Theory of Half-Space Light Absorption Enhancement for Leaky Mode Resonant Nanowires

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ABSTRACT: Semiconductor nanowires supporting leaky mode resonances have been used to increase light absorption in optoelectronic applications from solar cell to photodetector and sensor. The light conventionally illuminates these devices with a wide range of different incident angles from half space. Currently, most of the investigated nanowires have centrosymmetric geometry cross section, such as circle, hexagon, and rectangle. Here we show that the absorption capability of these symmetrical nanowires has an upper limit under the half-space illumination. Based on the temporal coupled-mode equation, we develop a reciprocity theory for leaky mode resonances in order to connect the angle-dependent absorption cross section and the radiation pattern. We show that in order to exceed such a half-space limit the radiation pattern should be noncentrosymmetric and dominate in the direction reciprocal to the illumination. As an example, we design a metal trough structure to achieve the desired radiation pattern for an embedded nanowire. In comparison to a single nanowire case the trough structure indeed overcomes the half-space limit and leads to 39% and 64% absorption enhancement in TM and TE polarizations, respectively. Also the trough structure enables the enhancement over a broad wavelength range.

KEYWORDS: Nanowire, absorption, solar cell, photodetector, coupled-mode theory, leaky mode resonance

Semiconductor nanowires have been explored extensively over the past decades due to their potential applications in high-performance optoelectronic devices, including photodetectors,1−10 solar cells,11−22 and sensors.23−27 The small size of nanowires provides opportunities for further miniaturization of these devices, which will lead to many advantages in terms of operation performance, signal-to-noise ratio, and power generation. However, due to the diffraction limit, the miniaturized devices cannot capture enough incident photons and result in unsatisfactory external quantum efficiency. Recently, it has been demonstrated that leaky mode resonances (LMRs) in nanowires can efficiently enhance light absorption.1 The resonant absorption enhancement has been successfully used in photodetectors1,2 and solar cells.11,12 Ca o et al. demonstrated that LMRs effectively enhance the external quantum efficiency of Ge nanowire photodetectors especially at the important 1.55 μm communication wavelength.2 The LMRs have also been utilized to increase absorption efficiency of solar cell, and about 25% increase in short circuit current has been observed as compared with the planar structure.12 A core-shell nanowire, which supports several nearly degenerate LMRs, has been demonstrated to lead to enhanced absorption in an a-Si shell.11

We note that in the applications of solar cells and photodetectors, nanowires typically operate in the configuration where light illuminates the nanowire with a wide range of different incident angles from half space (Figure 1a). Currently, most of the geometric cross sections of the investigated nanowires have centrosymmetry, such as circle,1,11,12,16 hexagon,12,22 and rectangle.12,28 However, we recently have shown that for nanowires supporting a single LMR, the integration of the absorption cross section over all angles, \[ \int_0^{2\pi} C_{\text{abs}}(\phi) \, d\phi \] has a maximum of \( \lambda \), where \( \lambda \) is the light wavelength in free space and \( \phi \) is the incident angle.29 For the nanowires with centrosymmetry, the incident-angle dependent absorption cross section satisfies \( C_{\text{abs}}(\phi) = C_{\text{abs}}(\phi + \pi) \). Therefore, the absorption cross section integrated over half space has an upper limit of \( \lambda/2 \) (i.e., the incident angle \( \phi \) from

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0 to π as illustrated in Figure 1a), which here we refer to as half-space limit.

In this Letter, based on the temporal coupled-mode theory (CMT) formalism,\textsuperscript{29−32} we develop a reciprocity theory for LMR nanowires. We show that the incident-angle dependent absorption cross section is determined by the radiation pattern of LMR. In order to exceed the half-space limit the radiation pattern of leaky mode resonances should be noncentrosymmetric and dominate in the directions that are reciprocal to the directions of illumination. As an example, we design a metal trough structure to achieve the desired radiation pattern for an LMR nanowire. The trough has a half-elliptical outer boundary. The thickness of the trough along the short and long axes is a and b, respectively.

