on profit is negative.

Proof: Since profit is a normal good, $d\pi/dy > 0$. To find an expression for $d\pi/dy$, I totally differentiate (4.2.M/1) - (4.2.M/3) to obtain

$$\begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix} \begin{vmatrix} d\pi \\ dx \\ d\lambda \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -1 \end{vmatrix} dy. \tag{4.2.M/10}$$

Applying Cramer's rule,

$$\frac{d\pi}{dy} = \frac{\begin{vmatrix} 0 & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ -1 & -1 & 0 \end{vmatrix}}{D}$$

or

$$\frac{d\pi}{dy} = \frac{U_{12} + B_1U_{22}}{D} \,. \qquad (4.2.M/11)$$

To find the income effect $(d\pi/d\beta)_y$, one first finds the total effect $d\pi/d\beta$, and subtracts the substitution effect $(d\pi/d\beta)_s$. The total effect $(d\pi/d\beta)$ may be found by applying Cramer's rule to system (4.2.M/4),

$$\frac{d\pi}{d\beta} = \frac{\begin{vmatrix} -\lambda B_{13} & U_{12} & B_1 \\ 0 & U_{22} & -1 \\ -B_3 & -1 & 0 \end{vmatrix}}{D}$$

$$= \frac{\lambda B_{13} + B_3[U_{12} + B_1U_{22}]}{D} \,.$$

Subtracting the substitution effect (4.2.M/8) yields the income effect,

$$\left(\frac{d\pi}{d\beta}\right)_y = \frac{B_3[U_{12} + B_1U_{22}]}{D},$$

or, substituting in (4.2.M/11),

$$\left(\frac{d\pi}{d\beta}\right)_y = B_3\left(\frac{d\pi}{dy}\right).$$

Since $B_3 < 0$ by Lemma 4.1.K, and $d\pi/dy > 0$ if profit is a normal good, $(d\pi/d\beta)_y$ must be less than zero.

Q.E.D.

Recall from Section 2.3.M that institutional costs are defined to be a "normal good" if their level would increase with a relaxation of the budget constraint.

Theorem 4.2.D: If institutional costs are a normal good, then the income effect $(dx/d\beta)_y$ of an increase in the required capital productivity on institutional costs is negative.

Proof: Since institutional costs are a normal good, the dx/dy obtained by solving (4.2.M/10) is positive. Applying Cramer's rule,

$$\frac{dx}{dy} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & 0 & B_1 \\ U_{12} & 0 & -1 \\ B_1 & -1 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{dy} = \frac{-(U_{11} + \lambda B_{11}) - U_{12}B_1}{D}. \qquad (4.2.M/12)$$

To find the income effect $(dx/d\beta)_y$, one finds the total effect $dx/d\beta$, and subtracts the substitution effect $(dx/d\beta)_s$. The total effect $dx/d\beta$ may be found by applying Cramer's rule to system (4.2.M/4),

$$\frac{dx}{d\beta} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{13} & B_1 \\ U_{12} & 0 & -1 \\ B_1 & -B_3 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{d\beta} = \frac{\lambda B_{13} B_1 + B_3[-(U_{11} + \lambda B_{11}) - U_{12} B_1]}{D}.$$

Subtracting the substitution effect (4.2.M/9), yields the income effect

$$\left(\frac{dx}{d\beta}\right)_y = \frac{B_3[-(U_{11} + \lambda B_{11}) - U_{12} B_1]}{D},$$

or, substituting (4.2.M/12),

$$\left(\frac{dx}{d\beta}\right)_y = \left(\frac{dx}{dy}\right) B_3.$$

Since $B_3 < 0$ by Lemma 4.1.K and $dx/dy > 0$ if institutional costs are a normal good, $(dx/d\beta)_y$ must be less than zero.

Q.E.D.

4.3 Effect of a Change in the Allowed Rate of Return

One might also be interested in how a change in the allowed rate of return affects profits and institutional costs when the firm is subject to a capital productivity requirement. It turns out that the results are exactly the same as when the firm is not subject to a binding capital productivity requirement, for exactly the same reasons.

In particular, the firm will continue to exhibit a tendency for institutional costs to increase as the allowed rate of return is reduced toward the cost of capital. To understand this tendency, one should look at the behavior of the firm when the allowed rate of return equals the cost of capital with the rate of return constraint assumed binding. Since the firm is prohibited from earning a positive profit in this situation, the managers would become institutional cost maximizers. They will choose to set prices and buy inputs as would a monopolist subject only to the capital productivity requirement, with the benefits being taken as institutional costs instead of profit. This corresponds to point A in Figure 32. Since institutional costs must be less than this if the firm is earning positive profits, institutional costs should be generally increasing as the allowed rate of return is lowered.

For small reductions in the allowed rate of return, the results shown in Table 1, Section 2.3, continue to hold under a capital productivity requirement. It should be noted, however, that in order to obtain the sign of the substitution effects for the firm subject to a capital productivity requirement, it is again necessary to assume a homogeneous production function with non-decreasing returns to scale.

4.3.M. Mathematical Formulation

The derivations of the effects of a change in the allowed rate of return given in Section 2.3. are easily extended to the firm subject to a rate of return constraint and capital productivity requirement with λ_2 and λ_3 greater than zero, as presented in Section 4.1. The following lemmas, concerning the solution to the managers' problem subject to a capital productivity requirement, are all that is really necessary.

Lemma 4.3.A: If the rate-of-return constraint and the capital productivity requirement are equalities, then

$$\frac{dg}{ds} = \left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{k}{s-r}\right).$$

This will be negative if $s > r$ and positive if $s < r$.

Proof: By the chain rule,

$$\frac{dg}{ds} = \left(\frac{dg}{dk}\right)\left(\frac{dk}{ds}\right).$$

Substituting, using Lemmas 4.1.C and 2.2.D, yields

$$\frac{dg}{ds} = \left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{k}{s-r}\right).$$

Since by (2.2.M/7), $z_{11}(k,g) = -R_{11}(k,g)$, which is positive by the strict concavity of $R(k,g)$, and since I assume $z_{12}(k,g) < 0$ (see Section 2.5.M), dg/ds will be negative if $s > r$ and positive if $s < r$.

Q.E.D.

Lemma 4.3.B: If the underlying production function $g = h(z,k)$ is homogeneous with non-decreasing returns to scale and the capital productivity requirement is an equality, then

$$\frac{d^2g}{dk^2} \geq 0.$$

Proof: If $g = h(z,k)$, then

$$\frac{dg}{dk} = h_1(z,k)\frac{dz}{dk} + h_2(z.k).$$

But $dz/dk = z/k$ under these assumptions by Lemma 4.1.H, so

$$\frac{dg}{dk} = h_1(z,k)\frac{z}{k} + h_2(z,k)$$

$$= \left(h_1(z,k)z + h_2(z,k)k\right)\frac{1}{k}.$$

But if $h(z,k)$ is homogeneous with non-decreasing returns to scale, then by Euler's Theorem,

$$h_1(z,k)z + h_2(z,k)k = gb,$$

where $b \geq 1$. So

$$\frac{dg}{dk} = \frac{gb}{k}.$$

Taking the derivitive,

$$\frac{d^2g}{dk^2} = \frac{b}{k}\frac{dg}{dk} - \frac{gb}{k^2} = \frac{b}{k}\left(\frac{gb}{k}\right) - \frac{gb}{k^2} = \frac{g}{k^2}\left(b^2 - b\right) \geq 0.$$

Q.E.D.

Lemma 4.3.C: If the rate-of-return constraint and capital productivity requirement are equalities with $\lambda_2 > 0$, and $s \neq r$, then

$$B_2(\pi,x,\beta) > 0.$$

Proof: By definition,

$$B(\pi,s,\beta) = \left(p(g) - f(g)\right)g - z(k,g) + c - rk - \pi.$$

Taking the derivitive,

$$B_2(\pi,s,\beta) = \left[p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right]\frac{dg}{ds}$$

$$- \left[z_1(k,g) + r\right]\frac{dk}{ds}. \qquad (4.3.M/1)$$

Substituting in dg/ds and dk/ds using Lemmas 2.2.D and 4.3.A yields

$$B_2(\pi,s\beta) = \left[p'(g)g + p(g)-f'(g)g-f(g)-z_2(k,g)\right]\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{k}{s-r}\right)$$

$$+ \left[z_1(k,g) + r\right]\left(\frac{k}{s-r}\right).$$

Substituting for $p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)$ and $z_1(k,g) + r$ using equations (4.1.M/1) and (4.1.M/2) yields

$$B_2(\pi,s,\beta) = \left(\lambda_3 z_{12}(k,g)\right)\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{k}{s-r}\right)$$

$$+ \left[\lambda_2(s-r) - \lambda_3 z_{11}(k,g)\right]\left(\frac{k}{s-r}\right) = \lambda_2 k > 0.$$

Q.E.D.

Lemma 4.3.D: If the rate-of-return constraint and capital productivity requirement are equalities with λ_2 and λ_3 greater than zero, and if the underlying production function is homogenous with non-decreasing returns to scale, and if $s > r$, then

$$B_{12}(\pi,s,\beta) > 0.$$

Proof: From (4.3.M/1) and the chain rule,

$$B_2(\pi,s,\beta) = \left[\left(p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right)\frac{dg}{dk} - \left(z_1(k,g) + r\right)\right]\frac{dk}{ds}.$$

Taking the derivitive with respect to π,

$$\begin{aligned} B_{12}(\pi,s,\beta) = &\Big[\frac{\partial}{\partial g}\left(p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)\right)\frac{dg}{d\pi}\frac{dg}{dk} \\ &- z_{12}(k,g)\frac{dk}{d\pi}\frac{dg}{dk} + \left(p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)\right)\frac{d^2g}{dkd\pi} \\ &- z_{12}(k,g)\frac{dg}{d\pi} - z_{11}(k,g)\frac{dk}{d\pi}\Big]\frac{dk}{ds} \\ &+ \Big[\left(p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)\right)\frac{dg}{dk} \\ &- \left(z_1(k,g)+r\right)\Big]\frac{d^2k}{dsd\pi}\ . \end{aligned} \qquad (4.3.M/2)$$

I shall now consider the sign of the first expression in brackets,

$$\begin{aligned} &\frac{\partial}{\partial g}\left(p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right)\frac{dg}{d\pi}\frac{dg}{dk} \\ &- z_{12}(k,g)\frac{dk}{d\pi}\frac{dg}{dk} + \left(p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)\right)\frac{d^2g}{dkd\pi} \\ &- z_{12}(k,g)\frac{dg}{d\pi} - z_{11}(k,g)\frac{dk}{d\pi}\ . \end{aligned} \qquad (4.3.M/3)$$

Using Lemmas 2.2.B and 4.1.D, the first two terms may be written

$$\frac{\partial}{\partial g}\left(p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)\right)\left(\frac{-z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{1}{s-r}\right)\frac{dg}{dk}$$

$$- z_{12}(k,g)\left(\frac{1}{s-r}\right)\frac{dg}{dk} \qquad (4.3.M/4)$$

Using (2.2.M/7) - (2.2.M/9) this expression may be written in terms of the revenue function as

$$\left[\frac{-R_{22}(k,g)R_{11}(k,g)}{R_{12}(k,g)} + R_{12}(k,g)\right]\left(\frac{1}{s-r}\right)\frac{dg}{dk}. \qquad (4.3.M/5)$$

Since $R_{11}(k,g)R_{22}(k,g) - \left(R_{12}(k,g)\right)^2 > 0$ by convexity of the revenue function, while $R_{12}(k,g) = - z_{12}(k,g)$ is positive by assumption (see Section 2.5.M), the term in brackets in (4.3.M/5) is negative. Since, byLemma 4.1.C, $dg/dk > 0$, (4.3.M/4) will be negative if $s > r$.

Using (4.1.M/1) to substitute for $p'(g)g+p(g)-f'(g)g-f(g)-z_2(k,g)$ and the chain rule, the third term in (4.3.M/3) may be written as

$$\lambda_3 z_{12}(k,g)\frac{d^2g}{dk^2}\frac{dk}{d\pi} .$$

Since $z_{12}(k,g) < 0$ by assumption (see Section 2.5.M), $d^2g/dk^2 \geq 0$ by Lemma 4.3.B, and $dk/d\pi > 0$ if $s > r$ by Lemma 2.2.B, this third term in (4.3.M/3) will be non-positive if $s > r$.

Using Lemmas 2.2.B and 4.1.D, the last two terms of (4.3.M/3) may be written as

$$- z_{12}(k,g)\left(\frac{-z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{1}{s-r}\right) - z_{11}(k,g)\left(\frac{1}{s-r}\right) = 0.$$

I have thus shown that the first expression in brackets in (4.3.M/2) is negative. Since $dk/ds < 0$ if $s > r$ by Lemma 2.2.D, the entire first term of (4.3.M/2) is positive if $s > r$.

