BARGAINING OVER OBJECTS THAT INFLUENCE FUTURE BARGAINING POWER*

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Abstract:

In a number of political and economic contexts, people bargain over the division of objects or resources that themselves contribute to future bargaining power. For example, in international politics states bargain over territory using threats to go to war, and gaining territory today can increase a state's military capabilities tomorrow. I consider a complete information game in which two states bargain over territory in successive periods, with the current division determining the states' military odds (and thus, the 'disagreement point') in the next period. The game has a unique subgame perfect equilibrium. Bargaining is efficient (war does not occur) provided that the function $p(x)$ from territory held to the probability of winning at war is continuous. If not, then war may occur due to a moderate imbalance of power and the states' inability to commit not to increase demands once they have grown stronger. When the states *are* able continuously to adjust the military odds through resource transfers, then gradual appeasement, or "salami tactics," occur. If small changes in resources imply a large change in the military odds (which some international relations scholars call "offense dominance"), the initially weaker state is nearly or completely eliminated over time. If the military odds are less sensitive to changes in territory ("defense dominance"), then almost any initial distribution will lead by a path of concessions to a stable division, or rough "balance of power." These basic results on equilibrium paths of demands generalize remarkably far – they depend hardly at all on the specific bargaining protocol employed in each period.

1 Introduction

In a number of contexts, people bargain about the division of objects or resources that themselves contribute to future bargaining power. The clearest examples come from international relations, where state leaders frequently bargain over territory using threats to go to war. Because territory is not just directly valuable but can also be used to generate military resources, a deal struck today may affect the terms of deals struck in the future. In particular, conceding territory to another state may allow it to grow stronger militarily and so to demand yet more in the future. This problem of appeasement almost surely appears in contexts other than international politics. For example, if firms in an oligopolistic market bargain implicitly or explicitly over market shares using threats to resort to a price war, and a firm does better in a price war the larger its market share, then the same issues might arise. Alternatively, political parties or ethnic groups in an unstable democracy or authoritarian regime may fear that conceding any political power to another group or to the regime will allow it to accumulate yet more, and this possibility might in principle frustrate negotiations over democratic institutions for sharing control of the government. The problem might arise, for example, between parties bargaining over the design of an electoral system, which will influence their relative strength in the legislature and thus their future bargaining leverage.

It is not obvious what sort of outcomes one should expect for bargaining problems of this sort. Two basic questions present themselves. First, would bargaining in this context be efficient? One might imagine, for instance, that the inability to commit not to increase demands after gaining a concession would imply that one small concession would lead to complete elimination or at least to many more concessions. Thus, if certain distributions of, say, territory are not stable because they are not enforceable, there might in principle be no enforceable distributions both sides would prefer to an inefficient war.[1] Second, if war is not the outcome, what would be? Would one side be completely eliminated over time or is there a stable range of distributions? Finally, what factors determine the nature of whatever outcomes obtain?

I begin below by considering a simple infinite horizon, complete information bargaining game in which two states bargain over territory, and "today's" distribution of territory determines "tomorrow's" military balance or odds of winning at war. In successive periods, one state unilaterally chooses the division of territory, presenting the second state with a fait accompli that it may either accept or go to war to reverse. War is modelled as a costly lottery that eliminates one of the two states with a probability determined by the distribution of territorial resources in the *prior* period. Thus, a distribution of resources agreed to in one period determines the disagreement point for bargaining in the next period.

The game proves to have a unique subgame perfect equilibrium for any given set of initial conditions. Interestingly, equilibrium is characterized by behavior that looks like "salami tactics" (Schelling 1960, 66ff.) – the holdings of one of the two states (and not necessarily the state choosing between war and acquiescence) are gradually reduced in a series of small concessions. Regarding the first question noted above, wars do not occur in equilibrium. Bargaining is efficient, provided that (1) states are risk-neutral or risk-averse over the resources in question, (2) the issues in dispute are infinitely divisible, and (3) the function mapping territorial (or other resources) distributions into probabilities of winning at war is continuous. While the first assumption is probably unproblematic in most applications, the second and third might conceivably be violated, depending on the specific context. Consider, for example, two states separated by a mountain range or river. It could be that controlling territory on the far side of the natural barrier gives a state a discontinuous jump in military advantage (thus violating (3)). I give an example showing that war can occur in situations of this sort as result of the combination of an

[1] Wagner (1994) suggests that this might be the case.

imbalance of power and the states' inability to commit themselves not to increase their demands once they control territory on the other side of the barrier. More generally, the results suggest a theoretically coherent explanation of why war might be more likely when relative power is "out of balance."

Regarding the second question, the results of the bargaining prove to depend crucially on what international relations scholars refer to as the "offense-defense balance," here interpreted as the impact that additional territory has on a state's probability of winning at war. When additional territory or resources have a small enough impact on a state's military odds ("defense dominance"), there is a stable distribution of territory that is the long-run result of bargaining almost regardless of the initial distribution and almost regardless of which state is structurally advantaged by the ability to make "take-it-or-leave-it" offers. By contrast, if changes in resources have a large enough effect on the odds of winning ("offense-dominance"), then the initial distribution matters. Loosely, under offense dominance the initially larger state will completely or virtually eliminate the smaller state over the course of (possibly very many rounds of) the bargaining. There may exist a stable distribution under offense dominance, but it will be *dynamically* unstable in the sense that a tiny change in the distribution of territory leads to the complete or near elimination of the less favored state.

As it turns out, these principal results on equilibrium paths of demands depend hardly at all on the specific bargaining protocol (take-it-or-leave-it offers) considered in sections 2 and 3. In section 4 I show that, if we restrict attention to equilibria involving monotonic sequences of territorial divisions, then essentially the same patterns will obtain for *any* bargaining protocol that allows players the option to fight in each period rather than accept any particular deal. Regardless of the specific bargaining protocol employed, under sufficient defense dominance any efficient and monotonic equilibrium sequence of demands must approach a limit within a central zone of roughly equal distributions. Under sufficient offense dominance, if the initial distribution is unequal enough, then all efficient and monotonic sequences of demands must lead to the elimination or near-elimination of the less favored state, regardless of the bargaining protocol.

In related work, Robert Powell (1995) has modelled the appeasement problem as a game of timing in which a "status quo state" decides when to stop making concessions and fight, while a "dissatisfied state" chooses when to stop making demands. In contrast to the model developed here, in the timing game there is no explicit bargaining process; rates of demands and concessions are exogenously fixed.[2] In a series of papers, Jack Hirshleifer (1988, 1989, 1991, 1995) has considered a family of contest models where players simultaneously allocate between productive resources and arms, and relative arms determine the final distribution of resources through a "contest success function."[3] Hirshleifer (1995) obtains a similar result that for sufficient offense dominance the only stable resource distributions involve just one state or player.[4] My approach here differs in that I explicitly characterize a dynamic bargaining game, while I do not model the choice to allocate between productive and predatory activities. Also, while the result concerning offense dominance is similar to Hirshleifer's, the logic behind it is quite different (see below). To my knowledge, this paper is the first to consider the dynamic problem explicitly and the first to model it using a Rubinstein-like noncooperative bargaining game.

Section 2 describes the model and characterizes the unique sub-game perfect equilibrium. Section 3 analyzes the behavior of the players in equilibrium, showing how the offense-defense balance matters.

[2]Powell's model also proves to have multiple equilibria, even with complete information. See also Kim and Morrow 1992 for an analysis of "power transitions" in which a rising state chooses when to challenge a declining state, and the latter chooses whether to fight back if challenged. Their model allows the declining state the option to attack only if it is challenged, which makes preventive war impossible by fiat; also, as in Powell's case there is no explicit bargaining in Kim and Morrow.

[3]See also Skaperdas (1992) and Garfinkel (1990). For a dynamic version which focuses on arms allocations to deter explicitly modelled military attacks, see Powell (1993).

[4]What I am calling "offense dominance" Hirshleifer calls the "decisiveness" of the contest success function.

Section 4 considers how the results generalize for a large class of bargaining protocols. Section 5 briefly examines the conditions that generate war as an equilibrium outcome in model, and section 6 concludes.

2 The Model

Two states, 1 and 2, will bargain in successive periods $t = 1, 2, 3, \ldots$, over the disposition of all territory held or controlled by both. A peaceful outcome in period t is given by a number $x_t \in [0, 1]$ which represents the fraction of territory controlled by state 1. If the outcome is $x_t = 0$ (or $x_t = 1$) then state 1 (or state 2) has been eliminated and the strategic interaction ends.

I will consider a simple "take-it-or-leave-it" bargaining protocol in which state 1 chooses the division x_t in each period t and state 2 then either acquiesces or goes to war to contest it. This protocol is worth considering both for tractability and because it represents the extreme where one side has all of the structural bargaining power. Further, as shown below, the basic features of subgame-perfect paths of demands in the take-it-or-leave-it case obtain for almost any more complicated bargaining protocol as well.[5]

If state 2 acquiesces to the division x_t in period t, then the states receive utilities $u_1(x_t)$ and $u_2(1-x_t)$ for controlling territory for this period. Without loss of generality we can normalize $u_i(z)$ ($i = 1, 2$) so that $u_i(0) = 0$ and $u_i(1) = 1$. Assume further that $u_i(z)$ is strictly increasing and weakly concave, which means that the states prefer more territory to less and have risk-neutral or risk-averse preferences for territory. The latter assumption is consistent with the common presumption among students of international politics that states (or, more precisely, their leaders) view elimination as *very* bad relative to gaining more territory.[6]

If in period t state 2 rejects the demand x_t and decides to fight, a war occurs that is modelled as a costly lottery with two possible outcomes – one of the two states wins, eliminating or taking over the loser. State 1 wins with probability $p(x_{t-1})$, where $p(\cdot)$ is a non-decreasing, continuous function defined on [0, 1] with $p(0) = 0$ and $p(1) = 1$. Both states pay positive costs for war, c_1 and c_2 (in utility). If a state wins at war, it receives $u_i(1) = 1$ in period t and in all subsequent periods, while if it loses it receives 0 in the current and all future periods.[7] The states are assumed to have a common discount factor $\delta \in [0, 1]$. Thus the expected utility of the war option for state 1 in period t is

$$p(x_{t-1})\frac{1}{1-\delta} + (1 - p(x_{t-1}))0 - c_1 = \frac{p(x_{t-1})}{1-\delta} - c_1.$$

Similarly, for state 2 it is $\frac{1-p(x_{t-1})}{1-\delta} - c_2$.