We first develop the reciprocity theory based on the temporal coupled-mode equation. Here we consider that a TM polarized wave, which has its magnetic field normal to the nanowire axis and its electric field only in the axial direction, is incident on the nanowire located at the origin. Similar derivation can be carried out for the TE polarized case, which has its electric field normal to the nanowire axis. For the TM polarization, the total electric field for the far-field can be written as

\[ E_{\text{total}}(\rho, \theta) = A_0 \frac{\exp(-i k \rho)}{\sqrt{\rho}} |\alpha^+(\theta)| + A_0 \frac{\exp(i k \rho)}{\sqrt{\rho}} |\alpha^-(\theta)| \]  

where \( (\rho, \theta) \) is the polar coordinates, \( k \) is the wave vector in the free space, and \( A_0 \) is a normalization coefficient. We take the convention that the field varies in time as \( \exp(-i \omega t) \). So \( |\alpha^+\rangle \) and \( |\alpha^-\rangle \) can be identified as the incoming and outgoing wave amplitudes, respectively. With the choice of \( A_0 = \left( \frac{4 \pi}{\omega c} \right)^{1/4} \), \( |\alpha^+\rangle \) and \( |\alpha^-\rangle \) represent the time-averaged power of the incoming and outgoing waves.\textsuperscript{30,31} Here \( \langle f | g \rangle \) denotes the integration \( \int_\theta f^* (\theta) g(\theta) \, d\theta \) over all angles.

When the nanowire supports a single LMR, the far field of the resonant mode can be written as

\[ E_{\text{LMR}}(\rho, \theta) = A_0 \frac{\exp(i k \rho)}{\sqrt{\rho}} |\alpha^a\rangle \]  

where the function \( \langle d | \theta \rangle \) represents the radiation pattern of the LMR. In this case, the scattering process can be described by the temporal coupled-mode theory with the equations:\textsuperscript{29,30,32}

\[ \frac{d c}{d t} = (−i \omega_0 + \gamma + \gamma_0) c + \langle d^a | a^a \rangle \]  

\[ |a^-\rangle = \hat{B} |a^+\rangle \]  

where \( c \) is the amplitude of the resonance, \( \omega_0 \) is the resonant frequency, \( \gamma_0 \) is the intrinsic loss rate due to absorption in the nanowire, \( \gamma \) is the external leakage rate due to the coupling of the resonance to the outgoing wave, \( \hat{B} \) is the scattering operator for a background scattering process that is independent of the resonance, and \( \hat{B}^* \hat{B} = 1 \). The amplitude \( c \) is normalized such that \( |d|^2 \) corresponds to the energy inside the nanowire.

According to the temporal coupled-mode theory,\textsuperscript{29,30,32} \( |d|^2 \) and \( \hat{B} \) are constraint due to the energy conversation and time-reversal symmetry, as a result

\[ \langle d | d \rangle = 2 \gamma \]  

\[ \hat{B} |d^a\rangle = -|d\rangle \]  

Now we define a scattering operator that connects the incoming and the outgoing wave: \( |a^-\rangle = \hat{S} |a^+\rangle \). From the temporal coupled-mode theory eq 3, we have

\[ \hat{S} = \hat{B} + \frac{|d\rangle \langle d|}{i(\omega_0 - \omega) + \gamma + \gamma_0} \]  

To evaluate the absorption cross section, we suppose a TM polarized incident plane wave \( E_{\text{inc}} = \exp(ik \cdot r) \). The absorbed power by the nanowire:

\[ P_{\text{abs}} = \langle a^\dagger a^+ \rangle - \langle a^- a^- \rangle = \langle a^\dagger | (1 - \hat{S}^* \hat{S}) |a^+ \rangle^2 \]  

Substituting eq 5 into eq 6, we can obtain the absorption cross section as

\[ C_{\text{abs}} = \frac{P_{\text{abs}}}{T} = \frac{\mu_0}{\epsilon_0} \frac{4 \gamma_0}{(\omega_0 - \omega)^2 + (\gamma + \gamma_0)^2} \]  

\[ |\alpha^+\rangle \]  

The trough has a half-elliptical outer boundary. The thickness of the trough along the short and long axes is a and b, respectively.
where \( I = \frac{1}{2} \sqrt{\frac{\epsilon}{\mu}} \) is the incident power. Equation 7 shows that generally the absorption cross section of a LMR nanowire exhibits a Lorentzian line shape around the resonant frequency.