So consider the sign of the second term in (4.3.M/2). Substituting for $p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g)$ and $z_1(k,g) + r$ using equations (4.1.M/1) and (4.1.M/2) and for dg/dk using Lemma 4.1.C allows me to write the second expression in brackets as

$$- [\lambda_3 z_{12}(k,g)]\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right) - [\lambda_2(s-r) - \lambda_3 z_{11}(k,g)] = - \lambda_2(s-r).$$

This is negative if $s > r$. By Lemma 2.2.E,

$$\frac{d^2k}{d\pi ds} = \frac{-1}{(s-r)^2},$$

which is negative if $s \neq r$. Therefore, the entire second term of (4.3.M/2) is positive if $s > r$. I have thus demonstrated that $B_{12}(\pi,s,\beta) > 0$ if $s > r$.

Q.E.D.

If one replaces $B(\pi,s)$ and its partials with $B(\pi,s,\beta)$ and its partials throughout Section 2.3, one obtains proofs of the comparative statics results for the case of a firm under a capital productivity requirement. Lemmas 4.1.J, 4.3.C and 4.3.D assure that the relevant partials continue to have the same signs. However, Theorems 2.3.M.A and 2.3.M.B require the additional assumption that the underlying production function is homogeneous with non-decreasing returns to scale in order to assure that $B_{12} > 0$.

4.4. Practical Implications

This chapter has proposed a third model of firm behavior under rate-of-return regulation based on the assumption that firm managers maximize a utility function of profit and institutional costs, instead of profit alone, and that there are regulatory restrictions on capital investment. The model was then used to examine the impacts of two policy instruments available to regulators under rate-of-return regulation: the setting of the required capital productivity and the setting of the allowed rate of return. The model is of value in demonstrating which of the results of the previous two chapters continue to apply under these more sophisticated and, I believe, more realistic assumptions

The main practical implication of Chapter 2 continues to hold for the firm subject to a capital productivity requirement. This is that setting an appropriate allowed rate of return is likely to involve a tradeoff--as profits are reduced, institutional costs tend to increase. Consumer prices as a function of the allowed rate of return will reflect this tradeoff, probably reaching a minimum somewhere between the cost of capital and the rate of return the firm would earn as a monopolist subject only to the capital productivity requirement. One could, therefore, again visualize the relationship between the allowed rate of return and consumer price as being like that shown in Figure 18, where r is the cost of capital, s* is the rate of return the firm would earn as a monopolist subject only to a capital productivity requirement and p_m the price that would be charged by such a monopolist. Section 4.3 demonstrated that the income and substitution effects of a small reduction in the allowed rate of return under a capital productivity requirement are the same as those without the capital productivity requirement, as derived in Section 2.3.

As in Chapter 3, the imposition of a capital productivity requirement would lead managers to purchase gas beyond the point where its marginal cost equals its marginal revenue product, so as to be able to justify additional capital investment. Unfortunately, the rules-of-thumb for how a price-minimizing or welfare-maximizing regulator would set the capital productivity requirement, as derived in Chapter 3, no longer apply to a firm with utility maximizing mangers. In Section 4.2, I was able to determine the signs of the income and substitution effects of a small change in the required capital productivity on profit and institutional costs. These expressions could be used in numerical simulations of actual firms.

The model described in this chapter is also of value in that it can be used to demonstrate how institutional costs might absorb some of the rents from wellhead price controls. This is the subject of the following chapter.

4.4.M Mathematical Formulation

In this subsection, I will show why the relationship between allowed rate-of-return and price for a firm subject to a capital productivity requirement is as shown in Figure 18. I first show that if $s = r$, the firm has the same output and price as a monopolist subject only to a capital productivity requirement. The cases of $s > r$ and $s < r$ will then be considered.

Theorem 4.4.M.A: If $s = r$, the values of g and k chosen by the managers are the same as they would choose if the firm were subject only to a capital productivity requirement. Since price is a function of g, the price charged by this firm $p(g)$ is the same as that charged by the monopolist subject only to a capital productivity requirement.

Proof: By Lemma 4.1.P, if $\pi \leq \pi^*$ then the solution for g and k on on the budget constraint in $\pi - x$ space are the same as they would be if the firm were subject only to a capital productivity requirement. But by Theorem 4.1.M.5.B, if $s = r$, the budget constraint is undefined for values of $\pi > \pi^*$. By Theorem 4.1.M.6.A, the solution chosen by the managers must lie on the budget constraint. Hence, the values of g and k chosen by the managers are the same as they would choose if the firm were subject only to a capital productivity requirement.

Q.E.D.

Lemma 4.4.A: The values of g and k always move in the same direction in response to a change in s.

Proof: I consider two cases, depending upon the value of λ_3.

i) If $\lambda_3 = 0$, then by (4.1.M/1),

$$p'(g)g + p(g) - f'(g)g - f(g) - z_2(k,g) = 0.$$

Differentiating yields

$$\frac{dg}{dk} = \frac{z_{12}(k,g)}{\frac{d}{dg}[p'(g)g + p(g) - f'(g)g - f(g)] - z_{22}(k,g)}.$$

Substituting, using (2.2.M/9), yields

$$\frac{dg}{dk} = \frac{z_{12}(k,g)}{R_{22}(k,g)}.$$

Since $z_{12}(k,g) < 0$ by assumption (see Section 2.5.M), while $R_{22}(k,g) < 0$ by the strict concavity of $R(k,g)$, it follows that $dg/dk > 0$.

ii) If $\lambda_3 > 0$, then the capital productivity requirement (4.1.M/4) is an equality. By Lemma 4.1.C, $dg/dk > 0$.

In both cases, $dg/dk > 0$, implying that g and k must always move in the same direction in response to a change in s.

Q.E.D.

Theorem 4.4.M.B: If $s > r$ and $\pi > \pi^*$ at the solution, then the g chosen by the managers must be greater than the g' they are assumed to choose if subject to the capital productivity requirement alone, and the consumer price p(g) is less than the price the managers would charge if subject to the capital productivity requirement alone. Similarly, if

$s < r$ and $\pi > \pi^*$ at the solution, then $g < g'$ and $p(g)$ is greater than the price the managers would charge if subject to the capital productivity requirement alone.

Proof: The rate-of-return constraint (4.1.M/3) requires that

$$\pi \leq (s-r)k.$$

By Lemma 4.1.0,

$$\pi^* = (s-r)k',$$

where k' is assumed to be the k the managers would choose if subject to the capital productivity requirement alone. Using these two expressions, and the assumption that $\pi > \pi^*$,

$$(s-r)k \geq \pi > \pi^* = (s-r)k',$$

or

$$(s-r)k > (s-r)k'.$$

This implies $k > k'$ if $s > r$, and $k < k'$ if $s < r$.

By Theorem 4.4.M.A, if $s = r$ then $k = k'$ and $g = g'$, where g' is assumed to be the g the managers would choose if subject only to the capital productivity requirement. Since Lemma 4.4.A requires g and k to move in the same direction in response to a change in s, it must be that $g > g'$ if $s > r$ and $g < g'$ if $s < r$. Since $p'(g) < 0$, the results concerning price follow immediately.

Q.E.D.

As in Section 2.5, I have not shown that price as a function of allowed rate of return necessarily has the concave shape shown in Figure 18.

CHAPTER 5

WELLHEAD PRICE CONTROLS

Changes in the allowed rate of return and capital productivity are not the only changes which affect gas firm profit and institutional costs; almost any change in regulations, taxes, or market environment should have an impact. In this chapter, I examine a third instrument available to gas firm regulators -- wellhead price controls. The principal result of this chapter is that instead of simply passing the savings in gas purchase costs from wellhead price controls through to consumers, gas firms may have a tendency to absorb some of the rents generated by wellhead price controls as institutional costs.

I consider two models of wellhead price controls. In Section 5.1, I assume the regulators set a fixed ceiling on the price of all gas. In Section 5.2, I assume the regulators only set a fixed ceiling on the price of certain categories of gas, thereby establishing a system of partial wellhead price controls. In both sections, I shall assume an allowed rate of return above the cost of capital, and that both the rate-of-return constraint and the capital productivity requirement are binding.

5.1 A Ceiling on the Price of All Gas

In this section, I consider a rather extreme policy, and obtain some rather extreme results. I assume the regulators set a fixed ceiling on the wellhead price of all gas. I will show that the gas firm with utility-maximizing managers would respond by charging the market-clearing price for whatever allocation of gas it has available. Institutional costs would absorb all rents on the sale of this gas beyond the profit ceiling established by the regulators. Consumers, as well as gas producers and the gas firm's stockholders, would actually be worse off under this system than they would be under a system with no wellhead price controls.

To establish these results, I first consider the situation with no wellhead price controls. In this case, the gas market could be visualized as in Figure 35. Curve S is the wellhead supply of gas, while

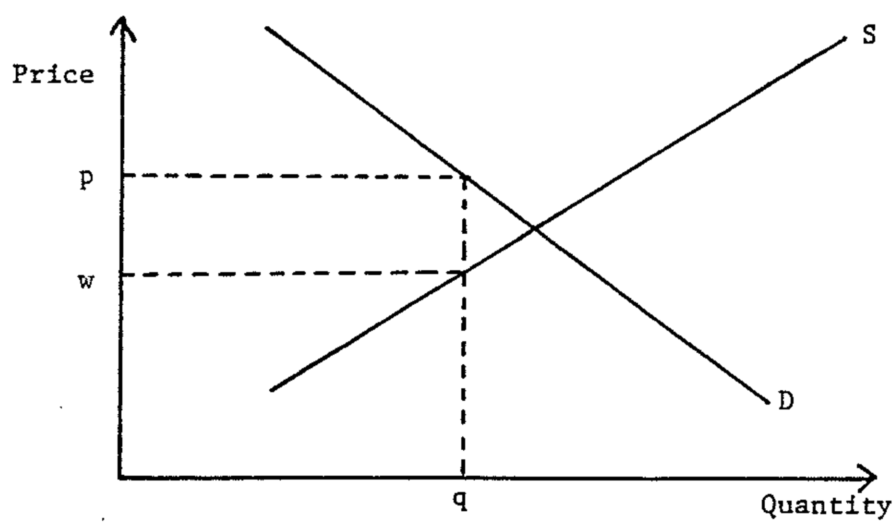

Figure 35
Gas Market without Wellhead Price Ceiling

curve D is the consumer demand for gas. The market will settle into an equilibrium where quantity q of gas is sold, with w being the wellhead price, p being the consumer price, and p - w gas firm markup.

If one were to neglect the response of pipelines and distribution companies and simply assume their markup is constant, one would expect the introduction of a wellhead price ceiling to produce a situation like that depicted in Figure 36. The wellhead price ceiling is set at w'. With a constant pipeline and distribution company markup, the resulting consumer price will be p'. The wellhead price controls have succeeded in reducing consumer prices from p to p', but also create a shortage of gas equal to q" - q', the difference between what consumers would like to buy at this new low price, and what producers are willing to produce.

If one assumes the model of gas firm behavior proposed in the preceding chapter, a different situation emerges, however. Since the gas price would be limited, the gas supply curve would fix a limit on available supply, as before. But, given a fixed supply, the capital productivity requirement would fix a limit on gas firm capital investment.

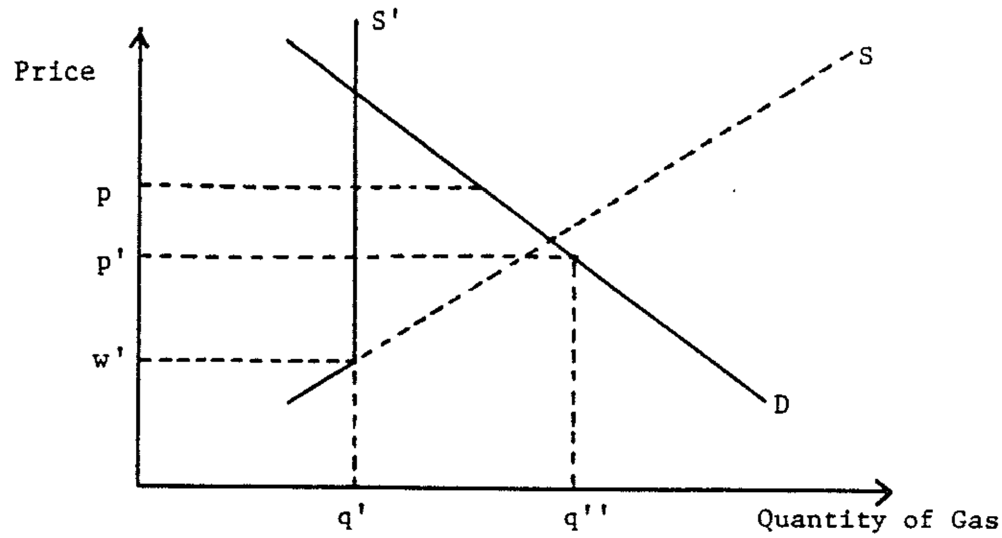

Figure 36
Gas Market with Wellhead Price Ceiling Given
Constant Gas Firm Markup

Given the limit on capital investment, the rate-of-return constraint would fix a limit on gas firm profits. Since there is no way the managers can do anything to increase profits, there is nothing to stop them from simply allowing institutional costs to rise until the gas price reaches its market-clearing level. The outcome would then be as shown in Figure 37.

