THE TRANSITION TO NONDEPLETABLE ENERGY: SOCIAL PLANNING AND MARKET MODELS OF CAPACITY EXPANSION

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ABSTRACT

Theoretical and empirical analyses of the long-run evolution of energy markets rely on the concept of a backstop energy source: a hypothetical source of unlimited quantities of energy, available at a constant cost. In this paper we first develop a model which determines the socially optimal rate of investment in backstop capacity and, simultaneously, the optimal rates of production of depletable and backstop energy, under the assumption that the costs of creating backstop capacity increase with the rate of investment. Using this model, it is shown that it is optimal to expand backstop capacity before depletion of conventional energy and keep it idle while conventional energy is cheap.

In the second part of the paper we formulate and study a two-player Stackelberg game model decribing the market interaction between the conventional energy cartel and a competitive backstop sector. In this game the cartel which is the price leader seeks a dynamic limit-pricing strategy against the backstop sector depicted as a follower possessing perfect foresight. Numerical examples show that the leader's strategy consists of an initial phase of low production and high prices, followed by a phase where price equals operating cost and backstop capacity is idle.

I. Introduction

Depletable energy sources, primarily crude oil and natural gas, have been the mainstay of world energy consumption for decades. As these types of energy become more scarce and their costs rise, other sources of energy and new technologies for utilizing energy will be developed. Unconventional, nondepletable energy sources will gradually replace conventional ones, and eventually the world energy economy will make the transition from reliance on depletable to reliance on nondepletable energy.

Uncertainty about future energy demand and supply (both conventional and unconventional) makes it difficult to predict the nature and timing of this transition. Many of the technologies that will eventually play a major role in energy production have not yet been invented. So it is impossible to describe in any detail the evolution of energy production in the long-run. In particular, predictions of the market shares of individual unconventional energy sources are subject to wide margins of error.

The concept of a backstop energy source was invented[1] to simplify the analysis of long-run energy issues. The backstop is a hypothetical energy source that can produce unlimited quantities of energy at a constant cost. That is to say, the unit cost of backstop energy is assumed to remain the same no matter how high the current rate of production or total cumulative production. The backstop represents in a

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[1]By William Nordhaus. For typical examples of the use of this concept in theoretical treatments of energy economics, see both Nordhaus [1979] and Dasgupta and Heal [1979].

highly simplified way the entire range of energy sources and technologies which must eventually replace depletable energy.

The backstop concept is powerful because it solves the problem of how best to plan consumption of the fixed stock of depletable energy. The backstop provides, in effect, a long-run steady-state for the energy economy: without it, available energy sources must inevitably be exhausted and economic collapse may be unavoidable. Since the supply of backstop energy is unlimited (by assumption), its unit cost sets a ceiling for the price of depletable energy. Typically, then, an efficient consumption plan for depletable energy is characterized by a price which rises monotonically to the backstop cost. At the moment the price reaches this level, the stock of depletable energy is exhausted and the transition to the backstop occurs.

The standard model of the transition is a social planning model in which the production rates of two homogenous resources (depletable and backstop) are determined over all time so as to maximize the present value of utility (net of extraction and production costs). In Figure 1 we sketch the results of this model. If depletable resources have a zero production cost (for simplicity), and the unit cost of backstop energy is c, the price (or marginal value) of energy rises monotonically until depletable resources are exhausted at date T. At that date price equals the backstop cost and backstop production begins. Its rate of production, which is constant for all time, equates price and production costs. The two most notable features of this model are, first, that energy prices never rise above the cost of the backstop and, second, that the transition from one source to the other occurs

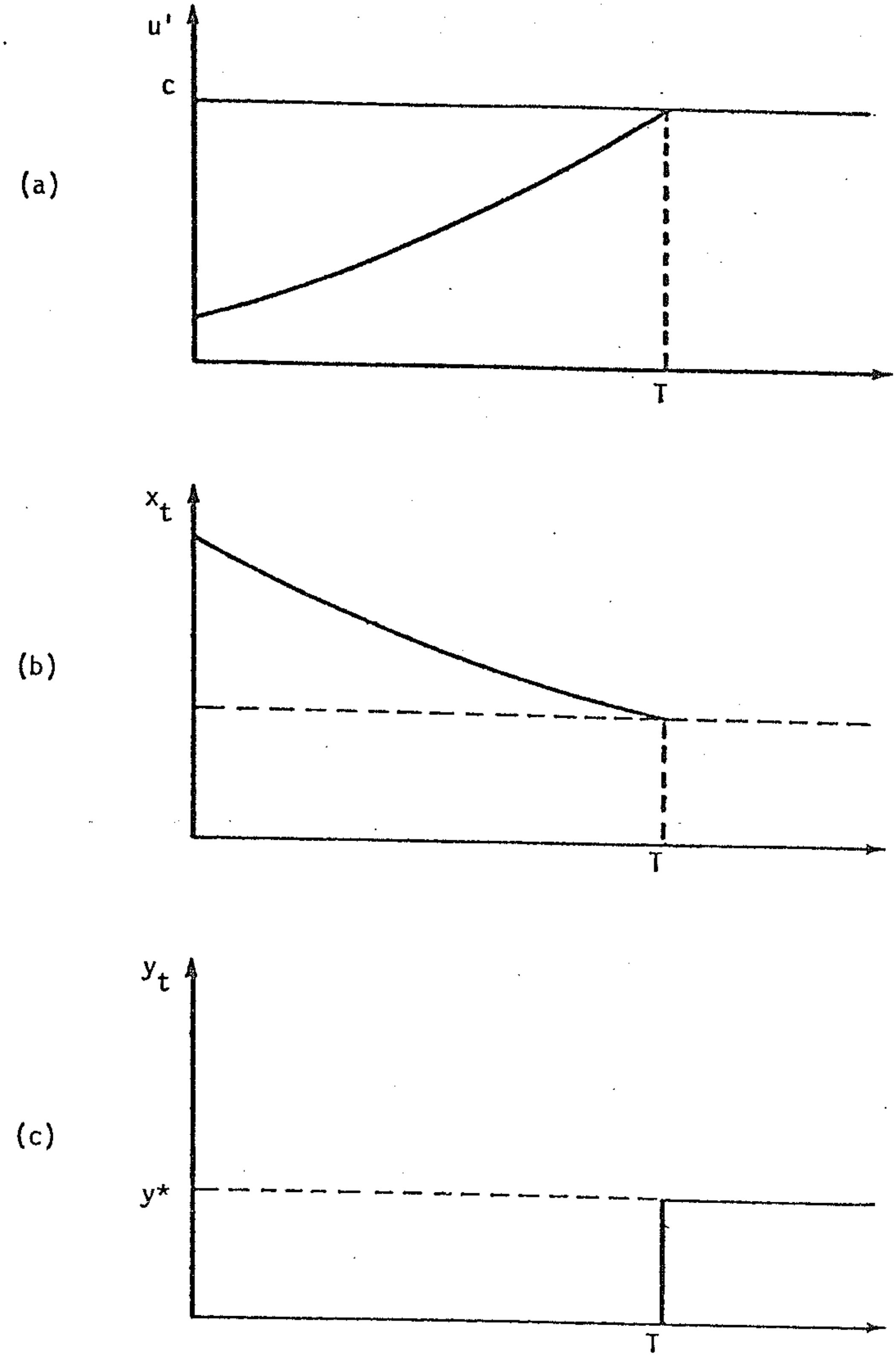

Figure 1

Solution to Standard Backstop Model

instantaneously; i.e., backstop output jumps from zero to its steady-state level at the transition date.

This model may be an adequate abstract description of the efficient allocation of depletable energy sources, but it is clearly inadequate as a description of the evolution of the unconventional energy sector.[2] In this paper we focus on two potentially important aspects of the interaction between conventional, depletable energy and unconventional, non-depletable energy. The first is the effect of capital costs on the evolution of unconventional energy production capacity. The available evidence indicates that capital costs for new energy technologies are high, and that they rise rapidly with the rate of investment. So even if the potential supply of backstop energy is unlimited, its cost may not be constant over successive vintages of capacity. One question we seek to answer, then, is what is the optimal rate of investment in backstop capacity, and what is the resulting price path of energy? The second issue we address is the strategic interaction between the established conventional energy sector and an unconventional sector whose potential for expansion constantly threatens the conventional sector's market share. Spokesmen for OPEC, for example, have repeatedly expressed their concern that high OPEC prices will encourage entry of new energy sources. And potential producers of unconventional energy (e.g., shale oil and tar sands) have similarly expressed their fear that OPEC might retaliate against them by driving the world oil price below their breakeven level. We model the interaction between these two

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[2]For an overview of empirical modeling of the backstop see EMF [1982].

sectors as a Stackelberg game, with the conventional sector the leader and the backstop sector a competitive follower.

The paper is organized as follows. In Section II we lay out the basic backstop capacity model, using a social planning framework. The analytic solution to this model is developed in detail. Then in Section III we develop a Stackelberg model in which the backstop capacity model is used for the follower, and the leader is the depletable resource sector. Finally, in Section IV we present the results of numerical implementations of these two models. These numerical examples allow us to compare the Stackelberg and social planning solutions, and to study the sensitivity of the models to the rate at which backstop capacity costs rise with investment. In Section V we summarize our results.

II. Backstop Capacity Expansion: Social Planning Model

1. Model Formulation

In this section we develop a basic social planning model for the evolution of backstop capacity and production. We assume a social planner controls the rate of production of all energy resources. These resources are of two types: depletable energy, which is costless[3] to produce but available in limited amount, and backstop energy, which has a constant production cost and unlimited (physical) availability. The two resources are perfect substitutes; that is, a unit of each provides the same gross utility to society.

Since our interest is primarily in the development of backstop capacity, we make a distinction between the production or operating cost of the backstop, and the cost of creating backstop capacity. In keeping with the spirit of the backstop concept we assume backstop operating costs are constant, regardless of the scale of production or the vintage of capital used. The cost of investment for the backstop, on the other hand, is not constant. Rather, we assume that beyond some critical rate of investment (per year), the marginal cost of backstop capacity increases.

The rationale for this crucial assumption is simple. Recall that the backstop represents an entire industry, one which is expected to grow very fast over a short time span. Such growth will place severe demands on the supply of skilled labor (e.g., engineers), and on scarce materials (e.g., steel pipe). At some rate of growth these demands will

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[3] Depletable energy is assumed here to have a zero extraction cost solely for simplicity of presentation. See Heal [1976], and Oren and Powell [1983], for a discussion of backstop models in which the cost of one or both energy sources varies with cumulative production.

begin to drive up the prices of these key inputs. Eventually, an absolute scarcity of some input may impose a hard constraint on the feasible rate of expansion of the backstop sector. For simplicity we assume here that the investment cost function is a continuous and monotonically increasing function of the rate of investment.

The social planner's problem is to choose the optimal rates of production of depletable and backstop energy, as well as the rate of investment in backstop capacity. We assume the existence of a concave social utility function defined on total energy production. Furthermore, we assume the planner's objective is to maximize the present value of social utility net of production and investment costs. The operative constraints are, first, that total production of depletable energy cannot exceed the fixed stock; second, that backstop capacity increases each period by the rate of backstop investment[4]; and third, that backstop production is limited each period by the current level of backstop capacity. Formally, the planner's problem can be written as follows:

Social Planning Model

$$\max_{\{x_t, y_t, v_t\}} \int_0^\infty e^{-rt}[u(x_t + y_t) - cy(t) - f(v_t)]dt \qquad (2.1)$$

[4]We assume zero depreciation of backstop capacity. Including it would complicate matters with no reward in terms of insight.

subject to

$$\dot{S}(t) = x(t);\; S(0) = 0,\; S(t) \leq \bar{S}$$

$$\dot{K}(t) = v(t);\; K(0) = 0$$

$$y(t) \leq K(t)$$

$$x(t) \geq 0,\; y(t) \geq 0,\; v(t) \geq 0.$$

where

$x(t)$ = depletable resource production

$y(t)$ = backstop production

$v(t)$ = backstop investment

$K(t)$ = backstop capacity

$S(t)$ = cumulative production of depletable energy

$\bar{S}$ = stock of depletable energy

$u(\cdot)$ = social utility function

$f(\cdot)$ = backstop investment cost function

c = backstop operating cost

r = social discount rate.