To obtain the incident-angle dependence of the absorption cross section, we now expand the plane wave into cylindrical waves as

\[
\exp(ik \cdot \mathbf{r}) = \sum_{m=-\infty}^{\infty} i^m \exp(-im\phi) \left( H_m^{(1)}(k\rho) + H_m^{(2)}(k\rho) \right) \frac{1}{2}
\times \exp(im\theta) \tag{8}
\]

where \( \phi \) is the incident angle of the plane wave illustrated in Figure 1a. In the far field, where \( k\rho \to \infty \), eq 8 can be rewritten as

\[
\exp(ik \cdot \mathbf{r}) \approx \frac{\exp(ik\rho)}{\sqrt{\rho}} \frac{1}{2\pi k} \exp\left(-\frac{i\pi}{4}\right) \sum_{m=-\infty}^{\infty} \exp\left[im(\theta - \phi)\right] + \exp\left[-ik\rho \right] \frac{1}{\sqrt{\rho}} \exp\left(-\frac{i\pi}{4}\right) \sum_{m=-\infty}^{\infty} \exp\left[im(\theta - \phi - \pi)\right] = \exp(ik\rho) \frac{2\pi}{\sqrt{\rho}} \exp\left(-\frac{i\pi}{4}\right) \delta(\theta - \phi) + \frac{2\pi}{\sqrt{\rho}} \exp\left(-\frac{i\pi}{4}\right) \delta(\theta - (\phi + \pi)) \tag{9}
\]

where \( \delta \) is the Dirac delta function, and we utilize the equation

\[
\delta(\theta) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \exp(im\theta). \tag{10}
\]

By applying eq 9, eq 11 becomes

\[
\int_0^{2\pi} C_{abs}(\phi) \, d\phi = \lambda \frac{2\gamma_0}{(\omega - \omega_0)^2 + (\gamma + \gamma_0)^2} \frac{d(\theta + \pi)^2}{(\theta + \pi)^2} \tag{12}
\]

where \( \omega_0, \gamma_0 \) and \( (\theta + \pi)^2 \) correspond to the resonant frequency, the external leaky rate, the intrinsic loss rate, and the radiation pattern of the two LMRs, respectively. In the case of double degenerate LMRs, the integration of the total absorption cross section over all angles has a maximum of \( 2\lambda \). For a nanowire having centrosymmetry that supports a pair of degenerate LMRs, the radiation pattern \( l_d(\theta) \) may not have the full symmetry of the structure. However, the radiation pattern of each degenerate mode still has centrosymmetry, i.e., \( l_d(\theta + \pi)^2 = l_d(\theta)^2 \).

Correspondingly, there is still an upper limit of \( \lambda \) for the total absorption cross section under half-space light. Therefore, in order to exceed such a limit, we need to break the centrosymmetry of the nanowire geometry even though the nanowire supports degenerate resonances.

Now we demonstrate the half-space absorption enhancement by breaking the centrosymmetric geometry with a wire-trough structure (Figure 1b). We have recently fabricated such a metal trough with standard evaporation techniques and experimentally demonstrated that a metallic trough network is a remarkable transparent electrode with great mechanical flexibility. Here we show that the trough structure can be used to control the radiation of LMR and enhance the light absorption of embedded nanowire for a wide range of different incident angles from half space.