It can be seen that the resulting consumer price p" would be higher than it would be without the wellhead price controls, due to the increase in gas firm markup. Gas firms would have lower profits under

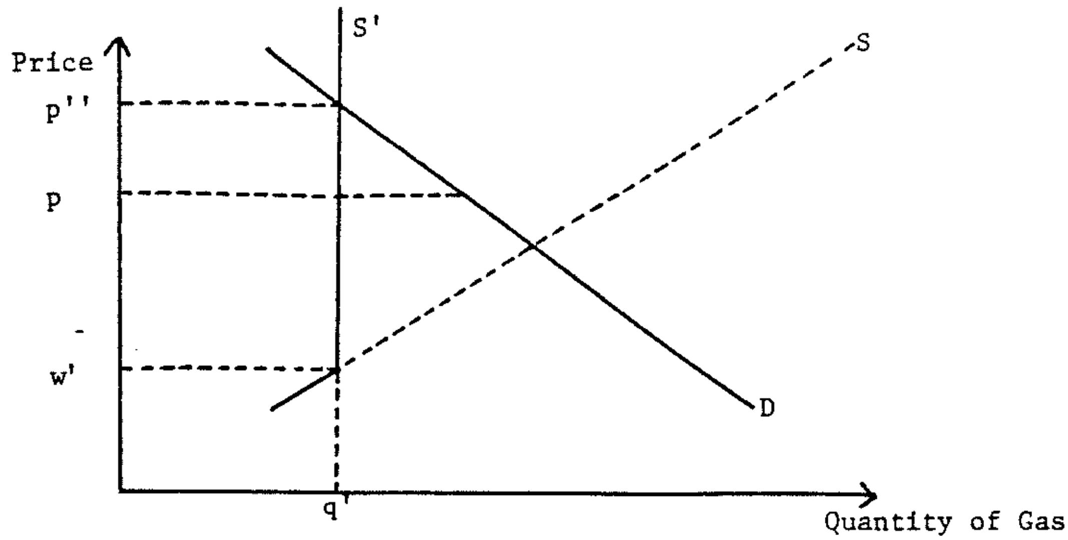

Figure 37
Gas Market with Wellhead Price Ceiling Given Gas
Firms as Modeled in Chapter 4

the wellhead price ceiling as well, since lower gas volume means lower capital investment by the capital productivity requirement, which means lower profit by the rate-of-return constraint. Naturally, gas producers would also be worse off under wellhead price controls. It should be added, however, that my model may tend to overstate the case, since it neglects regulatory lag, which generally gives gas firms some ability to increase short-term profits by reducing institutional costs.

The change in the gas firm's budget constraint with the introduction of the wellhead price ceiling is illustrated in Figure 38. The new constraint has a vertical drop since, as noted, a limit on the firm's gas supply fixes a limit on the firm's profits.

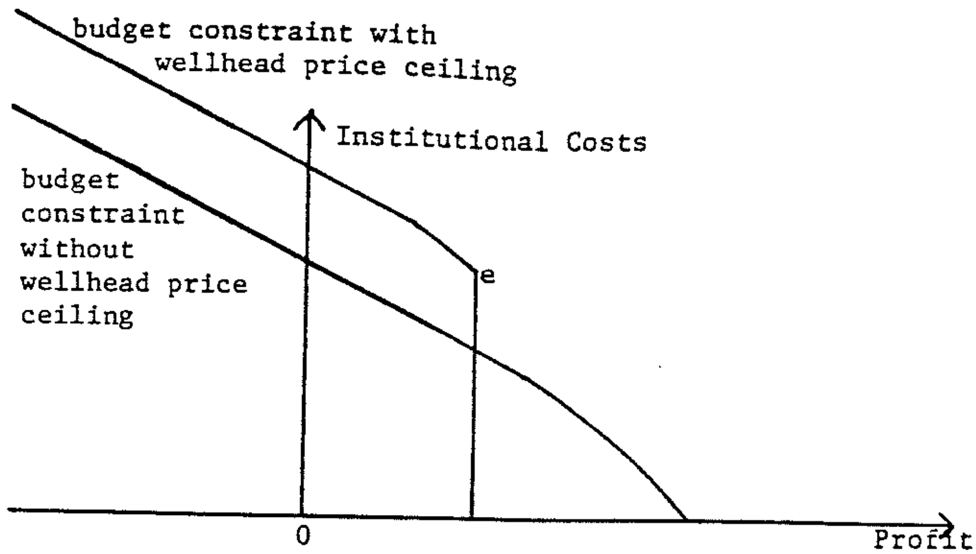

Figure 38
Effect of Wellhead Price Ceiling on the Gas Firm's
Budget Constraint

5.1.M Mathematical Formulation

In this subsection, I will demonstrate how a reduction in the wellhead ceiling price of gas is likely to lead to an increase in consumer prices, and a reduction in gas firm profit. I assume that there is a well-defined function $G(w)$, with $G'(w)>0$, giving the gas allocation available to the firm at each setting of the wellhead price ceiling w. If the firm is a monopsony in the wellhead gas market, then $G(w)$ is simply the gas supply function. Otherwise $G(w)$ is determined by some non-market mechanism, such as prior contracts or rationing by the regulators. The remainder of my notation and assumptions are the same as were used in Section 4.1.M.

The following theorem is crucial to determining how a reduction in the wellhead price ceiling affects consumers.

Theorem 5.1.M.A: The firm with utility-maximizing managers never sells gas for less than its market-clearing price.

Proof: Assume that the utility-maximizing solution did require the managers to sell gas for less than its market-clearing price. Given this solution, they could increase their level of institutional costs without changing the level of profit by simply increasing the price of gas. This would always be feasible under the rate-of-return constraint, the capital productivity requirement, and the firm's gas allocation under the wellhead price ceiling, since the price increase would not affect g or k at the solution. But, since institutional costs are assumed to have positive marginal utility, the managers' utility would be improved by this price increase. This is a contradiction, since a gas price lower than the market-clearing level was assumed to maximize utility. So the firm never sells gas for less than its market-clearing price.

Q.E.D.

The desired result concerning consumer prices then follows easily.

Theorem 5.1.M.B: If the gas allocation available to the firm under the wellhead price ceiling G(w) is a binding constraint on the firm's gas supply g, then a reduction in the wellhead price ceiling w, increases the price of gas to consumers p(g).

Proof: I must show $dp/dw < 0$. By the chain rule,

$$\frac{dp}{dw} = p'(g) \frac{dg}{dG} G'(w).$$

By assumption, $p'(g) < 0$ and $G'(w) > 0$. Since $G(w)$ is assumed to be a binding constraint on g, $dg/dG = 1$. So I have shown that $dp/dw < 0$.

Q.E.D.

The final theorem gives the desired result concerning gas firm profit.

<u>Theorem 5.1.M.C</u>: If the gas allocation available to the firm under the wellhead price ceiling $G(w)$, is a binding constraint on the firm's gas supply g, then a reduction in the wellhead price ceiling w, leads to a reduction in gas firm profits π.

Proof: I must show $d\pi/dw > 0$. By the chain rule,

$$\frac{d\pi}{dw} = \frac{dg}{dG} G'(w)/\frac{dg}{d\pi} \cdot$$

Since $G(w)$ is assumed to be a binding constraint on g, $dg/dG = 1$. By assumption, $G'(w) > 0$. By Lemma 4.1.D, if the rate-of-return constraint and capital productivity requirement are binding and s is not equal to r, as assumed throughout this chapter, then $dg/d\pi > 0$. So I have shown that $d\pi/dw > 0$.

Q.E.D.

5.2 Partial Wellhead Price Controls

An alternative to establishing a fixed ceiling on the wellhead price of all gas is to establish ceilings only on the price of gas from low-elasticity sources. Such a system of partial wellhead price controls can reduce the average wellhead price of gas, but, unlike a ceiling on the wellhead price of all gas, does not put a corresponding limit on the overall available supply of gas. Although such a system could reduce consumer prices, this section will show why it could also tend to increase the institutional costs of gas firms.

Under a system of partial wellhead price controls, the regulators set wellhead price ceilings on a discriminatory basis -- low price ceilings on low-elasticity sources, no price ceiling at all on high-elasticity sources. The regulators then let the gas firm's average-cost pricing system determine the outcome. Gas firms pay less than the free-market wellhead price for gas from low-elasticity sources, but generally end-up paying more than the free-market wellhead price for gas from high-elasticity sources. The higher production of gas from high-elasticity sources more than compensates for the lower production of gas from low-elasticity sources. The result can be greater gas production at a lower average wellhead price than under a system without wellhead price controls. The obvious appeal of this system to policymakers seeking to hold-down consumer gas prices may explain the adoption of the system by the U.S. Congress in the Natural Gas Policy Act of 1978.

The system does, however, create the potential for increased institutional costs in gas firms. To see this, one may note that a reduction in the price ceiling on price-controlled gas will probably have two impacts on the gas firm's budget constraint. First, assuming the reduction in the price ceiling succeeds in lowering the purchase cost of any given volume of gas to the firm, the budget constraint shifts upward. This is because there is a fixed volume of gas associated with any given level of profit, independent of its cost. This fixed volume is determined by the rate-of-return constraint, which fixes a required level of capital investment for the given level of profit, and the capital productivity requirement, which fixes a required gas volume associated with that required level of capital investment. Since the purchase cost of the fixed volume of gas associated with each level of profit is reduced, the firm can have higher institutional costs at each level of profit. The budget constraint, therefore, shifts upward.

Second, assuming the marginal cost of non-price-controlled gas increases as a result of the reduction in the price ceiling on price-controlled gas, the budget constraint becomes steeper. One would expect the marginal cost of non-price-controlled gas to increase as the price ceiling on price-controlled gas is reduced, since the smaller available quantity of price-controlled gas leads to increased demand for non-price-controlled gas. The increase in the marginal cost of non-price-controlled gas makes it more expensive for the managers to acquire the gas they need to justify an increase in capital investment under the capital productivity requirement, so as to increase their profit ceiling under the rate-of-return constraint. The budget constraint, therefore, becomes steeper.

The change in the budget constraint with the introduction of partial wellhead price controls is illustrated in Figure 39. The

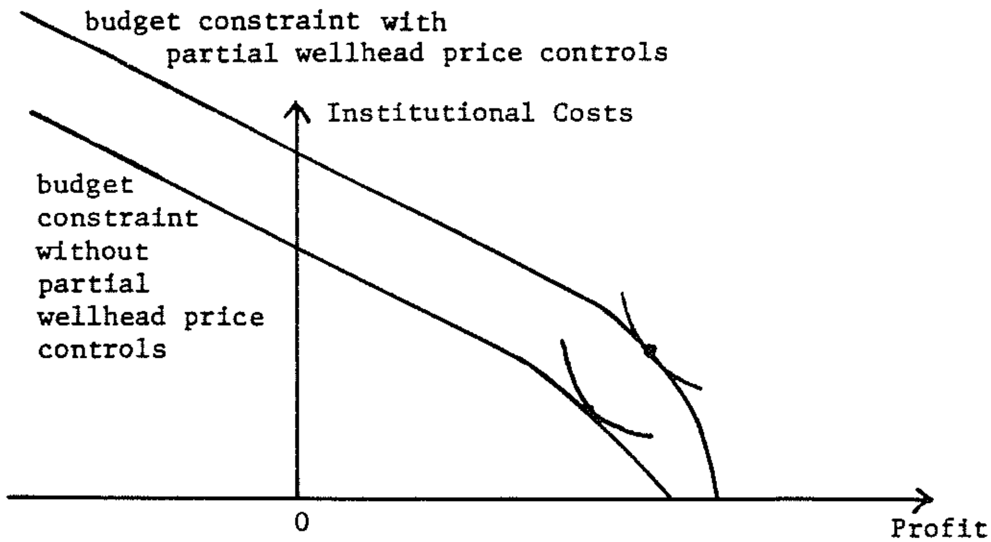

Figure 39
Effect of Partial Wellhead Price Controls on the
Gas Firm's Budget Constraint

shift in the budget constraint will lead to an income effect favoring higher institutional costs. At the same time, the increased steepness makes the profit/institutional costs tradeoff more favorable to institutional costs, so the substitution effect also favors higher institutional costs. The overall effect of discriminatory wellhead price ceilings is thus to increase gas firm institutional costs, just as in the case of a fixed ceiling on the wellhead price of all gas.