Notice in particular that the distribution of territory (or resources) in the previous period $t-1$ determines the military balance in the present period t, and thus the present odds at war. For concreteness

[5]If we were to consider an application in which a dictator bargains with an opposition group over the division of state-generated resources or laws on civil liberties – think of war in this case as a civil war initiated by the opposition – then giving one player (the dictator) the power to make take-it-or-leave-it offers might be the most substantively appropriate modelling choice.

[6]See, for an example, Waltz (1979). In fact, risk-neutrality/risk-aversion is stronger than needed for virtually every result given in the paper. A weaker assumption sufficient for proposition 1, the lemma, and with some appropriate modifications, the theorem, is that for all $x \in [0, 1]$, $u_1(x) + u_2(1 - x) \geq 1$.

[7]An interesting alternative interpretation of $p(x)$ would consider this the specific territorial outcome following a war. Thus if state 2 chooses to reject the demand x_t and go to war, the outcome 'on the ground' for period t is then $x_t = p(x_{t-1})$, and the states continue bargaining from here. I show in work in progress that very similar results obtain for this version of the problem.

one might imagine that state 1's "demand" x_t is a military grab that attempts to present state 2 with a fait accompli. If state 2 attacks immediately its chances are determined by the military balance based on the prior distribution of resources, since state 1 has not had time to exploit its present gains. If state 2 acquiesces, however, state 1 will have time to exploit the new resources militarily and will enter the next period in a stronger position.

Let $h_t = (x_1, x_2, \ldots, x_{t-1})$ be a "history to period t," and H_t be the set of all such histories. A pure strategy for state 1 is then a sequence of maps $s_1^t : H_t \to [0, 1]$. A pure strategy for state 2 is a sequence of maps $s_2^t : H_{t+1} \to \{\text{fight, acquiesce}\}$. Payoffs for the game are defined as discounted sums of the payoffs in each period. For example, if state 2 rejects state 1's demand in period t, state 1's payoff is

$$\left(\sum_{i=1}^{t-1} u_1(x_i)\delta^{i-1}\right) + \delta^{t-1}\left(\frac{p(x_{t-1})}{1-\delta} - c_1\right).$$

Let $G(x_0, \delta)$ be the extensive form game so defined, where $x_0 \in (0, 1)$ is the initial distribution of territory.

The game $G(x_0, \delta)$ captures in a simple fashion a bargaining problem where today's agreement determines the value of the "disagreement point" for tomorrow's bargaining. Below, Proposition 1 asserts that $G(x_0, \delta)$ has a unique subgame perfect (SGP) equilibrium and characterizes the equilibrium strategies. Before proceeding to these, however, some preliminary definitions and observations will prove helpful.

Let $V_2(x_t, h_t)$ be state 2's continuation payoff in period $t+1$ in some SGP equilibrium if 2 accepts an offer of x_t in period t following the history h_t. Faced with the demand x_t in period t, state 2 is choosing between fighting now, which yields $\frac{1-p(x_{t-1})}{1-\delta} - c_2$, and accepting x_t, which yields $u_2(1-x_t) + \delta V_2(x_t, h_t)$. Clearly, in any SGP equilibrium state 2 cannot credibly threaten to reject any x_t such that

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 < u_2(1-x_t) + \delta V_2(x_t, h_t).$$

An important observation is that in *any* SGP equilibrium the continuation payoff $V_2(x_t, h_t)$ must be greater than or equal to $\frac{1-p(x_t)}{1-\delta} - c_2$, because state 2 can always choose to fight in period $t+1$. The proof of Proposition 1 below will show that this further implies that in equilibrium state 2 will accept any x_t such that

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 \leq u_2(1-x_t) + \delta\left(\frac{1-p(x_t)}{1-\delta} - c_2\right). \tag{1}$$

The proof further shows that under the assumptions on $p(\cdot)$ and $u_i(\cdot)$, state 1 will always want to offer x_t such that (1) holds with equality. That is, given x_{t-1}, x_t^* is defined as the implicit solution to

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 = u_2(1-x_t^*) + \delta\left(\frac{1-p(x_t^*)}{1-\delta} - c_2\right). \tag{2}$$

Given an initial distribution of resources x_0, equation (2) can be used to generate an equilibrium path of demands $x_1^*, x_2^*, x_3^*, \ldots, x_t^* \ldots$[8] To see that if an x_t^* solves (2) it is unique, consider the function $f(x)$ defined as the right-hand side of (2), with x rather than x_t^* as the argument. $f(x)$ is strictly decreasing because $1 - p(x)$ is weakly decreasing and $u_2(1-x)$ is strictly decreasing. Further,

[8] Provided, of course, that x_t remains in [0, 1]. If not then one of the states is eliminated, as discussed below.

$f(0) = \frac{1}{1-\delta} - \delta c_2$, which is always greater than the left-hand side of (2). Therefore, provided that $f(1)$ is less than the left-hand side of (2), there must be a unique demand x_t^* that solves (2), since both $u_2(\cdot)$ and $p(\cdot)$ are continuous. Finally, $f(1) = -\delta c_2$, which is less than the left-hand side provided that $p(x_{t-1}) < 1 - c_2(1-\delta)^2$. If this last condition fails, then state 1 can demand $x_t^* = 1$ and state 2 will accept, so that there will be a unique equilibrium demand in all cases.

Proposition 1: $G(x_0, \delta)$ *has a unique subgame perfect equilibrium with strategies determined as follows. Let* $x_t^*(x_{t-1})$ *be defined as the implicit solution to equation (2), with* $x_t^*(x_{t-1}) \equiv 1$ *for* $x_{t-1} \in [0,1]$ *such that no solution exists. In any period* $t = 1, 2, \ldots$, *after any history, state 1 demands* $x_t^*(x_{t-1})$, *while state 2 accepts any* $x_t \leq x_t^*(x_{t-1})$ *and fights otherwise.*[9]

Thus in the game's unique subgame perfect equilibrium war does not occur − the bargaining is efficient despite the states' inability to commit themselves not to increase their demands once they have grown stronger.[10] The keys to this conclusion are the assumed availability of infinite set of possible divisions in X and the continuity of $p(x)$, which together imply that state 1 can always find a demand x_t^* that leaves state 2 just willing to grant (or, it turns out, accept) the concession rather than fight. In effect, granting a concession to state 1 has both a good side and a bad side for state 2. On the good side, state 2 gets a period of peaceful enjoyment of the territory $1 - x_t$ while on the bad side it may be less able to resist subsequent demands. Continuity of $p(x)$ ensures that there is some (possibly small or even negative) demand by state 1 such that the good and bad sides together making granting the concession just as attractive as fighting now. In turn, the fact that war is inefficient proves to imply that state 1 will always prefer to make this demand rather than start a war by demanding more than it would make sense for state 2 to yield. When state 1 is demanding territory from state 2, we would say that appeasement is the optimal strategy for state 2, provided that state 1 demands just x_t^* and no more.

The intuition for and implications of Proposition 1 become clearer when we consider specific functional forms for $p(x)$ in the next section. That analysis, however, is facilitated by three more general observations that follow from the proposition.

First, it is apparent that if there is a territorial division x^* such that $x^* = x_t^*(x^*)$, then this division is *stable* in the sense if it is ever reached (or if $x_0 = x^*$) the subsequent sequence of equilibrium demands will be $\{x^*, x^*, x^*, \ldots\}$. From (2), any such stable division x^* must satisfy

$$1 - p(x^*) - c_2(1-\delta) = u_2(1 - x^*), \tag{3}$$

which says state 2 would be indifferent between fighting at $p(x^*)$ and living for ever with $u_2(1-x^*)$ each period. Rewritten as $p(x^*) = 1 - u_2(1-x^*) - c_2(1-\delta)$, equation (3) also provides a graphical way of locating stable divisions for any given $p(x)$ and $u_2(1-x)$ – see figure 1b for an example involving a risk neutral state 2 and what will be described below as a "defense dominant" technology.

Second, as state 2 puts more weight on future payoffs (that is, as δ approaches 1), the amount of territory transferred in each period approaches zero.[11] Intuitively, the more states value future payoffs, the less territory they are willing to give up today due to the consequences of these concessions tomorrow. An interesting implication of this observation is that if the set of feasible issue resolutions is discrete

[9] All proofs are in the appendix.

[10] In state 2's case, 2 may be unable to commit itself to accept small enough concessions from state 1 once 2 has grown stronger.