To show the half-space absorption enhancement, we compare the wire-trough structure to a single nanorod with a circular geometric cross section. According to C. Garnet’s study, a single a-Si nanorod has the optimal absorptive performance for the solar spectrum when the nanorod has a diameter around 110 nm. Such a rod has two degenerate TM111 resonances reaching the critical coupling at 654 nm wavelength. Here we use a commercial finite element method (FEM) software (COMSOL) to design a silver trough structure and assume that the trough has a half-elliptical outer boundary (Figure 1b). The frequency-dependent dielectric constants of a-Si and silver are taken from the experiments. The silver
trench is characterized by two thicknesses $a$ and $b$, as shown in Figure 1b. To realize LMRs resonating in the vicinity of 650 nm, we first set $a = b = 30$ nm and modify the diameter of the embedded nanowire while keeping the wire and the trench touching. Our simulation shows that an embedded nanowire with a 125 nm diameter supports two nearly degenerate TM$_{11}$ resonances at 658 nm (Figure 2a). According to the mirror symmetry around the $y$-axis of the wire-trench structure, the two TM$_{11}$ resonances can be classified into an even and an odd mode (the inset in Figure 2a). In order to maximize the absorption cross section, we need to ensure that the two LMRs are not only degenerate but also working close to the critical coupling, i.e., $\gamma_0 = \gamma$. For this purpose, we fix $a = 30$ nm and modify $b$. Figure 2b plots that $\gamma$ and $\gamma_0$ of both resonances vary as functions of $b$. Here we choose $b = 60$ nm, and in this case the two LMRs are nearly critically coupled.

Next we show that the trench structure allows a desired radiation pattern to enhance the half-space absorption. We plot the radiation patterns for both resonances in Figure 3a at the wavelength of 658 nm. Especially it shows that the radiation pattern of the even LMR dominates in the angles from $\theta = 0$ to $\theta = \pi$ (from $\phi = 0$ to $\phi = \pi$). To show the half-space absorption enhancement, we also plot the optimal single nanowire case, the absorption cross section of the 110 nm diameter single nanowire at the resonant frequency of the TM$_{11}$ resonance in Figure 3b. We also calculate the effective absorption cross section after excluding the parasitic absorption by the metal trough. The black dashed line corresponds to the isotropic absorption cross section of the 110 nm diameter single nanowire calculated by FEM.

Our simulation shows that the TE$_{01}$ mode resonates at 664 nm. The far-field of the scattering field in the TE polarized case (Figure 4a) dominates in angles from $\theta = \pi/6$ to $\theta = 11\pi/6$. We also plot the incident-angle dependent absorption cross section of the wire-trench structure (the black solid line in Figure 4b). Indeed the dominant different incident angles (the cross line in Figure 3b). Indeed it shows an excellent agreement between the temporal coupled-mode theory and the numerical calculation.

Figure 3 illustrates that the wire-trench structure strongly enhances the broad-angle absorption cross section for one side (from $\phi = 0$ to $\phi = \pi$). To show the half-space absorption enhancement, we also plot the optimal single nanowire case, the absorption cross section of the 110 nm diameter single nanowire at the resonant frequency of the TM$_{11}$ resonance in Figure 3b. We also calculate the effective absorption cross section after excluding the parasitic absorption by the metal trough (the blue solid line in Figure 3b). At the normal incident angle, the absorption cross section of the wire-trench structure shows 37% enhancement as compared with that of the single nanowire. Moreover, the enhancement covers a wide range of incident angle from $\phi = \pi/6$ to $\phi = 5\pi/6$, and the average enhancement is 39%.

Now we consider the half-space absorption enhancement in the case of TE polarization. Our simulation shows that the TE$_{01}$ mode resonates at 664 nm. The far-field of the scattering field in the TE polarized case (Figure 4a) dominates in angles from $\theta = \pi/6$ to $\theta = 11\pi/6$. We also plot the incident-angle dependent absorption cross section of the wire-trench structure (the black solid line in Figure 4b). Indeed the dominant

![Figure 2](image2.png)

**Figure 2.** (a) Resonant frequency of the even and odd LMRs in the wire-trench structure, schematically shown in Figure 1b. The inset shows the electric field distribution of two modes. (b) The external leaky rate $\gamma_{0(e)}$ and the intrinsic loss rate $\gamma_{0(o)}$ of the even and the odd LMRs vary as a function of $b$, where the thickness $a = 30$ nm.