5.2.M Mathematical Formulation

In this subsection, I examine the situation if not all gas sources are subject to a wellhead price ceiling. I will demonstrate how a reduction in the wellhead price ceiling could lead to increased gas firm institutional costs. I assume the firm's total gas purchase costs are a function of the firm's total gas purchases g and the ceiling price on price-controlled gas w. Let $d(g,w)$ be this total gas purchase cost function. The remainder of my notation and assumptions are the same as were used in Section 4.1.M.

I assume $d_2(g,w) > 0$. This implies that a reduction in the wellhead price ceiling succeeds in lowering the purchase cost of any given volume of gas to the firm. I also assume $d_{12}(g,w) < 0$. This implies that a reduction in the wellhead price ceiling increases the firm's marginal gas purchase costs.

To see why it should be that $d_{12}(g,w) < 0$, let $G(w)$ again give the allocation of price-controlled gas available to the firm at each setting of the wellhead price ceiling w, with $G'(w) > 0$. Also let:

$g_o(w)$ = total gas volume purchased by all other firms ($g_o'(w) \leq 0$);

$G_o(w)$ = allocation of price-controlled gas to all other firms ($G_o'(w) \geq 0$);

$f\big(g-G(w)+g_o(w)-G_o(w)\big)$ = wellhead price of non-price-controlled gas ($f'(\ ) > 0$).

The following theorem may then be used to help justify the assumption.

<u>Theorem 5.2.M.A</u>: If the marginal cost of gas to the firm is a non-decreasing function of gas purchases by other firms, then $d_{12}(g,w) < 0$.

Proof: The firm's gas purchase costs are

$$d(g,w) = wG(w) + \big(g - G(w)\big)f\big(g-G(w)+g_o(w)-G_o(w)\big).$$

The firm's marginal cost of gas is the derivitive of this expression with respect to g,

$$d_1(g,w) = f\big(g-G(w)+g_o(w)-G_o(w)\big)$$

$$+ \big(g - G(w)\big)f'\big(g-G(w)+g_o(w)-G_o(w)\big), \qquad (5.2.M/1)$$

from which it can be seen that

$$d_{12}(g,w) = \frac{\partial d_1(g,w)}{\partial g_o(w)} g'_o(w) + \frac{\partial d_1(g,w)}{\partial G_o(w)} G'_o(w) + \frac{\partial d_1(g,w)}{\partial G(w)} G'(w)$$

(5.2.M/2)

To determine the sign of this expression, I must determine the signs of the partials of $d_1(g,w)$. Since I assume the marginal cost of gas to the firm is a non-decreasing function of gas volume purchased by other firms, $\partial d_1(g,w)/\partial g_o(w) \geq 0$. It will be useful to have an explicit expression for $\partial d_1(g,w)/\partial g_o(w)$, which may be obtained by taking the partial of (5.2.M/1) with respect to $g_o(w)$,

$$\frac{\partial d_1(g,w)}{\partial g_o(w)} = f'\big(g-G(w)+g_o(w)-G_o(w)\big)$$

$$+ \big(g - G(w)\big)f''\big(g-G(w)+g_o(w)-G_o(w)\big). \qquad (5.2.M/3)$$

Taking the partial of (5.2.M/1) with respect to $G_o(w)$ yields

$$\frac{\partial d_1(g,w)}{\partial G_o(w)} = -f'\big(g-G(w)+g_o(w)-G_o(w)\big)$$

$$- \big(g - G(w)\big)f''\big(g-G(w)+g_o(w)-G_o(w)\big),$$

which, by (5.2.M/3), is equal to $-\partial d_1(g,w)/\partial g_o(w)$. Hence, $\partial d_1(g,w)/\partial G_o(w) \leq 0$.

Taking the partial of (5.2.M/1) with respect to G(w) yields

$$\frac{\partial d_1(g,w)}{\partial G(w)} = -f'\big(g-G(w)+g_o(w)-G_o(w)\big)$$

$$- \big(g - G(w)\big)f''\big(g-G(w)+g_o(w)-G_o(w)\big)$$

$$- f'\big(g-G(w)+g_o(w)-G_o(w)\big).$$

Substituting, using (5.2.M/3),

$$\frac{\partial d_1(g,w)}{\partial G(w)} = \frac{-\partial d_1(g,w)}{\partial g_o(w)} - f'\big(g-G(w)+g_o(w)-G_o(w)\big).$$

Since $\partial d_1(g,w)/\partial g_o(w) \geq 0$ and $f'(\ ) > 0$ by assumption, $\partial d_1(g,w)/\partial G(w) < 0$.

Using these results concerning the signs of the partials of $d_1(g,w)$ and the assumptions that $g_o'(w) \leq 0$, $G_o'(w) \geq 0$, and $G'(w) > 0$, one can obtain the signs of each term of (5.2.M/2). The first and second terms are non-positive while the third term is negative, so I have shown that $d_{12}(g,w) < 0$ as claimed.

Q.E.D.

The budget constraint in π - x space faced by the managers of the firm under partial wellhead price controls may be obtained by maximizing

$$p(g)g - d(g,w) - z(k,g) - rk - \pi,$$

subject to the rate-of-return constraint

$$(s-r)k - \pi \geq 0,$$

and the capital productivity requirement

$$-z_1(k,g) - \beta \geq 0.$$

Maximization is over g and k, where both must be non-negative. The Lagrangian will be

$$L = p(g)g - d(g,w) - z(k,g) - rk - \pi + \lambda_2\big((s-r)k - \pi\big)$$
$$- \lambda_3\big(z_1(k,g) + \beta\big).$$

The first-order conditions require that:

$$\frac{\partial L}{\partial g} = p'(g)g + p(g) - d_1(g,w) - z_2(k,g) - \lambda_3 z_{12}(k,g) \leq 0;$$
$$\frac{\partial L}{\partial g} g = 0; \qquad g \geq 0; \qquad (5.2.M/4)$$

$$\frac{\partial L}{\partial k} = -z_1(k,g) - r + \lambda_2(s-r) - \lambda_3 z_{11}(k,g) \leq 0; \qquad (5.2.M/5)$$
$$\frac{\partial L}{\partial k} k = 0; \qquad k \geq 0;$$

$$\frac{\partial L}{\partial \lambda_2} = (s-r)k - \pi \geq 0; \qquad (5.2.M/6)$$
$$\frac{\partial L}{\partial \lambda_2}\lambda_2 = 0; \qquad \lambda_2 \geq 0;$$

$$\frac{\partial L}{\partial \lambda_3} = -z_1(k,g) - \beta \geq 0; \qquad (5.2.M/7)$$
$$\frac{\partial L}{\partial \lambda_3}\lambda_3 = 0; \qquad \lambda_3 \geq 0.$$

I shall assume that g and k are greater than zero at the solution, which implies that (5.2.M/4) and (5.2.M/5) must be equalities.

For notational convenience, define the budget constraint in π - x space, obtained by solving the optimization problem above, as

$$x = B(\pi, s, \beta, w).$$

It will be useful throughout the remainder of this section to know some of the partials of this function, assuming the constraints are binding. The following series of lemmas gives the needed partials. Note, first, that the rate-of-return constraint (5.2.M/6) is identical to the rate-of-return constraint (2.2.M.2/3), while the capital productivity requirement (5.2.M/7) is identical to the capital productivity requirement (4.1.M/4). This implies that lemmas 2.2.B and 4.1.D also apply to the firm under partial wellhead price controls.

Lemma 5.2.A: If the rate-of-return constraint and the capital productivity requirement are equalities and $s \neq r$, then

$$B_1(\pi, s, \beta, w) \leq -1.$$

Proof: By definition,

$$B(\pi, s, \beta, w) = p(g)g - d(g,w) - z(k,g) - rk - \pi,$$

so

$$B_1(\pi, s, \beta, w) = [p'(g)g + p(g) - d_1(g,w) - z_2(k,g)]\frac{dg}{d\pi} - [z_1(k,g) + r]\frac{dk}{d\pi} - 1.$$

Substituting for $dg/d\pi$ and $dk/d\pi$ using Lemmas 4.1.D and 2.2.B yields

$$B_1(\pi, s, \beta, w) = -[p'(g)g+p(g)-d_1(g,w)-z_2(k,g)]\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{1}{s-r}\right) - [z_1(k,g) + r]\frac{1}{s-r} - 1.$$

But because g and k lie on the budget constraint in π - x space, (5.2.M/4) and (5.2.M/5) may be solved simultaneously to eliminate λ_3, yielding

$$-[p'(g)g + p(g) - d_1(g,w) - z_2(k,g)]\left(\frac{z_{11}(k,g)}{z_{12}(k,g)}\right)\left(\frac{1}{s-r}\right)$$
$$- [z_1(k,g) + r]\frac{1}{s-r} = -\lambda_2.$$

So,

$$B_1(\pi,s,\beta,w) = -\lambda_2 - 1.$$

Since $\lambda_2 \geq 0$, I have shown $B_1(\pi,s,\beta,w) \leq -1$.

Q.E.D.

<u>Lemma 5.2.B</u>: If the rate-of-return constraint and the capital productivity requirement are equalities and $s \neq r$, then total gas purchases g and capital k must be independent of the price ceiling w for any given level of profit π.

Proof: If the rate-of-return constraint (5.2.M/6) is binding with $s \neq r$, it may be solved to determine k as a function of exogenous variables s and π, but not w. So k is independent of w. Since I assume $z_{12}(k,g) < 0$ (see Section 2.5.M), $-z_1(k,g)$ is a monotonically increasing function of g for any given k. If the capital productivity requirement (5.2.M/7) is an equality, it will therefore fix a unique level of g for any given k, so g must independent of w as well.

Q.E.D.

Lemma 5.2.C: If the rate-of-return constraint and the capital productivity requirement are equalities and $s \neq r$, then

$$B_4(\pi,s,\beta,w) < 0.$$

Proof: By definition

$$B(\pi,s,\beta,w) = p(g)g - d(g,w) - z(k,g) - rk - \pi.$$

Since, by the preceding lemma, g and k are independent to w,

$$B_4(\pi,s,\beta,w) = -d_2(g,w). \quad (5.2.M/8)$$

But $d_2(g,w) > 0$ by assumption, so $B_4(\pi,s,\beta,w) < 0$.

Q.E.D.

Lemma 5.2.D: If the rate-of-return constraint and the capital productivity requirement are equalities and $s > r$, then

$$B_{14}(\pi,s,\beta,w) > 0.$$

Proof: By (5.2.M/8)

$$B_4(\pi,s,\beta,w) = -d_2(g,w),$$

so

$$B_{14}(\pi,s,\beta,w) = -d_{12}(g,w)\frac{dg}{d\pi}. \quad (5.2.M/9)$$

By Lemma 4.1.D, $dg/d\pi > 0$ if $s > r$, while I assume $d_{12}(g,w) < 0$. So $B_{14}(\pi,s,\beta,w) > 0$.

Q.E.D.

If one assumes that both the rate-of-return constraint and the capital productivity requirement are equalities at the solution, the manager's problem is to maximize utility

$$U(\pi,x),$$

subject to the budget constraint,

$$B(\pi,s,\beta,w) - x \geq 0.$$

Maximization is over π and x, where x, but not π, must be non-negative. The Lagrangian will be

$$L = U(\pi,x) + \lambda[B(\pi,s,\beta,w) - x].$$

The first-order conditions require that:

$$\frac{\partial L}{\partial \pi} = U_1(\pi,x) + \lambda[B_1(\pi,s,\beta,w)] = 0; \qquad (5.2.M/10)$$

$$\frac{\partial L}{\partial x} = U_2(\pi,x) - \lambda \leq 0; \qquad (5.2.M/11)$$

$$\frac{\partial L}{\partial x}\, x = 0; \qquad x \geq 0;$$

$$\frac{\partial L}{\partial \lambda} = B(\pi,s,\beta,w) - x \geq 0; \qquad (5.2.M/12)$$

$$\frac{\partial L}{\partial \lambda}\, \lambda = 0; \qquad \lambda \geq 0.$$

If one assumes $U_2(\pi,x) > 0$ at the solution, that is the managers are not satiated in institutional costs, then λ must be greater than zero by

(5.2.M/11). If one also assumes that $x > 0$ at the solution, then both (5.2.M/11) and (5.2.M/12) become equalities.

The second-order conditions require that the determinant

$$D = \begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix}$$

be greater than zero. Note that I have dropped the parameters of the partials of $U(\pi,x)$ and $B(\pi,s,\beta,w)$ for compactness of notation.