[11] To see this, rearrange equation (2) as $1 - p(x_{t-1}) - c_2(1-\delta)^2 = (1-\delta)u_2(1-x_t^*) + \delta(1 - p(x_t^*))$, and then notice that as $\delta \to 1$ it must be the case that $x_t^* \to x_{t-1}$.

rather than continuous – suppose, for example, that $X = \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{k}{n}, \ldots, \frac{n-1}{n}, 1\}$ – then for large enough δ a state can prefer to fight rather than make the smallest possible discrete concession. As shown below in section 5, this can have the effect of either "freezing the status quo," or of generating a war if the state pressing demands prefer to fight at such an x_t.[12]

Third, it is straightforward to use Proposition 1 to show that state 1 can never be completely eliminated while for a sufficiently unfavorable distribution of territory, state 2 can always be eliminated. This is a consequence of state 1's structural bargaining power (its ability to make take-it-or-leave-it offers) which implies that when it is strong enough relative to state 2 (i.e., x_t close enough to 1), it can confront state 2 with a choice between a war it will almost surely lose and simply conceding its sovereignty. Since war is costly, at some point state 2 prefers just to 'go out of business' quietly rather than suffer the punishment it would face in a war.[13] On the other hand, for state 2 to eliminate state 1 it would have to be that state 2 could credibly threaten to reject any positive demand $x_t > 0$ by state 1. But again because of the continuity of $p(x)$ and the fact that war is costly, there will always be some amount of territory so small that it is not worth state 2's while to pay the costs of crushing the tiny upstart in a war.[14]

3 Equilibrium Dynamics and the Offense-Defense Balance

The function $p(x)$, which summarizes how relative resources translate into relative military power, chiefly determines the path of aggression and appeasement in equilibrium. In this section I consider some examples of specific functional forms for $p(x)$ and their implications for equilibrium paths of demands under the take-it-or-leave-it protocol.

3.1 The offense-defense balance and $p(x)$

The function $p(x)$ has close analogues in the literature on strategic behavior in contests, situations where two or more players choose how much effort to exert towards winning a 'prize' and the probability of winning is determined jointly by the effort levels chosen.[15] Whereas contest models focus on the question of how much effort or resources a player should invest in competing, here $p(x)$ simply summarizes how much military power a player can generate given that she holds fraction x of the total resources in

[12]It should also be noted, however, that the usual rationale for focusing on what happens as δ approaches one does not apply here, since the time between periods reflects not only the time necessary to respond to an offer (as in the Rubinstein bargaining game), but also the time necessary to exploit last period's resource allocation for current military benefit.

[13]It is an interesting question why small states sometimes fight wars against major powers that they can be expected to lose with near certainty (e.g., Denmark vs. Prussia in 1863, Finland vs. USSR in 1939, Poland vs. Germany 1939, Chechnya vs. Russia 1994). In some cases the answer may be reputation building, in others rational or irrational hopes of intervention by other major powers, in others the fact that the small state's leaders face different incentives than do the citizens.

[14]In an empirical study of war termination, Kecskemeti (1958) observed that paradoxically the losing side in a war frequently retains bargaining power in negotiations with the winners, even after its military forces have been destroyed. The result in the model captures the logic behind this phenomenon. As in Kecskemeti's examples, in the model a 'state' with no military resources can take unilateral actions that confront the winners with a choice between a costly use of force to reverse them, or just acquiescing. If the losing side's demands are sufficiently small relative to the costs of compellence, the winning side is better off acquiescing.

[15]Contest models have been used in the study of patent races, oligopolistic competition, intrafirm incentive schemes, electoral campaigns, political and economic rent-seeking by lobbying, legal disputes, interstate arms competitions, and competition among animals over territory or mates. See, for some examples, Dixit (1987), Garfinkel (1990), Hirshleifer (1988), Nalebuff and Stiglitz (1983), Skaperdas (1992), Dasgupta and Stiglitz (1980), Tullock (1980).

the system. Thus decisions about allocating between consumption and arms or effort are implicitly suppressed or assumed to have already been made optimally. $p(x)$ simply answers questions such as "if state 1's resources are half as large as state 2's (i.e., $x = 1/3$), how likely is state 1 to prevail in a war?"

In terms more common in the international relations literature, $p(x)$ describes how different "force ratios" ($\frac{x}{1-x}$) translate into military odds. Put in these terms, the shape of $p(x)$ can be related to what a number of international relations scholars call the "offense-defense balance." While this concept is not clearly defined, it always refers to some measure of whether technological, organizational, and geographical factors make attacking and taking territory relatively easy ("offense dominance") or hard ("defense dominance").[16] One type of definition identifies the offense-defense balance with the force ratio needed to defend successfully against offense or to 'offend' successfully against defense.[17] For example, the balance is said to be 3:1 if it takes three times the defender's resources for an attacker to prosecute a successful offensive.

In one sense the function $p(x)$ generalizes this idea, since it describes how different force ratios translate into different probabilities of battlefield victory. The slope of $p(x)$ around $x = 1/2$ can be regarded as one natural measure of relative offense or defense dominance, since this slope indicates how responsive the odds of winning are to changes in relative territory or resources near equality. When small changes make for large shifts in the probability of winning, offense is relatively favored, since a small favorable change in the force ratio makes a state much more likely prevail. Likewise, when small changes in territory do not have a large effect on a state's odds at war, defense is relatively favored. For instance, if the slope of the function is fairly flat for much of the range [0, 1], then even a relatively small state can attain a reasonably good chance of success against a larger state. This corresponds to the idea that defense is relatively easy, or that a quite large force advantage is needed to attain a high chance of victory.[18]

The literature on contests has tended to favor *ratio forms* for the analogues to $p(x)$, largely because they are simple and have some attractive mathematical properties (namely, concavity in effort). For the present model, the most natural ratio form is $p(x) = x$, which says that a state's probability of winning at war is exactly equal to the fraction of territory or resources it controls. However, numerous other functional forms are conceivable for $p(x)$, and, as the analysis below suggests, it is misleading to privilege the ratio form. Several possibilities are graphed in Figures 1a-e.

Figure 1a gives the simple ratio form. 1b and 1c represent "linear technologies," where a percentage increase in territory or resources results (almost everywhere) in a constant proportional increase in a state's probability of military victory.[19] The technology described in 1c is "offense dominant" relative to 1a and 1b, since the odds of victory at $x = 1/2$ are more sensitive to changes in territory. Figures

[16] See Jervis (1978) for the classic formulation and Glaser and Kaufmann (1995) for a recent attempt to develop a clear definition. For critiques see Levy (1981) and Fearon (1995a).

[17] See Glaser and Kaufmann (1995).

[18] I should stress that the international relations literature is unclear on the definition of the "offense-defense balance," and has included many different ideas under the heading. For example, neither first-strike advantages nor tactical advantages or disadvantages that accrue to the side trying to take rather than defend territory are comprehended in the formulation in the text, but these often appear in statements of the offense-defense balance. These aspects of "the balance" can be incorporated in the terms of the formal framework developed here by allowing a state's military odds to depend not only x but also on whether the state is the attacker (the side trying to take territory) or the defender. Let $p_A(x)$ be state 1's probability of winning given resource balance x and if state 1 is the attacker and state 2 is the defender. Let $p_D(x)$ be state 1's probability of winning given x if 1 is the defender, and 2 is the attacker (in the symmetric case, $p_D(x) = 1 - p_A(1-x)$). Then the difference $p_A(x) - p_D(x)$ is a measure of the size of "offense dominance given the resource balance x" in a different sense from that given in the text. The analysis below can be modified to incorporate this sense as well; see footnote 32 below.

[19] Strictly speaking, 1b violates the continuity assumption made earlier about $p(x)$.

1d and 1e give smoother versions of 1b and 1c.[20] Note that in each case $p(1/2) = 1/2$, which represents the assumption that given equal resource bases the two states are equally likely to win in a war. This assumption can be relaxed (and will be below, briefly) to represent cases where one side is more efficient at generating military power from a given amount of resources.

3.2 Examples of equilibrium paths

If we assume that state 2 is risk-neutral ($u_2(1-x) = 1-x$), then equation (2) can be solved explicitly for cases where $p(x)$ is largely linear.[21]

In the simplest case, the ratio form $p(x) = x$, solving equation (2) yields $x_t^* = x_{t-1} + c_2(1-\delta)^2$. This means that from any given initial distribution $x_0 < 1 - c_2(1-\delta)^2$, state 1 demands and is granted an additional "chunk" of territory of size $c_2(1-\delta)^2$ in each period, until state 2 is completely eliminated. (Note also that for this case there is no stable division x^* solving equation (3).)

So in this case "salami tactics" work to the point that state 2 eventually loses sovereignty. State 1 unilaterally grabs the same small amount of territory in each period, just enough that it is not quite worthwhile for state 2 to go to war rather than accept the slightly diminished bargaining power it will have in the next period. The size of each slice of "salami" depends on two factors – state 2's costs for fighting and its discount factor, or regard for future payoffs. Greater costs for war allow state 1 to get away with more aggressive demands each period and thus make for the more rapid elimination of state 2.[22] By contrast, as state 2 grows more patient, putting more weight on future payoffs ($\delta \to 1$), state 1's demands grow smaller and smaller, and the number of periods it takes to eliminate state 2 approaches infinity. Intuitively, caring more about future payoffs strengthens state 2's resolve not to make concessions now that will lead to lower payoffs later, and this increases state 2's bargaining power each period.[23]

Surprisingly, matters are quite different for the related linear technologies illustrated in 1b and 1c. These are defined by the general linear probability function $p(x) = \text{median}\{0, \frac{1}{2}(1-b) + bx, 1\}$ where the coefficient $b > 0$ gives the slope of $p(x)$ through $x = 1/2$. $b > 1$ corresponds to the offense-dominant case 1c and $b < 1$ to the defense-dominant case 1b. ($b = 1$ is of course the special case $p(x) = x$.)

