Coupling of Modes Analysis of Resonant Channel Add–Drop Filters

C. Manolatou, M. J. Khan, Shanhui Fan, Pierre R. Villeneuve, H. A. Haus, *Life Fellow, IEEE,* and J. D. Joannopoulos

***Abstract*— The operation principle of resonant channel add–drop filters based on degenerate symmetric and antisymmetric standing-wave modes has been described elsewhere using group theoretical arguments. In this paper, the analysis is carried out using coupling of modes in time. A possible implementation of such a filter is a four-port system utilizing a pair of identical single-mode standing wave resonators. The analysis allows a simple derivation of the constraints imposed on the design parameters in order to establish degeneracy. Numerical simulations of wave propagation through such a filter are also shown, as idealized by a two-dimensional geometry.**

***Index Terms*—Coupled-mode analysis, FDTD method, optical filters, optical waveguides, resonators, wavelength division multiplexing.**

I. Introduction

The wide use of optical wavelength division multiplexing (WDM) calls for compact, convenient channel add–drop filters. The "Dragone" filter [1] provides a means of simultaneously separating all the channels, which can then be dropped and/or added individually. After recombination via an inverse filter, the full WDM distribution is restored. This type of filter is now widely used. Resonators have also been considered for channel dropping devices. If the resonators are small enough so that the spacing of the resonant frequencies accommodates the set of WDM channels within the communications window, the goal of dropping one channel by one filter without affecting the other channels is achieved. One proposed version uses distributed feedback (DFB) resonators side-coupled to the signal bus [2]. In order to remove all of the power in one channel, two such resonators are required. Another version uses ring resonators between two optical waveguides, one guide acting as the signal bus and the other as the receiving waveguide. This structure has the advantage that a single resonator can remove all of the power in one channel [3]. The filter responses of these structures are Lorentzian (single pole). By combining a number of resonators with appropriate coupling, more sophisticated transfer characteristics could be achieved [3], [4]. This concept has already been studied in the context of microwave circuit design.

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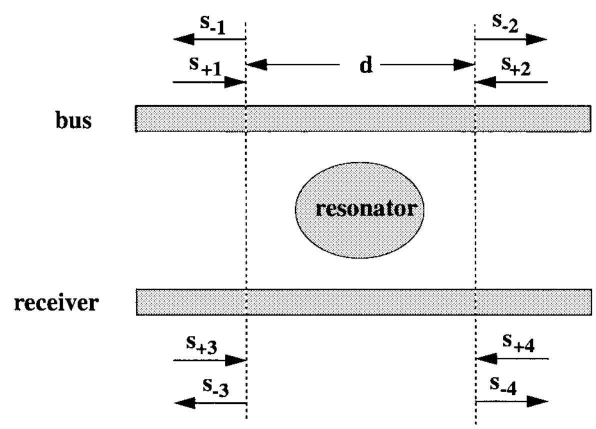


Fig. 1. General four-port system consisting of a resonator between two waveguides.

While a ring resonator between two optical waveguides provides an ideal basic structure for removal of a channel from the signal bus, the performance of ring resonator filters can be affected adversely by the coupling between counterpropagating waves caused by surface roughness [5]. Smooth surfaces are required of a high quality not yet achievable with existing fabrication technology. This fact raises the question as to whether the performance of a ring channel dropping filter could be realized with a resonant structure not as sensitive to surface roughness. The principle of operation of such a structure was explained using group theoretical arguments in [6]–[8]. Here we recast the description and the explanation of its operation into coupled-mode theory (CMT) in time.

Briefly summarized, we show that an optical resonator with degenerate symmetric and antisymmetric modes side-coupled to two waveguides performs the same function as a ring resonator. For a symmetric system consisting of two identical coupled resonators between two waveguides, the expected splitting of the degeneracy can be counteracted by proper coupling to the waveguides. This concept is also demonstrated by finite-difference time-domain (FDTD) simulations of wave propagation through such a filter.

II. Four-Port System Employing a Single-Mode Resonator

The basic implementation of the channel add–drop filters considered in this paper is a four-port system that consists of a resonator placed between two waveguides. A schematic is shown in Fig. 1. The resonator is evanescently coupled to the waveguides which we shall refer to as the bus and the receiver, respectively. In general, the resonator modes interact with the forward and backward propagating modes of the waveguides over a finite length. The interaction length in each waveguide is the region where the fields of the resonator modes overlap with the waveguide fields, and it is assumed to be fully contained between the input/output reference planes, defined on either side of the resonator, as shown in the schematic. In this section, we consider the case of a resonator that supports only one mode in the frequency range of interest, with amplitude denoted by a. The squared magnitude of this amplitude is equal to the energy in the mode. The waveguides are assumed to be single mode and the waveguide dispersion is ignored in our analysis. This simplification is justified if the resonance peak is narrow. The amplitudes of the incoming (outgoing) waves in the bus are denoted by s_{+1} (s_{-1}) and s_{+2} (s_{-2}) and in the receiver waveguide by s_{+3} (s_{-3}) and s_{+4} (s_{-4}), respectively. The squared magnitude of these amplitudes is equal to the power in the waveguide mode. The equation for the evolution of the resonator mode in time is given by

$$\frac{da}{dt} = \left(j\omega_o - \frac{1}{\tau_o} - \frac{1}{\tau_e} - \frac{1}{\tau_e'}\right) a + \kappa_1 s_{+1} + \kappa_2 s_{+2} + \kappa_3 s_{+3} + \kappa_4 s_{+4} \quad (1)$$

where ω_o is the resonant frequency, $1/\tau_o$ is the decay rate due to loss, $1/\tau_e$ and $1/\tau_e'$ are the rates of decay into the bus and the receiver, respectively, κ_1 and κ_2 are the input coupling coefficients associated with the forward and backward propagating modes in the bus, and κ_3 and κ_4 are similarly defined for the receiver. The decay rates are related to the unloaded quality factor Q_o and the external quality factors Q_e and Q_e' of the resonator by $Q_o = \omega_o \tau_o/2$, $Q_e = \omega_o \tau_e/2$, and $Q_e' = \omega_o \tau_e'/2$.