2. Solution to Social Planning Model[5]

The solution to the planning problem (2.1) is based on the current-value Hamiltonian

[5]Further details concerning the solution are available in Powell [1983].

$$H = u(x_t + y_t) - cy_t - f(v_t) + \lambda_s(t)x_t + \lambda_k(t)v_t + \mu(t)[K_t - y_t] \qquad (2.2)$$

The adjoint variables $\lambda_s(t)$ and $\lambda_k(t)$ measure the effect on the objective of changes in the cumulative extraction of depletable resources and backstop capacity, respectively. The variable $\mu(t)$ reflects the impact of the constraint that backstop production cannot exceed capacity.

Each of the control variables $x(t)$, $y(t)$, and $v(t)$ must be non-negative. Following the maximum principle we maximize (2.2) under these constraints. This yields the first order optimality conditions:

$$\begin{aligned} u'(x_t + y_t) + \lambda_s(t) &= 0 \quad \text{if } x_t > 0 \\ &\leq 0 \quad \text{if } x_t = 0 , \end{aligned} \qquad (2.3)$$

$$\begin{aligned} u'(x_t + y_t) - c - \mu(t) &= 0 \quad \text{if } y_t > 0 \\ &\leq 0 \quad \text{if } y_t = 0 \end{aligned} \qquad (2.4)$$

$$\begin{aligned} f'(v_t) &= \lambda_k(t) \quad \text{if } v_t > 0 \\ &\geq \lambda_k(t) \quad \text{if } v_t = 0 . \end{aligned} \qquad (2.5)$$

The necessary conditions for an optimal solution are completed with the adjoint equations

$$\dot{\lambda}_s(t) = r\lambda_s(t) \qquad (2.6)$$

$$\dot{\lambda}_k(t) = r\lambda_k(t) - \mu(t) , \qquad (2.7)$$

and the complementary slackness conditions

$$\mu(t) \geq 0 \ , \ \mu(t)[K_t - y_t] = 0 \ , \ y_t \leq K_t . \qquad (2.8)$$

We begin our analysis of the backstop capacity model (2.1) by observing that the necessary conditions (2.3) and (2.6) together imply that whenever the production of depletable energy is positive, marginal utility must rise at the rate of interest. That is, when $x(t) > 0$

$$u'(x_t + y_t) = -\lambda_s(0)e^{rt} \qquad (2.9)$$

A number of properties of the optimal path of depletable energy production follow from this result. First, we can see that production of depletable energy must begin at time zero, since it is costless and all units of backstop energy cost at least c. Next, we note that production of depletable energy cannot extend over an infinite horizon, since the marginal utility of energy (from (2.9)) would eventually exceed the cost of backstop energy. We denote the date of exhaustion of the stock $\bar{S}$ of depletable energy by T. Finally, it can be shown that the marginal utility of energy must exceed c at T, and some backstop capacity must be created before T. Both these properties follow from the necessity of continuity in the path of $u'(\cdot)$.

We turn now to analyze the behavior of backstop production, $y(t)$. We have seen that marginal utility rises exponentially on $[0,T]$, while depletable energy is being produced. At some time during this interval $u'(\cdot)$ crosses c from below. We denote this date by T_y. Assume for the moment that $K(T_y) > 0$. We will show that backstop output must equal capacity after T_y, when $u'(\cdot) > c$, and it must be zero before that date[6].

The rate of production of backstop energy is governed by (2.4) and (2.8). The complementarity condition (2.8) implies that the multiplier $\mu(t)$ is positive when $y(t) = K(t)$, and zero when $y(t) < K(t)$. Is it possible for the marginal utility of energy to be below c and backstop production to be positive? From (2.4), if $y(t) > 0$, $u'(\cdot) - c = \mu(t)$. But if $u'(\cdot) < c$, it follows that $\mu(t) < 0$, a contradiction of (2.8). Thus, whenever $u'(\cdot) < c$, $y(t) = 0$. Again, is it possible when $u'(\cdot) > c$ that $y(t) = 0$ If $y(t) = 0$, and $K(t) > 0$, then (2.8) implies $\mu(t) = 0$. But then from (2.4) $u'(\cdot) - c < 0$, a contradiction of our original supposition. The only possibility, therefore, when $u'(\cdot) > c$ is that $y(t) = K(t)$. Thus, whenever $u'(\cdot) > c$, backstop output equals capacity. Finally, if $u'(\cdot) = c$, (2.4) tells us nothing about $y(t)$ beyond the fact that it lies between zero and the current level of capacity.

The third control variable in (2.1) is $v(t)$, the rate of backstop investment. We have already remarked that the adjoint variable $\lambda_k(t)$ measures the marginal value of a unit of backstop capacity at time T. Equation (2.5), which determines $v(t)$, simply requires that (for $v_t > 0$) the marginal cost of investment ($f'(\cdot)$) equals its marginal value $\lambda_k(t)$. And if the marginal cost of the smallest unit of investment exceeds its return, investment is zero.

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[6]Note that T_y could be zero; that is, $u'(0) > c$. In this case $u'(\cdot)$ could rise exponentially as long as any depletable energy remained, and backstop production would always equal capacity.

To see how the rate of investment changes over time we must analyze the behavior of $\lambda_k(t)$, using (2.7) and (2.8). First we define

$$\eta(t) = \mu(t) - u'(x_t + y_t) + c. \qquad (2.10)$$

From our earlier discussion, we know that when $u'(\cdot) > c$ (and $K_t) > 0$), $y(t) > 0$ and $u'(\cdot) - c - \mu(t) = 0$. Thus, when $u'(\cdot) > c$, $\eta(t) = 0$. Similarly, when $u'(\cdot) < c$, $y(t) = 0$ and $u'(\cdot) - c - \mu(t) \leq 0$. So in this case, $\eta(t) \geq 0$; thus in all cases $\eta(t) \geq 0$.

Now we can recast (2.7) in a more meaningful form, first by integrating it forward in time, and then by eliminating $\mu(t)$ using (2.10). The first operation yields

$$\lambda_k(t) = \int_t^\infty e^{r(t-\tau)} \mu(\tau) d\tau .$$

The second gives

$$\lambda_k(t) = \int_t^\infty e^{r(t-\tau)}[u'(x_\tau + y_\tau) - c + \eta(\tau)]d\tau. \qquad (2.11)$$

Equation (2.11) shows that $\lambda_k(t)$, the marginal value of a unit of backstop capacity at t, has two components. One is the present value of net utility, $u'(\cdot) - c$, measured over the infinite lifetime of a unit of capacity created at t. The other component relates to the multiplier $\mu(t)$. From our earlier discussion we know $\mu(t) = 0$ when $u'(\cdot) > c$, and $\mu(t) \geq 0$ when $u'(\cdot) < c$. Thus, when $u'(\cdot) > c$, the value of $\lambda_k(t)$ comes solely from the utility component. But backstop investment may have value even when $u'(\cdot) < c$; i.e., when the direct utility gained from backstop investment is negative. This value arises from the fact that the convex investment cost function penalizes too-rapid rates of investment. The social planner will choose to create backstop capacity even when its direct value is negative, and current capacity is idle, because the cost of delaying investment and creating capacity more rapidly at a later date is higher.

We can now establish the key property of this model: that backstop investment begins before T_y, the date when marginal utility first reaches the operating cost of the backstop. Assume to the contrary that T_v, the date backstop investment begins, is later than T_y. If investment begins at T_v, then $\lambda_k(t) < f'(0)$ for $t < T_v$ and $\lambda_k(T_v) \geq f'(0)$ at T_v. But turning to (2.11) we see that $\dot{\lambda}_k(t) < 0$ for $t > T_y$ since $u'(\cdot) > c$ and $\mu(t) \geq 0$. Thus, if $\lambda_k(T_v) \geq f'(0)$, $\lambda_k(t) > f'(0)$ over some interval prior to T_v, which contradicts the assumption that investment begins at T_v.

Summarizing our results to this point, we have shown that marginal utility rises exponentially on [0,T] while depletable energy is being produced. At some date T_v, when $u'(\cdot) < c$, backstop investment begins. At a subsequent date T_y, when $u'(\cdot) = c$, backstop production turns positive and equals backstop capacity thereafter. From T_v to T_y backstop capacity is positive and backstop investment continues, but capacity sits idle.

Two issues remain to be discussed. The first is the behavior of the model after exhaustion of the depletable resource at T. Since $u'(T) > c$ and $K(T) > 0$, it follows that marginal utility can only fall after T, since backstop capacity never depreciates. In fact it can be shown using (2.11) that backstop investment remains positive after T, as does backstop production, and the marginal value of energy approaches a lower asymptote given by

$$u'(K_t) = c + r\ f'(0).$$

In effect this says that the last unit of backstop capacity built must return an amount that covers operating costs plus the amortized cost of capacity.

The second issue concerns the path followed by backstop investment. From (2.11) we can easily show that the rate of investment must be declining after T_y, since $\mu(t) = 0$, and $u'(\cdot) - c$ is decreasing. But from T_v to T_y $\mu(t)$ is decreasing while $u'(\cdot) - c$ is increasing. Thus we cannot use (2.11) directly to infer the behavior of the rate of investment. The following indirect argument establishes that backstop investment is increasing up to T_y. Since investment increases up to T_y and decreases thereafter, it must reach its maximum rate at that date.

We know that some backstop capacity is created during the interval from T_v to T_y. We also know that backstop output is zero up to T_y and equal to capacity thereafter. We can, therefore view the planner's problem as that of attempting to choose the optimal endowment of backstop capacity for society at T_y. This endowment, along with the depletable resource stock $S(T_y)$, determines the value society will derive from energy consumption after T_y, when backstop output equals capacity. This argument implies that an equivalent formulation of the backstop capacity model (2.1) would be to choose $x(t)$, $y(t)$, and $v(t)$ optimally from T_y forward, given that society inherits the optimal capital stock $K(T_y)$, and resource stock $S(T_y)$, at T_y. Formally, this equivalent model is

$$\underset{\{x_t, y_t, v_t\}}{\text{Max}} \int_{T_y}^{\infty} e^{-rt}[u(x_t + K_t) - cK_t - f(v_t)]\, dt + \phi[K(T_y), S(T_y), T_y]$$

subject to

$$\dot{S}(t) = x(t) \;;\; S(T_y) = S_{T_y} \;,\; S(t) \leq \bar{S}$$

$$\dot{K}(t) = v(t) \;;\; K(T_y) = K_{T_y} \quad ;$$

where

$$\phi[K(T_y), S(T_y), T_y] = \underset{\{x_t, v_t\}}{\text{Max}} \int_0^{T_y} e^{-rt}[u(x_t) - f(v_t)]\, dt$$

subject to

$$\dot{S}(t) = x(t) \;;\; S(0) = 0,\; S(t) \leq S_{T_y},$$

$$\dot{K}(t) = v(t) \;;\; K(0) = 0,\; K(T_y) = K_{T_y}.$$

Here ϕ measures the optimal value of energy consumption given that $y(t) = 0$ up to T_y, and the optimal capital endowment $K(T_y)$ is created by T_y.

Now ϕ is separable in $x(t)$ and $y(t)$. In effect, the optimal rate of investment depends only on the cost of investment, and in no way on the rate of extraction of depletable energy. Thus we can solve for $v(t)$ in ϕ by solving

$$\underset{\{v_t\}}{\text{Max}} \int_0^{T_y} e^{-rt} f(v_t)\, dt$$

subject to

$$\dot{K}(t) = v(t) \; ; \; K(0) = 0, \; K(T_y) = K_{T_y} .$$

The necessary conditions here are

$$f'(v_t) = \lambda_k(t)$$

and

$$\lambda_k(t) = r\lambda_k(t).$$

From these conditions we can conclude that the marginal value of backstop investment rises exponentially on $[0,T_y]$. Therefore, once $\lambda_k(t)$ reaches the level where $v(t) > 0$, investment itself rises monotonically. This is as one would expect: given discounting, the cheapest way to invest so that a given level of capacity is reached at T_y is to start slowly and raise the rate of investment as the terminal date approaches. We have thus proven that the rate of backstop investment rises monotonically from T_v to T_y, and falls monotonically thereafter.

The solution to the backstop capacity model can be summarized as follows.

(a) On an initial phase, from 0 to T, depletable energy extraction is positive and the marginal utility of energy rises exponentially.