Solving for $x_t^*(x_{t-1})$ in equation 2 yields $x_t^*(x_{t-1}) = A + Bx_{t-1}$, where

$$A = \frac{\frac{1}{2}(1-b)(1-\delta) + c_2(1-\delta)^2}{1-\delta+\delta b} \text{ and } B = \frac{b}{1-\delta+\delta b}.$$

So, neglecting for the moment problems created by the two non-linear 'kinks' in $p(x)$ when $b > 1$, this result says that state 1's demand x_t is a linear function of the current status quo x_{t-1}. This implies

[20] A functional form for generating the symmetric curves in 1d and 1e is $p(x) = \frac{x^a}{x^a+(1-x)^a}$. When $a \in (0,1)$, the curve looks like 1d; when $a > 1$ it looks like 1e. To incorporate the effect of military advantages that depend on whether a state is attacking or defending (see footnote 18), we can modify this so that the probability state 1 wins if it is the attacker is $p_A(x) = \frac{bx^a}{bx^a+(1-x)^a}$, where $b \in (0,1)$ means that the side defending territory has an advantage, and $b > 1$ means the attacking side has an advantage.

[21] Very similar results hold for cases 1d and 1e, which can be solved with the aid of a computer but not analytically.

[22] Obviously, this particular result is driven by the fact that one state has the power to make take-it-or-leave-it offers.

[23] Since I have assumed that the costs of war are paid in a single period, increasing the discount factor not only makes state 2 more concerned about future payoffs but also has the effect of reducing its time-average costs of war, $c_2(1-\delta)$. Note, however, that even if we assumed that war results in a permanent reduction in welfare – for example, let $c_2 = c_2'/(1-\delta)$ – the result would still hold. In this case, state 1 would demand an additional $c_2'(1-\delta)$ in each period, which also approaches zero as δ approaches one.

that the equilibrium path of demands will be described by a linear dynamical system in which

$$x_t^* = A\left(\frac{1-B^t}{1-B}\right) + B^t x_0.$$

The system has a stable division at

$$x^* = \frac{A}{1-B} = \frac{1}{2} + c_2\frac{1-\delta}{1-b},$$

and x^* is dynamically stable when $B < 1$ and dynamically unstable when $B > 1$.[24] Algebra indicates that $B > 1$ if and only if offense dominance obtains ($b > 1$), and $B < 1$ if and only if defense dominance obtains ($b < 1$).

Using algebra it can further be shown that under offense dominance ($b > 1$), if the initial distribution of territory favors state 1 – formally, if $x_0 > x^*$ – then the sequence of state 1's demands is steadily increasing ($x_0 < x_1 < x_2 < \ldots < x_t < \ldots$) until state 2 acquiesces to loss of sovereignty. On the other hand, if state 2 is favored by the initial distribution ($x_0 < x^*$), then *state 1 offers a series of concessions that lead to its own near-elimination* (i.e., $x_0 > x_1 > x_2 > \ldots > x_t > \ldots$). This is perhaps surprising given that state 1 would appear to have so much bargaining power conferred on it by the take-or-leave-it structure of the game. But when offense dominates and state 2 is relatively favored in military resources, state 1 is forced to appease state 2 by making one unilateral concession after another. State 1's structural advantage manifests itself mainly in the fact that state 1 can never be completely eliminated. Instead, the sequence of demands will approach the dynamically stable equilibrium $\underline{x}^*$ in figure 2c, which approaches zero as δ approaches one.

Under defense dominance ($b < 1$), by contrast, the equilibrium path of 1's demands will converge towards x^* for any initial distribution $x_0 \in (0, 1-\epsilon(\delta))$, where $\epsilon(\delta)$ is a positive number that approaches zero as δ approaches 1.[25] Note that under defense dominance, the only manifest advantage for state 1 that comes from proposal power is that the equilibrium division x^* always favors it slightly, since $\frac{1}{2} + c_2\frac{1-\delta}{1-b} > \frac{1}{2}$ when $b < 1$.

The sensitivity of the military odds to changes in resources around an even distribution thus proves to be the crucial determinant of outcomes in this kind of bargaining problem. When small changes in the force ratio have a large effect on the players' value for the disagreement outcome (offense dominance), the long-run outcome of bargaining is "history dependent" in the sense that the initial distribution of relative power determines which player effectively eliminates the other. When small changes in resources have less of an impact (defense dominance), history does not matter in that any initial distribution of power yields a path of appeasement leading to the same final outcome, a stable balance of power.

The rationale appears to be something like this: Except at equilibrium divisions, either state 1 can exploit state 2's impatience and fear of the costs of war by demanding a bit more than the status quo, or state 1 must cede a little ground to prevent state 2 from wanting to fight, given its current level of power and satisfaction with the status quo. Under offense dominance, such small changes have a big effect on relative power, and they make the new situation the same as the old only worse – either state 1 can now exploit state 2's fears even more, or state 2 requires an even bigger concession from state 1 to

[24] "Dynamically stable" here means that divisions x close to x^* generate sequences approaching x^*, while dynamically unstable means that such sequences will lead away from x^*.

[25] Exactly what happens for x_0 very close to 1 depends of course on what is assumed about $p(x)$ for x near 1. I assume here that $p(x) = (1/2)(1-b) + bx$ for $x \in (0,1)$, $p(0) = 0$, and $p(1) = 1$. Substantively this means that holding zero resources implies no chance of military victory, while holding any positive amount above zero implies a discontinuous jump in one's chances. In this case the claim could also be put as follows: If $b < 1 - 2c_2(1-\delta)$, then the equilibrium path of demands converges to x^* for all $x_0 \in (0,1)$.

be appeased. By contrast, under defense dominance the party making concessions tends to grow more resolved as more is given away, because its relative power is not shrinking as fast as its marginal utility for territory is increasing.[26]

4 Generalizing to Other Bargaining Protocols

While the take-it-or-leave-it bargaining protocol employed above makes the game analytically tractable, it is natural to wonder if the principal results will generalize when we consider other, less one-sided structures of bargaining. In this section I show that some of the crucial results concerning equilibrium paths of demands in fact generalize remarkably far – to *any* bargaining protocol in which the players have the option to fight in each period rather than accept any particular deal. More specifically, I show that under defense dominance and some weak conditions on other parameters, if the initial distribution of resources is not too extreme, then all efficient and monotonic equilibrium sequences of demands must approach a stable, centrally located "balance of power," regardless of the bargaining protocol. By contrast, under offense dominance, if the initial distribution is not within a central interval, then all efficient and monotonic equilibrium sequences lead towards the elimination or near-elimination of the less-favored player (again, regardless of bargaining protocol).[27]

Consider modifying the game described in section 2 as follows. In each successive period $t = 0, 1, 2, \ldots$, the states will play some finite extensive form bargaining game that has the property that both states have the opportunity in every period to fight rather than accept any proposed division of the territory. This is the only restriction placed on the bargaining protocols for each period (note that it is not even necessary that the protocol be the same from period to period). An agreement x_t reached in a period yields payoffs $(u_1(x_t), u_2(1-x_t))$ for this period, while war in period t concludes the game with expected payoffs $(\frac{p(x_{t-1})}{1-\delta} - c_1, \frac{1-p(x_{t-1})}{1-\delta} - c_2)$, as before.[28] Call the set of all such games Γ.

Some notation and definitions follow. Let $h_t^* = \{x_0, x_1, x_2, \ldots, x_t\}$ represent a sequence of demands that can be supported in some pure-strategy subgame perfect equilibrium of some game in Γ. This will be taken to include three cases: (i) $t = \infty$, which represents an efficient (i.e., peaceful) sequence of demands in which neither state is ever eliminated; (ii) t finite, and $x_t = 1$ or $x_t = 0$, which represents efficient sequences that lead to the elimination of one of the states; and (iii) t finite with war occuring in period t, an inefficient sequence. An equilibrium sequence of demands h_t^* is called *monotonic* if either $x_i \leq x_{i+1}$ for all $i < t$ or $x_i \geq x_{i+1}$ for all $i < t$. Let H^* be the set of all equilibrium sequences of demands h_t^* (for all $t > 0$, including $t = \infty$) that can be supported in some subgame perfect equilibrium of some game in Γ.

[26] Goemans (1995, 51-52) suggests a similar argument informally. Though the conclusions are similar, the logic behind the result seems to differ markedly from that in Hirshleifer (1995). As noted, Hirshleifer considers a contest model in which players simultaneously choose how to allocate resources between arms and productive investment, with relative arms determining the final distribution of resources through a "contest success function." The contest success function might be interpreted as a sort of "reduced-form" bargaining process. Equilibrium requires that the marginal returns from spending on arms equal the marginal cost of reducing the amount of consumption goods available to divide up. With a highly offense-dominant contest success function, the marginal gain to spending on arms will be large at any symmetric profile. But this is unsupportable, since both sides would then want to spend everything on arms, leaving no pie to divide. Thus large enough offense dominance rules out the possibility of a stable equilibrium with two players.

[27] The only real price for these more general results is the restriction of attention to equilibrium sequences that are monotonic, which does not seem very large in substantive terms; for a discussion, see below.

[28] We could also allow for a finer time structure and discounting within periods, introducing the possibility of bargaining protocols with equilibria with inefficient delay within periods. All the results given in this section hold *a fortiori* for such cases as well.