By power conservation, the outgoing waves are (see the Appendix)

$$s_{-1} = e^{-j\beta d}(s_{+2} - \kappa_2^* a) \quad (2)$$

$$s_{-2} = e^{-j\beta d}(s_{+1} - \kappa_1^* a) \quad (3)$$

$$s_{-3} = e^{-j\beta' d}(s_{+4} - \kappa_4^* a) \quad (4)$$

$$s_{-4} = e^{-j\beta' d}(s_{+3} - \kappa_3^* a) \quad (5)$$

where β and β' are the propagation constants in the bus and the receiver, respectively. Equations (2)–(5) show that if the resonator is not excited then the incident waves appear at the output undisturbed, with a phase shift that is due to the finite distance d between the reference planes, for simplicity taken to be the same in both waveguides. The coupling coefficients are found in the Appendix following a treatment similar to [9]. Their squared magnitudes are equal to the respective decay rates into the waveguides due to power conservation, and their phases are related to the phase mismatch between the waveguide and resonator modes and the choice of reference planes. So we can write

$$\kappa_i = \sqrt{\frac{1}{\tau_{e_i}}}\, e^{j\theta_i}, \qquad i = 1, \cdots, 4 \quad (6)$$

with $1/\tau_{e1,3}$ and $1/\tau_{e2,4}$ defined as the decay rates in the forward and backward direction, respectively, satisfying

$$\frac{1}{\tau_{e1}} + \frac{1}{\tau_{e2}} = \frac{2}{\tau_e} \quad (7)$$

$$\frac{1}{\tau_{e3}} + \frac{1}{\tau_{e4}} = \frac{2}{\tau_e'} \quad (8)$$

and θ_i are the respective phases. We choose 1 as the input port and we set s_{+2}, s_{+3}, and s_{+4} to zero. If s_{+1} has a $e^{j\omega t}$ time dependence, then we find from (1) at steady state

$$a = \frac{\sqrt{\frac{1}{\tau_{e1}}}\, e^{j\theta_1} s_{+1}}{j(\omega - \omega_o) + \frac{1}{\tau_o} + \frac{1}{\tau_e} + \frac{1}{\tau_e'}}. \quad (9)$$

Substituting a from (9) into (2)–(5), we get the filter response of the system

$$\frac{s_{-1}}{s_{+1}} \equiv R = -\, e^{-j\beta d} \frac{\sqrt{\frac{1}{\tau_{e1}\tau_{e2}}}\, e^{j(\theta_1 - \theta_2)}}{j(\omega - \omega_o) + \frac{1}{\tau_o} + \frac{1}{\tau_e} + \frac{1}{\tau_e'}} \quad (10)$$

$$\frac{s_{-2}}{s_{+1}} \equiv T = e^{-j\beta d}\left(1 - \frac{\frac{1}{\tau_{e1}}}{j(\omega - \omega_o) + \frac{1}{\tau_o} + \frac{1}{\tau_e} + \frac{1}{\tau_e'}}\right) \quad (11)$$

$$\frac{s_{-3}}{s_{+1}} \equiv D_L = -\, e^{-j\beta' d} \frac{\sqrt{\frac{1}{\tau_{e1}\tau_{e4}}}\, e^{j(\theta_1 - \theta_4)}}{j(\omega - \omega_o) + \frac{1}{\tau_o} + \frac{1}{\tau_e} + \frac{1}{\tau_e'}} \quad (12)$$

$$\frac{s_{-4}}{s_{+1}} \equiv D_R = -\, e^{-j\beta' d} \frac{\sqrt{\frac{1}{\tau_{e1}\tau_{e3}}}\, e^{j(\theta_1 - \theta_3)}}{j(\omega - \omega_o) + \frac{1}{\tau_o} + \frac{1}{\tau_e} + \frac{1}{\tau_e'}} \quad (13)$$

where R is the reflection from the input port, T is the transmission through the bus, and D_L and D_R represent the transmission (channel dropping) into the left and right ports of the receiver, respectively. Using (10)–(13), we can show the different behavior of a traveling wave and a standing-wave mode in this configuration.

In a traveling-wave mode, such as the mode supported by a ring or a disk resonator, the power flows continuously in only one direction in the resonator. For example, the forward traveling mode of the bus waveguide excites the clockwise propagating mode of a ring or disk, as shown in Fig. 2(a). Then, from (7) and (8), we have $1/\tau_{e2} = 1/\tau_{e3} = 0$, $1/\tau_{e1} = 2/\tau_e$, and $1/\tau_{e4} = 2/\tau_e'$, and (10) and (13) give $R = D_R = 0$, over the entire bandwidth. At $\omega = \omega_o$, the incident power in the bus in the forward direction is partially transferred to the receiver in the backward direction, limited only by the loss. If, in addition

$$\frac{1}{\tau_e} = \frac{1}{\tau_e'} + \frac{1}{\tau_o} \quad (14)$$

then, at resonance, (11) and (12) give $T = 0$ and $|D_L|^2 = (1 - \tau_e/\tau_o)$, so the input signal power at ω_o is completely removed from the bus and is dropped into port 3 of the receiver, reduced by a fraction τ_e/τ_o due to loss. Thus, the system operates as a channel dropping filter.

If the resonator mode consists of a pure standing wave, such as the mode of a quarter-wave-shifted DFB resonator

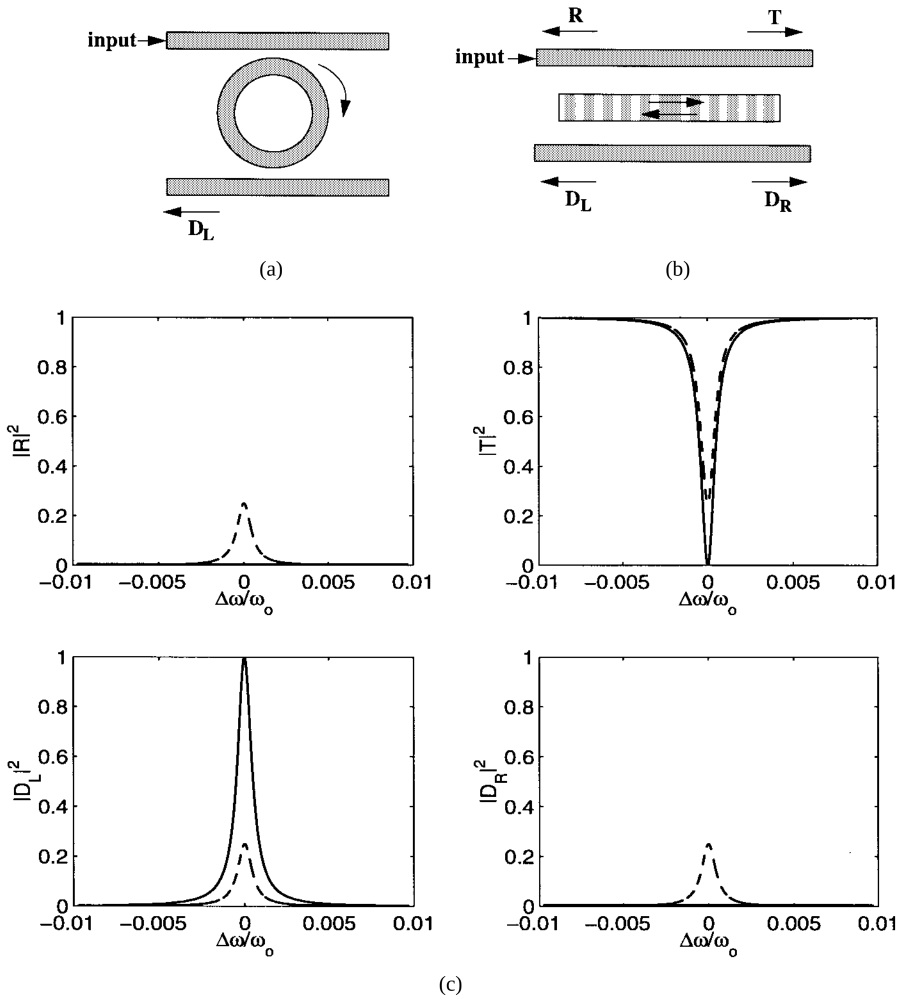


Fig. 2. Four-port systems using (a) a traveling wave, (b) a standing wave resonator, and (c) an example of the corresponding filter response with $Q_e = Q_e' = 2000$ and negligible loss. Solid line: traveling wave; dashed line: standing wave.