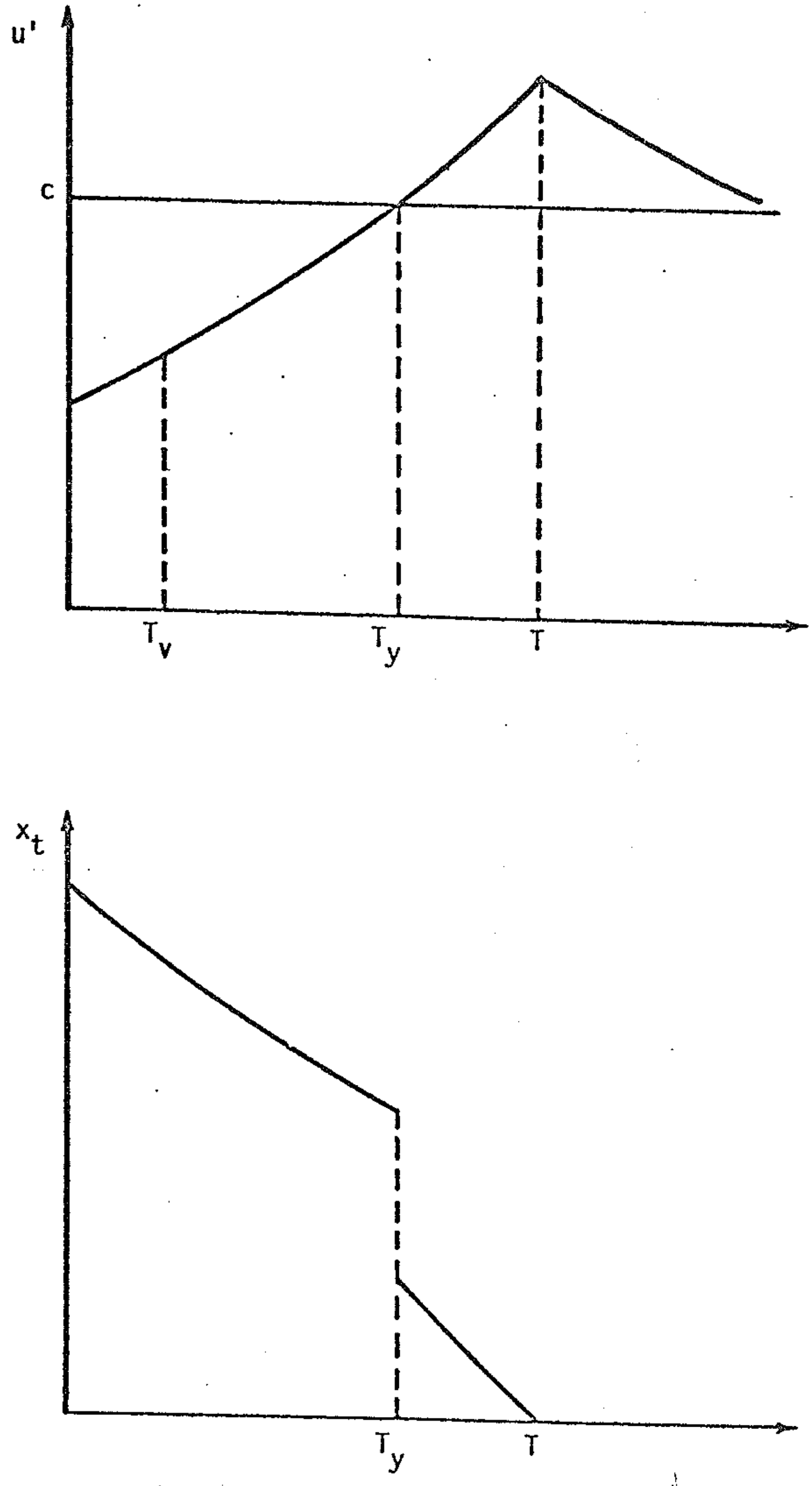
(b) Backstop investment begins <u>before</u> the date T_y when the marginal utility of energy equals c. The rate of investment rises to a maximum at T_y and declines thereafter.

(c) Backstop production is zero up to T_y. After that date, production equals capacity.

(d) In the long-run, backstop capacity approaches a steady-state level where marginal utility equals $c+rf'(0)$.

We illustrate the solution to Model (2.1) in Figures 2a through d. We see first that depletable energy is consumed on $[0,T]$, (Figure 2b), and marginal utility rises exponentially over this period (Figure 2a). Backstop investment (Figure 2c) begins at some date T_v, before exhaustion of the depletable resource stock and before the date T_y when $u'(\cdot) = c$. Investment reaches its maximum rate at T_y and thereafter tapers off. Backstop capacity (Figure 2d) rises at an increasing rate between T_v and T_y, and at a decreasing rate thereafter. Finally, backstop production is zero up to T_y, and equal to capacity thereafter. Since backstop production increases discontinuously at T_y, we can infer that extraction of the depletable resource must fall discontinuously at that date, so as to preserve the continuity of marginal utility.

(a)



(b)

Figure 2

Backstop Capacity Model

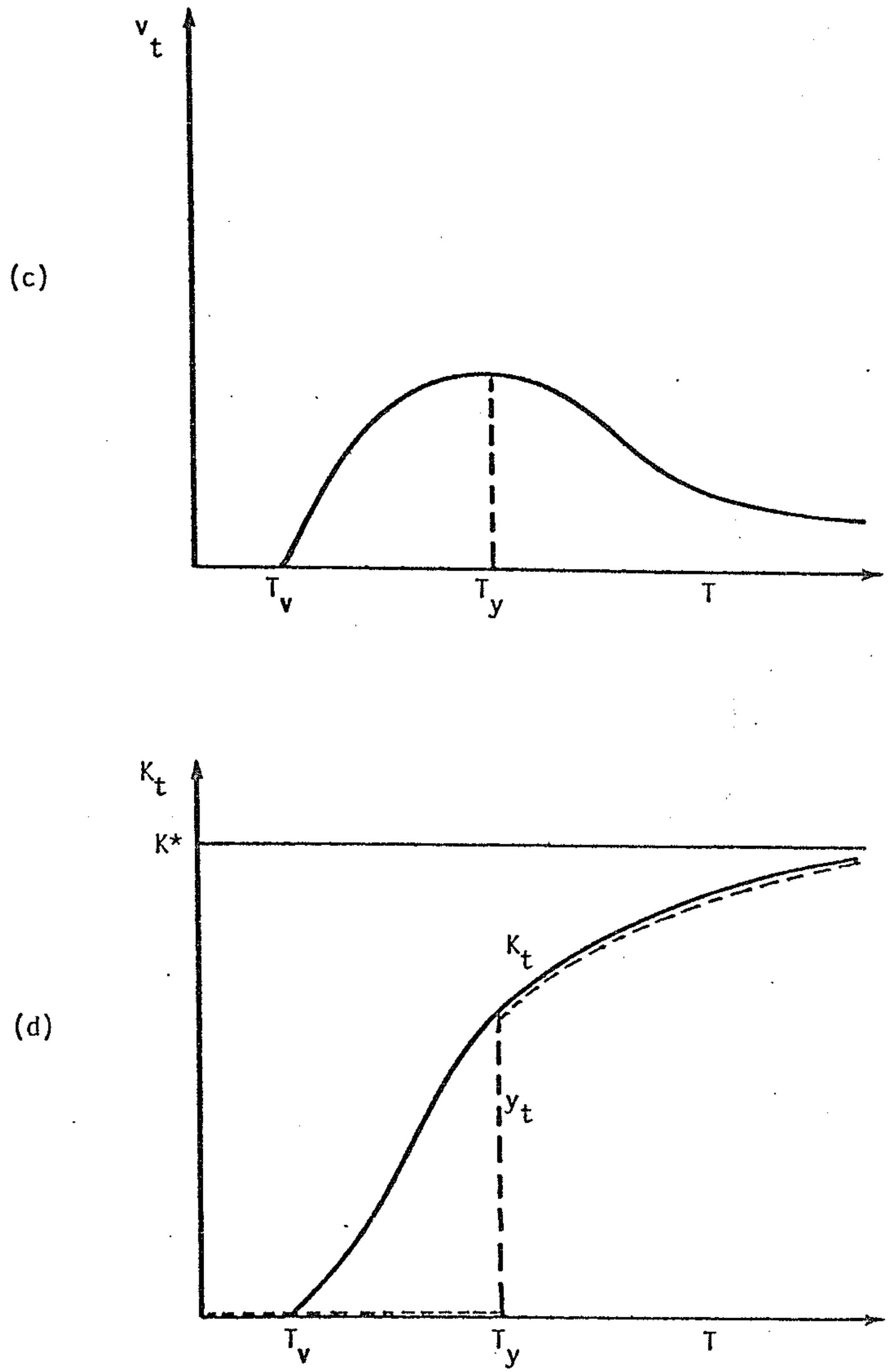


Figure 2 (Cont.)

Backstop Capacity Model

III. Backstop Capacity Expansion: Market Model

1. Model Formulation

In the previous section we analyzed a model in which the socially optimal rates of investment and production of backstop energy were determined along with the rate of production of depletable energy. All energy sources were assumed to be under the control of a single, benevolent agent. Now we turn to a model in which the same variables are determined by a market in which several agents compete. We assume depletable resources are owned by a single agent with the power to influence market price, and backstop resources are owned by a multitude of small agents who individually lack market power. To reflect the differences in power between these two competing sectors we model their interaction as a Stackelberg game[7], with the depletable energy sector as the leader and the backstop sector as the follower. Both players are assumed to have perfect foresight. The crucial difference between them is that the backstop sector, as the follower, takes as given the depletable energy sector's production rate (or, equivalently, the price of energy), while the depletable energy sector takes into account both the effect its production has on price through the demand curve, and the effect it has on backstop investment and production.

The leader in this model is pulled in two directions. Its monopoly power leads it to choose a low rate of production so as to garner high

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[7] Stackelberg models for depletable resource markets have been proposed by Marshalla [1978] and Gilbert [1978]. Both develop models in which a depletable resource cartel is the leader and all remaining depletable resource producers (the "competitive fringe") together are the follower. Gilbert's analysis is particularly interesting since he studies the effects of capacity constraints on fringe production. Our model differs from his in that the follower is the backstop sector (there is no competitive fringe), and backstop capacity is endogenous.

revenues. But high prices convey a signal to the backstop sector that the return to investment is high. And the higher backstop capacity and output, the lower are the leader's profits. So the existence of the backstop sector forces the leader to choose higher output rates and lower revenues than it otherwise would.

The backstop sector here, as the follower, is essentially controlled by the leader. Whatever price the leader sets determines backstop investment and output. The backstop sector is passive in this model, while the depletable energy sector is free to choose whatever price path it can sustain, subject to the constraints imposed by market demand and the backstop sector.

This model rests on five basic assumptions. First, we assume that all energy resources, depletable and nondepletable, are controlled by either the backstop or depletable energy sectors. Thus, we ignore the so-called competitive fringe, producers of depletable energy who act as price-takers[8]. Second, we assume the depletable energy sector has market power, while the backstop sector acts as a price-taker. Third, as the Stackelberg leader the depletable energy sector takes into account the reaction of the backstop sector to its own plans. The backstop sector, on the other hand, acts as if its decisions have no effect on the leader. Fourth, we assume both players have perfect foresight. Finally, we make the technical assumption that the depletable energy

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[8] In Gilbert [1978] exogenous capacity constraints on the competitive fringe play a central role. Gilbert shows that when fringe capacity is expanding, the price set by the depletable resource sector may fall over time. In Section IV we will show how backstop operating costs set a floor for the price path, which typically falls from time zero toward the level of backstop operating costs, then for a period of time stays constant at that level, and finally rises again as depletable resources near exhaustion.

sector acts as the leader only up to an exhaustion date of its choosing. This is not restrictive, since the exhaustion date can be as early or as late as the leader chooses. Once the leader has exhausted its resources, it leaves the market to the backstop sector, which carries on as a competitive industry.

We base our model of the follower on the backstop model developed in Section II. The backstop sector's objective is to maximize the present value of profits, with the future price of energy taken as given (i.e., fixed by the leader). A constant operating cost and convex investment cost function are assumed. Finally, we assume the leader depletes its resources entirely by some date T. After that date the backstop sector is left on its own, with whatever capacity it inherited at T.