![Figure 3](image3.png)

**Figure 3.** (a) Black and blue lines correspond to radiation pattern $d(\theta)$ of the even and odd LMRs, respectively. (b) Black solid line and red cross line correspond to the incident-angle dependent absorption cross section of the wire-trench structure calculated by the temporal coupled-mode theory (eq 13) and the numerical method (FEM), respectively. The blue solid line represents the effective absorption cross section of the wire-trench structure after excluding the parasitic absorption by the metal trough. The black dashed line corresponds to the isotropic absorption cross section of the 110 nm diameter single nanowire calculated by FEM.

![Figure 4](image4.png)

**Figure 4.** (a) Scattering field under the illumination of a 664 nm TE polarized wave. (b) The black solid line corresponds to absorption cross section of the wire-trench structure in TE polarized case. The blue solid line represents the effective absorption cross section of the wire-trench structure after excluding the parasitic absorption by the metal trough. The black dashed line corresponds to the isotropic absorption cross section of the 110 nm diameter single nanowire calculated by FEM.
absorptive angles have a $\pi$ rotation from the far field, which agrees with the theoretical prediction. Compared with the optimal nanowire case (the dotted line in Figure 4b), the wire-trench structure exhibits strong absorption enhancement for the incident angle varying from $\phi = \pi/6$ to $\phi = 5\pi/6$. We also calculate the effective absorption by the embedded wire after excluding the parasitic absorption in the metal (the blue solid line in Figure 4b). At the normal incident angle, the absorption cross section of the embedded nanowire is enhanced 89% by the metallic trench structure.

Now let us show that half-space absorption enhancement can cover a broad wavelength range. We define the effective absorption as the integration of absorption cross section from $\phi = \pi/6$ to $\phi = 5\pi/6$. Figure 5 plots the effective absorption in the wavelength ranging from 400 to 760 nm. For the TM polarized case (Figure 5a) the wire-trench structure shows 39% enhancement, and the average enhancement from 400 to 760 nm is 2% in comparison to the optimal single nanowire case. In the TE polarized case (Figure 5b), the wire-trench structure shows a broadband enhancement over the visible spectrum. At 664 nm, the wire-trench structure shows 64% enhancement, and the average enhancement from 400 to 760 nm is 26% as compared with the optimal single nanorod case.

In conclusion, we develop a framework for the half-space light absorption enhancement for leaky mode resonant nanowires. We show that nanowires with centrosymmetric geometry cross section have an upper limit under the half-space illumination. Based on the temporal coupled-mode equation, a reciprocity theory is developed for leaky mode resonances in order to connect the angle-dependent absorption cross section and the radiation pattern. To exceed the half-space limit it needs to break the centrosymmetry, and a desired radiation pattern should be noncentrosymmetric and dominate in the direction reciprocal to the illumination. We note that when the half-space light illuminates obliquely off the axis the developed reciprocity theory is still valid. In this case, the off-axis component needs to be taken into account when controlling the LMR radiation pattern.

We also numerically demonstrate the half-space absorption enhancement with a wire-trench structure. We show that the radiation pattern of the nanowire is strongly modified by the metallic trough and exhibits a desired profile for half-space absorption. As expected, at the resonant frequency, the trough structure enhances 39% and 64% for the TM and TE polarized cases in comparison to an optimal single nanowire. Moreover, the trough structure enables the enhancement over a broad wavelength range for both the TM and the TE polarizations. We note that it has been experimentally demonstrated that a metal trough network is a remarkable transparent electrode with great mechanical flexibility. Therefore, by combining the functionality of both absorption enhancement and transparent electrode, the trough structure is expected to augment the performance of applications including nanowire photodetectors, solar cells, and optical sensors.

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Notes
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