[Fig. 2(b)], there is no net power flowing in either direction in the resonator. Thus, the resonant mode decays equally into the forward and the backward propagating waveguide mode, so from (7) and (8), we have $1/\tau_{e1} = 1/\tau_{e2} = 1/\tau_e$ and $1/\tau_{e3} = 1/\tau_{e4} = 1/\tau_e'$. If (14) is satisfied, the power transfer into the receiver at resonance is maximized, and from (10)–(13) we find $|R|^2 = |T|^2 = 0.25$ and $|D_L|^2 = |D_R|^2 = 0.25(1 - \tau_e/\tau_o)$. That is, at best, half the input power at frequency ω_o remains in the bus and is equally distributed into ports 1 and 2 while the other half, reduced by a fraction τ_e/τ_o due to loss, is equally distributed into ports 3 and 4 of the receiver.

An example of the filter response for the two cases described above is shown in Fig. 2(c) as a function of normalized frequency, with $Q_e = Q_e' = 2000$ and negligible loss.

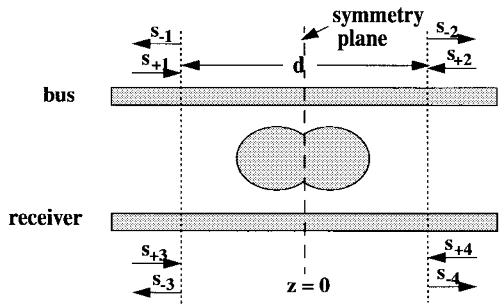
Clearly, a single-mode traveling wave resonator side-coupled to the bus and the receiver can fully transfer a channel at the resonance frequency from the bus to the receiver while a single-mode standing wave resonator is not adequate for channel dropping. However, as we show next, it is possible to get the response of a single-mode traveling-wave resonant filter using two standing-wave modes.

III. Symmetric Standing-Wave Channel Add–Drop Filter

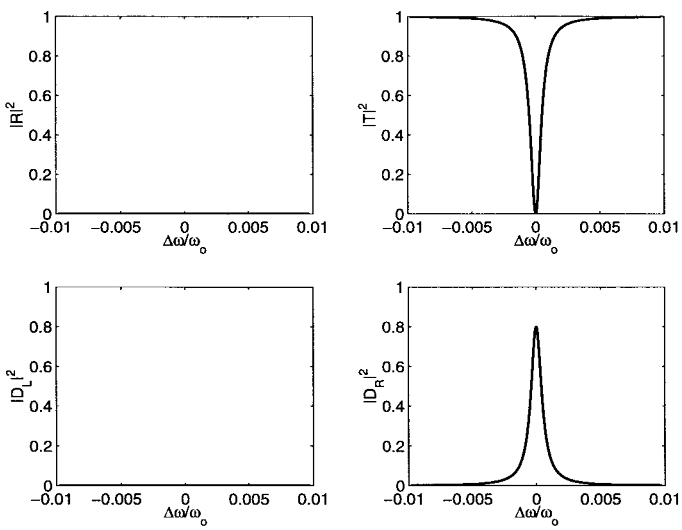
We now consider a resonant structure with two standing-wave modes placed between the bus and the receiver, with a symmetry plane perpendicular to the waveguides, at $z = 0$ [Fig. 3(a)]. The two modes of the system are symmetric and antisymmetric with respect to this plane. With the reference planes defined at $z = \pm d/2$, the phases of the coupling coefficients differ by even (odd) multiples of π in the case of the symmetric (antisymmetric) modes. Thus, the forward and backward incident waves couple into the symmetric mode in phase and into the antisymmetric mode out of phase. The symmetric mode has amplitude a_s and the antisymmetric mode has amplitude a_a. Using the analysis of the previous section simplified for the case of a symmetric structure, we have

$$\frac{da_s}{dt} = \left(j\omega_s - \frac{1}{\tau_{e_s}} - \frac{1}{\tau_{e_s}'} - \frac{1}{\tau_{o_s}}\right) a_s + \kappa_s(s_{+1} + s_{+2}) + \kappa_s'(s_{+3} + s_{+4}) \quad (15)$$

$$\frac{da_a}{dt} = \left(j\omega_a - \frac{1}{\tau_{e_a}} - \frac{1}{\tau_{e_a}'} - \frac{1}{\tau_{o_a}}\right) a_s + \kappa_a(s_{+1} - s_{+2}) + \kappa_a'(s_{+3} - s_{+4}) \quad (16)$$



(a)



(b)

Fig. 3. (a) Channel dropping filter using a resonant structure with a symmetry plane perpendicular to the waveguides. (b) Filter response when a pair of degenerate symmetric and antisymmetric standing-wave modes is excited with $Q_o = 5000$, $Q_e = 2000$, and $1/Q_e' = 1/Q_e - 1/Q_o$ for maximum power transfer.

where $\omega_{s,a}$ are the resonant frequencies $1/\tau_{o_{e,a}}$ are the decay rates due to loss, $1/\tau_{e_{s,a}}$ and $1/\tau'_{e_{s,a}}$ are the rates of decay into the signal bus and the receiver, respectively, and $\kappa_{s,a}$ and $\kappa'_{s,a}$ are the input coupling coefficients associated with the bus and the receiver, respectively. The amplitudes of the outgoing waves are found by generalizing (2)–(5) to the case of two excited modes

$$s_{-1} = e^{-j\beta d}(s_{+2} - \kappa_s^* a_s + \kappa_a^* a_a) \tag{17}$$

$$s_{-2} = e^{-j\beta d}(s_{+1} - \kappa_s^* a_s - \kappa_a^* a_a) \tag{18}$$

$$s_{-3} = e^{-j\beta' d}(s_{+4} - \kappa_s'^* a_s + \kappa_a'^* a_a) \tag{19}$$

$$s_{-4} = e^{-j\beta' d}(s_{+3} - \kappa_s'^* a_s - \kappa_a'^* a_a). \tag{20}$$

In analogy with (6), the input coupling coefficients can be written as

$$\kappa_{s,a} = \sqrt{\frac{1}{\tau_{e_{s,a}}}}\, e^{j\theta_{s,a}}$$
$$\kappa'_{s,a} = \sqrt{\frac{1}{\tau'_{e_{s,a}}}}\, e^{j\theta'_{s,a}}. \tag{21}$$