The formal statement of the follower's problem is

Stackelberg Model--Follower

$$\max_{\{y_t, v_t\}} \int_0^T e^{-rt}[(P_t - c)y_t - f(v_t)]dt + \Phi[T, K(T)] \qquad (3.1)$$

subject to

$$\dot{K}(t) = v(t) \quad ; \quad K(0) = 0$$

$$0 \leq y(t) \leq K(t) \ , \quad v(t) \geq 0$$

where

$$\Phi[T, K(T)] = \max_{\{y_t, v_t\}} \int_T^\infty e^{-rt}[(P_t - c)y_t - f(v_t)]dt$$

subject to

$$\dot{K}(t) = v(t) \ , \ K(T) = K_T$$

$$P(t) = P(y_t) \ .$$

The leader's problem can be written very simply if we leave the constraints imposed by the follower implicit. Thus the leader in this model solves:

Stackelberg Model--Leader

$$\underset{\{x_t\},T}{\text{Max}} \int_0^T e^{-rt}[P(x_t + y_t)x_t]dt \tag{3.2}$$

subject to

$$\dot{S}(t) = x(t) \ , \ S(0) = 0, \ S(t) = \bar{S}$$

and the constraints imposed by the backstop sector.

The usual procedure for solving Stackelberg models of this type is to determine the first-order conditions for the follower's problem, with the leader's control variables taken as given. These equations are then added as constraints to the leader's problem. For reasons which we will discuss fully in Section 3 below, this procedure cannot be used here to solve the leader's problem analytically. Instead, we will analyze the leader's behavior using a numerical version of the model. In the following section we discuss the solution to the follower's problem. Following that we discuss the difficulties in achieving an anlytic solution to the leader's problem, and offer some conjectures as to general properties it might have. In Section IV we will examine the results of a numerical version of the full model, and compare the Stackelberg model with the social planning model of Section II.

2. Solution for Backstop Sector

The follower's problem in the Stackelberg model is closely akin to Model 2.1. We will, therefore, suppress some of the details of the solution. The Hamiltonian for the follower is

$$H = (P_t - c)y_t - f(v_t) + \lambda_k^f(t)v_t + \mu(t)[K_t - y_t] \,. \tag{3.3}$$

Here we have added the superscript f to the adjoint variable corresponding to the investment constraint as a reminder that this variable measures the marginal value of capacity to the follower. The multiplier $\mu(t)$, of course, reflects the value of being able to produce below capacity.

The first-order conditions arising from maximization of (3.3) are as follows:

$$\begin{aligned} P(t) - c - \mu(t) &= 0 \qquad \text{when } y(t) > 0\,, \\ &\leq 0 \qquad \text{when } y(t) = 0. \end{aligned} \tag{3.4}$$

$$\begin{aligned} f'(v_t) &= \lambda_k^f(t)\,, \qquad \text{when } v(t) > 0\,, \\ &\geq 0\,, \qquad \text{when } v(t) = 0. \end{aligned} \tag{3.5}$$

$$\dot{\lambda}_k^f(t) = r\lambda_k^f(t) - \mu(t), \tag{3.6}$$

$$\mu(t) \geq 0, \quad \mu(t)[K_t - y_t] = 0 \,. \tag{3.7}$$

$$y(t) \leq K(t) \tag{3.8}$$

We first turn to the analysis of the follower's production plan. We examine three cases:

(a) $P(t) < c$

(b) $P(t) > c$

(c) $P(t) = c$.

First, if $P(t) < c$, it stands to reason that backstop production should be zero since revenues fail to cover operating costs. Formally, this is a consequence of (3.4) and (3.7), since $\mu(t) \geq 0$ and $P(t)-c-\mu(t) = 0$ when $y(t) > 0$. Thus it is a contradiction to suppose $y(t) > 0$ and $P(t) < c$. A similar argument shows that $y(t) = K(t)$ when $P(t) > c$. For suppose instead that $P(t) > c$ and $y(t) < K(t)$. Then from the complementarity condition (3.7) $\mu(t) = 0$. But if $\mu(t) = 0$ and $y(t) \geq 0$, (3.4) implies $P(t) = c$, a contradiction. Finally, suppose $P(t) = c$. In this case (3.4) tells us nothing about $y(t)$. All that we can say is that $0 \leq y(t) \leq K(t)$; that is, when price exactly covers operating costs, the backstop sector is indifferent as to its rate of output. Summarizing this analysis, we can see that the output of the backstop sector at any one time is completely determined by the relationship between the price of energy and the operating cost c. When P exceeds c all backstop capacity is fully utilized; when P is less than c all capacity is idle, and if P should equal c exactly, backstop output is indeterminate between zero and capacity.

We now turn to the determination of backstop investment. Equation (3.5) has the usual interpretation: the optimal rate of investment at time t equates the marginal cost of investment to its marginal value, $\lambda_k^f(t)$. Now $\lambda_k^f(t)$ is determined from (3.6). Recalling our earlier discussion, we see that if $P(t) > c$, $y(t) = K(t)$ and $\mu(t) = P(t) - c$. On the other hand, if $P(t) < c$, $y(t) = 0$ and by (3.7) $\mu(t) = 0$. Thus we can replace $\mu(t)$ by $P(t) - c$ in (3.6) and integrate forward, giving an explicit expression for $\lambda_k^f(t)$:

$$\lambda_k^f\,(t) = \int_t^T e^{r(t-\tau)}[p_\tau - c]d\tau + \lambda_k^f(T). \tag{3.9}$$

Once again, we can interpret $\lambda_k^f(t)$ as the present value of profits resulting from a marginal increase in capacity at time t. The multiplier $\mu(t)$ automatically reflects the fact that when $P(t) < c$ the backstop sector shuts down and incurs no loss, so the value of capacity includes only the profits made when $P(t) > c$.

As stated previously, the behavior of the backstop sector in this Stackelberg model is entirely determined by the leader. The leader sets the price path from time zero to exhaustion at T (and implicitly for all later periods, since the behavior of the backstop is determined by the capital stock it inherits at T and the assumption of competitive behavior thereafter), and backstop investment and output are determined as described above. The leader's optimal production path must be chosen in the light of the follower's reaction. We turn to this problem in the following sub-section.

3. Solution for Depletable Energy Sector

In Section 1 we described the procedure for solving dynamic Stackelberg games analytically: the response of the follower is summarized by the first-order necessary conditions, and these equations are added to the leader's problem as constraints. In our model this procedure runs into two sorts of difficulties. One arises from the indeterminacy of backstop production when $P(t) = c$; the other from the non-differentiability of some of the constraints which represent the follower. We will discuss these difficulties in turn.

In Section 2 we noted that if the leader should choose to set the price equal to backstop operating cost, the actual rate of backstop production is indeterminate between zero and $K(t)$. Technically,this indeterminacy is a violation of the basic Stackelberg paradigm.[9] If the response of the follower to any one of the leader's possible strategies is not unique, then there is indeterminacy in the attainable values of the leader's objective. Consequently, the leader's optimization problem cannot be solved. What is missing in this case is an agreement between the leader and the follower on how to divide the market during any interval over which $P(t) = c$. If, for example, we knew that backstop output would always equal capacity, the leader could unambiguously predict the follower's behavior. Alternatively, we could assume backstop output drops to zero whenever $P(t) = c$. Neither of these possibilities is required by the model as stated; both are, however, consistent with it. A similar problem of indeterminacy has arisen in other applications of the Stackelberg paradigm to resource market models (e.g., Marshalla [1978]).

The second source of difficulty with the model of the follower is that the constraints which specify both backstop output and investment are not differentiable. Thus investment is determined by the relations

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[9] See Basar and Olsder [1982], p. 126.

$$f'(v_t) = \lambda_k^f(t) \qquad \text{when } v_t > 0$$

and

$$f'(v_t) \geq \lambda_k^f(t) \qquad \text{when } v_t = 0.$$

An equivalent expression for the relationship between $v(t)$ and $\lambda_k^f(t)$ is

$$v_t = \text{Max}[0, (f')^{-1}[\lambda_k^f(t)]] \quad , \qquad (3.10)$$

where $(f')^{-1}$ denotes the inverse function of $f'(\cdot)$.

Now for the leader, $v(t)$ is a control variable and $\lambda_k^f(t)$ is a state variable. Thus (3.10) is a constraint involving both state and control variables, and it is not generally differentiable at $v(t) = 0$. By the same token, the condition which determines backstop output, (3.4), is a non-differentiable relationship between the output rates of the leader and follower. The presence of these two non-differentiable constraints rules out use of the Maximum Principle in solving the leader's problem analytically. Consequently, we resort to numerical methods the results of which are discussed in the following section.

Before turning to these results, we offer some conjectures as to general properties that the solution to the Stackelberg model may have, based partly on educated intuition and partly on the numerical results themselves. First, we note that for the leader, choosing prices in the vicinity of backstop operating costs is crucial for determining the response of the backstop sector. As we will see subsequently, the leader's strategy can include a phase where it sets the price equal to backstop operating cost and drives backstop output to zero. We suggest that this is a general property of this model: that the leader will control backstop investment in early years by threatening to lower prices drastically in later years. The other side of this policy is to allow prices to be high in early years, so as to reap maximum revenues early. This occurs in our numerical examples, and we suspect it is a general property except in the following circumstance. Recall that in a depletable resource problem with elastic demand, when a monopoly owns the resource, equilibrium requires marginal revenue to rise at the rate of interest. Now if in our model the leader controlled a very large resource stock, it might choose an initial marginal revenue, and possibly price as well, below the backstop operating cost. Here the leader prices below backstop costs not to delay entry of the backstop, but to dispose of the large resource stock optimally. Now what happens when prices rise to the level of backstop operating costs? There are two possibilities: either prices continue to rise, or they stay at the level of backstop operating costs for some time. We suggest on the basis of our numerical results that the latter case is more likely, but this remains to be proved.

Finally, we offer a comment on how this model might change if the competitive fringe were taken into account. The competitive fringe consists of producers of depletable energy with no market power. In Stackelberg models related to ours, Marshalla's for example, the equilibrium conditions for the fringe impose an exponential price path on the leader in early years, before fringe reserves are exhausted. In our model we would expect the presence of the fringe to smooth out some of the discontinuities in the leader's strategy. Thus if the leader faced both a competitive fringe and a backstop sector constituted as in our model, it would have less power to stave off the backstop and would most likely not be able to drive it out of the market completely. Again, these general properties, if true, remain to be established.

IV. Numerical Examples

1. Formulation

In order to provide illustrations of the quantitative behavior of the social planning and market models, we have formulated equivalent discrete-time versions of the continuous-time models analyzed earlier. These problems were solved numerically using the general constrained nonlinear optimization program MINOS/Augmented[10].

A common set of parameters was used for all examples. We took for market demand the simple linear function

$$P(t) = 100 - .5\,(x_t + y_t).$$

The social planner's objective function was taken to be the social surplus generated by this demand curve. The stock of depletable resources was set at 1000. Finally, the backstop operating cost was set at 30 and the quadratic backstop investment cost function $f(v_t) = 1/2\ \alpha\ v_t^2$ was used, with the slope parameter α taking on the values 0.1, 1.0, 5.0, and 10.0 in the sensitivity experiments reported below.[11]

The formulation of the social planning problem in discrete time is straightforward. The results reported below are based on the solution of the following non-linear program.

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[10] Developed by Murtagh and Saunders [1977], [1980].