Next, state 1 will be described as *dissatisfied at* $x \in [0,1]$ if it prefers going to war with military resources x to living with this division forever. Formally, state 1 is dissatisfied at $x \in [0,1]$ if $\frac{p(x)}{1-\delta} - c_1 > \frac{u_1(x)}{1-\delta}$, or $p(x) > u_1(x) + c_1(1-\delta)$. Likewise, state 2 is said to be disatisfied at $x \in [0,1]$ if $\frac{1-p(x)}{1-\delta} - c_2 > \frac{u_2(1-x)}{1-\delta}$, or $p(x) < 1 - u_2(1-x) - c_2(1-\delta)$. Note that both states can never be dissatisfied at the same division x since this would imply that $u_1(x) + u_2(1-x) < 1 - (1-\delta)(c_1 + c_2)$, which contradicts risk-aversion/risk-neutrality.

The lemma below establishes that if a state is dissatisfied at some division x_i, then, for all games in Γ, any monotonic sequence of equilibrium demands that includes x_i must be headed in the direction that favors the dissatisfied state (increasing x_i for state 1, decreasing for state 2). The intuition for this result is straightforward – it cannot be that a dissatisfied state expects to accept a sequence of demands that makes it increasingly *worse* off, since then it would do better (in any equilibrium) to fight right away at x_i, since its odds of winning would only decrease. Thus, no such paths are possible, independent of the specific bargaining protocols that would produce them.

Lemma: *If state 1 is dissatisfied at* $x_i \in (0,1)$, $1 \leq i < t$, *then for all monotonic* h_t^* *that include* x_i *(and for all* $t > 1$*),* $x_{j+1} \geq x_j$ *for all* $j < t$ *and with a strict inequality for some* $j < t$*. The same is true for state 2 if the weak inequality is reversed.*

For a given success function $p(x)$ (and other exogenous parameters), the lemma allows us to draw some strong implications regarding monotone equilibrium sequences of demands. Though the analysis that follows is easily generalized to any given $p(x)$, it makes both substantive and heuristic sense to begin with two classes of success functions – offense dominant and defense dominant – described as follows.

Consider functions $p(x)$ on [0, 1] such that $p(0) = 0$, $p(1) = 1$, p(1/2) = 1/2, $p'(x) \geq 0$ for all $x \in [0,1]$ and $p(x)$ is twice differentiable for $x \in (0,1)$. Call such a function *defense dominant* if $p''(x) < 0$ for all $x \in (0,1/2)$, and $p''(x) > 0$ for all $x \in (1/2,1)$. Call such a function *offense dominant* if $p''(x) > 0$ for all $x \in (0,1/2)$ and $p''(x) < 0$ for all $x \in (1/2,1)$. Figures 1d and 1e picture typical defense dominant and offense dominant $p(x)$'s, respectively.

Let D_i be the set of divisions $x \in [0,1]$ such that state i is dissatisfied at x. Thus,

$$D_1 = \{x : p(x) > u_1(x) + c_1(1-\delta)\}, \text{ and}$$

$$D_2 = \{x : p(x) < 1 - u_2(1-x) - c_2(1-\delta)\}.$$

Given the assumptions on $p(x)$ and $u_i(\cdot)$, it is easy to see (graphically) that D_i is either empty or is an open interval contained in [0, 1]. See figures 1d, 1e, and 2 for illustration. In substantive terms, $D_i = \emptyset$ means that for state i, there is no territorial division x such that the state prefers to fight at $p(x)$ rather than to live with this division forever – this is a case where either war is a bad option for the state because the time-average costs, $c_i(1-\delta)$ are large, or because the state is quite risk averse. Note that as δ approaches one, risk aversion is the only reason that D_i could remain empty; if the states are risk-neutral, for instance, D_1 and D_2 cannot be empty for δ sufficiently close to 1.

For cases where $p(x)$, the costs of fighting, and the degree of risk aversion are such that neither state is ever dissatisfied (i.e., $D_1 = D_2 = \emptyset$), we can say nothing about equilibrium sequences of demands without a more precise specification of the bargaining protocol. Recall the earlier example of the ratio form, $p(x) = x$, which implies that neither state is ever dissatisfied regardless of fighting costs or the degree of risk aversion (i.e., $p(x) = x$ implies $D_1 = D_2 = \emptyset$). Thus, in the take-it-or-leave-it model, whichever state has the power to make offers has all the bargaining power, and can gradually eliminate the state not favored by the bargaining protocol. A different protocol, however, could produce very different results in this case. For example, consider a Nash-demand game version where each period the states simultaneously make demands, and compatible demands ($x_t + y_t = 1$) are implemented while incompatible demands yield war. If neither state is ever disatisfied, then this protocol can support virtually *any* path of demands, whether it leads to war, elimination, or peace, provided that the path moves slowly enough from division to division. So there is almost nothing special or interesting about the problem of bargaining over objects that influence future bargaining power in cases where neither party is ever dissatisfied in the sense given above. In these cases, the specifics of the bargaining protocol will matter in ways already largely predictable from existing bargaining theory.[29]

By contrast, if either D_1, D_2, or both are nonempty, then some much stronger implications can be drawn. Consider cases where *both* states can be dissatisfied for different divisions in X.[30] For such cases, define $\underline{d}_1$ and $\bar{d}_1$ such that $D_1 = (\underline{d}_1, \bar{d}_1)$ and $\underline{d}_2$ and $\bar{d}_2$ such that $D_2 = (\underline{d}_2, \bar{d}_2)$.[31] Thus, these are the divisions that demarcate the intervals in X for which the states are dissatisfied. It is easy to see (graphically) that defense dominance, $\bar{d}_1 < \underline{d}_2$, while under offense dominance, $\bar{d}_2 < \underline{d}_1$.

Theorem: *(a) Consider all games in Γ such that $p(x)$ satisfies the conditions for defense dominance in the text, and such that $D_1 \neq \emptyset$ and $D_2 \neq \emptyset$. For all such games,*

(i) every efficient monotonic sequence of demands $h_t^* \in H^*$ with $x_0 \in (\underline{d}_1, \bar{d}_2)$ has $t = \infty$ (i.e., neither player is peacefully eliminated);

(ii) every efficient monotonic sequence of demands $h_t^* \in H^*$ with $x_0 \in (\underline{d}_1, \bar{d}_2)$ has a limit $x^* \in [\bar{d}_1, \underline{d}_2]$ (i.e., all such sequences approach a division in the central region between D_1 and D_2);

(iii) if both states are risk neutral, then, in the limit as either c_1 and c_2 approach 0 or as δ approaches 1, every efficient monotonic sequence of demands $h_t^* \in H^*$ with $x_0 \in (0, 1)$ approaches the long-run equilibrium division $x^* = 1/2$.

(b) Consider all games in Γ such that $p(x)$ satisfies the conditions for offense dominance in the text, and such that $D_1 \neq \emptyset$ and $D_2 \neq \emptyset$. For all such games,

(i) every efficient monotonic sequence of demands $h_t^* \in H^*$ with $x_0 \in (\underline{d}_1, \bar{d}_1)$ has a limit $x^* \in [\bar{d}_1, 1]$, which may include cases where state 2 is peacefully eliminated;

(ii) every efficient monotonic sequence of demands $h_t^* \in H^*$ with $x_0 \in (\underline{d}_2, \bar{d}_2)$ has a limit $x^* \in [0, \underline{d}_2]$, which may include cases where state 1 is peacefully eliminated;

(iii) if both states are risk neutral, then in the limit as either c_1 and c_2 approach 0 or as δ approaches 1, every efficient monotonic sequence of demands with $x_0 \in (0, 1)$ except for those with $x_0 = 1/2$ leads to the elimination or near-elimination of the less-favored state.

[29] I say "almost nothing special" because there remains the issue of *how quickly* one can move from one division to another.

[30] Again, it is not difficult to establish related but somewhat weaker conclusions for cases where either $D_1 = \emptyset$ or $D_2 = \emptyset$ but not both.

[31] That is, $\underline{d}_1$ and $\bar{d}_1$ solve $p(x) = u_1(x) + c_1(1 - \delta)$, where $\underline{d}_1 < \bar{d}_1$, and similarly for $\underline{d}_2$ and $\bar{d}_2$.

In words, under defense dominance, if the initial distribution of resources is not extreme, then regardless of the specific bargaining protocol all efficient and monotonic equilibrium sequences of demands lead into (or stay in) a central set of roughly equal divisions. Further, states cannot be peacefully eliminated unless they begin the game with very few resources. By contrast, under offense dominance, if the initial distribution of resources falls outside of the central, roughly equal zone, then any efficient and monotonic sequence of equilibrium demands leads to the elimination or near-elimination of the weaker state. At bottom, these results are driven by the fact that with defense dominance, there are in effect "decreasing returns" in converting resources into military strength, while under offense dominance there are increasing returns.[32]

It is worth pointing out that under offense dominance, roughly equal initial distributions of resources (i.e., $x_0 \in [\bar{d}_2, \underline{d}_1]$) are "potentially stable" in the sense that both sides would be willing to live with such divisions forever rather than go to war. However, the specifics of the bargaining protocol will determine whether these roughly equal distributions are *in fact* stable in the long run. In such cases, it is perfectly possible for an equilibrium sequence to begin with rough equality but then lead out of the central zone and towards the elimination of one of the players (as in the take-it-or-leave-it model). While we cannot say whether this will occur or in which direction without asking about the specific bargaining protocol, we can say that once an efficient monotone sequence of demands leads out of the central zone, it must continue in the direction of eliminating (or nearly eliminating) the weaker side.[33]

While the restriction to monotonic sequences greatly facilitates obtaining the above results, it seems likely that even this can be weakened somewhat. The chief difficulty raised by non-monotonic sequences is that they introduce the theoretical possibility of sequences of demands that alternative or "bounce back and forth" between divisions on either side of $x = 1/2$.[34] One is inclined to dismiss these on the grounds of substantive implausibility or the fact that they will be inefficient if the states are risk-averse. Alternatively, if we restricted attention to protocols that generate equilibrium sequences that are "incremental" – that is, the distance between bargained settlements from period to period is sufficiently small – I would expect results along the lines of the theorem for these as well.