With s_{+1} as the input signal at frequency ω, we find the filter response at the four ports of the system, as defined in the previous section:

$$R = e^{-j\beta d}\left(-\frac{\frac{1}{\tau_{e_s}}}{j(\omega-\omega_s)+\frac{1}{\tau_{e_s}}+\frac{1}{\tau'_{e_s}}+\frac{1}{\tau_{o_s}}} + \frac{\frac{1}{\tau_{e_a}}}{j(\omega-\omega_a)+\frac{1}{\tau_{e_a}}+\frac{1}{\tau'_{e_a}}+\frac{1}{\tau_{o_a}}}\right) \tag{22}$$

$$T = e^{-j\beta d}\left(1-\frac{\frac{1}{\tau_{e_s}}}{j(\omega-\omega_s)+\frac{1}{\tau_{e_s}}+\frac{1}{\tau'_{e_s}}+\frac{1}{\tau_{o_s}}} - \frac{\frac{1}{\tau_{e_a}}}{j(\omega-\omega_a)+\frac{1}{\tau_{e_a}}+\frac{1}{\tau'_{e_a}}+\frac{1}{\tau_{o_a}}}\right) \tag{23}$$

$$D_L = e^{-j\beta' d}\left(-\frac{\sqrt{\frac{1}{\tau_{e_s}\tau'_{e_s}}}\, e^{j(\theta_s-\theta'_s)}}{j(\omega-\omega_s)+\frac{1}{\tau_{e_s}}+\frac{1}{\tau'_{e_s}}+\frac{1}{\tau_{o_s}}} + \frac{\sqrt{\frac{1}{\tau_{e_a}\tau'_{e_a}}}\, e^{j(\theta_a-\theta'_a)}}{j(\omega-\omega_a)+\frac{1}{\tau_{e_a}}+\frac{1}{\tau'_{e_a}}+\frac{1}{\tau_{o_a}}}\right) \tag{24}$$

$$D_R = e^{-j\beta' d}\left(-\frac{\sqrt{\frac{1}{\tau_{e_s}\tau'_{e_s}}}\, e^{j(\theta_s-\theta'_s)}}{j(\omega-\omega_s)+\frac{1}{\tau_{e_s}}+\frac{1}{\tau'_{e_s}}+\frac{1}{\tau_{o_s}}} - \frac{\sqrt{\frac{1}{\tau_{e_a}\tau'_{e_a}}}\, e^{j(\theta_a-\theta'_a)}}{j(\omega-\omega_a)+\frac{1}{\tau_{e_a}}+\frac{1}{\tau'_{e_a}}+\frac{1}{\tau_{o_a}}}\right). \tag{25}$$

The two resonant modes are degenerate if they have equal frequencies and equal decay rates

$$\omega_s = \omega_a = \omega_o \tag{26}$$

$$\tau_{o_s} = \tau_{o_a} = \tau_o \tag{27}$$

$$\tau_{e_s} = \tau_{e_a} = \tau_e \tag{28}$$

$$\tau'_{e_s} = \tau'_{e_a} = \tau'_e. \tag{29}$$

Under these conditions, (22) gives $R = 0$ over the entire bandwidth of the resonator and (23)–(25) become

$$T = e^{-j\beta d}\left(1-\frac{\frac{2}{\tau_e}}{j(\omega-\omega_o)+\frac{1}{\tau_e}+\frac{1}{\tau'_e}+\frac{1}{\tau_o}}\right) \tag{30}$$

$$D_L = -e^{j(((\theta_s+\theta_a)-(\theta_s'+\theta_a'))/2-\beta' d)} \cdot \frac{\frac{2}{\sqrt{\tau_e \tau_e'}}}{j(\omega-\omega_o)+\frac{1}{\tau_e}+\frac{1}{\tau_e'}+\frac{1}{\tau_o}} \sin\left(\frac{\Delta\theta}{2}\right) \tag{31}$$

$$D_R = -j\, e^{j(((\theta_s+\theta_a)-(\theta_s'+\theta_a'))/2-\beta' d)} \cdot \frac{\frac{2}{\sqrt{\tau_e \tau_e'}}}{j(\omega-\omega_o)+\frac{1}{\tau_e}+\frac{1}{\tau_e'}+\frac{1}{\tau_o}} \cos\left(\frac{\Delta\theta}{2}\right) \tag{32}$$

where

$$\Delta\theta = (\theta_s - \theta_a) - (\theta_s' - \theta_a'). \tag{33}$$

At the resonance frequency ω_o, from (30) the transmission through the bus is

$$T = e^{-j\beta l} \frac{-\frac{1}{\tau_e}+\frac{1}{\tau_e'}+\frac{1}{\tau_o}}{\frac{1}{\tau_e}+\frac{1}{\tau_e'}+\frac{1}{\tau_o}}. \tag{34}$$

Thus, as in the case of a traveling wave resonator discussed in Section II, if the decay rates satisfy (14), the input signal power is completely removed from the bus and transferred to the receiver reduced by a fraction τ_e/τ_o due to loss. Under this condition, the bandwidth of the Lorentzian response is determined entirely by the coupling to the bus waveguide, and its peak is set by the ratio τ_e/τ_o. As we can see in (31) and (32), the distribution of the dropped signal power into the left and the right port of the receiver is determined by the phase difference $\Delta\theta$.

1) If $\Delta\theta = 2n\pi$, where n is an integer, then $D_L = 0$ for all frequencies so the channel is dropped in the forward direction. This means that if the resonator has a horizontal symmetry plane as well, i.e., parallel to the waveguides, the symmetric and antisymmetric modes have the same symmetry (even or odd) with respect to this plane. An example for this case is a composite system made up of two identical standing wave resonators, as we will see in the next section.
2) If $\Delta\theta = (2n+1)\pi$, then $D_R = 0$ for all frequencies so the propagation in the receiver waveguide is only in the backward direction. This means that if the resonator has a horizontal symmetry plane as well, the symmetric mode has even (odd) symmetry and the antisymmetric mode has odd (even) symmetry with respect to this plane. An example for this case is a ring resonator, if we consider its traveling-wave modes as superpositions of degenerate symmetric and antisymmetric standing-wave modes that are excited with a $\pi/2$-phase difference.
3) In any other case, both D_R and D_L are nonzero.

In Fig. 3(b), we show an example of the filter response of case 1), with $Q_o = 5000$, $Q_e = 2000$, and $Q_e' = 1/(1/Q_e - 1/Q_o)$ for maximum power transfer.

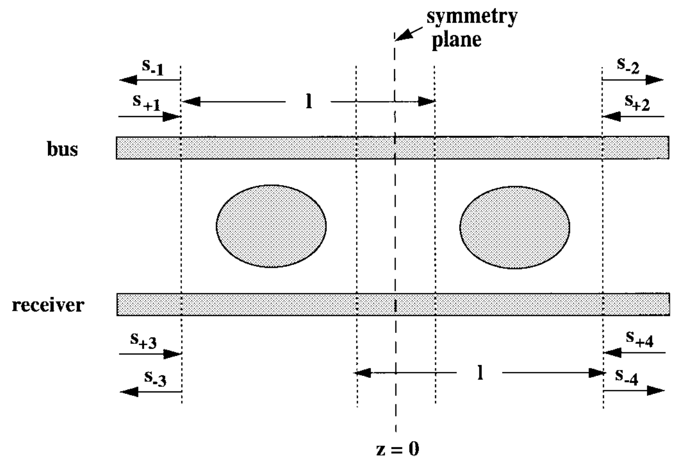


Fig. 4. Symmetric channel dropping filter based on two coupled identical single-mode resonators.