[11] Assuming a quadratic investment cost function probably overstates the penalty associated with rapid backstop capacity expansion. However, our results show that the actual rates of backstop investment chosen in the model are generally well above zero, suggesting that the properties of the investment cost function at low rates of investment may not be critical.

$$\underset{\{x_t, y_t, v_t\}}{\text{Max}} \sum_{t=1}^{T} (1+r)^{-t} \, [u(x_t + y_t) - c \cdot y_t - f(v_t)] \qquad (4.1)$$

subject to

$$\sum_{t=1}^{T} x(t) = \bar{S}$$

$$K(t+1) = K(t) + v(t); \qquad t = 1, \ldots, T-1$$

$$y(t) \leq K(t); \qquad t = 1, \ldots, T$$

$$x(t),\ y(t),\ v(t) \geq 0; \qquad t = 1, \ldots, T$$

In contrast to the simpler social planning model, converting the Stackelberg model to discrete time involves a number of steps and several additional assumptions. The first step is to solve an appropriate discrete-time version of the follower's problem (4.1). The first-order conditions from this solution must then become constraints in the leader's problem. The follower's problem can be stated as follows:

$$\underset{\{y_t, v_t\}}{\text{Max}} \sum_{t=1}^{T} (1+r)^{-t} \, [(P_t - c)y_t - f(v_t)] \qquad (4.2)$$

subject to

$$K(t+1) = K(t) + v(t) \; ; \qquad t = 1, \ldots, T-1$$

$$0 \leq y(t) \leq K(t) \; ; \qquad t = 1, \ldots, T$$

$$v(t) \geq 0 \; ; \qquad t = 1, \ldots, T.$$

If we introduce the multipliers $\mu(t)$ and $\eta(t)$ corresponding to the constraints $y(t) \leq K(t)$, and $y(t) \geq 0$, respectively, we can write the first-order conditions for this problem as

$$P(t) - c - \mu(t) + \eta(t) = 0 \tag{4.3}$$

$$f'(v_t) = \lambda(t+1) \tag{4.4}$$

$$\lambda(t+1) = (1+r)\lambda(t) - \mu(t) \tag{4.5}$$

$$\mu(t) \geq 0,\ \mu(t)[K_t - y_t] = 0 \tag{4.6}$$

$$\eta(t) \geq 0,\ \eta(t) \cdot y(t) = 0, \tag{4.7}$$

together with the constraints in (4.2).

Now it can be shown that when $P(t) > c$, $y(t) = K(t)$, and when $P(t) < c$, $y(t) = 0$. And should $P(t) = c$, $y(t)$ is indeterminate. This is just what we would expect from our discussion in Section III.2. But unless this indeterminacy in the follower's output rate is resolved, there is no feasible method for solving the model numerically. Two extreme approaches suggest themselves. One is to require the backstop to operate at full capacity when $P(t) = c$, the other is to require it to shut down completely. Since the depletable energy sector is the more powerful in our model, we adopt the second assumption here.

One difficulty remains in translating the first-order conditions (4.3) - (4.7) into explicit constraints on the leader's optimization. This is that the value taken by $y(t)$, whether zero or $K(t)$, depends on the sign of $P(t) - c$. That is, if $P(t) > c$, $y(t) = K(t)$, and if $P(t) \leq c$, $y(t) = 0$. These relationships are not in a form which can be used in MINOS/Augmented. But we can cast them in a useable form if we make the additional assumption that at prices equal to or below the backstop operating cost demand elasticity is such that the leader's revenues rise with increasing prices. This is in fact a condition which is likely to hold, since operating costs are generally a small fraction of the total costs of producing alternative energy. Under this assumption the leader would never choose to set a price below c, because its revenues would be greater at higher prices, and hence no competing backstop production will come on the market until $P(t) > c$.

We now proceed to simplify the follower's first-order conditions. First, we can eliminate $\eta(t)$ since $y(t) > 0$ is guaranteed. Then we use (4.3) to eliminate $\mu(t)$ in (4.5). The resulting first-order conditions for backstop investment are

$$f'(v_t) = \lambda(t+1) \tag{4.8}$$

$$\lambda(t+1) = (1+r)\lambda(t) - (P_t - c) \tag{4.9}$$

Finally, we guarantee that $y(t) = 0$ when $P(t) = c$, and $y(t) = K(t)$ otherwise, by imposing the following pair of constraints:

$$P(t) > c$$

$$(y_t - K_t)(P_t - c) = 0. \tag{4.10}$$

Now we come to the implementation of the leader's problem. The leader's objective is simply

$$\underset{\{x_t\}}{\text{Max}} \sum_{t=1}^{T} (1+r)^{-t} P(x_t + y_t)x_t . \tag{4.11}$$

The leader faces the standard resource constraint $\sum_{t=1}^{T} x(t) < \bar{S}$, as well as the constraints which summarize the reaction of the backstop sector to the leader's behavior. The leader's problem is stated in full below:

$$\max_{\{x_t\}} \sum_{t=1}^{T} (1+r)^{-t} P(x_t + y_t)x_t \tag{4.12}$$

subject to

$$\sum_{t=1}^{T} x(t) = \bar{S}$$

$$K(t+1) = K(t) + v(t) \qquad t = 1, \ldots, T-1$$

$$f'(v_t) = \lambda(t+1) \qquad t = 1, \ldots, T-1$$

$$\lambda(t+1) = (1+r)\lambda(t) - P(x_t + y_t) + c \qquad t = 1, \ldots, T-1$$

$$P(x_t + y_t) > c \qquad t = 1, \ldots, T$$

$$y(t) \leq K(t) \qquad t = 1, \ldots, T$$

$$(y_t - K_t)(P_t - c) = 0 \qquad t = 1. \ldots, T.$$

2. Base Case

Base case results for the social planning and Stackelberg models are shown in Figures 3 and 4. In both cases the backstop operating cost is 30 and the investment cost function is $f(v_t) = 1/2v_t^2$.

Examining the results for the social planning model first, we see in Figure 3a that the price (or marginal utility) of energy rises from an initial level around 22 to a maximum of 37 by period 12. As we would expect on the basis of our earlier discussion, this is approximately the date at which the depletable resource stock is exhausted. Figure 3b shows depletable energy output declining gradually for the first six periods from an initial level of 155, then declining steeply to zero by period 11. Investment in backstop capacity (Figure 3c) begins in the first period, and reaches its maximum at approximately the date that price equals operating cost.

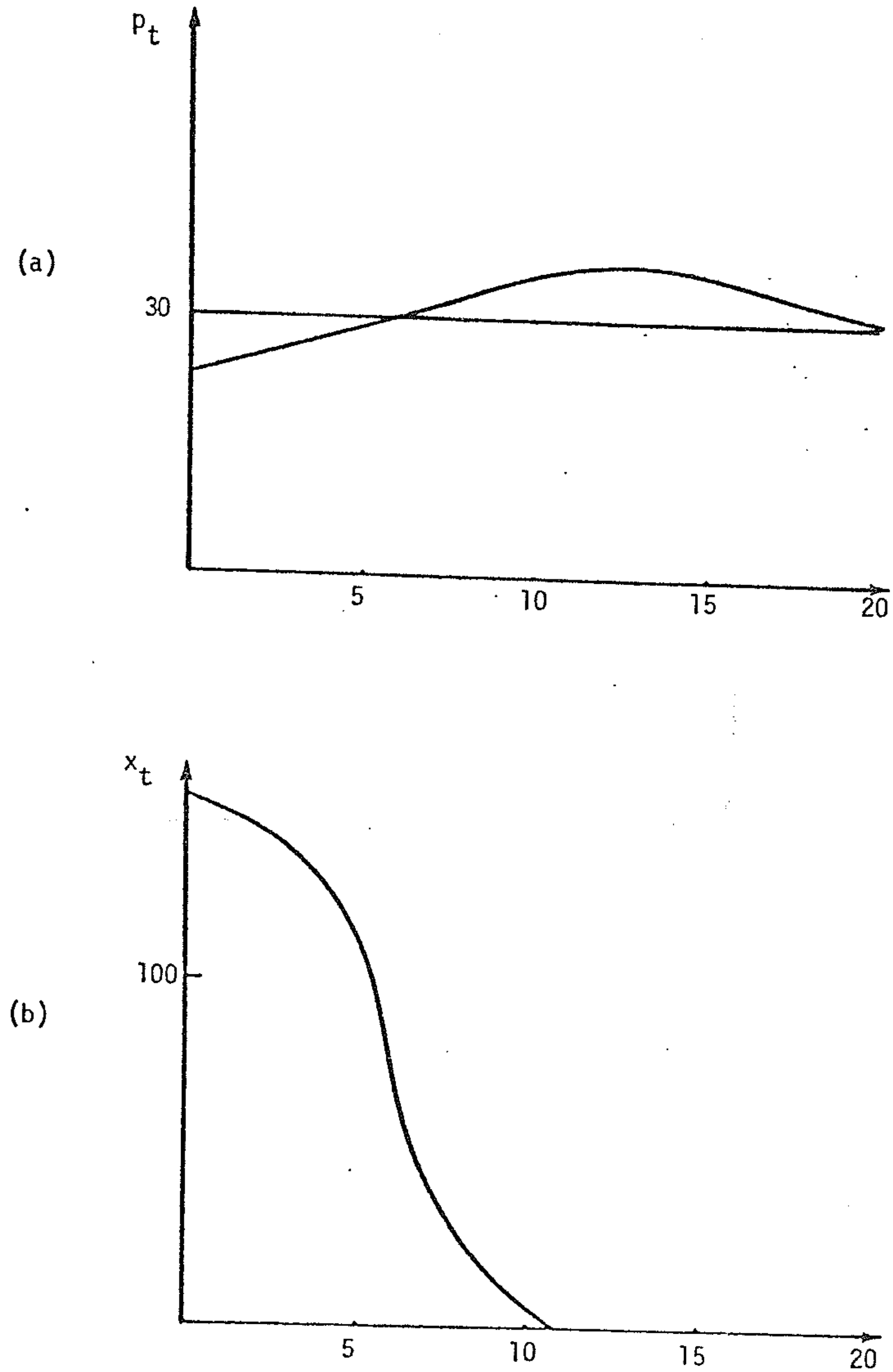
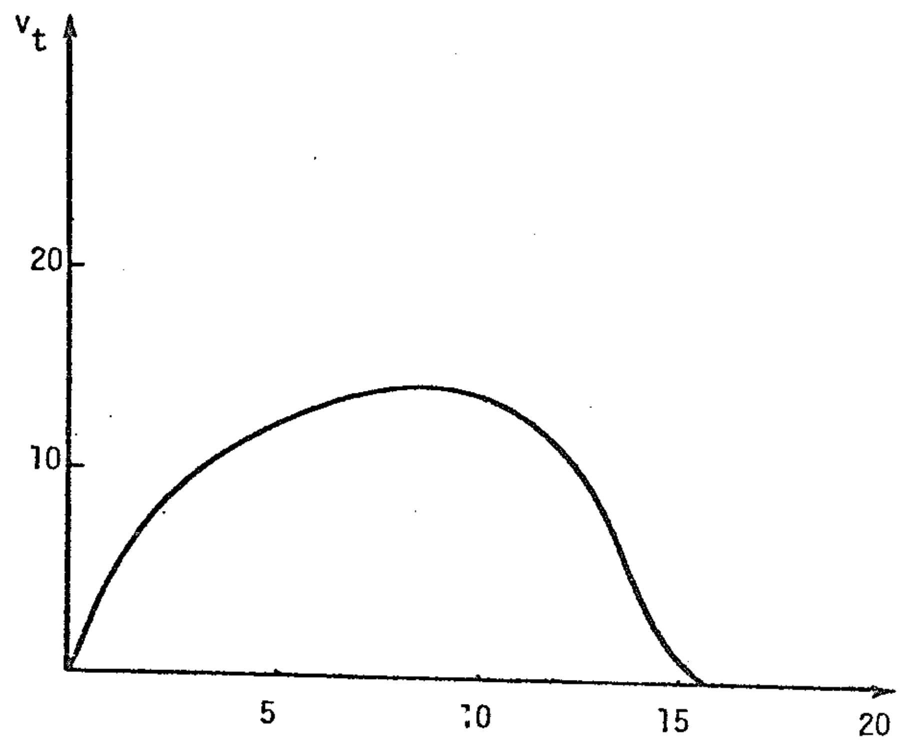


Figure 3

Social Planning Model: Base Case

(c)



(d)

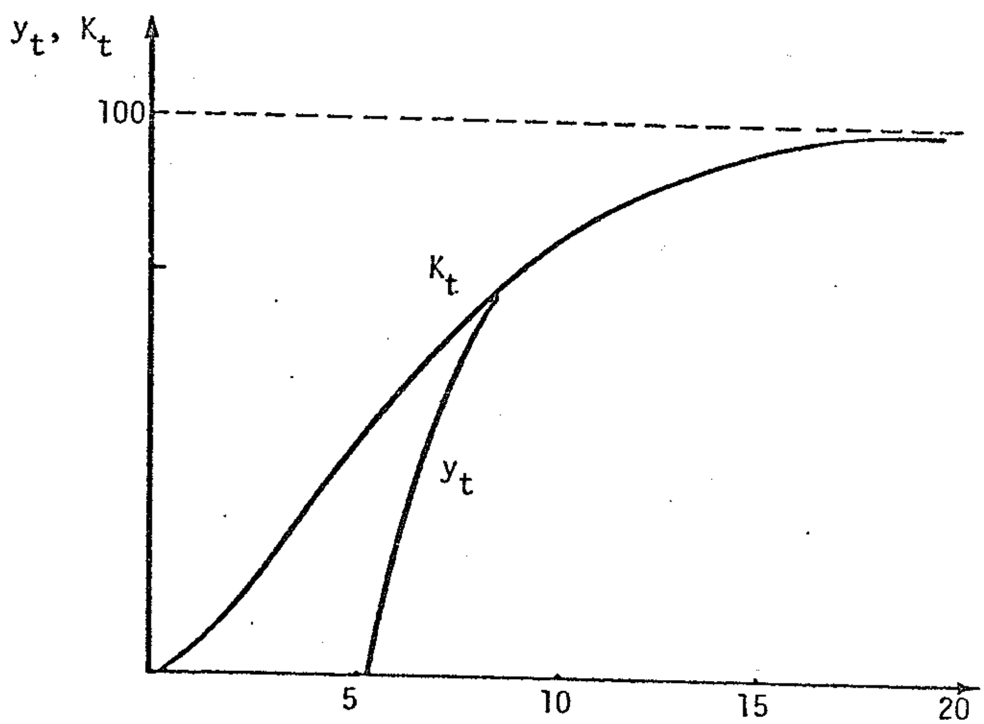


Figure 3 (cont.)

