# Wireless energy transfer with the presence of metallic planes

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We numerically demonstrated that efficient wireless energy transfer can be achieved between two high Q resonators in a complex electromagnetic environment. In particular, in the close proximity of metallic planes, efficient wireless energy transfer can be achieved with proper system designs.
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Wireless energy transfer has seen a renewed interest in recent years.[1–12] In particular, it was shown[6–9] that efficient mid-range wireless power transfer can be accomplished in a system schematically shown in Fig. 1. The configuration consists of two high-quality factor (Q-factor) LC resonators, acting as the source and the receiver, respectively. Each resonator consists of an inductor and a capacitor. As a concrete example, in Fig. 1, the inductor consists of a square-shaped planar single-loop coil of wire, which generates a magnetic dipole moment perpendicular to the coil plane. The capacitor is formed with two parallel metallic plates. Power transfer between the resonators occurs in the near-field regime through the magnetic field. The use of magnetic field as the coupling mechanism is important for safety reasons and also minimizes interference effect by off-resonant external dielectric objects.

Most of the previous analysis and experiments[6–9] involve transferring between a source and a receiver both in free space as shown in Fig. 1. However, in certain applications, there is also an interest in transferring energy into a receiver that is placed in a more complex electromagnetic environment.

In this paper, we consider the optimal design of power transfer to a receiver that is placed in a close proximity of a metallic ground plane. We show that high transfer efficiency can be accomplished in a configuration as shown in Fig. 2(a). In this configuration, a metallic plane is put behind the source resonator as well. Also, unlike the configuration shown in Fig. 1, where the electric and magnetic dipole moments are perpendicular to each other, here the resonators are in a "twisted" configuration, where the electric and magnetic dipole moments are parallel to each other, both being perpendicular to the metallic plane.

The mechanism for the wireless energy transfer can be described by the coupled mode theory. For a system consists of a source resonator (denoted with subscript $s$) and a receiver resonator (denoted with subscript $r$), we have the following equations:[7,13]

$$\dot{a}_s(t) = (i\omega_s - \gamma_s)a_s(t) + i\kappa a_r(t), \quad (1)$$

$$\dot{a}_r(t) = (i\omega_r - \gamma_r - \gamma_w)a_r(t) + i\kappa a_s(t), \quad (2)$$

where $|a_{s,r}|^2$ corresponds to the energy stored in the resonator. $\omega_{s,r}$ is the resonant frequency. $\gamma_{s,r}$ is the intrinsic (absorption, radiation, etc.) loss rate. $\gamma_w$ is the work extraction rate due to an output load that couples to the receiving resonator. $\kappa$ is the coupling coefficient.

In practice, when using such a wireless power transfer system, one typically operates in a continuous-wave (CW) mode, where the source resonator couples to a CW input source.[6,8,10] One can assume that the CW source oscillates at the frequency $\omega_s$ such that the coupling between the CW source and the source resonator is maximized. As a result, the fraction of the total power that is dissipated at the load is

$$f = \frac{\gamma_w|a_r^2|}{\gamma_s|a_s^2| + (\gamma_r+\gamma_w)|a_r^2|} = \frac{\frac{\kappa^2}{\gamma_s\gamma_r}\frac{\gamma_w}{\gamma_r}}{\left(1+\frac{\gamma_w}{\gamma_r}\right)\frac{\kappa^2}{\gamma_r\gamma_s} + \left(\frac{\omega_s-\omega_r}{\gamma_r}\right)^2 + \left(1+\frac{\gamma_w}{\gamma_r}\right)^2}. \quad (3)$$

This fraction $f$ depends on the load through $\gamma_w$ and is maximized when $\gamma_w^{max}/\gamma_r = [1+\kappa^2/(\gamma_s\gamma_r) + (\omega_s-\omega_d)^2/\gamma_r^2]^{1/2}$. Below, we define a transfer efficiency

$$\eta = f(\gamma_w^{max}) = \frac{\frac{\kappa^2}{\gamma_s\gamma_r}}{2+\frac{\kappa^2}{\gamma_s\gamma_r} + 2\sqrt{1+\frac{\kappa^2}{\gamma_s\gamma_r} + \left(\frac{\omega_s-\omega_r}{\gamma_r}\right)^2}}. \quad (4)$$

The transfer efficiency, therefore, is only determined by the intrinsic parameters of the coupled resonator system. We see that it is advantageous to have $\omega_r = \omega_s$ and to operate in the strong coupling regime with $\kappa >> \gamma_{s,r}$.