Thus, in agreement with [6]–[8], we found that, in order for a resonant system to operate as a channel dropping filter employing standing-wave modes, the excitation of two degenerate modes is necessary. The superposition of these modes with the appropriate phase relation gives the behavior of a traveling-wave mode.

IV. Symmetric System Using Two Identical Single-Mode Resonators

The symmetric resonant system analyzed in the previous section can be implemented using two identical coupled resonators, each supporting only one standing-wave mode in the frequency range of interest. The resonator pair is placed between the bus and the receiver waveguides, so that the total system has a symmetry plane at $z = 0$. A schematic is shown in Fig. 4. Normally, the mutual coupling of the two resonators would split the resonant frequencies, lifting the degeneracy. In this section, we show that the coupling to the waveguides can be designed to cancel the effect of frequency splitting due to the mutual coupling and reestablish the degeneracy.

The mode amplitudes of the resonator on the left and on the right of the symmetry plane are denoted by a_L and a_R, respectively. The resonant frequency, decay rates, and coupling constants for the left resonator are defined as in Section II and for the right resonator are found by mirror symmetry.

The resonator on the left is excited from the left by s_{+1} and s_{+3} and from the right by the outputs of the right resonator. The resonator on the right is excited from the right by s_{+2} and s_{+4} and from the left by the outputs of the left resonator. The distance from the left (right) reference plane of the resonator on the left of the symmetry plane to the left (right) reference plane of the resonator on the right of the symmetry plane is denoted by l, as shown in Fig. 4, and for simplicity is the same in both waveguides. The equations for the mode amplitudes of the two resonators are

$$\frac{da_L}{dt} = \left(j\omega_o - \frac{1}{\tau_e} - \frac{1}{\tau_e'} - \frac{1}{\tau_o}\right)a_L - j\mu a_R + \sqrt{\frac{1}{\tau_e}}\, e^{j\theta_1}\, s_{+1} + \sqrt{\frac{1}{\tau_e}}\, e^{j\theta_2}\, e^{-j\beta l} \cdot \left(s_{+2} - \sqrt{\frac{1}{\tau_e}}\, e^{-j\theta_1} a_R\right) + \sqrt{\frac{1}{\tau_e'}}\, e^{j\theta_3} s_{+3} + \sqrt{\frac{1}{\tau_e'}}\, e^{j\theta_4}\, e^{-j\beta' l}\left(s_{+4} - \sqrt{\frac{1}{\tau_e'}}\, e^{-j\theta_3} a_L\right) \quad (35)$$

$$\frac{da_R}{dt} = \left(j\omega_o - \frac{1}{\tau_e} - \frac{1}{\tau_e'} - \frac{1}{\tau_o}\right)a_R - j\mu a_L + \sqrt{\frac{1}{\tau_e}}\, e^{j\theta_1} s_{+2} + \sqrt{\frac{1}{\tau_e}}\, e^{j\theta_2}\, e^{-j\beta l} \cdot \left(s_{+1} - \sqrt{\frac{1}{\tau_e}}\, e^{-j\theta_1} a_R\right) + \sqrt{\frac{1}{\tau_e'}}\, e^{j\theta_3} s_{+4} + \sqrt{\frac{1}{\tau_e'}}\, e^{j\theta_4}\, e^{-j\beta' l}\left(s_{+3} - \sqrt{\frac{1}{\tau_e'}}\, e^{-j\theta_3} a_R\right) \quad (36)$$

where μ is the mutual coupling coefficient between the resonators and is real by power conservation. For the decay rates, we have used the fact that a standing-wave mode decays equally into both directions in the waveguide as discussed in Section II. Expressions analogous to (2)–(5) have been used for the outgoing waves of the left (right) resonator that appear as inputs to the right (left) resonator. In (35) and (36), we can see that, in addition to the direct coupling expressed by μ, the two resonators are also indirectly coupled through the waveguides. We define the amplitude of the symmetric and antisymmetric modes of the total system as

$$a_s = \frac{a_L + a_R}{\sqrt{2}} \quad (37)$$

$$a_a = \frac{a_L - a_R}{\sqrt{2}} \quad (38)$$

which, due to (35) and (36), satisfy

$$\frac{d}{dt}\begin{Bmatrix} a_s \\ a_a \end{Bmatrix} = j\left[\omega_o \mp \left(\mu - \frac{1}{\tau_e}\sin\phi - \frac{1}{\tau_e'}\sin\phi'\right)\right]\begin{Bmatrix} a_s \\ a_a \end{Bmatrix} - \left[\frac{1}{\tau_o} + \frac{1}{\tau_e}(1 \pm \cos\phi) + \frac{1}{\tau_e'}(1 \pm \cos\phi')\right]\begin{Bmatrix} a_s \\ a_a \end{Bmatrix} + \sqrt{\frac{2}{\tau_e}}\, e^{j((\theta_1+\theta_2-\beta l)/2)}\begin{Bmatrix} \cos\frac{\phi}{2} \\ j\sin\frac{\phi}{2} \end{Bmatrix}(s_{+1} \pm s_{+2}) + \sqrt{\frac{2}{\tau_e'}}\, e^{j((\theta_3+\theta_4-\beta' l)/2)}\begin{Bmatrix} \cos\frac{\phi'}{2} \\ j\sin\frac{\phi'}{2} \end{Bmatrix}(s_{+3} \pm s_{+4}) \quad (39)$$

where

$$\phi = \beta l + \theta_1 - \theta_2 \quad (40)$$

$$\phi' = \beta' l + \theta_3 - \theta_4. \quad (41)$$

Comparing (39) with (15) and (16), we have

$$\begin{Bmatrix} \omega_s \\ \omega_a \end{Bmatrix} = \omega_o \mp \left(\mu - \frac{1}{\tau_e}\sin\phi - \frac{1}{\tau_e'}\sin\phi'\right) \quad (42)$$

$$\frac{1}{\tau_{o_s}} = \frac{1}{\tau_{o_a}} = \frac{1}{\tau_o} \quad (43)$$

$$\begin{Bmatrix} \kappa_s \\ \kappa_a \end{Bmatrix} = \sqrt{\frac{2}{\tau_e}}\, e^{j((\theta_1+\theta_2-\beta l)/2)}\begin{Bmatrix} \cos\frac{\phi}{2} \\ j\sin\frac{\phi}{2} \end{Bmatrix} \quad (44)$$

$$\begin{Bmatrix} \kappa_s' \\ \kappa_a' \end{Bmatrix} = \sqrt{\frac{2}{\tau_e'}}\, e^{j((\theta_3+\theta_4-\beta' l)/2)}\begin{Bmatrix} \cos\frac{\phi'}{2} \\ j\sin\frac{\phi'}{2} \end{Bmatrix} \quad (45)$$

$$\frac{1}{\tau_{e_{s,a}}} = |\kappa_{s,a}|^2 \quad (46)$$

$$\frac{1}{\tau_{e_{s,a}}'} = |\kappa_{s,a}'|^2. \quad (47)$$

From (44) and (45), we can see that the symmetric and the antisymmetric excitations couple into the system with a $\pi/2$ phase difference. In the special case that ϕ and ϕ' are even (odd) multiples of π, only the symmetric (antisymmetric) mode is excited, leading to the behavior of the standing wave resonant system described in Section II, with decay rates $2/\tau_e$ and $2/\tau_e'$ into the bus and the receiver, respectively, and $1/\tau_o$ due to loss.