Social Planning Model: Base Case

Figure 3d shows that idle capacity of over 62 units is in place by the time backstop output begins. Of course, backstop output is zero because the price is below operating costs during this early period. At period 6 backstop production begins and it reaches the level of capacity by the subsequent period. Thereafter production is always at capacity since the price exceeds operating cost. And capacity continues to expand, driving the price back down to the level of operating costs by period 16. With some allowances made for the discrete nature of this example, it is evident that all of the essential properties of the social palnning model are exhibited here.

In Figure 4 we show the corresponding results for the Stackelberg model. Figure 4d shows that the leader chooses a price path which falls from an initially high level to the level of backstop costs at period 4. For the next four periods price is pegged at 30: the leader chooses to stave off backstop production and delay investment by keeping the prices at the level of operating costs for a substantial period of time. After period 8, prices are allowed to rise a few units above 30; by the last period the expansion of backstop capacity and output has brought the price back to 30. The production path the leader chooses which generates this price path is shown in Figure 4b. For the first three periods leader production is low, around 75 units. Then output jumps to 140 units for periods 4 through 8: this is the interval during which the leader takes the whole market and backstop output is zero. By period 8 most of the leader's resources are exhausted; production falls to zero by period 12. The leader's strategy, clearly, consists of an initial quick-kill in which high revenues are secured by low rates of production, followed by a period of limit-pricing in which the backstop

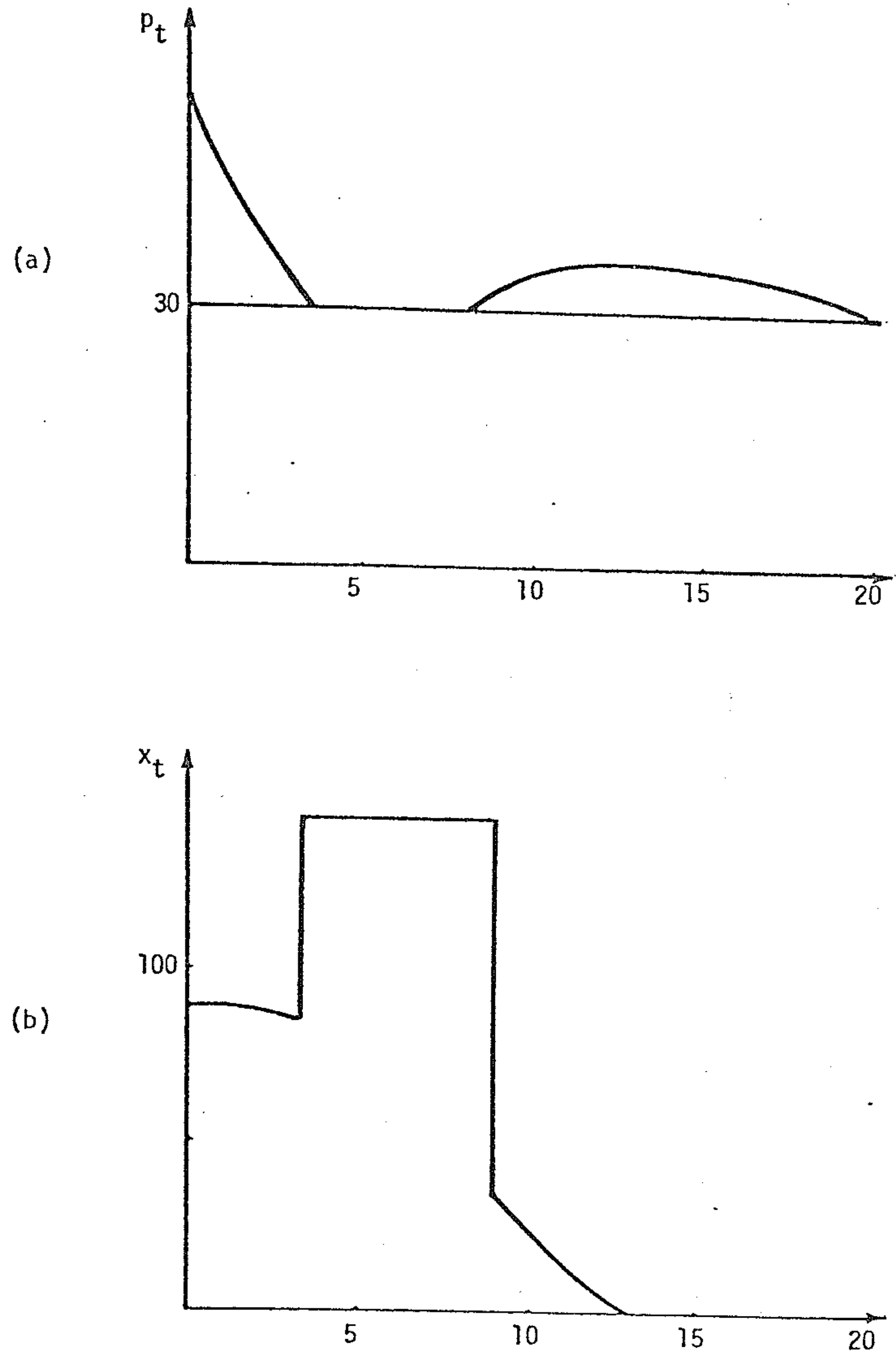
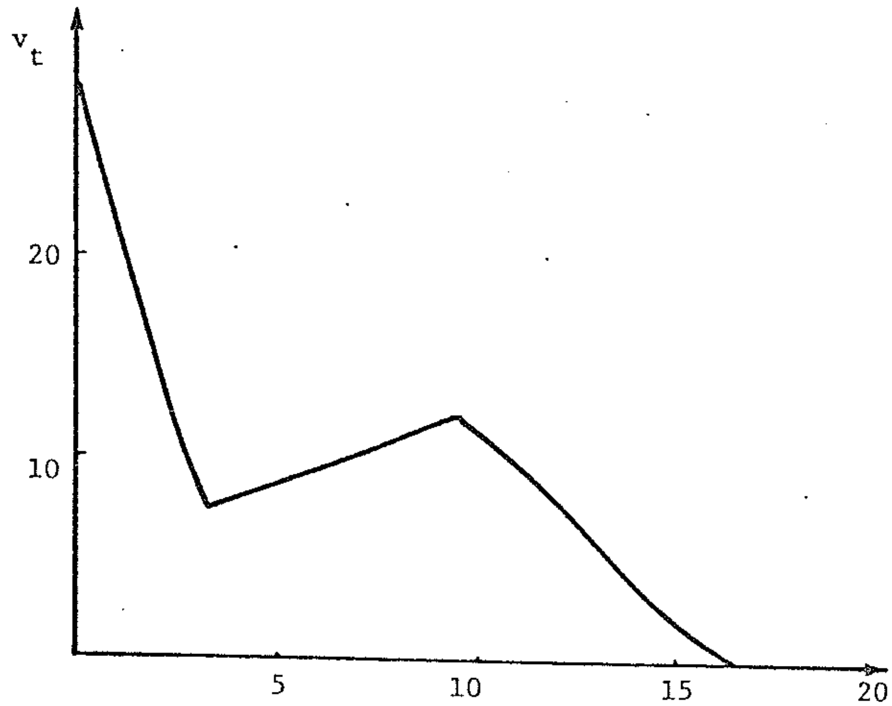


Figure 4

Stackelberg Model: Base Case

(c)



(d)

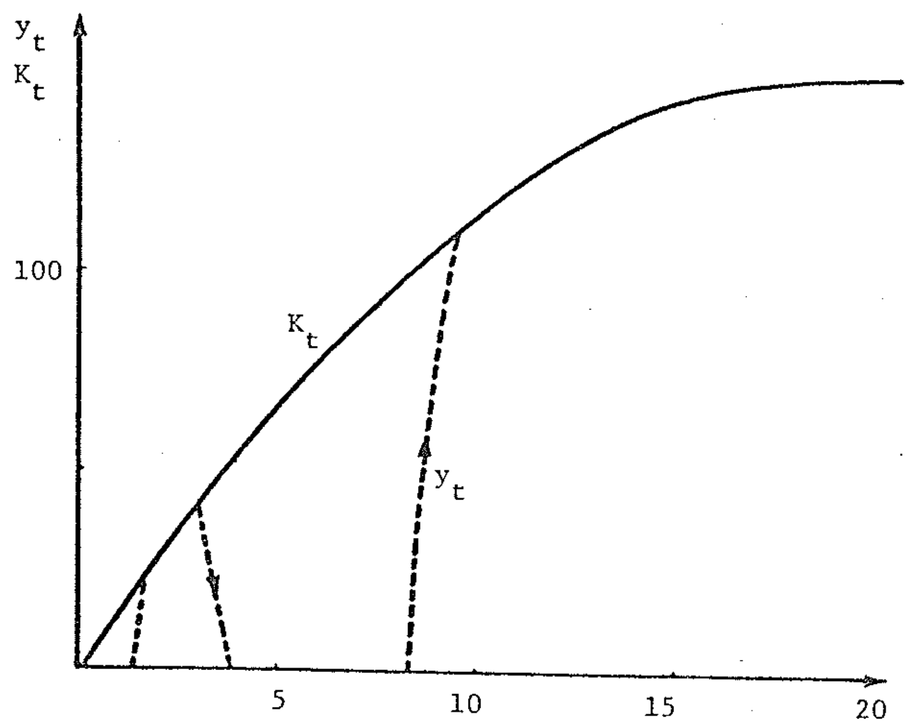


Figure 4 (cont.)

Stackelberg Model: Base Case

sector is totally shut out of the market. The leader can afford to allow prices to be high in early years because the threat of a subsequent period of zero profits causes the backstop sector to invest in capacity cautiously.

Backstop investment is shown in Figure 4c. The rate of investment in this case is very high but falling in the first two periods; thereafter, investment increases slowly from about 8 to 10.5 units per year. At period 8 investment reaches its maximum; it drops to zero by period 15. The most notable conclusion we can draw from these results is that backstop investment is positive until the steady-state level of capacity is achieved, even when the leader excludes backstop production from the market for a lengthy period. The return to backstop investment can be positive even when backstop output is zero. This is a result, again, of the rising cost of backstop investment. The leader cannot stave off the backstop forever, and when the leader's reserves run out, the price of energy will rise above the level of backstop operating costs. The return on backstop investment will then be positive, but since there is a cost penalty for investing too fast, it pays the backstop sector to increase its capacity before it is actually needed; i.e., to add to capacity even when existing capacity is standing idle.

The paths of backstop capacity and output which are (indirectly) determined by the leader are shown in Figure 4d. Since backstop investment is positive until very near the terminal date, it follows that backstop capacity grows steadily towards its asymptote. But backstop <u>output</u> is not steady. In periods two and three, when a small amount of capacity is in place and prices are high, backstop output is at capacity. By period four the leader has driven the price down to backstop operating costs, and backstop capacity lies idle until period 8. Capacity increases during this period, as we have remarked. So when the leader relinquishes control at period 8, backstop output jumps from zero to over 100 units. Thereafter, as price remains above cost, output remains at capacity.