To find the effect of a small change in G, dG, conditions (5.2.M/10) - (5.2.M/12) may be totally differentiated. This yields a system of simultaneous equations in $d\pi$, dx, $d\lambda$, and dw. Dropping the parameters of $U(\pi,x)$ and $B(\pi,s,\beta,w)$, the system may be expressed in matrix notation as

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{14} \\ 0 \\ -B_4 \end{bmatrix} dw. \tag{5.2.M/13}$$

I am now in a position to show why, under these assumptions, institutional costs increase when the wellhead price ceiling is lowered.

Theorem 5.2.M.A: The substitution effect of an increase in the wellhead price ceiling on institutional costs $(dx/dw)_s$ is negative if $s > r$.

Proof: To derive the substitution effect, one wants to add some compensation, call it y, to the budget constraint (5.2.M/12) so as to hold the solution to the same isoutility curve when w changes. If this were done, (5.2.M/13) could be rewritten as

$$\begin{vmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{vmatrix} \begin{vmatrix} d\pi \\ dx \\ d\lambda \end{vmatrix} = \begin{vmatrix} -\lambda B_{14} \\ 0 \\ -B_4 - \frac{dy}{dw} \end{vmatrix} dw.$$

(5.2.M.14)

If one knew dy/dw, this system could be solved for the substitution effect $(d\pi/dw)_s$. But is possible to solve for dy/dw. For by the third equation of this system,

$$B_1 d\pi - dx = (-B_4 - \frac{dy}{dw})dw.$$

Now, by (5.2.M/10) and (5.2.M/11),

$$\frac{U_1}{U_2} = B_1,$$

so substituting back,

$$-\frac{U_1}{U_2} d\pi - dx = (-B_4 - \frac{dy}{dw})dw. \qquad (5.2.M/15)$$

If the solution is to be held to the same isoutility curve $U(\pi,x)$ = constant, total differentiation may be used to show that

$$\frac{dx}{d\pi} = -\frac{U_1}{U_2},$$

which implies that the left side of (5.2.M/15) is equal to zero. Thus, it must be that

$$\frac{dy}{dw} = -B_4$$

if the solution is to be held to a given isoutility curve.

Substituting into (5.2.M/14) yields the system

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} -\lambda B_{14} \\ 0 \\ 0 \end{bmatrix} dw, \qquad (5.2.M/16)$$

which may be solved for the substitution effect $(dx/dw)_s$. Applying Cramer's rule,

$$\left(\frac{dx}{dw}\right)_s = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{14} & B_1 \\ U_{12} & 0 & -1 \\ B_1 & 0 & 0 \end{vmatrix}}{D},$$

or

$$\left(\frac{dx}{dw}\right)_s = \frac{\lambda B_{14} B_1}{D}. \qquad (5.2.M/17)$$

Since $\lambda = U_2(\pi,x) > 0$ by assumption, while $B_{14}(\pi,s,\beta,w) > 0$ if $s > r$ by Lemma 5.2.D, and $B_1(\pi,s,\beta,w) \leq 0$ by Lemma 5.2.A, I have shown that

$$\left(\frac{dx}{dw}\right)_s < 0.$$

Q.E.D.

Recall from Section 2.3.M that institutional costs are defined to be a "normal good" if their level would increase with a relaxation of the budget constraint. One could relax the budget constraint by adding a positive y to the left side of (5.2.M/12). Hence, if a small increase in y produces an increase in institutional costs x, that is if $dx/dy > 0$, then profit is a normal good.

<u>Theorem 5.2.M.B</u>: If institutional costs are a normal good, then the income effect $(dx/dw)_y$ of an increase in the wellhead price ceiling on institutional costs is negative.

Proof: Since institutional costs are a normal good, $dx/dy > 0$. To find an expression for dx/dy, I totally differentiate (5.2.M/10) - (5.2.M.12),

$$\begin{bmatrix} U_{11} + \lambda B_{11} & U_{12} & B_1 \\ U_{12} & U_{22} & -1 \\ B_1 & -1 & 0 \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} dy. \qquad (5.2.M/18)$$

Applying Cramer's rule,

$$\frac{dx}{dy} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & 0 & B_1 \\ U_{12} & 0 & -1 \\ B_1 & -1 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{dy} = \frac{-(U_{11} + \lambda B_{11}) - U_{12}B_1}{D} \qquad (5.2.M/19)$$

To find the income effect $(dx/dw)_y$, one finds the total effect dx/dw, and subtracts the substitution effect $(dx/dw)_s$. The total effect dx/dw may be found by applying Cramer's rule to system (5.2.M/13),

$$\frac{dx}{dw} = \frac{\begin{vmatrix} U_{11} + \lambda B_{11} & -\lambda B_{14} & B_1 \\ U_{12} & 0 & -1 \\ B_1 & -B_4 & 0 \end{vmatrix}}{D},$$

or

$$\frac{dx}{dw} = \frac{\lambda B_{14}B_1 + B_4[-(U_{11} + \lambda B_{11}) - U_{12}B_1]}{D}.$$

Subtracting the substitution effect (5.2.M/17), yields the income effect

$$\left(\frac{dx}{dw}\right)_y = \frac{B_4[-(U_{11} + \lambda B_{11}) - U_{12}B_1]}{D},$$

or, substituting from (5.2.M/19),

$$\left(\frac{dx}{dw}\right)_y = B_4\left(\frac{dx}{dy}\right).$$

Since $B_4 < 0$ by Lemma 5.2.C, while $dx/dy > 0$ if institutional costs are a normal good, $(dx/dw)_y$ must be less than zero.

Q.E.D.

I am now in a position to prove the principal result of this section.

<u>Theorem 5.2.M.C</u>: If institutional costs x are a normal good and $s > r$, then institutional costs must increase overall if the wellhead price ceiling w is lowered.

Proof: By Theorem 5.2.M.A, the substitution effect $(dx/dw)_s$ is negative if $s > r$, while by Theorem 5.2.M.B, the income effect $(dx/dw)_y$ is negative as long as x is a normal good. So the overall effect dx/dw is negative.

Q.E.D.

CHAPTER 6

EMPIRICAL DISCUSSION

The theory proposed in the preceding chapters suggests three hypotheses about the behavior of firms under rate-of-return regulation which should be empirically testable. These are that gas firms should be selling gas with a marginal cost exceeding its marginal revenue product; that institutional costs should tend to increase as the allowed rate of return is reduced; and that wellhead price controls should produce increased institutional costs. Unfortunately, a full empirical investigation of these hypotheses is beyond the scope of this study. This chapter will, however, survey some of the available data and studies for suggestive evidence related to these hypotheses. The chapter also discusses some of the difficulties to be encountered in empirical tests, and approaches for dealing with them.

The first section presents evidence that gas firms are selling gas with a marginal cost exceeding its marginal revenue product. The second section discusses several studies dealing with institutional costs in regulated firms. One of these studies provides some tentative support for the hypothesis that institutional costs tend to increase as the allowed rate of return is reduced. The third section presents an overview of gas industry cost trends during the period 1973-1981, when rapid rises in the prices of competing fuels, but smaller rises in wellhead price ceilings, could have produced an environment favorable to higher institutional costs. The trends show rapid increases in gas industry costs in a pattern consistent with increasing institutional costs.

6.1. Gas Firm Pricing

In Chapters 3 and 4, I argued that regulatory restrictions on capital investment might force gas firms to sell gas with a marginal cost exceeding its marginal revenue product. This result distinguishes my model, with its restrictions on capital investment, from others, such as the Averch-Johnson model, which predict that gas firms would set the marginal cost of gas equal to its marginal revenue product.

Wellhead gas purchase price data for late 1981 have been published for 19 of the 20 largest interstate pipeline firms having purchased gas adjustment (P.G.A.) clauses in their tariffs. A P.G.A. clause permits a pipeline firm to adjust its tariffs for changes in its cost of purchased gas without formal regulatory review. The data were taken from statements of wellhead gas purchases which these companies file periodically with FERC in support of these tariff adjustments. In 1981, these firms accounted for 82% of all gas sold for resale by class A and B FERC-regulated pipelines[1], and 75% of all gas sold for resale in the United States.[2]

Table 4 may be used to demonstrate that all 19 pipelines were selling gas with a marginal cost exceeding its marginal revenue product. The first column gives the average wellhead price paid by each firm for gas from sources classified as "high-cost" under Section 107 of the Natural Gas Policy Act of 1978. The wellhead price of gas from these sources is either completely unregulated, or subject to a substantially higher price ceiling than other sources. Since these are average high-cost gas prices rather than marginal costs, they are certainly conservative estimates of each firm's marginal cost of gas. The second column gives my calculation of the highest tariff each firm was charging for gas. Since these are prices rather than marginal revenues, and since these prices must cover transmission costs as well as gas purchase costs, they are certainly over-estimates of each firm's marginal revenue product of gas.

It can be seen that for every firm except Texas Eastern the average wellhead price for high-cost gas exceeded the highest tariff the firm was charging, generally by a substantial margin. In the case of Texas

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[1]Volume of gas sold for resale by each FERC-regulaged pipeline and by all class A and B FERC-regulated pipelines are given in U.S. Energy Information Administration, <u>Statistics of Interstate Natural Gas Pipeline Companies 1981</u>, DOE/EIA-0145(81) (Washington, DC: Government Printing Office, October 1982), pp. 236-252. Class A and B pipelines are those having annual gas operating revenues of $1,000,000 or more.

[2]Total volume of gas sold for resale in the United States is given in American Gas Association, <u>Gas Facts 1981</u> (Arlington, VA: 1982), p. 85.

TABLE 4
AVERAGE WELLHEAD PRICE FOR HIGH-COST GAS VS. HIGHEST TARIFF
19 MAJOR INTERSTATE PIPELINES
LATE 1981

Pipeline Firm	Average Wellhead Price for High-Cost Gas ($/m.c.f.)	Highest Tariff[1] ($/m.c.f.)
Southern	8.04	3.32
Transco	6.89	4.56
United	5.03	3.41
Trunkline	7.88	4.37
Colorado Interstate	7.06	3.74
Columbia	7.76	3.91
Northwest	3.63	3.40
Michigan-Wisconsin	4.96	3.52
El Paso	4.20	2.44
Transwestern	5.81	3.14
Florida Gas	5.84	2.88
Texas Gas	8.81	2.99
Natural Gas Pipeline	3.75	2.80
Tennessee	8.25	3.34
Kansas-Nebraska	6.39	2.31
Northern Natural	3.59	3.32
Panhandle Eastern	4.80	3.27
Cities Service	4.02	3.32
Texas Eastern	3.92	5.08[2]

SOURCES: Average high-cost gas wellhead prices were taken from U.S. Energy Information Administration (E.I.A.), The Current State of the Natural Gas Market, DOE/EIA-0313 (Washington, D.C.: Government Printing Office, December, 1981), p.66.

(continued)

TABLE 4-Continued

Tariffs were taken from H. Zinder and Associates, Summary of Rate Schedules of Natural Gas Pipeline Companies, 52nd edition (Washington, DC: September 15, 1981), with the exception of tariffs for Colorado Interstate, Transwestern, Florida Gas, and Cities Service which were taken from the 53rd edition of the same source (March 15, 1982).

NOTES: Gas wellhead price data were developed from the companies' most recent Purchase Gas Adjustment Filing with FERC as of October, 1981. Filing dates for each company are shown in E.I.A., Current State, p. 143. Each tariff quoted went into effect on or after the date of the cited P.G.A. filing for the company.

A 20th firm, Consolidated Gas Supply, was included in the E.I.A.'s sample, but no average high cost gas purchase price was published for this firm, since the firm purchased little or no high cost gas. Consolidated is a Northeastern regional system which purchases its gas primarily from other pipelines.

[1]All tariff calculations include both commodity charge and demand charge, where applicable. For demand/commodity tariffs the buyer was assumed to take 50% of the maximum contractually available gas volume, unless the tariff specified a higher take-or-pay requirement, in which case the required take was assumed. For tariffs quoted in decatherms, a 1.0225 decatherm/m.c.f. conversion factor was used.

[2]This tariff is an "overrun charge" for gas purchased during the winter-period in excess of the maximum daily volume the seller has contractually agreed to make available. The next highest tariff was \$3.45/m.c.f.

Eastern, the $5.09 per m.c.f. tariff would have been charged only under circumstances too unusual for it to be taken as representative of the firm's marginal revenue. The next highest tariff is probably more representative, and at $3.45 per m.c.f. is comfortably below Texas Eastern's average high-cost gas purchase price of $3.92 per m.c.f.

One may object that these firms' high cost gas purchases are merely indicative of an industry trying to meet its contractual service obligations in the face of a gas shortage, rather than the result of constraints on capital investment. However, in abstract terms, service obligations merely require the firm to charge a market-clearing price. It is the regulatory constraint on capital investment which explains why firm managers do not alternatively make some, perhaps useless, addition to their rate base. They could then raise their rates to the market-clearing level and increase the regulatory ceiling on profits at the same time.