The conclusions derived for the filter response of the symmetric system shown in Section III apply to this system as well: the system can operate as a channel add–drop filter if its symmetric and antisymmetric modes satisfy the degeneracy conditions (26)–(29). The decay rates due to loss are already equal, as seen in (43). From (42), the condition for frequency degeneracy is satisfied if

$$\mu - \frac{1}{\tau_e}\sin\phi - \frac{1}{\tau_e'}\sin\phi' = 0. \quad (48)$$

From (44) to (47), the conditions for equal decay rates are satisfied if

$$\cos\phi = 0 \quad (49)$$

$$\cos\phi' = 0. \quad (50)$$

Therefore, there are two degrees of freedom in designing this system: knowing the propagation constants β and β' and the phase differences $\theta_1 - \theta_2$ and $\theta_3 - \theta_4$, we must choose the distance l so that the symmetric and antisymmetric modes have decay rates equal to those of the individual resonators $1/\tau_e$ and $1/\tau_e'$. Then, by varying the coupling between the waveguides and the resonators, we must make $1/\tau_e$ and $1/\tau_e'$ such that the splitting of the resonance frequencies due to direct coupling between the two resonators is cancelled.

The signal power at resonance is completely removed from the bus if the degenerate decay rates satisfy the maximum power transfer condition (14). In this special case, the bandwidth of the Lorentzian response is then set by τ_e and the peak power at the output ports of the receiver by the ratio τ_e/τ_o.

As discussed in Section III, the direction of the channel dropping is determined by the phase difference $\Delta\theta = (\theta_s - \theta_a) - (\theta'_s - \theta'_a)$. Here, $\theta_{s,a}$ and $\theta'_{s,a}$ are the phases of the coupling constants defined in (44) and (45), respectively. From (44) and (45), we can see that

$$\theta_s - \theta_a = \begin{cases} -\frac{\pi}{2}, & \text{if } \sin\phi > 0 \\ \frac{\pi}{2}, & \text{if } \sin\phi < 0 \end{cases} \tag{51}$$

$$\theta'_s - \theta'_a = \begin{cases} -\frac{\pi}{2}, & \text{if } \sin\phi' > 0 \\ \frac{\pi}{2}, & \text{if } \sin\phi' < 0. \end{cases} \tag{52}$$

Therefore, $\Delta\theta = 0$, if the degeneracy conditions are satisfied with $\phi - \phi' = 2n\pi$, where n is an integer, and the channel is dropped in the forward direction, i.e., in port 4. If $\phi - \phi' = (2n+1)\pi$, then $\Delta\theta = \pm\pi$, and the channel is dropped backward, i.e., in port 3. In this particular case, it is possible to satisfy (48) even if the resonators are not directly coupled ($\mu = 0$) provided that $1/\tau_e = 1/\tau'_e$.

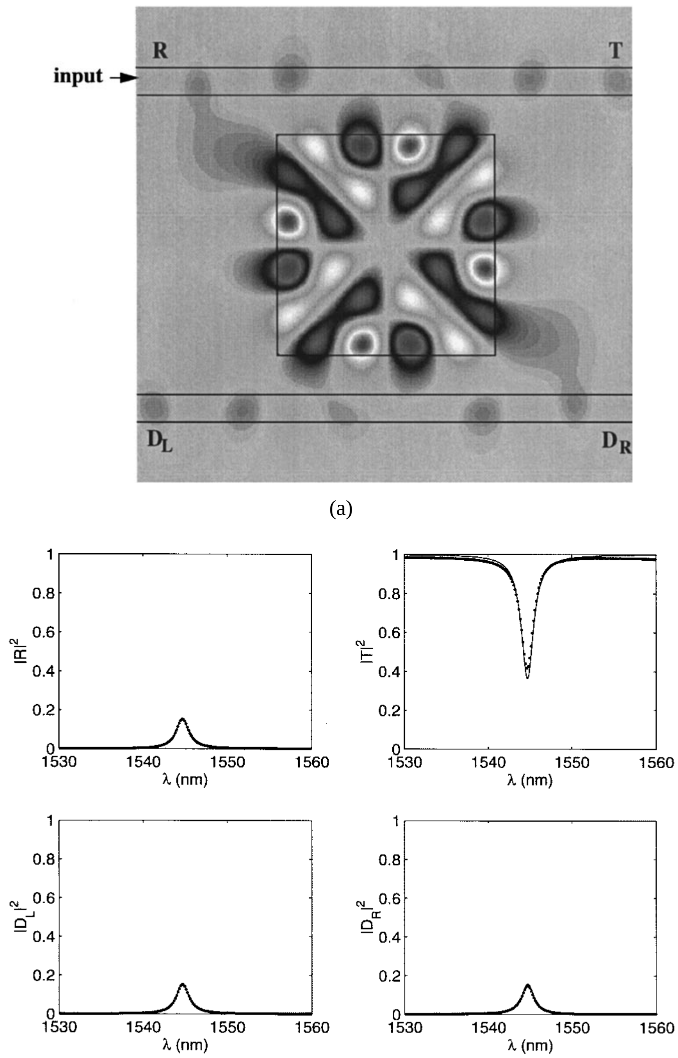
The design of the filter is simplified when the two resonators are individually symmetric. With the reference planes defined symmetrically on either side of each resonator as in Section III, the distance l is equal to the distance between the individual symmetry planes. In addition, we have $\theta_1 - \theta_2 = 0\ (\pm\pi)$ and $\theta_3 - \theta_4 = 0\ (\pm\pi)$ if the mode supported by each resonator is symmetric (antisymmetric). So, the conditions for degeneracy become

$$\mu \mp \frac{1}{\tau_e}\sin(\beta l) \mp \frac{1}{\tau'_e}\sin(\beta' l) = 0 \tag{53}$$

$$\cos(\beta l) = \cos(\beta' l) = 0. \tag{54}$$

We can see that, in this case, the choice of l depends on the symmetry of the individual modes that make up the symmetric and antisymmetric modes of the system. For example, in the case that $\tau_e = \tau'_e$ and $\beta = \beta'$, we can see from (53) that the resonance frequencies are degenerate only if $l = (n + 1/4)\lambda_g$ for symmetric individual modes and only if $l = (n + 3/4)\lambda_g$ for antisymmetric individual modes, where $\lambda_g = 2\pi/\beta$ is the guided wavelength.