5 How Bargaining Might Fail and War Occur

War can occur on the equilibrium path if any of three conditions fail.[35] First, if one or both of the states is sufficiently risk acceptant, war may be the equilibrium outcome. This is an uninteresting case, however, since risk-acceptance tends to render war efficient. It also seems substantively implausible that many states have been headed by leaders with such preferences, given that the survival of the regime

[32]If we make a state's military odds conditional on whether it is attacking or defending, then a similar analysis can be carried out by defining D_1 in terms of $p_A(x)$ rather than $p(x)$, and D_2 in terms of $1 - p_D(x)$ (2's probability of winning if it is the attacker; see footnote 18). Increasing "offense dominance" in this second sense (i.e., increasing the difference $p_A(x) - p_D(x)$ for all $x \in (0, 1)$) will tend to narrow or even eliminate the set of potentially stable allocations between D_2 and D_2, while increasing "defense dominance" will increase the range of potentially stable allocations in the center.

[33]I would conjecture that for bargaining protocols in which both sides must agree in order to change the existing territorial division, any x_t such that neither side is dissatisfied will be long-run stable in any efficient equilibrium, since proposals to change the terms can be vetoed by the side that would be made worse off. (Alternation between two or more divisions would remain a possibility, but would likely be inefficient under risk aversion and also seems substantively strange.) If this is correct, then, under offense dominance, divisions in $(\bar{d}_2, \underline{d}_1)$ would be long-run stable, while under defense dominance sequences of demands with starting points in D_1 and D_2 would approach, respectively, $\bar{d}_1$ and $\underline{d}_2$.

[34]For example, using a Nash-demand protocol as described in the text, it may be possible to support equilibrium sequences that alternate between a division in D_1 and a division in D_2.

[35]Of course, introducing incomplete information in any of several ways might also create a risk of war in equilibrium; I do not explore these possibilities here, and in fact doubt whether incomplete information would give rise to issues or dynamics not already familiar in the literature on bargaining under incomplete information.

and territorial integrity tend to be paramount goals.

Second, the set of issue resolutions over which the players are bargaining might be effectively discrete rather than continuous. For various technological or political reasons, contested stretches of territory might be effectively indivisible. Deep nationalist attachments, for example, might imply that a government gains only if the whole of some stretch of territory is controlled, whereas controlling a part of it is no better than controlling none at all.[36] If we consider parties bargaining over electoral systems rather states bargaining over territory, then the set of feasible electoral systems could be finite rather than infinite, creating effective indivisibilities.[37]

Formally, suppose $X = \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{k}{n}, \ldots, \frac{n-1}{n}, 1\}$, $n > 1$. Let $x^k = \frac{k}{n}$, and consider the game with the take-it-or-leave-it protocol considered in section 2. It is perfectly possible here to have an issue resolution x^k such that one state is dissatisfied, but the other side prefers fighting to giving up the discrete amount of resources necessary to appease the dissatisfied party. For example, suppose $x_{t-1}^{k-1} \in D_1$ and

$$1 - p(x_{t-1}^{k-1}) - c_2(1-\delta) > (1-\delta)u_2(1-x_t^k) + \delta(1 - p(x_t^k) - c_2(1-\delta)), \tag{4}$$

which says that state 2 prefers fighting at $p(x_{t-1}^{k-1})$ to making the smallest possible discrete concession, x_t^k. War in period t would then be the equilibrium outcome.

An argument that "explains" war by stipulating that no compromise settlements are available says little, but we can go further than this here. Notice that no matter how large n is – no matter how fine we make the smallest possible discrete offer – if state 2 puts enough weight on future payoffs ($\delta \to 1$), then condition (4) holds and war will occur whenever state 1 is dissatisfied. This result stems from the fact that with discrete issue resolutions any concession yields a discontinuous change in a state's future military prospects, which in turn determines how the state fares in future bargaining. As δ approaches one, the long-run consequences of any fixed concession are not worth it, even if the concession is very small. Put differently, if state leaders care strongly enough about their future territory or resources, and if resources are for whatever reason not infinitely divisible, then they may reject "appeasement" in preference for war.[38]

It follows that if that the set of feasible issue resolutions is finite, then for large enough δ, equilibrium implies immediate war for any $x_0 \in D_1$ or $x_0 \in D_2$, and a "frozen" status quo for x_0 anywhere else. Thus, for functions $p(x)$ like those considered in the theorem, (1) rough "balances of power" will be stable, both in the sense that no war occurs and territory does not change hands; (2) moderate imbalances of power will make for war; and (3) extreme imbalances of power will be stable and peaceful. This will be true under either offensive dominance or defense dominance, with the only difference being that under offense dominance the larger states will press the demands that lead to war, while under defense dominance the smaller states will appear as the "aggressors" or "revisionists."

The final condition that can make for war in the game's equilibrium – and possibly the most interesting – is a discontinuity in $p(x)$. Suppose that $p(x)$ is discontinuous at $\hat{x}$, with

$$\lim_{x \to \hat{x}^+} p(x) \equiv p^+(\hat{x}) > p(\hat{x}) = \lim_{x \to \hat{x}^-} p(x).$$

Assume that state 1 is dissatisfied at $\hat{x}$, and that

$$1 - p(\hat{x}) - c_2(1-\delta) > (1-\delta)u_2(1-\hat{x}) + \delta(1 - p^+(\hat{x}) - c_2(1-\delta)),$$

[36] Wagner (1996, 3) offers the example of states bargaining over the possession of set of islands, where the ocean space between them has no value, and for technological reasons it is not feasible to share control of a single island.

[37] For a discussion of issue indivisibilities as a source of conflict, see Fearon 1995b.

[38] This result does not depend at all on the assumption in the current set up that state 2's time-average costs for war go to zero as δ approaches one (see footnote 23).

which necessarily holds for δ sufficiently close to one. Then an inefficient war is the equilibrium outcome of any sequence involving $\hat{x}$.

In this case, the states in effect face a problem of credible commitment brought about by the discontinuity in $p(x)$. State 1 would ideally like to avoid war by committing itself not to exploit the jump in bargaining leverage it gets if state 2 yields any resources beyond the division $\hat{x}$. If it could do this, then both sides would be made better off by a peaceful path of demands in which state 1 demands less than it can get away with after state 2 makes a concession $x > \hat{x}$. But both sides know that state 1 will have every incentive to exploit the increased leverage it would get from the jump in military power, and the fear of this can lead state 2 to prefer war to making any concessions whatsoever.

As in the case where the issues in dispute are not infinitely divisible, this commitment problem can only operate to produce war for resource divisions such that one of the two states is dissatisfied. Thus, once again, if the two states are equally efficient at generating military power from a given amount of resources, then war can result from this mechanism only when power is moderately out of balance (i.e., $x_0 \in D_1$ or $x_0 \in D_2$).

These results and the logic behind them are interesting in light of a long tradition of argument on the relationship between relative power and the likelihood of interstate war. For several hundred years, diplomats and state leaders have commonly maintained that imbalances of power cause wars. The classical argument has been that if a state grows too strong, disturbing the balance, it will be tempted towards aggression against neighboring states, who will resist. By itself, however, this argument is either wrong or incomplete, as a number of scholars have pointed out. Georg Simmel (1906), A.F.K. Organski (1958), and Geoffrey Blainey (1973) have all argued that war is more likely when relative power is relatively *balanced*, on the grounds that imbalances will lead the weak to concede peacefully to the strong while a rough balance fosters miscalculation.[39] Others have considered simple bargaining models in which the risk of war is independent or largely insensitive to changes in relative power, because while stronger parties demand more, weaker parties are correspondingly more willing to make concessions.[40]

The results given above suggests a theoretically coherent argument that fills out the classical intuition. The counterarguments to the proposition "imbalances of power foster war" ask, in effect, "why shouldn't the weaker party offer concessions that would leave both sides better off than fighting?" For conventional bargaining problems where the objects in dispute are *not* determinants of future bargaining power, this is a paralyzing objection. But if the objects in dispute do influence future bargaining power, then the commitment problem described above may operate in some circumstances. If a state would suffer a discontinuously large drop in bargaining leverage for making a small territorial concession (for example, over "strategic territory," such as the Golan Heights), then it might prefer fighting now to appeasement leading to extortion or elimination later. The problem would not operate if the dissatisfied state could commit to restrain its demands once it gains a small territorial concession. This full logic is not comprehended in traditional discussions of the relationship between the balance of power and war.[41]

[39] Thomas Hobbes (1985[1651], 200) had already suggested this in the 17th century: "in the condition of meer Nature, the inequality of Power is not discerned but by the event of Battell." For Hobbes, peace is assured only by a tremendous inequality of power, hence his desire for an all-powerful sovereign. For John Locke (1980[1690], chap. 3), by contrast, inequalities of power are dangerous and a balance conduces to peace; he views a great imbalance as tantamount to a state of war, because the weaker party is hopelessly insecure and cannot know what the stronger intends to do with him.

[40] See Wittman (1979) for the observation, and Fearon (1992) for a take-it-or-leave-it bargaining model with incomplete information in which the probability of war does not vary with relative power. Powell (1996) considers a model in which both sides must agree to any revisions in the status quo, and shows that in this case war is most likely when power shifts so that relative power does not match the existing distribution of benefits or resources. See Wagner (1994) for an excellent informal statement and analysis of the various arguments on how relative power is related to the risk of violent conflict.