Of course, Table 4 is only suggestive. A more thorough analysis of industry gas purchase policies will be needed to determine whether and how restrictions on capital investment have affected the industry in practice. However, the data presented here areconsistent with the hypothesis that restrictions on capital investment have forced gas firms to sell gas with a marginal cost exceeding its marginal revenue product.

6.2 Regulation and Institutional Costs

Institutional costs are difficult to investigate empirically since they are inherently unobservable. There have, however, been several studies attempting to show how a regulated environment produces higher institutional costs. This section will discuss these studies, as well as one other study which may be reinterpreted as a test of my hypothesis that a reduction in the allowed rate of return leads to an increase in institutional costs.

Institutional costs are not, unfortunately, a line item in the accounts of any firm. In fact, as discussed in Chapter 2, they exist only because of the imperfect information which the firm owners have about the managers' performance. The researcher seeking to investigate institutional costs empirically must operate in this same environment of imperfect information. The approach which the researcher must use is to carefully try to take into account all of the other factors which should explain variations in cost between firms or explain variations in cost for a single firm over time. One may then infer that differences in institutional costs are part of the unexplained residual variation. If this residual is correlated with factors which should theoretically explain differences in institutional costs, then one has obtained some empirical evidence supporting the theory. Interfirm comparisons of this nature are easier, and the results more convincing, if one focuses on some specific cost element which should be reasonably comparable for all firms. The number of variables needed to explain differences in cost is then kept to a minimum.

Three published studies have used this approach to examine whether regulatory structures which result in less competition lead to higher costs. A study of the banking industry by Franklin R. Edwards[3] found that a high three-bank concentration ratio in a Standard Metropolitan Statistical Area (S.M.S.A.) was a significant variable in explaining both the number of bank employees and total bank wages in the S.M.S.A., despite the inclusion of other variables in the regression equations designed to measure the size of the banking market and the costs of operating in the S.M.S.A.

Turning specifically to utilities, Walter Primeaux, Jr.[4] identified a sample of 23 cities where municipally-owned electric utilities compete directly with investor-owned utilities. Each competitive municipally-owned utility was paired as nearly as possible with a similar monopoly municipally-owned utility based on location state, annual electricity sales, and generation power sources. Primeaux found that overall operating costs were significantly lower for the competitive municipals. His regression equations included variables designed to measure capacity utilization, fuel cost, purchased power costs, economies of scale, market density, and differences in customer sizes and types.

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[3] Franklin R. Edwards, "Managerial Objectives in Regulated Industries: Expense Preference Behavior in Banking", Journal of Political Economy, 85 (February, 1977): 147-162.

[4] Walter J. Primeaux, Jr., "A Reexamination of the Monopoly Market Structure for Electric Utilities" in Promoting Competition in Regulated Markets, ed. Almarin Phillips (Washington, DC: The Brookings Institution, 1975), pp. 175-200.

Rodney Stevenson[5] compared electricity generating costs of a sample of 25 combination electric/gas utilities with those of 54 straight electric utilities. He found the generating costs for the combination utilities to be significantly greater than for the straight electric utilities. He credits this result to more efficient use of inputs by the straight electrics, who face competition from gas utilities. His regression equations included variables designed to measure factor input prices, economies of scale, and capacity utilization.

The published study which most directly addresses a hypothesis of this study is one by Wallace Hendricks[6]. Although he did not intend to address the relationship of allowed rate of return to institutional costs, it is possible to reinterpret his results as providing some evidence on the issue. Hendricks collected data on journeyman lineman's wages in a sample of 106 unionized electric utilities--this classification is considered by the major industry unions to be a "benchmark" job. Hendricks cites studies which show that in most industries one observes the most profitable firms paying the highest wages. Yet when Hendricks divided his sample into high-, medium-, and low-profit firms based on profit levels over the preceding eleven years, he found that it was the low-profit electric utilities which were paying the highest

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[5] Rodney Stevenson, "X-Inefficiency and Interfirm Rivalry: Evidence from the Electric Utility Industry", Land Economics, 58 (February, 1982): 52-66.

[6] Wallace Hendricks, "The Effect of Regulation on Collective Bargaining in Electric Utilities," Bell Journal of Economics, 6 (Autumn, 1975): 451-465.

wages. He obtained this result despite the inclusion of other variables in his regression equations reflecting local wage levels and firm size.

Hendricks argues that his results are consistent with a Joskow-type model of regulatory stickiness[7]--low-profit firms will be less resistant to paying higher wages, since they are more likely to be able to use the higher wage costs to justify an immediate rate increase. This explanation leaves me unsatisfied. A firm which is close to the threshold of justifying a rate increase may be less resistant to paying higher wages than one which is not. However, I see no reason why firms which have had consistently lower profits over an eleven-year period should be less resistant, unless perhaps these firms had more rapid cost increases than other firms, forcing them to file for rate increases more frequently. Low long-term profits may reflect longer lags in approving rate increases in some states than in others, but one would expect utilities subject to long lags to more vigorously resist paying higher wages.

A more plausible explanation is that the lowest-profit firms were those with the lowest allowed rate of return. Consistent with the theory proposed in this study, these firms had higher institutional costs. One form these institutional costs took was the payment of excessively high wages, as a result of managers bargaining less vigorously then they could have in their union contract negotiations. It

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[7]See Paul Joskow, "Pricing Decisions of Regulated Firms: A Behavioral Approach", <u>Bell Journal of Economics and Management Science</u>, 4 (Spring, 1973): 118-140.

would be useful to see Hendrick's study extended to include an analysis of why the low-profit firms had low profits. If a principal reason proved to be a low allowed rate of return, this would tend to support the hypothesis that a low allowed rate of return leads to higher institutional costs.

6.3 Wellhead Price Controls and Institutional Costs

One would expect the dramatic increases in oil prices of 1973-74 to have had an effect on gas firms similar to the hypothesized effect of a lowering of the wellhead price ceiling, which I discussed in Section 5.2. The budget constraint would shift upward because wellhead price controls hold down the average cost of gas to the firm in the face of increased demand. This shift would produce an income effect favoring higher institutional costs. At the same time, the budget constraint would become more steep as the price of marginal gas supplies, such as the "high cost" gas discussed in Section 6.1, Canadian gas, synthetic gas, and imported liquefied natural gas, was bid up. This increased steepness would produce a substitution effect favoring higher institutional costs. So one would expect to see an increase in institutional costs resulting from the oil price increases.

While an elaborate analysis, such as the ones discussed in the preceding section, of gas industry costs is beyond the scope of this study, this section will present a brief overview of cost trends in the post-embargo period. The data I will present show increases in many categories of gas industry expense which were well above general inflation.

While I will discuss several alternative explanations, the data leave open the possibility that at least part of the increase was due to increasing institutional costs.

The first subsection examines these cost trends for all investor-owned gas firms, whether pipelines or distribution companies. The second subsection repeats the analysis for FERC-regulated pipelines only. Better data on these latter firms are available then on all investor-owned gas firms, including breakdowns of transmission expense and general and administrative expense.

6.3.1 Costs of All Investor-Owned Gas Utilities

The principal published data on the entire investor-owned gas utility industry are the figures compiled by the American Gas Association (A.G.A.) and published in their annual Gas Facts. Although this source is lacking in detail, it is sufficient to provide an overview of gas industry cost trends. Table 5 gives a breakdown of investor-owned gas utility costs, in nominal dollars, for the years 1973 and 1981. The A.G.A. has not compiled statistics on the volume of gas sales by these investor-owned companies prior to 1974. It can, however, be determined that in both 1974 and 1981 investor-owned companies accounted for about 95% of all gas industry final sales[8] and that between 1973 and 1981 all

[8] American Gas Association, Gas Facts 1978, Table 67, p. 86 and Gas Facts 1981, Table 70, p. 90.

TABLE 5
BREAKDOWN OF COST
ALL INVESTOR-OWNED GAS UTILITIES

	1973 (million $)	1981 (million $)	% change
Gross Operating Revenues	20,585	102,138	+396%
Cost of Gas Purchased or Produced	11,247	79,391	+606%
Value Added	9,338	22,747	+144%
Operating Expenses			
Transmission	564	3,211	+469%
Storage	151	446	+195%
Distribution	558	1,078	+ 93%
Customer Accounts	435	1,122	+158%
Sales	131	169	+ 29%
Customer Service	-	199	-
General and Administrative	976	2,729	+180%
Administrative and General	2,815	8,954	+218%
Maintenance Expenses			
Transmission	132	421	+219%
Storage	23	82	+257%
Distribution	346	705	+104%
Other	19	49	+158%
	520	1,257	+142%
Depreciation	1,381	2,768	+100%
Taxes	2,043	5,070	+148%
Operating Income	2,579	4,698	+ 82%
Producer Price Index (1967 = 100)	134.7	293.4	+118%

SOURCES: 1973 cost data from American Gas Association, Gas Facts 1973, Tables 99, p. 124, and 109, p. 134; 1981 cost data from American Gas Association, Gas Facts 1981, Tables 133, p. 160, and 138, p. 165; producer price index data from Bureau of the Census, Statistical Abstract of the United States 1982-83, p. 454.

gas industry final sales declined from 16.1 to 15.0 billion cubic feet, or 6.7 percent.[9]

Despite the decline in gas volume, Table 5 shows that many categories of gas industry expense increased faster than general inflation during this period, which was 118 percent as measured by the producer price index.[10] This as one would expect in an environment of increasing institutional costs. There are, however, other possible explanations which deserve further examination.

The large increase in transmission operating expense is explained, at least in part, by the fact that a major portion of this expense is for compressor fuel, which increased rapidly in price during this period. It may also be partly explained by a trend toward utilities contracting with each other for gas transmission and compression. Contracting would cause the expense of moving a given unit of gas to be double-counted in the data, once as payments by a contractee and once as actual cost of transmission by a contractor. Hence a trend toward more contracting would cause the data to overstate the actual increase in transmission operating expense. Unfortunately, the A.G.A. did not collect a breakdown of transmission expense which would allow one to isolate these two factors.

Another possible explanation is that the rising value of energy made it economical for gas firms to increase expenditures on efforts to

[9] American Gas Association, Gas Facts 1981, Table 67, p. 87. A 1.0225 quadrillion b.t.u. per billion cubic feet conversion factor was assumed.

[10] Calculated from the producer price index reported in Bureau of the Census, Statistical Abstract of the United States 1982-83 (Washington, DC: Government Printing Office, 1982), p. 454.

conserve compressor fuel and reduce system leakage. This might help to explain not only the increase in transmission expense, but the increases in the maintenance categories as well.

The increase in storage expense may be at least partly attributable to the cost of operating new storage facilities built by gas utilities in the late 1970's in the face of a shift in their market away from industrial customers toward residential and commercial customers, whose demands fluctuate more. One should not, however, overstate the effect of this shift: residential and commercial customers accounted for 44.1 percent of gas utility final sales volume in 1973, while the same figure was 45.3 percent in 1981.[11] Both storage and distribution operating expense also include a certain amount of compressor fuel cost which is not broken out in the A.G.A. data.

Finally, the large increase in administrative and general expense could be due to rising gas prices, which justified more careful analysis of management decisions. The more complicated regulatory environment of the late 1970's may also have increased the administrative burdens on managers.

6.3.2. Costs of FERC-Regulated Pipelines

FERC collects a variety of statistics on the pipelines they regulate, which is published each year in the Statistics of Interstate Natural Gas Pipeline Companies. Table 6 presents a breakdown of costs for all class A and B FERC-regulated pipelines for the years 1973 and

[11] American Gas Association, Gas Facts 1981, Table 67, p. 87.

TABLE 6
BREAKDOWN OF COST
ALL CLASS A AND B FERC-REGULATED PIPELINES

	1973 (million $)	1981 (million $)	% change
Gross Operating Revenues	9,870	55,710	+464%
Cost of Gas Purchased or Produced	5,296	43,733	+727%
Value Added	4,574	11,977	+162%
Operating Expenses			
Transmission			
Compressor Fuel	164	1,125	+586%
Transmission by Others	200	1,115	+457%
Other Transmission Expense	235	684	+191%
	599	2,924	+388%
Storage			
Compressor Fuel	6	47	+683%
Other Storage	79	269	+241%
	85	316	+272%
Distribution	77	190	+147%
Customer Accounts	53	213	+302%
Sales	20	17	- 15%
Customer Service	-	24	-
Administrative and General	402	1,364	+239%
	1,236	5,048	+308%
Maintenance			
Transmission	119	355	+198%
Storage	12	52	+333%
Distribution	43	122	+184%
	174	528	+204%
Depreciation	823	1,660	+102%
Taxes	915	2,096	+129%
Operating Income	1,428	2,646	+ 85%
Producer Price Index (1967 = 100)	134.7	293.4	+118%

SOURCES: 1973 cost data from Federal Power Commission, Statistics of Interstate Natural Gas Pipeline Companies 1973, pages 101 and 501-501A; 1981 cost data from U.S. Energy Information Administration, Statistics of Interstate Natural Gas Pipeline Companies 1981, pages 84 and 144-146; producer price index data from Bureau of the Census, Statistical Abstract of the United States 1982-83, p. 454.