In [6] and [7], the concept discussed in this section was demonstrated numerically by FDTD simulations in a two-dimensional (2-D) photonic crystal made of dielectric rods in air. In that case, the two waveguides and two single-mode microcavities were made from defects and the coupling constants were controlled by varying the refractive index of specific rods. In this paper, we perform two-dimensional FDTD simulations using conventional high index-contrast waveguides and square resonators. In these simulations, the structures are viewed as infinite in the third dimension, and the electric field polarization is perpendicular to the paper. Starting from the actual three-dimensional structure, the dependence on the third dimension could be taken into account by the effective index method. However, this method is of limited accuracy for high index-contrast structures or near cut-off, so it is not used here. The computational domain is discretized into a uniform orthogonal mesh with a cell size of 20 nm.

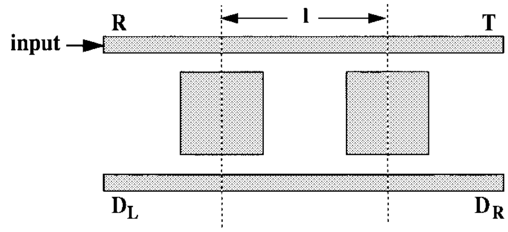


(a)

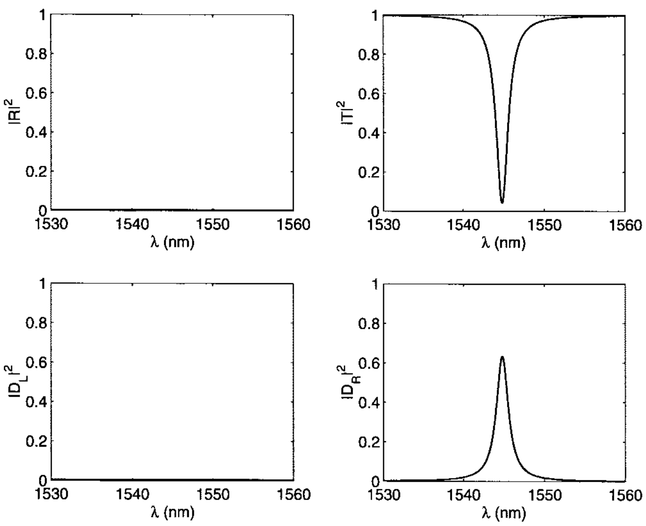
(b)

Fig. 5. (a) Electric field amplitude distribution in a square resonator side coupled to the bus and receiver. (b) Filter response calculated by CMT (solid line) and by FDTD simulation (dotted line).

The system is excited using a source located at the bus waveguide with the spatial profile of the fundamental waveguide mode and a wide Gaussian spectrum centered at $\lambda = 1550$ nm. The frequency response of the system is obtained by calculating the discrete Fourier transforms of the fields and integrating the power flux over the waveguide cross section at the four ports of the system. A square resonator supports standing-wave modes which have high Q if the nulls of the electric field are along the diagonals. Fig. 5(a) shows the amplitude of the electric field (polarized perpendicular to the paper) in a square resonator of side 1.54 μm and refractive index 3.2 placed symmetrically between two waveguides of width 0.2 μm and index 3.2 that are 2.32 μm apart center to center. The width of the waveguides was chosen to ensure their single-mode operation in the 2-D FDTD model over the entire bandwidth of the excitation. The theoretical response obtained using (10)–(13) with $Q_e = Q'_e = 2185$ and $Q_o = 4250$ and a resonance wavelength $\lambda_0 = 1545$ nm fits very well the numerical results [Fig. 5(b)]. As expected by the theory



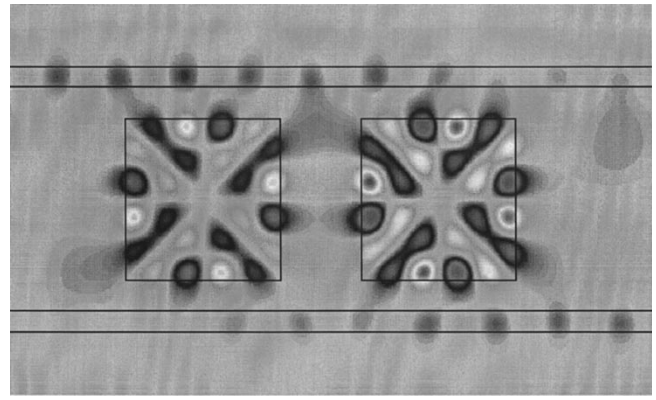
(a)



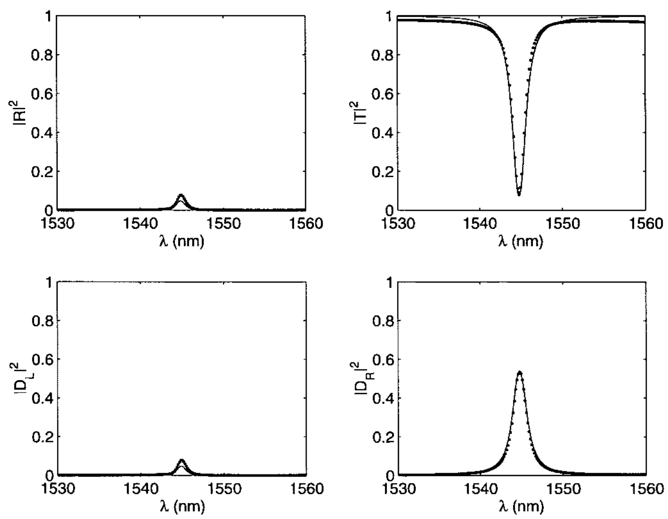
(b)

Fig. 6. (a) Implementation of a symmetric channel dropping filter using two square resonators. (b) Filter response when all degeneracy conditions are satisfied, as calculated by CMT.

in Section II, the output at all four ports is below 25%. The electric field pattern of the resonator mode is antisymmetric with respect to the vertical symmetry plane; therefore, in order to achieve degenerate decay rates, the distance between the two square centers must be $l = (n + 3/4)\lambda_g$. In this case, we choose $n = 3$. Note that, in this example, we do not have as many degrees of freedom as in [6] and [7] for manipulating the coupling between the two resonators and the waveguides. Having fixed the distance l, we can only vary the separation between the waveguides and the resonators, which determines the external Q's, by the step size of our FDTD mesh (20 nm). With the separation of the two waveguides as above, we find $Q_e = Q_e' = 2185$, and, assuming that the degeneracy conditions are satisfied, the filter response obtained by CMT is shown in Fig. 6. However, the numerical results suggest that there is a remaining splitting of the frequencies that deteriorates the performance of the channel dropping filter. In the field pattern in Fig 7(a), it can be seen that, although most of the power is dropped forward at the resonant frequency, there is still some power in the remaining three outputs. To account for the small splitting of the frequencies, we choose μ in (42) so that $(\omega_a - \omega_s)/\omega_o = 0.35 \cdot 10^{-3}$, estimated by the lowering of the received peak power. Then, the theoretical response matches well the numerical results [Fig. 7(b)]. Any remaining discrepancies are due to the fact that the radiation losses for a symmetric and an antisymmetric mode are not necessarily the same, as we have assumed here



(a)



(b)

Fig. 7. (a) Electric field amplitude distribution in a symmetric channel dropping filter using two square resonators identical to the one of Fig. 5. (b) Filter response calculated by CMT (solid line) and by FDTD simulation (dotted line).