We conclude this section with some observations on the comparison of the Stackelberg and socially optimal results. First, prices in the socially optimal case start out below backstop operating costs and overshoot it only in later periods. Prices under the Stackelberg regime, by contrast, start out high, fall to the level of backstop costs during an intermediate phase and then drift higher. In later years prices under the socially optimal regime are above the prices determined by the Stackelberg leader. Backstop investment is also higher in the social optimal case, at least for the first half of the total time covered. As a result, capacity is higher in this early phase in the social optimum case. Finally, backstop output in the social planning model is zero up to period 6, at which point it jumps permanently to the level of capacity. Backstop output in the Stackelberg model is more erratic, being positive at periods 2 and 3, and finally rising to the level of capacity permanently at period 8.

3. Sensitivity to Backstop Investment Costs

The basic assumption behind our backstop models is that investment costs rise more than linearly with the rate of investment. Thus there is a penalty for creating backstop capacity too fast. Since depletable energy is finite, the steady-state level of backstop production must eventually be reached. What our social planning model determines is the optimal rate at which to approach this steady-state. The optimal rate balances the cost of investing with the value of backstop output. Our Stackelberg model, by contrast, determines the investment path of an optimizing backstop industry under the control of a dominant depletable energy sector.

To study the sensitivity of the models to changes in investment costs, we have made four parallel runs for each, based on the quadratic investment cost function $f(v_t) = 1/2\ \alpha v_t^2$, with α taking on the values 0.1, 1.0, 5.0, and 10.0. As the parameter α increases, the slope of the investment cost function rises, and with it the cost penalty associated with high rates of investment. The results of these experiments are shown in Figure 5 for the social planning model, and 6 for the Stackelberg model.

Figure 5a shows the price paths associated with these four cases for the social planning model. Clearly, higher investment costs lead to higher prices. Also, the date at which price exceeds operating cost ($c = 30$) comes sooner in each successive run. The optimal extraction paths of depletable energy (Figure 5b) show a lower rate of consumption early and a later exhaustion date. The rate of investment (Figure 5c) also is lower as the slope of $f(\cdot)$ increases, and the period during which investment is positive is longer. We also see in Figure 5c confirmation of the result proved earlier: the rate of investment rises to a maximum when $p(\cdot) = c$, and thereafter falls. Finally, in Figure 5d we show the backstop capacity and output paths for these runs. Although the uultimate steady-state capacity level is not affected by changes in the slope of $f(\cdot)$, in these experiments 20 time periods is not enough to reach the steady-state capacity level for high investment costs. In

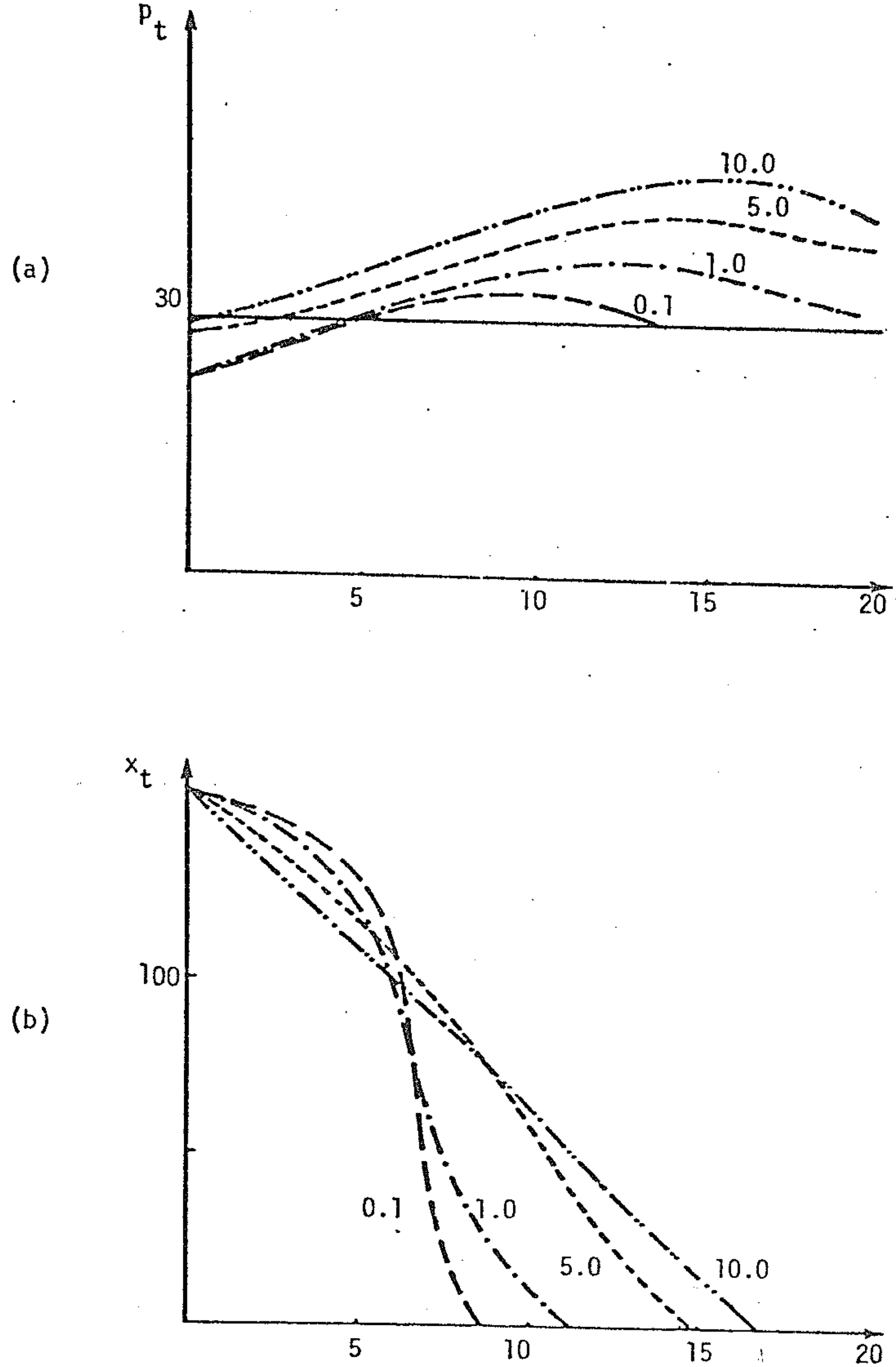


Figure 5

Social Planning Model: Sensitivity

to Backstop Investment Costs

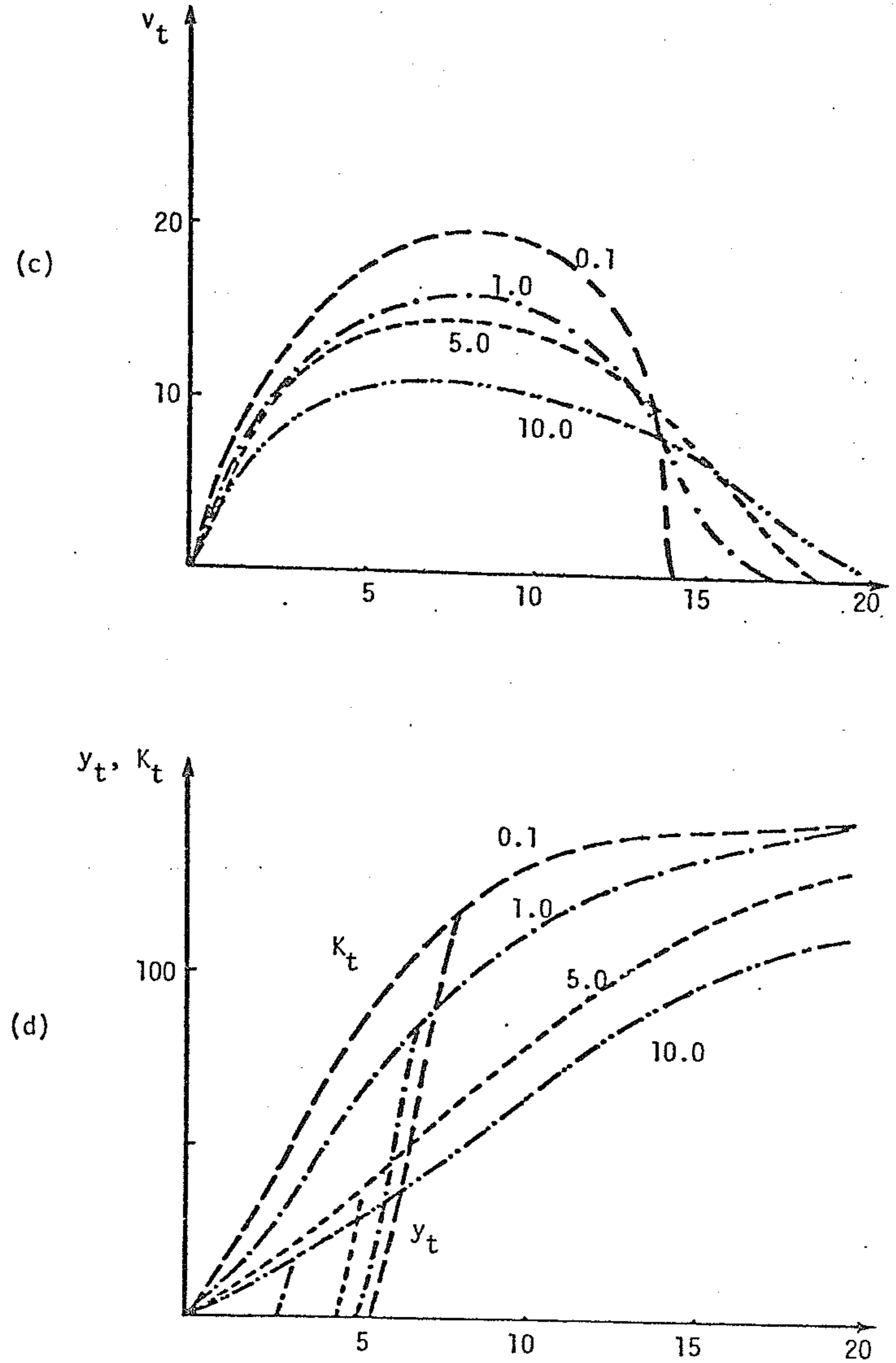


Figure 5 (cont.)

Social Planning Model: Sensitivity
to Backstop Investment Costs

these cases (α = 5.0 and 10.0, for example) we see that investment begins at time zero but at relatively low rates, and backstop production begins early, before any significant amount of capacity is installed. As the cost of investment declines, however, the rate of expansion increases. And, what is more, the date at which the idle capacity is turned on is delayed. So in the extreme case, where $\alpha = 0.1$, capacity is half-way to its steady-state level by period 6 when it is first put into use. This case, of course, is closest to the standard backstop model, since investment costs are almost negligible. So, as one would expect, the transition to the backstop is abrupt in the sense that backstop output is zero until very nearly the date of exhaustion of the stock of depletable energy. But there is a difference. Backstop capacity is created at a steady rate right from the start, and kept idle until the moment the price rises above operating costs. This is in contrast to the standard model, in which backstop capacity expands to its steady-state level at the moment of transition.

The corresponding results for the Stackelberg model are shown in Figure 6. Figure 6a shows the price paths generated by these four cases. All show the same pattern: prices decline from an initial level above operating costs, follow $P(t) = c$ for an interval, and finally rise above c to the horizon. Generally speaking, the higher the parameter α the quicker prices drop to the level of operating costs and the higher prices rise above costs in the final phase. The limit-pricing interval is longer when backstop investment is cheap. This is as expected: when backstop capacity is costly, it provides an additional constraint on the backstop sector so the leader is free to set a price above operating cost.