NOTE: Figures may not add due to rounding.

1981. Class A and B pipelines are those having more than one million dollars per year in operating revenues. During the period from 1973 to 1981, total natural gas sales by these pipelines declined from 16.5 to 15.2 billion cubic feet, or 8 percent.[12] Like the data for all privately-owned gas utilities, the data for FERC-regulated pipelines shows that many categories of expense increased substantially faster than general inflation during this period. It will be recalled that inflation, as measured by the producer price index, was 118 percent between 1973 and 1981.

Again, the largest increase, as measured in both dollars and percent, was in transmission operating expense. In this case, however, the data do allow one to break-out the cost of compressor fuel and payments to other gas utilities for transmission and compression performed under contract. Table 6 shows that even when these two explained sources of cost increase are excluded from transmission operating expense, the remaining costs ("other transmission expense") still increased by 191 percent between 1973 and 1981. These remaining costs consisted principally of supervision, labor, and supplies needed for routine cleaning, lubrication, monitoring, and control of transmission facilities.[13] Increasing energy conservation expenditures are one

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[12] U.S. Energy Information Administration, Gas, p. 30. Note that most gas pipeline sales are for resale. The pipeline sales volumes quoted exceed the final sales volumes for the entire gas industry quoted in Section 6.3.1 because some gas may be resold by several pipelines prior to final sale.

[13] Detailed descriptions of each expense classification are contained in Code of Federal Regulations, title 18, part 201, items 700-932 (1983).

possible explanation for the increase, increasing institutional costs are another. In a similar manner, one can break-out the cost of compressor fuel used in the operation of gas storage facilities. Table 6 shows that even when this cost is excluded, the remaining gas storage operating expense ("other storage") still increased 241 percent between 1973 and 1981.

The FERC data also give a breakdown of administrative and general expense, as shown in Table 7. One cannot rule out the possibility that a justifiable need for more careful analysis of management decisions or an increasingly complicated regulatory environment explain these increases. However, it is interesting to note that three categories of administrative and general expense which might be expected to contribute to management utility showed especially large increases. These are office supplies and expenses (which includes communication services and travel expenses), outside services employed, and rents. This would suggest the possibility of increasing institutional costs.

The author tried two other approaches to the analysis of gas pipeline administrative and general expense. These were testing for a correlation between the gas acquisition costs and the administrative and general expenses of individual pipelines, and a comparison of the increases in administrative and general expenses of gas pipelines with those of oil companies and electric utilities. These approaches did not produce definitive results, and are therefore discussed in the appendix.

TABLE 7
BREAKDOWN OF ADMINISTRATIVE AND GENERAL EXPENSE
CLASS A AND B FERC-REGULATED PIPELINES

	1973 (million $)	1981 (million $)	% change
Salaries	124.2	407.3	+227%
Office Supplies and Expenses	52.8	221.4	+319%
Administrative Expenses Transferred Credit	(17.4)	(100.5)	-
Outside Services Employed	37.2	177.1	+376%
Property Insurance	15.8	37.3	+136%
Injuries and Damage	12.8	40.1	+213%
Employee Pensions and Benefits	102.3	353.2	+245%
Franchise Requirements	.7	2.1	+200%
Regulatory Commission Expenses	7.8	21.0	+169%
Duplicate Charges-Credit	(.4)	(3.5)	-
General Advertising	-	7.7	-
Miscellaneous General Expenses	44.4	122.6	+176%
Rents	17.3	62.2	+260%
Maintenance of General Plant	4.3	15.5	+260%
TOTAL	401.9	1363.6	+239%

SOURCES: 1973 data from U.S. Federal Power Commission, Statistics of Interstate Natural Gas Pipeline Companies 1973, p. 501A; 1981 data from U.S. Energy Information Administration, Statistics of Interstate Natural Gas Pipeline Companies 1981, p. 147.

NOTE: Figures may not add due to rounding.

6.4 Conclusion

This chapter has briefly surveyed some of the available data and studies for evidence which might relate to the hypotheses proposed in the earlier chapters. The available data strongly suggest that major gas pipelines are buying gas with a marginal cost exceeding its marginal revenue product, as one would expect under a regulatory restriction on capital investment. Evidence on the effect of rate-of-return regulation and wellhead price controls on institutional costs is more tentative, due to difficulties inherent in attempting to measure institutional costs, and the limited amount of empirical research addressing the issue of institutional costs in regulated firms. The data and studies discussed in this chapter are, however, consistent with the hypotheses that institutional costs can be affected by regulation, and that, in particular, a reduction in the allowed rate of return or wellhead price controls lead to higher institutional costs.

6.A Appendix--Further Discussion of Administrative and General Expenses

6.A.1 Individual Pipeline Data

Under the theory presented earlier, if all else were held the same, a pipeline's institutional costs would vary inversely with the pipeline's average wellhead gas purchase cost. Hence, if it were observed that those pipelines with the lowest average gas purchase costs had the highest administrative and general expenses per unit of gas sold, or that those pipelines with the smallest increases in average gas purchase costs in the 1973-81 period had the largest increases in administrative and general expenses per unit of gas sold in this period, this would be evidence tending to support the theory. The principal problem with the approach is that there could be other explanations for such an inverse relationship as well. For example, a pipeline which purchases gas through a middleman might have a higher gas cost, but lower administrative and general expenses, than one which purchases directly from producers.

Table 8 shows the 1973 and 1981 administrative and general expenses per thousand cubic feet (m.c.f.) of gas sold, and the 1973 and 1981 average wellhead gas costs per m.c.f. for the 20 largest interstate gas pipeline companies as measured by 1981 revenues. A regression of 1981 administrative and general expense per m.c.f. on 1981 average wellhead gas cost per m.c.f. yields the following result:

$$\text{A and G Expense} = \underset{(.0089)}{-.0200}(\text{Gas Cost}) + .1123$$

20 Observations $\qquad R^2 = .220.$

The standard error of the coefficient of gas cost is shown in parenthesis. A t-test indicates that this coefficient is significant at the 95% confidence level.

A regression of the 1973-1981 increase in administrative and general expense per m.c.f. on the 1973-1981 increase in average wellhead gas cost per m.c.f. yields the following result:

$$\Delta\text{A and G Expense} = \underset{(.0081)}{-.0139}(\Delta\text{Gas Cost}) + .0753$$

20 Observations $\qquad R^2 = .142.$

A t-test indicates that the coefficient of gas cost is not significant at the 95% confidence level.

TABLE 8
ADMINISTRATIVE AND GENERAL EXPENSE VS. GAS COST
20 LARGEST INTERSTATE PIPELINES

	A and G per m.c.f.		Gas Cost per m.c.f.[1]	
	1973	1981	1973	1981
El Paso + Northwest[2]	.0259	.0982	.27	2.41
Columbia	.0142	.0443	.33	2.54
Transco	.0204	.0617	.25	2.88
Tenneco	.0143	.0531	.21	2.20
United	.0104	.0590	.26	2.52
Natural Gas Pipeline	.0256	.0788	.23	2.03
Texas Eastern	.0210	.0534	.26	1.97
Michigan-Wisconsin	.0225	.1291	.28	2.46
Northern Natural	.0253	.1070	.21	1.84
Southern	.0208	.0597	.25	2.50
Panhandle Eastern	.0246	.0756	.27	2.16
Consolidated	.0300	.0669	.43	2.52
Texas Gas	.0136	.0431	.27	2.11
Trunkline	.0182	.0436	.24	2.43
Pacific Gas Transmission	.0051	.0233	.35	4.82
Midwestern	.0065	.0243	.33	3.35
Colorado Interstate	.0244	.0831	.19	2.50
Cities Service	.0119	.0536	.20	2.15
Transwestern	.0058	.0459	.25	2.36
Mississippi River	.0163	.0323	.38	2.67

SOURCES: 1973 data from U.S. Federal Power Commission, Statistics of Interstate Natural Gas Pipeline Companies 1973, pp. 501-511A and pp. 301-311; 1981 data from U.S. Energy Information Administration, Statistics of Interstate Natural Gas Pipeline Companies 1981, pp. 144-194 and pp. 124-140.

NOTES:

[1]Gas cost per m.c.f. is "total production expenses" as defined by the FERC divided by "total natural gas sales." "Total production expenses" includes the cost of both produced and purchased gas.

[2]El Paso Natural Gas spun-off Northwest Pipelines in 1974.

The negative sign of both coefficients is as one would expect if institutional costs were subject to an income effect. Although the sign of the first coefficient is significant, I believe it would be prudent not to put too much weight on this result without a more thorough investigation of other factors which might influence administrative and general expense.

6.A.2 Comparison with Other Industries

The post-embargo increases in gas firm administrative and general expenses may be compared to those of other firms whose products also rose significantly in price during this period. If the increases in administrative and general expenses tended to be larger for gas firms, this would be evidence that the increases for gas firms were due to something unique about the gas industry, such as regulation-induced increases in institutional costs. In this section I compare the increases in gas firm administrative and general expenses to those of electric utilities and oil companies.

Both the gas and electric utility industries were subject to similar increases in the value of their product in the 1970's, and both were subject to similar systems of rate-of-return regulation. However, only gas firms were subject to wellhead price controls. Hence, it is tempting to argue that the administrative and general expenses of gas utilities should have increased more quickly than those of electric utilities under the theory presented here. The problem with the argument is that the costs of generating electricity in existing facilities did not increase as quickly as energy costs generally. The costs of hydro-generation were virtually immune to energy price increases; the costs of gas and oil generation were held down by government price regulation; even increases in the costs of coal and nuclear generation were frequently delayed by long-term contracts. The availability of low-cost electricity from these sources could have had an impact on electric utilities much like the impact of wellhead price controls on gas utilities.

It is easy to make such a comparison of gas and electric utilities. FERC collects statistics on major privately-owned electric utilities in a format essentially the same as that used for gas utilities. These statistics show that administrative and general expense for class A and B privately-owned electric utilities increased from $1.681 billion in 1973 to $5.170 billion in 1981.[14] This is a 208 percent increase, or a 145 percent increase after adjustment for the 26 percent increase in sales (in megawatt-hours) over this period.[15] It will be recalled that this compares to a 180 percent increase for all privately-owned gas utilities, or a 200 percent increase after adjustment for the roughly 7 percent decline in sales (in billion cubic feet). It also compares to a 239 percent increase for all class A and B FERC-regulated pipelines, or a 268 percent increase after adjustment for the 8 percent decline in sales.

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[14]U.S. Energy Information Administration, Statistics of Privately-Owned Electric Utilities, 1981, DOE/EIA-0044(81) (Washington, DC: Government Printing Office, June 1983), p. 32.

[15]ibid, p. 29.

One might similarly be tempted to argue that the theory presented here would lead one to expect larger increases in administrative and general expenses for gas firms than for oil companies, since oil companies are less-closely regulated. Unfortunately, the only available data on oil company administrative and general expenses are derived from an income account item--"selling, general and administrative expenses"--which the companies are required to provide in their 10-K reports to the U.S. Securities Exchange Commission (S.E.C.). Table 9 shows this expense in 1973 and 1981, and the percent change, for the 19 largest U.S. oil companies ranked in order of 1981 revenues. The table shows that the increases tended to be substantially smaller than the 180 percent increase in administrative and general expense for all privately-owned gas utilities, the 239 percent increase for FERC-regulated pipelines, or the increases of most individual pipelines (see Table 8).

Although these results appear to be consistent with the theory, there are some problems. The most obvious of these is the inclusion of selling expenses in the oil company figures. It is hard to say how this would have affected the data. In fact, it is even hard to say what are meant by selling expenses; since the applicable S.E.C. regulations[16] give no definition, the companies are presumably free to define selling expenses as they see fit. A second problem is that oil companies themselves may have had their expenditures distorted by the price regulation to which they were subject during most of this period, although I cannot

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[16] _Code of Federal Regulations_, title 17, part 210.5-03 (1983).