[41] To my knowledge, R. Harrison Wagner (1994, 598) is the first to have thought about the problem in this way. The problem is also discussed in Fearon (1995b, 408-9). Goemans (1995, chap. 2) argues that this commitment problem is an important obstacle to the conclusion of wars once they have started.

It remains, however, to provide a substantive interpretation for a discontinuity in the function $p(x)$. In the introduction I suggested that geographic barriers such as mountains and rivers might have this effect, as could any system of fixed border fortifications. Alternatively, discontinuities might arise for "strategic" territory, meaning territory that is positioned or shaped in such a way as to confer large military advantages on whoever controls it.

To give an extreme example, imagine two states with "capitol cities" at $x = 0$ and $x = 1$, and a river at $x = 1/2$. Suppose that controlling all territory on the same side of the river as one's capitol provides a significant defensive advantage, whereas losing control of one's own bank allows the other side a rapid and successful offensive conquest. Thus, let $p(x) = 0$ for $x \in [0, 1/2)$, $p(1/2) \in [1/2, 1)$, and $p(x) = 1$ for $x \in (1/2, 1]$. If the two states in such an example were not equally efficient at producing military power from a given amount of resources – for instance, suppose $p(1/2) > 1/2$ – then it is easily shown that state 1 can prefer war to a status quo of $x_0 = 1/2$, but the inability to commit itself to limit its demands once $x_1 > 1/2$ can make for war. Thus, a shift in relative power from $p(1/2) \approx 1/2$ to some $p(1/2) > 1/2$ can engender the commitment problem that leads to war.[42]

6 Conclusion

The models analyzed in this paper formalize a type of bargaining problem in which the objects bargained over are themselves determinants of future bargaining power. This type of problem – call it "bargaining over power" – probably appears in a wide range of political and economic settings, but may be most common in politics. It is clearly central to much of the bargaining that takes place between states in international relations, although until Wagner (1994) informal analyses of international relations had never isolated or clearly characterized it.[43] Particularly since Chamberlain caved in to Hitler's demands in the meetings leading up to Munich, dilemmas of appeasement have been much studied historically. But they have never been seen or analyzed as an instance of a particular class of bargaining problems.

Without a model, it is not at all obvious what to expect for such bargaining problems, because plausible partial arguments can be offered to support a variety of contradictory claims. For example, (1) one might predict war as result of the players' inability to commit not to increase demands after growing stronger. Or (2) perhaps the status quo would be highly robust and peace likely, because each side's resolve not to make concessions would be strengthened by the anticipated long-run consequences of doing so. And (3) one might even expect that offense-dominance – greater sensitivity of military odds to changes in resources – would make such bargaining processes *more* rather than less stable. Greater offense dominance might make each side's commitment to "stand firm" more credible by making the long-run consequences of a small concession much worse, for example.

The models considered here are quite useful for assessing such partial arguments and for gaining a more complete theoretical understanding of the nature of this type of bargaining problem. Regarding (1), as we have seen, war due to the players' inability to commit not to increase demands after growing stronger can occur, although this requires both that power be somewhat imbalanced and that a change in resources produces a discontinously large change in military prospects. Otherwise, the bargaining in the model will be efficient in equilibrium. War does not occur, while gradual appeasement does (except at stable territorial divisions). Appeasement occurs because a state can typically exploit an opponent's

[42] A second possibility source of discontinuities in $p(x)$ (suggested to me by Harrison Wagner) is that people simply do not think in terms of continuous subjective probability functions. They may think, "if we have this much, we have a chance, but if we concede any more, we would surely be crushed in a war." If so, then the mechanism described here could operate to yield war.

[43] See also Fearon (1995b) and Goemans (1995, chapter 2)

fear of the costs of war and discounting of future payoffs, provided that it does not demand too much. In other words, salami tactics generally work, because a state cannot credibly threaten to go to war over a small enough piece of salami.

Regarding partial argument (2), it is in one sense correct to say that the long-run consequences of conceding a power-generating resource stiffens a players' resolve not to make concessions: When bargaining over an object that influences future bargaining power, players are willing to reject offers that they surely would accept if not for the impact on future power. Consider the risk-neutral case where $u_2(1-x) = 1-x$. Suppose the states' relative power is exogenously fixed at $p = x_0$ and is wholly independent of the object x in dispute – that is, in this case the states are bargaining over an object that does *not* affect relative power. Here, it is straightforward to show that state 1's unique equilibrium demand is $x^* = x_0 + c_2(1-\delta)$ in every period, which is always accepted by state 2. By contrast, when x influences relative power, say, according to $p(x) = x$, state 1's first period demand is $x_1^* = x_0 + c_2(1-\delta)^2$, as shown above. Note that the concession $c_2(1-\delta)^2$ is much smaller than $c_2(1-\delta)$. In fact, the number of periods it will take before state 2 cedes as much as it would if x did not affect relative power is $\frac{c_2(1-\delta)}{c_2(1-\delta)^2}$, or $\frac{1}{1-\delta}$, which approaches infinity as δ approaches 1. Thus the future implications of ceding objects that influence relative power *do* stiffen a player's resolve not to make large concessions, and result in much smaller demands than would otherwise be the case. However, the status quo is not thereby frozen, at least not where players can "slice the salami" in fine enough slices.

Regarding intuition (3), we have seen that the reverse obtains – greater offensive dominance favors the elimination of smaller states through gradual appeasement rather than a freezing of the status quo. States in general cannot credibly threaten to reject small demands by dissatisfied states, and under offense dominance this implies that they will be left in a position where they are even more subject to extortion, since a small concession more than proportionately increases the strength of the party receiving it. By contrast, under defense dominance power equalizes rather than accumulating as concessions are made, leading to a stable balance.

While I think the main benefits of the paper's analysis are theoretical – the specification of a significant and unstudied class of bargaining problems and an investigation of their strategic dynamics – at least one very broad-gauged empirical implication can be drawn for the special case of states bargaining over territory. The theoretical results would predict a long-term trend in favor of increasing concentration in the international system, as periods of offense dominance in military technology will tend to favor the elimination of smaller states and perhaps the consolidation of larger federations. Periods of defense dominance could see a counter-trend towards deconcentration in the form of successful secessionist movements – note that defense dominance means better prospects for guerrilla and other types of warfare of the "weak". However, if states are better able to prevent disintegration under defense dominance than they are conquest under offense dominance, then a long-term alternation of defense- and offense-dominance would produce, in the model, a trend towards a smaller number of larger, more or less equally sized units.[44] This is consistent with some broad-gauged historical interpretations of the long-run trend towards concentration in the European states system, some of which have located the major causes in periods of offense-dominant military technology (e.g., Wright 1965, Osgood and Tucker 1967). If, as many have argued, the nuclear revolution makes for a condition of defense dominance at least among the major powers (e.g., Jervis 1978 and 1989), then the results here would imply greater pressures towards disintegration of existing states and federations rather than towards consolidation.

[44]Of course, this is taking some liberty in interpreting the model, since it represents a world of two states rather than many, and more complicated coalitional dynamics may result with more than two states. See, for example, Niou, Ordeshook, and Rose 1989.

7 Appendix

Proof of Proposition 1

The first step is to show that state 2 cannot credibly threaten to reject any $x_t < x_t^*(x_{t-1})$ in any SGP equilibrium after any history. Fix an equilibrium and an h_t, and suppose state 2 is offered $x_t < x_t^*(x_{t-1})$. (By the argument in the text $x_t^*(x_{t-1})$ exists and is unique, provided that $p(x)$ is continuous.) State 2 cannot rationally reject x_t if

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 < u_2(1-x_t) + \delta V_2(x_t, h_t).$$

The following argument shows that this inequality must hold. By definition,

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 = u_2(1-x_t^*(x_{t-1})) + \delta\left(\frac{1-p(x_t^*(x_{t-1}))}{1-\delta} - c_2\right).$$

Because $u_2(1-x_t) + \delta(\frac{1-p(x_t)}{1-\delta} - c_2)$ strictly increases as x_t decreases, it follows that

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 < u_2(1-x_t) + \delta\left(\frac{1-p(x_t)}{1-\delta} - c_2\right)$$

for $x_t < x_t^*(x_{t-1})$. Finally, since state 2 can always choose to fight in period $t+1$, in any SGP equilibrium $V_2(x_t, h_t) \geq \frac{1-p(x_t)}{1-\delta} - c_2$, which implies that

$$\frac{1-p(x_{t-1})}{1-\delta} - c_2 < u_2(1-x_t) + \delta\left(\frac{1-p(x_t)}{1-\delta} - c_2\right) \leq u_2(1-x_t) + \delta V_2(x_t, h_t).$$

This demonstrates that state 2 will always accept $x_t < x_t^*(x_{t-1})$ in any SGP equilibrium after any history.

It follows that if state 1 wishes, it can guarantee acceptance in each period from t forward by setting $x_t = x_t^*(x_{t-1})$.[45] This implies that $V_2(x_t, h_t)$ is at most equal to $\frac{1-p(x_t)}{1-\delta} - c_2$, which implies further that in any SGP equilibrium state 2 will after any history reject any $x_t > x_t^*(x_{t-1})$, since $f(x_t)$ (defined as the central expression in last set of inequalities above) is decreasing in x_t. This establishes that state 2 has the unique SGP equilibrium strategy given in the Proposition.