In the simulation that follows, we will seek to determine these intrinsic parameters ($\omega_{s,r}$, $\gamma_{s,r}$, $\kappa$) by simulating the system in the absence of the load (i.e., $\gamma_w = 0$). We initiate the system such that $a_s(t=0) = 1$, $a_r(t=0) = 0$. From Eqs. (1) and (2), we then have

$$a_s(t) = \frac{\omega_s - \omega_2 + i\gamma_s}{\omega_1-\omega_2}e^{i\omega_1 t} + \frac{\omega_1-\omega_s - i\gamma_s}{\omega_1-\omega_2}e^{i\omega_2 t}, \quad (5)$$

$$a_r(t) = \frac{\kappa}{\omega_1-\omega_2}(e^{i\omega_1 t} - e^{i\omega_2 t}), \quad (6)$$

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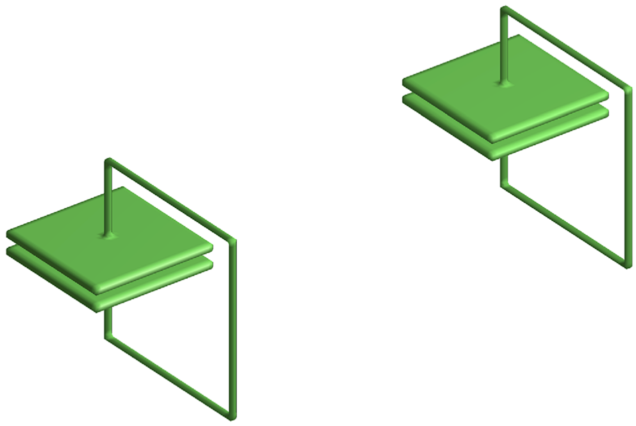

FIG. 1. (Color online) Simplified schematic of the wireless energy transfer system in free space, as demonstrated in Ref. 7.

where

$$\omega_{1,2} = \frac{1}{2}(\omega_s + \omega_r + i\gamma_s + i\gamma_r) \pm \frac{1}{2}[(\omega_s + \omega_r + i\gamma_s + i\gamma_r)^2 + 4(\kappa^2 + \gamma_s\gamma_r - i\gamma_r\omega_s - i\gamma_s\omega_r - \omega_s\omega_r)]^{\frac{1}{2}}. \tag{7}$$

As a result, the energy oscillates back and forth between the resonators. From such oscillations, we determine the intrinsic parameters and, hence, the transfer efficiency using Eq. (4).

We simulate the energy transfer systems shown in Fig. 2 with the finite-different-time-domain (FDTD) method. All materials are assumed to be copper with conductivity[14] $\sigma = 5.8 \times 10^7$ S/m. The central operating frequency is 10 MHz, corresponding to a wavelength of 30 m. For the inductor coil, the size of the coil is 0.8 m × 0.8 m, the cross section of the wire is square (0.04 m × 0.04 m). For the capacitor, the plates have the size of 0.6 m × 0.6 m. The distance between the plates of the capacitor is 0.04 m. The systems operate in a regime such that both the size of the resonators and the distance between them are deep subwavelength. The distance between the coil and its nearest metallic plane is 0.6 m.

In the simulation, we excite a magnetic dipole source at the middle of the source coil. The source has a Gaussian-like profile in time and its central frequency is the resonant frequency of the resonators. We record the magnetic field (with a direction perpendicular to the coil plane) at the monitor points which are placed near the source and receiver coil in a symmetrical way.