TABLE 9
CHANGE IN OIL COMPANY
"SELLING, GENERAL AND ADMINISTRATIVE EXPENSES"
1973-1981
AS REPORTED TO THE U.S. SECURITIES EXCHANGE COMMISSION

	1973 (million $)	1981 (million $)	% change
Exxon	2,277	5,231	+130%
Mobil Oil	1,209	2,921	+142%
Texaco	787	1,293	+ 64%
Standard Oil (Cal.)	519	1,011	+ 95%
Standard Oil (Ind.)	700	1,261	+ 80%
Gulf	N.A.	4,945	-
Atlantic Richfield	339	1,628	+380%
Shell	369	566	+ 53%
Conoco	279	N.A.	-
Phillips Petroleum	291	491	+ 69%
Sun	231	917	+297%
Occidental Petroleum	239	1,016	+325%
Standard Oil (Ohio)	N.A.	N.A.	-
Getty Oil	107	283	+164%
Union Oil	231	482	+109%
Ashland Oil	216	621	+188%
Amerada Hess	113	286	+153%
Marathon Oil	98	362	+269%
Cities Service	144	255	+ 77%

SOURCES: 1981 data are from Moody's Industrial Manual, 1982 edition. 1973 data are from the most recent edition of Moody's Industrial Manual reporting 1973 data for the particular company; this was the 1980 edition except for Mobil Oil (1978), Texaco (1978), Standard Oil (Cal) (1979), Phillips Petroleum (1979); Sun (1979), Occidental Petroleum (1978), Getty Oil (1974), Amerada-Hess (1974), and Cities Service (1979).

say in which direction. The historical profit regulation to which oil companies were subject might have induced higher institutional costs, but the resulting environment of shortages would have discouraged selling expenditures. Although oil price regulation ended in early 1981, the industry's adjustment to the deregulated environment was undoubtedly not instantaneous. For these reasons, one must read the evidence in Table 9 with a good deal of skepticism.

CHAPTER 7

SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

Previous literature on rate-of-return regulation has focused primarily on the possible incentives for an improper mix of capital and other inputs, the Averch-Johnson effect. The possible lack of incentives for minimizing operating costs has received less attention. At the same time, previous literature on the effects of wellhead price controls has focused primarily on producers and consumers, with little attention given to their effect on gas utilities. This study has therefore focused on how rate-of-return regulation might affect the operating efficiency of gas utilities, including an analysis of how wellhead price controls might affect operating efficiency.

The classic Averch-Johnson model of the rate-of-return regulated firm produces two results which do not accord with the observed behavior of gas utilities. First, the model results become indeterminate if the allowed rate of return set by the regulators equals the cost of capital, while the firm would simply go out of business if the allowed rate of return is below the cost of capital. Yet in recent years, high interest rates have raised the cost of capital to many utilities above their allowed rates of return without a noticeable discontinuity in their behavior. Second, the model tells us that the firm should purchase all non-capital inputs only up to the point where their marginal revenue product equals marginal cost. Yet, it is common for gas utilities to sell gas at a price less than the cost of the most expensive gas they purchase. The new models proposed in this study were designed to explain these anomalous results.

To set up my first model, I argued that a firm could be thought of as a set of contracts between individuals, where the contracts are enforced in an environment of imperfect information. The managers of such a firm have some ability to pursue objectives of their own, which differ from simple profit-maximization. For my purposes, the managers can be thought of as maximizing a utility function of profits and the institutional costs arising from their pursuit of other objectives, subject to a budget constraint. The managers will choose to operate at the point on the budget constraint which is tangent to the highest possible isoutility curve.

I showed how lowering the allowed rate of return alters the shape of the budget constraint. This results in an income effect favoring lower profits and lower institutional costs, as well as a substitution effect favoring lower profits and higher institutional costs, assuming an allowed rate of return above the cost of capital. With an allowed rate of return equal to the cost of capital, the managers become institutional costs maximizers, who set consumer prices as if the firm were unregulated. This raises the possibility that the regulator may face a tradeoff: as the allowed rate of return is lowered from the unregulated level to the cost of capital the firm's profits decline, but institutional costs tend to increase. The budget constraint is also defined for allowed rates of return less than the cost of capital. In this new model, there is no discontinuity in the firm's behavior at allowed rates of return equal to or less than the cost of capital.

I then argued that, in addition to the setting of the allowed rate of return, regulators have a second instrument of control available: their power to approve or deny new capital investments. A second model assumed that the firm managers were strictly profit-maximizing, but that the regulators set a minimum marginal internal rate of return, or capital productivity, requirement for new capital investments. It was shown how this would force the firm to purchase non-capital inputs, such as gas, beyond the point where their marginal revenue product equals marginal cost. Rules-of-thumb for how a price-minimizing or welfare-maximizing regulator might set this capital productivity requirement were derived.

The model-building concluded with a discussion of a third model, combining both utility-maximizing managers and a capital productivity requirement. I showed how increasing the capital productivity requirement altered the shape of the budget constraint. This produces an income effect favoring lower profits and lower institutional costs, as well as a substitution effect favoring lower profits and higher institutional costs, assuming an allowed rate of return above the cost of capital. The effects of changing the allowed rate of return are like those in the first model. As in the second model, the firm is still forced to purchase non-capital inputs beyond the point where their marginal revenue product equals marginal costs. There are, however, no simple rules-of-thumb for setting the capital productivity requirement which a price-minimizing or welfare-maximizing regulator might follow.

I then turned to examine how a firm, as represented by this third model, would respond to a third instrument used in gas utility regulation: wellhead price controls. Assuming a binding ceiling on the wellhead price of all gas, I showed how the gas firm's institutional costs tend to consume all rents on the available supply of gas beyond the regulatory profit ceiling. Consumers, producers, and the firm's stockholders would all be worse off than they would be in the absence of the wellhead price ceiling. If only some gas supplies are subject to a wellhead price ceiling, but lowering the price ceiling is effective in reducing the average cost of gas to the firm, I showed how the resulting income and substitution effects both favor higher institutional costs. In short, gas firms may have a tendency to absorb some of the rents generated by wellhead price controls as institutional costs.

I concluded with a brief discussion of the empirical evidence for these results. The theory suggests that gas utilities should be buying gas with a marginal revenue product less than marginal cost. Available evidence for 19 major FERC-regulated pipeline firms suggests that each of these firms is paying more for high cost gas than the price it is charging its consumers for this gas. The theory also suggests that institutional costs should rise as the allowed rate of return is reduced. This is consistent with the results of a published study showing that electric utilities with the lowest profits paid the highest wages, since the payment of unncessarily high wages could be one form of institutional costs. Finally, the theory suggests that institutional costs should rise as wellhead price ceilings are lowered. A survey of gas utility industry cost trends indicates that many categories of cost increased more quickly than inflation in the 1973-81 period, when the market value of energy rose more quickly than natural gas wellhead price ceilings.

I believe this study has demonstrated some previously unrecognized drawbacks of rate-of-return regulation and wellhead price controls. It has also identified some potential tradeoffs faced by regulators between gas firm capital investment, consumer prices, profit, and institutional costs. The study has, however, necessarily been limited in three respects, which need to be addressed in future research. First, the view of gas firms and their regulation taken here has been a highly abstract one; many important characteristics of the industry and its regulation could affect the results and, therefore, deserve to be examined. Second, no attempt has been made to quantify the tradeoffs which have been suggested, yet such quantification will be necessary if the model is to provide sound guidance to policymakers. Third, the study has been limited to a rate-of-return regulated gas utility industry, even though there is no shortage of alternative regulatory systems which might be considered, and even though the approach taken here is not applicable only to gas utility regulation.

More specifically, there are six characteristics of the gas firms and their regulation which I feel especially deserve further examination. The first of these is regulatory lag. By providing the firm with a short-term one-to-one tradeoff between institutional costs and before- and before-tax profit, the introduction of regulatory lag into the model would probably moderate, but not eliminate, the incentives for higher institutional costs discussed here. Bailey and Coleman's study (see Section 1.6) of the effects of regulatory lag on the Averch-Johnson model provides an example of how regulatory lag might be introduced into a model of rate-of-return regulation.

A second characteristic deserving further examination is the difference between marginal and average costs of capital and allowed rates of return. The difference between the average and marginal costs of capital develops as a result of the firm's embedded cost of debt remaining fixed in the face of a fluctuating market cost of capital. The difference between the marginal and average allowed rates of return develops out of deliberate regulatory policy, either in response to the difference between the average and marginal cost of capital, or to give the firm an incentive to do something, such as invest additional capital. It would be especially interesting to examine the latter motive critically by using a model to ask what kinds of incentives would result from a change in the marginal allowed rate of return, without a corresponding change in the average allowed rate of return.

A third characteristic deserving further examination is the presence of institutional costs in capital investments. In this study, I have assumed that institutional costs add only to the firms operating costs, thus making an implicit assumption that capital projects are accomplished at minimum cost. Clearly, institutional costs exist in capital projects as well, and rate of return regulation could have an important impact on the managers' incentives to reduce these costs.

A fourth characteristic deserving further examination is the long life and low salvage value of most gas firm capital. This implies that the managers cannot freely reduce capital investment in response to a lower than expected allowed rate of return on sunk capital. Even though, as I indicated in Section 2.4.1, the allowed rate of return used by the managers in their decisionmaking is prospective over the life of the investment, and therefore unlikely to change quickly, the constraint on capital liquidation may occasionally become binding. The mere possibility that this could happen adds a new element of uncertainty to gas firm capital investment decisionmaking, which could affect the results of the model, probably moderating the incentives for capital investment.

A fifth characteristic deserving further examination is the monitoring and incentive structure under which the managers operate. I have assumed a fixed monitoring and incentive structure, yielding a single management utility function. Firm owners may actually find it in their interest to modify their monitoring and incentive structure to suit the regulatory and market environment. This could have some effect on the model results, probably moderating the managers' incentives for non-profit-maximizing behavior of all kinds. With an explicit representation of the monitoring and incentive structure it is also possible that one might be able to derive restrictions on the shape of the managers' utility function, which could be used to strengthen some of the model results.

A sixth characteristic deserving further examination is the structure of wellhead gas markets. Section 5.2 discussed the impacts of a generalized system of partial wellhead price controls on gas firm institutional costs. A more thorough analysis is needed to determine the impacts of partial wellhead price controls on consumer prices and gas firm profits. Such an analysis might also examine the impacts of specific price-control policies. For example, how do the effects of a scheme which sets a ceiling on the price of most gas compare to one which sets a ceiling on the price of only a small fraction of the gas consumed? A broader analysis might consider the impacts of various alternative ways structuring the wellhead gas market. For example, how might gas pipeline firm behavior change if these firms operate as contract carriers rather than buyers and sellers of gas?

Two approaches could be used in conjunction with each other to quantify the tradeoffs suggested by the model. The first is to conduct numerical simulations. Even a simple numerical version of the model presented here could give one an estimate of the budget constraint faced by the managers at various settings of the allowed rate of return, capital productivity requirement, and wellhead price ceilings. Such a model would give one an idea of how sensitive profits, institutional costs, capital investment, and consumer prices are likely to be to these instruments. One could also construct more elaborate models, designed to represent the situation faced by the managers of an actual firm in some detail. Using an assumed utility function, one could try to predict the behavior of the firm. It is conceivable that some day regulators might be able to use such models directly in their decision-making.

The second approach to quantifying the tradeoffs is through additional empirical studies of actual gas firms. The most immediate need is for empirical work to demonstrate that institutional costs respond to changes in the allowed rate of return and wellhead price ceilings as hypothesized here. The most promising approach would seem to be that used by the studies cited in Section 6.2. Those studies constructed regression equations to explain some specific element of industry cost. If the residual is correlated with differences in the allowed rate of return or wellhead price ceilings, then one has some evidence for the hypothesis. Specifically, it would be useful to see Hendrick's study of electric utility industry wages extended to analyze the relationship between wage levels and allowed rate of return. It would also be useful to extend my analysis of the general and administrative expenses of individual pipelines (see Appendix 6.A.1) to determine whether the negative correlation between general and administrative expense and gas purchase cost continues to hold even when differences in the type of gas suppliers each firm deals with have been taken into account. More advanced future work might use the numerical simulation models to develop empirically testable hypothesis, and use empirical tests to estimate the parameters of the numerical simulation models.

On a broader level, it would be useful to step back from the focus of this study on rate-of-return regulation as now practiced. A new study might consider how public utility regulation could be modified to reduce the inefficiencies which have been identified in this study. The agency literature discussed in Section 2.1 has been producing some promising work on principles for regulation under imperfect information. Most of this work is highly abstract in nature. A further study might examine how these principles could be applied to produce concrete proposals for regulatory reforms. The models proposed here could be modified, and used to examine these reforms.

The basic principle of this model, the utility-maximizing managers who have institutional costs as one objective, could be applied to the analysis of policy in many areas. The approach is especially appropriate where the impacts of a policy on an organizations's operating efficiency is an issue. As noted in Section 1.1 , the model as it stands is generally applicable to any rate-of-return regulated firm. With suitable choice of the utility function, the model could probably be adapted to any situation where charges are cost-based, such as health care and government contracting. Finally, similar models might provide a new approach to analyzing the efficiency effects of various subsidies and taxes.