Given 2's strategy, if state 1 does not want war in period t then the best it can do to maximize $u_1(x_t)$ is to set $x_t = x_t^*(x_{t-1})$ (since $u_1(\cdot)$ is strictly increasing). It now remains to show that state 1 would want to do so. Notice that by 2's strategy, state 1 can assure rejection and a payoff of $\frac{p(x_{t-1})}{1-\delta} - c_1$ by setting $x_t > x_t^*(x_{t-1})$ when $x_t^*(x_{t-1}) < 1$. (If $x_t^*(x_{t-1}) = 1$ then by setting $x_t = 1$ state 1 gets $1/(1-\delta)$, its largest possible payoff.) Thus state 1's payoff for choosing $x_t = x_t^*(x_{t-1})$ is at least $u_1(x_t^*(x_{t-1})) + \delta(\frac{p(x_t^*(x_{t-1}))}{1-\delta} - c_1)$. If we can show that this is greater than state 1's payoff for fighting in period t, $\frac{p(x_{t-1})}{1-\delta} - c_1$, we will have shown that 1's best reply is to set $x_t = x_t^*(x_{t-1})$.

Equation (2) can be manipulated as follows:

$$\begin{aligned}
\frac{1}{1-\delta} - \frac{p(x_{t-1})}{1-\delta} &= u_2(1-x_t^*) + \frac{\delta(1-p(x_t^*))}{1-\delta} + c_2(1-\delta) \\
\frac{p(x_{t-1})}{1-\delta} &= 1 - u_2(1-x_t^*) - c_2(1-\delta) + \frac{\delta p(x_t^*)}{1-\delta}
\end{aligned}$$

[45] By the usual logic in Rubinstein-like bargaining games with an infinitely divisible 'pie,' equilibrium requires that the receiver of a proposal be exactly indifferent between accepting and rejecting an offer and so willing to accept with probability one.

So the question is whether

$$1 - u_2(1 - x_t^*) - c_2(1-\delta) + \frac{\delta p(x_t^*)}{1-\delta} - c_1 < u_1(x_t^*) + \delta(\frac{p(x_t^*)}{1-\delta} - c_1).$$

Simplifying yields $1 - (c_1 + c_2)(1-\delta) < u_1(x_t^*) + u_2(1 - x_t^*)$, which must be true since c_1 and c_2 are both greater than zero and risk-neutrality/risk-aversion implies that $u_1(x_t^*) \geq x_t^*$ and $u_2(1 - x_t^*) \geq 1 - x_t^*$.

QED.

Proof of the Lemma

Consider the case of state 1. Suppose to the contrary that we can have state 1 dissatisfied at x_i and $x_{i+1} \leq x_i$ for some h_t^*, where $1 \leq i < t$. Monotonicity implies that for all $j \geq i$, state 1's equilibrium payoff from period i forward is $\sum_{j=i}^{t} u_1(x_j)\delta^{j-i}$ if either $t = \infty$ or 1 is eliminated in period $t < \infty$. (Note that state 2 cannot be eliminated since the hypothesized sequence is monotone decreasing.) Call this case A. If there is a war in period t (call this case B), then state 1's payoff is

$$\sum_{j=i}^{t-1} u_1(x_j)\delta^{j-i} + \delta^{t-i}\left(\frac{p(x_{t-1})}{1-\delta} - c_1\right).$$

By contrast, if state 1 fights in period i, it receives $\frac{p(x_{i-1})}{1-\delta} - c_1$ in either case. For case A, we have

$$\frac{p(x_{i-1})}{1-\delta} - c_1 \geq \frac{p(x_i)}{1-\delta} - c_1 > \frac{u_1(x_i)}{1-\delta},$$

where the first inequality holds by monotonicity and the second the fact that state 1 is presumed to be dissatisfied at x_i. Notice, however, that

$$\frac{u_1(x_i)}{1-\delta} = \sum_{j=i}^{\infty} u_1(x_i)\delta^{j-i} \geq \sum_{j=i}^{t} u_1(x_j)\delta^{j-i},$$

since monotonicity and $u_1(x_i) > 0$ imply that the left-hand sum is weakly greater than the right-hand sum term-by-term. Thus state 1 would choose to fight in period i, contradicting the assumption that $i < t$.

For case B, rewrite state 1's payoff for fighting at x_i, $\frac{p(x_{i-1})}{1-\delta} - c_1$, as

$$\sum_{j=i}^{\infty}(p(x_{i-1}) - c_1(1-\delta))\delta^{j-i},$$

which is greater term-by-term than 1's proposed equilibrium path payoff

$$\sum_{j=1}^{t-1} u_1(x_j)\delta^{j-i} + \delta^{t-i}(\frac{p(x_{t-1})}{1-\delta} - c_1) = \sum_{j=1}^{t-1} u_1(x_j)\delta^{j-i} + \sum_{j=t}^{\infty}(p(x_t) - c_1(1-\delta))\delta^{j-i}.$$

Thus state 1 would choose to fight at x_i in either case, so the contrary cannot hold.

Q.E.D.

Proof of the Theorem

To see part (a)(i), consider three cases. First, if $x_0 \in D_2$ then by the Lemma all monotone equilibrium sequences beginning with x_0 must be decreasing, which implies that state 2 cannot be eliminated. For state 1 to be eliminated would then require either that there is some division $x_i \in D_1$ in an equilibrium sequence, which would contradict the Lemma, or the sequence "skips over" divisions $x_i \in D_1$ and proceeds to the elimination of state 1. But this is impossible, by the following argument. If it were true, then there must be some i such that $x_i \geq \bar{d}_1$ and $x_{i+1} \leq \underline{d}_1$. Further, peacefulness and subgame perfection require that

$$\frac{p(x_i)}{1-\delta} - c_1 \leq u_1(x_{i+1}) + \delta u_1(x_{i+2}) + \ldots + 0 + 0 + \ldots.$$

Choose any $x' \in D_1$, and note that $x' > x_{i+1}$. Thus

$$u_1(x_{i+1}) + \sum_{j=i+2}^{\infty} u_1(x_j)\delta^{j-i-1} < u_1(x') + \sum_{j+2}^{\infty} u_1(x_j)\delta^{j-i-1}.$$

But by the definition of D_1 the right-hand side is strictly less than $\frac{p(x')}{1-\delta} - c_1$, which entails a contradition since this is strictly less than $\frac{p(x_i)}{1-\delta} - c_1$.

This argument takes care of cases in which $x_0 \in D_2$, and an exactly parallel argument works for $x_0 \in D_1$. Suppose, finally that $x_0 \in [\bar{d}_1, \underline{d}_2]$. Once again, a monotone equilibrium sequence leading to the elimination of either state would either have to have some division $x_i \in D_1$ or $x_i \in D_2$, implying a contradiction of the Lemma, or would have to "skip over" either D_1 or D_2, in which case the argument used above applies. This proves (a)(i).

For part (a)(ii), there are again three cases. In the easiest case, if $x_0 \in [\bar{d}_1, \underline{d}_2]$, then a monotone equilibrium sequence cannot have any $x_i \in D_1$ or $x_i \in D_2$, since this would contradict either the Lemma (no decreasing sequences through D_1, no increasing sequences through D_2), or monotonicity (a monotonic sequence could not leave $(\bar{d}_1, \underline{d}_2)$ and then return to it). Further, by the argument given for (a)(i) above, we could not have a monotone sequence that "skipped over" either D_1 or D_2. Thus all efficient monotone $h_t^* \in H^*$ with $x_0 \in [\bar{d}_1, \underline{d}_2]$ must have a limit in $[\bar{d}_1, \underline{d}_2]$, since all bounded monotone sequences have limits.

Now consider cases where $x_0 \in D_1$. The Lemma implies that all monotone equilibrium sequences with $x_0 \in D_1$ must be increasing, so it will suffice to show that no such peaceful sequence can have a limit $z < \bar{d}_1$. (A limit $z > \underline{d}_2$ would contradict the Lemma or a "no skipping" argument.) Suppose, then, to the contrary there is such a limit $z < \bar{d}_1$. Peacefullness (efficiency) implies that for all $i > 0$,

$$\frac{p(x_i)}{1-\delta} - c_i \leq \sum_{j=i+1}^{\infty} u_1(x_j)\delta^{j-i-1}.$$

And since z bounds the sequence from above, the right-hand side of this inequality must be weakly less than $\frac{u_1(z)}{1-\delta}$, which in turn must be strictly less than $\frac{p(z)}{1-\delta} - c_1$ by the fact that $z \in D_1$ and the nature of dissatisfaction. Putting all this together yields

$$\frac{p(x_i)}{1-\delta} - c_i \leq \sum_{j=i+1}^{\infty} u_1(x_j)\delta^{j-i-1} \leq \frac{u_1(z)}{1-\delta} < \frac{p(z)}{1-\delta} - c_1.$$

But this cannot hold for all i, because

$$\lim_{i\to\infty} \frac{p(x_i)}{1-\delta} - c_1 = \frac{p(z)}{1-\delta} - c_1,$$

which implies that it cannot be the case that $\frac{p(x_i)}{1-\delta} - c_1 \leq \frac{u_1(z)}{1-\delta} < \frac{p(z)}{1-\delta} - c_1$ for all i, a contradiction. A parallel argument establishes that from an $x_0 \in D_2$ an efficient monotone equilibrium sequence cannot have a limit $z \in D_2$, which suffices to prove part (a)(ii).

Part (a)(iii) follows straightforwardly from (a)(ii) and the implied effects on D_1 and D_2 in the case where both states are risk neutral.

Parts (b)(i) and (b)(ii) follow directly from the Lemma and from an argument, exactly parallel to that given in the proof of (a)(ii) above, to the effect that it cannot be that an efficient monotonic sequence approaches a limiting division $x^* \in D_1$ or $x^* \in D_2$. Part (b)(iii) follows from (b)(i) and (b)(ii) and the implied effects on D_1 and D_2 in the case where both states are risk neutral.

Q.E.D.

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