We consider the optimized structure in Fig. 2(a) first. In Fig. 3(a), we show the field amplitudes as a function of time in the source and receiving resonators when the distance between the source and the receiver coil is 2 m. We see nearly complete energy exchange between the resonators. The envelope of these amplitudes fits very well to the coupled mode theory. In the coupled mode theory, for a symmetrical structure, $T \equiv \pi/2\kappa$ describes the time it takes for the energy to go from the source to the receiver. In this simulation, we see that $T \approx 7.5\,\mu$s. A coupled mode theory fit to Fig. 3 indicates that the resonator in the presence of the metallic planes has a lifetime of $T_r = 1134\,\mu$s. Since $T \ll T_r$, we are in the strong coupling regime. Using Eq. (4), the transfer efficiency is shown to be 97%, which is contrasted with a transfer efficiency of 95.4% when both metallic plates are absent.

Fig. 4 is the transfer efficiency as a function of coil to coil spacing for the wireless energy transfer system shown in Fig. 2. The efficiency of our optimal configuration (red curve

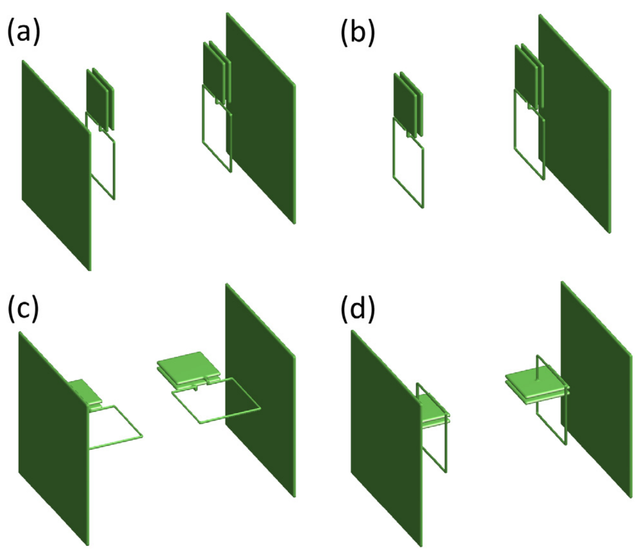


FIG. 2. (Color online) A variety of the resonator structures for the wireless energy transfer. (a) Our current optimal design with a "twisted" geometry. The distance between the coils $\approx\lambda/15$. Two metallic ground planes are placed near the system symmetrically. (b) Same system as (a) without the ground plane at the source coil side. (c) Same system as (a) but with the coil planes perpendicular to the ground planes. (d) Same system as (a) but with the capacitors at a different orientation.

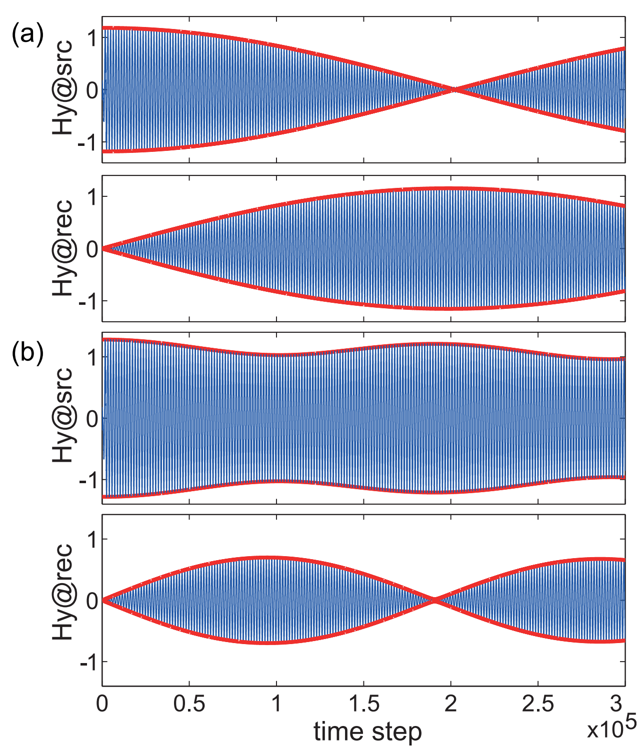


FIG. 3. (Color online) Magnetic field (perpendicular to the metallic plane) as a function of time at monitor points near the source and the receiver resonator for structures (a) and (b) in Fig. 2, respectively. The thick lines are the fitted curves from the coupled mode theory.