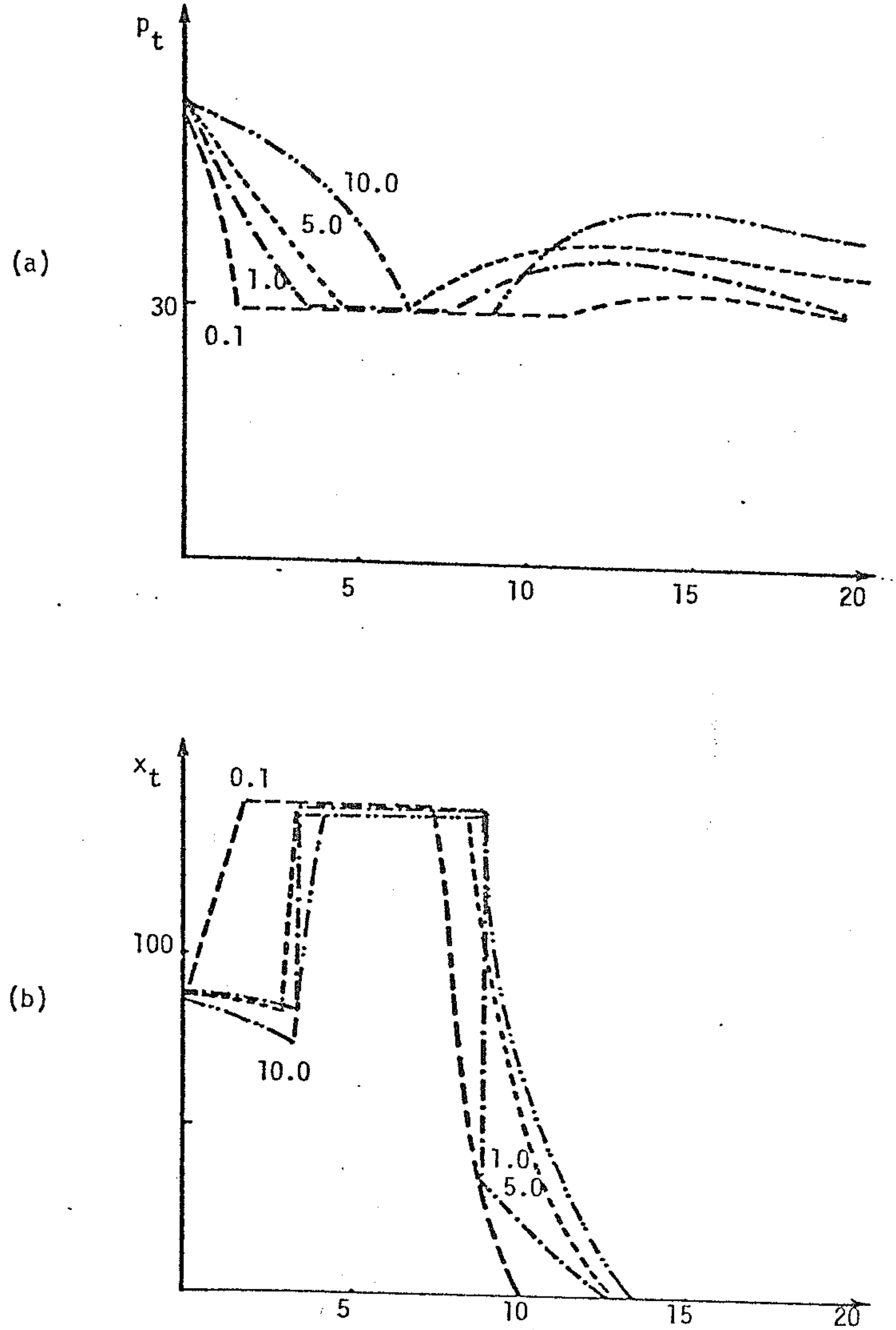
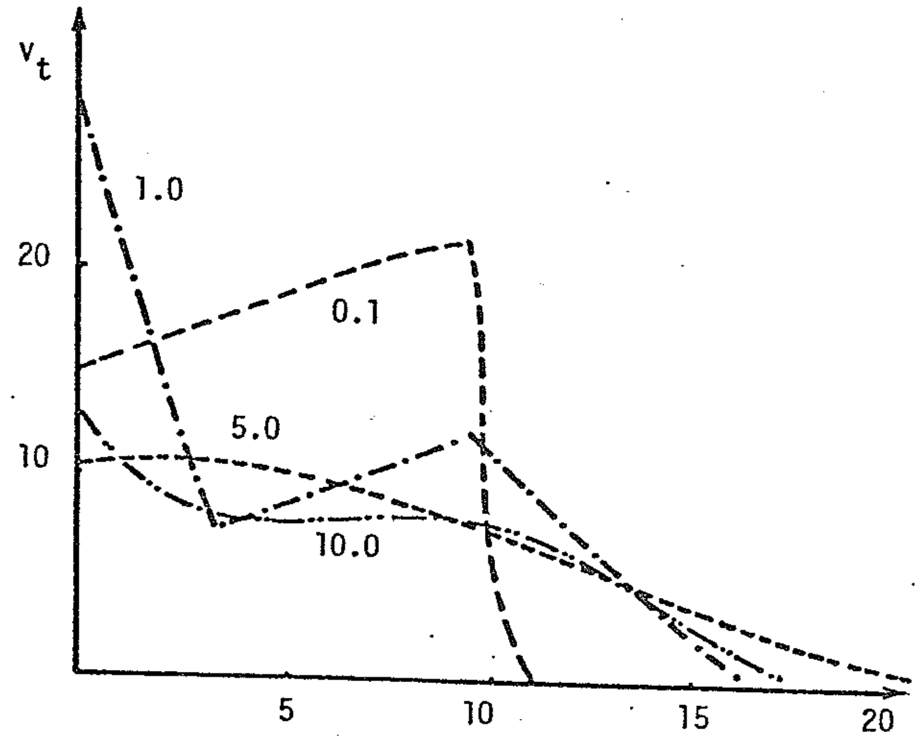


Figure 6

Stackelberg Model: Sensitivity
to Backstop Investment Costs

(c)



(d)

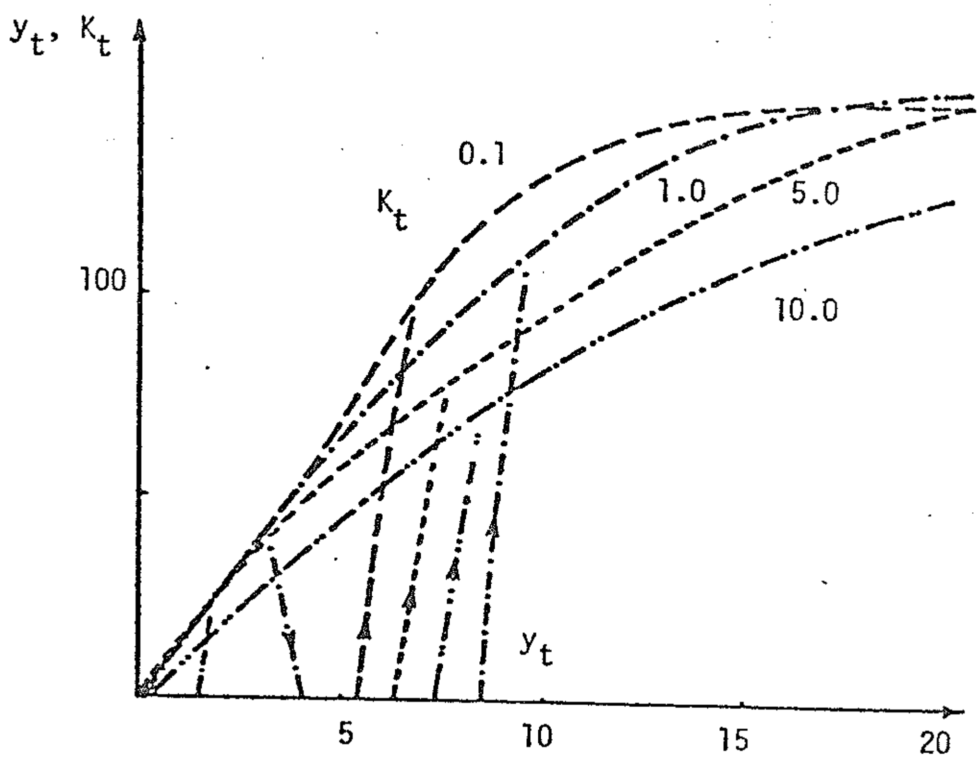


Figure 6 (cont.)

Stackelberg Model: Sensitivity

to Backstop Investment Costs

In Figure 6b we show the output paths chosen by the leader. All start around 70 units and rise to the limit-pricing level (140 units) within a few periods. Leader output drops quickly around period 10; we notice that the higher α is the more resources the leader holds for later periods. Backstop investment is shown in Figure 6c. When investment costs are low ($\alpha = 0.1$), investment rises gradually to a maximum rate near the date of exhaustion of depletable resources. This is the closest case to the standard backstop model with no investment cost. As we will see, backstop output is essentially zero up to the transition date. But in our model backstop capacity, rather than being created all at once, is created gradually over a substantial time interval. Finally, we observe that as investment costs increase, the rate of investment slows down and the period of capacity expansion is lengthened.

Figure 6d shows the associated paths for backstop output and capacity. Backstop capacity is generally lower in all time periods with higher values of α. The exception is the case $\alpha = 1.0$, in which substantial investment occurs in periods 2 and 3. Backstop output in this case is positive in those early periods, before it is driven to zero by the leader. This is also true for the case $\alpha = 0.1$. We see, then, that the relationship between backstop output and capacity is complex in these cases. While a limit-pricing phase is observed in each case, in some cases output is positive before this phase. And the date at which backstop output reaches capacity for good is not systematically related to the cost parameter α.

V. Summary

The standard abstract model of the transition from depletable to nondepletable resources predicts that energy prices will never rise above the cost of nondepletable energy, and the transition from one source to the other will occur instantaneously. In this paper we first constructed a social planning model in which the rate of investment in the nondepletable energy sector was determined along with the optimal rates of production of depletable and nondepletable energy. We showed that both resources were used simultaneously, and that the price of energy will rise above the operating cost of nondepletable sources. Investment in capacity in the nondepletable sector begins prior to the date when price equals operating cost, but capacity remains idle until this point is reached.

We then used this model as the foundation for a Stackelberg market model, in which all depletable resources are owned by the dominant player, and nondepletable resources are owned by a group of competitive followers. Numerical solutions of this model showed that the leader's strategy is to allow high prices initially, but to control backstop capacity expansion by threatening the follower by setting price at the operating cost of the nondepletable sector during an intermediate period. Since the nondepletable sector makes no profit during this period, its rate of capacity expansion in early years when prices are high is limited. In the Stackelberg model prices are generally above the socially optimum level in early years, and below in later years. Capacity in the nondepletable energy sector expands initially at a faster rate than is socially optimal, but output from this sector is delayed longer.

Bibliography

1. Basar, T. and Olsder, G. Dynamic Noncooperative Game Theory. London: Academic Press (1982).

2. Dasgupta, P.S. and Heal, G.M. Economic Theory and Exhaustible Resources. London: Cambridge University Press, (1979).

3. Energy Modeling Forum, World Oil: Summary Report. Stanford University, (1982).

4. Gilbert, R. "Dominant Firm Pricing Policy in a Market for an Exhaustible Resource." Bell Journal of Economics, (Autumn 1978), pp. 385-395.

5. Heal, G. "The Relationship Between Price and Extraction Cost for a Resource with a Backstop Technology." Bell Journal of Economics, (Autumn 1976), pp. 371-378.

6. Marshalla, R. "An Analysis of Cartelized Market Structures for Nonrenewable Resources." Ph.D. Thesis, Department of Engineering-Economic Systems, Stanford University, (1978).

7. Murtagh, B. and Saunders, M. "MINOS User's Guide". Technical Report SOL 77-9, Systems Optimization Lab, Stanford University, (February 1977).

8. Murtagh, B. and Saunders, M. "MINOS/Augmented User's Manual." Technical Report SOL 80-14, Systems Optimization Lab, Stanford University, (June 1980).

9. Nordhaus, W. The Efficient Use of Energy Resources. New Haven: Yale University Press, (1979).

10. Oren, S. and Powell, S. "Optimal Supply of a Depletable Resource with a Backstop Technology: Heal's Theorem Revisited". Energy Modeling Forum Occasional Paper 13, (1983).

11. Powell, S. "The Transition to Nondepletable Energy." Ph.D. Thesis, Department of Engineering-Economic Systems, Stanford University, (1983).