(the antisymmetric mode in our case is expected to have a higher radiation Q), while a small deviation of l from $15\lambda_g/4$ is also possible due to the finite spatial resolution of the FDTD mesh.

V. Discussion

Using coupling of modes in time as an alternative to the approach presented in [6]–[8], we have shown that a resonant structure with two degenerate modes that are symmetric and antisymmetric with respect to its symmetry plane functions as a channel add–drop filter. With this simple approach, we have also shown that the splitting of the degeneracy produced by the mutual coupling of two resonators can be removed by adjusting the coupling of the resonators to the adjacent waveguides and by properly choosing the phase shift introduced by the waveguide section between the coupling regions. The principle of operation of such filters was demonstrated using the FDTD method. The numerical simulations verify our theoretical predictions but also reveal the sensitivity of the filter performance to fabrication errors as small as our mesh discretization, here 20 nm, especially in the case of high index-contrast structures. In the case of evanescent coupling, the coupling coefficients have an exponential dependence on the separation between waveguides and resonators. The phase shift needed to ensure degeneracy of both the resonant frequencies and the decay rates is strongly dependent on the length and propagation constants of the waveguide sections between the two resonators. For weak mutual coupling of the two resonators, as was the case in our example, the phase shift is the parameter that most strongly affects the filter response. In addition, since, in general, the radiation Q's of the symmetric and antisymmetric modes are not equal, it is very important that the radiation Q's of the resonators be much higher than the coupling Q's in order to keep the mismatch of the total Q's of the two modes to a minimum.

APPENDIX

In order to calculate the coupling coefficients involved in the four-port system of Fig. 1, we examine the coupling to one waveguide at a time starting with the bus waveguide and following a treatment similar to [9]. This approach is valid if we assume weak coupling and small index discontinuity. Under the same assumption, the spatial variation of the waveguide mode amplitudes can be described by coupled mode equations of the form

$$\frac{d}{dz}\, b_{\pm}(z) = \mp j\beta b_{\pm}(z) + \kappa_{\pm}(z)a \tag{55}$$

where $b_{\pm}(z)$ is the amplitude of the forward/backward waveguide mode in the bus, β is the propagation constant, and $\kappa_{\pm}(z)$ describes the distributed coupling to the resonator. For reference planes located at $z = z_1$ and $z = z_2$, we have

$$b_{\pm}(z_1) = s_{\pm 1} \tag{56}$$

$$b_{\pm}(z_2) = s_{\mp 2}. \tag{57}$$

The rate of change of the waveguide mode power along z is equal to the power coupled per unit length to the polarization current due to the index perturbation that the resonator mode experiences in the presence of the waveguide. The assumed electric field distribution in the waveguide is $b_+(z)\vec{e}_+(x, y) + b_-(z)\vec{e}_-(x, y)$ where $\vec{e}_{\pm}(x, y)$ is the unperturbed forward/backward mode profile normalized to *unit power*. In the resonator, the assumed electric field distribution is $a(t)\vec{e}_r(x, y, z)$ where $\vec{e}_r(x, y, z)$ is the uncoupled resonator mode field normalized to *unit energy*. Using Poynting's theorem at steady state, we have

$$\pm\frac{d}{dz}\, |b_{\pm}|^2 = -j\,\frac{\omega\epsilon_o}{4} \iint dx\, dy(n^2 - n_r^2)\vec{e}_r \cdot \vec{e}_{\pm}^{\,*} ab^* + \text{c.c.} \tag{58}$$

where $n(x, y, z)$ and $n_r(x, y, z)$ are the index distributions of the background and the resonator, respectively. In this equation, we have neglected a z-dependent self-coupling term

$$-j\,\frac{\omega\epsilon_o}{4} \iint dx\, dy(n^2 - n_r^2)\vec{e}_{\pm} \cdot \vec{e}_{\pm}^{\,*} ab^*$$

that modifies the propagation constant. From (55), we have

$$\frac{d}{dz}\, |b_{\pm}|^2 = \kappa_{\pm} ab^* + \text{c.c.} \tag{59}$$

Comparing (58) and (59), we have

$$\kappa_{\pm}(z) = \mp j\,\frac{\omega\epsilon_o}{4} \iint dx\, dy(n^2 - n_r^2)\vec{e}_r \cdot \vec{e}_{\pm}^{\,*}. \tag{60}$$

We integrate (55) using the boundary conditions (56) and (57) along with (60) to find the amplitudes of the outgoing waves as

$$s_{-1} = e^{-j\beta(z_2 - z_1)}\Big(s_{+2} - j\,\frac{\omega\epsilon_o}{4}\int_{z_1}^{z_2} dz \iint dx\, dy \cdot (n^2 - n_r^2)\vec{e}_r \cdot \vec{e}_{+}^{\,*}\, e^{j\beta(z - z_1)} a\Big) \tag{61}$$

$$s_{-2} = e^{-j\beta(z_2 - z_1)}\Big(s_{+1} - j\,\frac{\omega\epsilon_o}{4}\int_{z_1}^{z_2} dz \iint dx dy \cdot (n^2 - n_r^2)\vec{e}_r \cdot \vec{e}_{-}^{\,*}\, e^{-j\beta(z - z_2)} a\Big). \tag{62}$$

The input coupling coefficients $\kappa_{1,2}$ can be found by power conservation. Neglecting the loss, the rate of change of the energy in the resonator mode must be equal to the difference between the incoming and outgoing power

$$\frac{d|a|^2}{dt} = |s_{+1}|^2 + |s_{+2}|^2 - |s_{-1}|^2 - |s_{-2}|^2. \tag{63}$$

Also, from (1) with $1/\tau_e' = 0$ and $\kappa_3 = \kappa_4 = 0$, we have

$$\frac{d}{dt}\, |a|^2 = -\frac{2}{\tau_e}\, |a|^2 + (\kappa_1 s_{+1} a^* + \text{c.c.}) + (\kappa_2 s_{+2} a^* + \text{c.c.}). \tag{64}$$

Substituting (61) and (62) into (63) and comparing with (64), we have

$$\kappa_1 = -j\,\frac{\omega\epsilon_o}{4}\int_{z_1}^{z_2} dz \iint dx\, dy(n^2 - n_r^2)\vec{e}_r^{\,*}\vec{e}_+ e^{-j\beta(z - z_1)}$$

$$\kappa_2 = -j\,\frac{\omega\epsilon_o}{4}\int_{z_1}^{z_2} dz \iint dx\, dy(n^2 - n_r^2)\vec{e}_r^{\,*}\vec{e}_- e^{j\beta(z - z_2)}$$

and

$$|\kappa_1|^2 + |\kappa_2|^2 = \frac{2}{\tau_e}.$$

With $z_2 - z_1 = l$, the outgoing waves can be now written as

$$s_{-1} = e^{-j\beta l}(s_{+2} - \kappa_2^* a)$$

$$s_{-2} = e^{-j\beta l}(s_{+1} - \kappa_1^* a).$$

The same analysis yields analogous expressions for the input and output coupling coefficients related to the receiver waveguide.

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