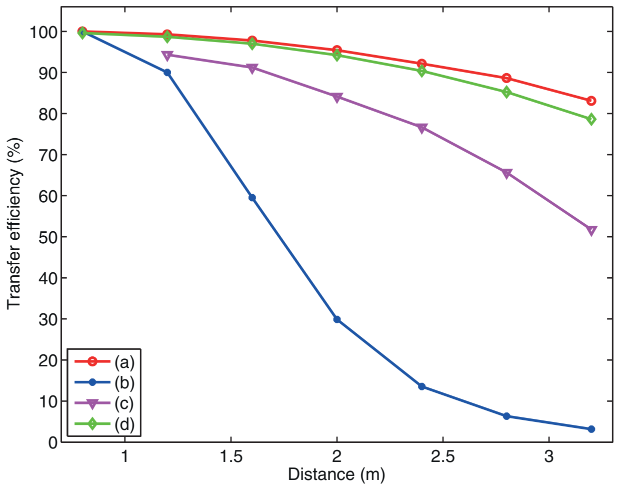


FIG. 4. (Color online) Transfer efficiencies as functions of the coil to coil distance for different structures shown in Fig. 2, assuming a working wavelength $\lambda = 30$ m.

in Fig. 4) decays slowly as a function of distance and remains above 90% for a distance shorter than 2.8 m.

We now illustrate some of the physics that enable efficient energy transfer in Fig. 2(a), by considering various alternative structures as show in Figs. 2(b)–2(d).

First, the most critical component is the use of a metallic plane on the source side. As an illustration, we consider the structure in Fig. 2(b), where there is no metallic plane on the source side. The results from the FDTD simulation are shown in Fig. 3(b). We see that, unlike the case in Fig. 2(a), here there is no complete energy exchange between the source and the receiving resonators. We again fit the FDTD results with coupled mode theory (curve (a) in Fig. 3(b)). The coupled mode theory indicated that there is a substantial frequency mismatch between $\omega_s$ and $\omega_r$. At a distance of 2 m, using Eq. (4), we obtain a transfer efficiency of 82%, which is substantially lower than the optimal configuration of the same transfer distance. For this configuration, the transfer efficiency as a function of coil to coil distance is shown as the curve (b) in Fig. 4. The transfer efficiency decreases rapidly as the distance increases. Comparing the structures in Figs. 2(a) and 2(b), we conclude that maintaining the symmetry between the source and the receiver in general is beneficial for high transfer efficiency since it automatically guarantees the frequency matching between the source and the receiver, in consistency with the coupled mode theory.

Additionally, the presence of metallic planes also reduces radiation loss. In the absence of planes, the lifetime of the resonator is 245 $\mu$s, which is in contrast of the lifetime of 1134 $\mu$s in the presence of the planes as quoted above. Thus, placing the metallic planes should also be beneficial to the non-resonant transfer scheme as discussed in Refs. 4 and 5 due to the reduction of radiation loss.

Second, the orientation of the coils also strongly affects the efficiency. Starting from the optimal configuration in Fig. 4(a), if we rotate the coils by 90° as shown in Fig. 2(c), the coils are no longer aligned for the maximum magnetic field coupling. The transfer efficiency again drops more rapidly with the increasing transfer distance (curve (c) in Fig. 4).

Third, the capacitor orientation also matters but for a lesser degree. Aligning the capacitor plate to be parallel to the ground plane improves the transfer efficiency slightly. The curve (d) in Fig. 4 corresponds to the structure (d) which is the same as in Fig. 2(a), except that the capacitor plates are rotated by 90°. The transfer efficiency is slightly less than the efficiency of our optimal design with a "twisted" geometry (structure (a)). Moreover, the "twisted" geometry in our optimal design is more compact in terms of space.

In conclusion, we study the wireless energy transfer in a complex electromagnetic environment and propose an optimal system design for the case when a metallic ground plane needs to be in a close proximity of the receiver resonator. Transfer efficiency as high as 97% can be achieved when the transfer distance is about $\lambda/15$. For an operating frequency of 10 MHz, this corresponds to a transfer distance of 2 m. We believe that the transfer efficiency can be further increased by fine tuning the system design,[9–12] for example, increasing the size of the metallic plane will result in a slightly higher transfer efficiency.

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