CHUNKY SHOES

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Abstract

Imagine a large population of individuals who will be drawn at random, one by one, to make a choice from a set of styles of consumer goods (such as shoes). Assume that individuals vary continuously in their disposition to "stand out from" or "blend in with the crowd," meaning that each has preferences over the number of others wearing the style she currently wears. More nonconformist types want to wear a style that relatively few others wear; more conformist types want to wear relatively popular styles. I characterize the dynamically stable equilibria of this process, and show that fashion trends can occur with the introduction or innovation of a new style. New styles are adopted first by relative nonconformists, but as they grow in popularity they become increasingly desirable for "wanna be's" and later for conformists – under certain conditions, the process involves "tipping" in the sense of Schelling (1980). Fashion cycles can occur if people have idiosyncratic style preferences in addition to preferences over conformity and distinction. The consequences of free choice of styles for social welfare are analyzed, along with the effects of price competition. Finally, the model and analysis suggest an explanation for "image" advertising that makes little or no reference to the price or intrinsic qualities of a product. Such advertising may serve as a public device for coordinating beliefs about the social meaning or signal sent by a product, assuring targeted consumers that the "right type" of others will be buying it.

1 Introduction

As little as five years ago, a woman who chose to wear shoes with thick, chunky heels was making a definite fashion statement. Few other woman were wearing this style at the time, so chunky shoes stood out. In addition, the style suggested an early 70s look, which had been discarded (and reviled) for some time. Now, in 1998, a young woman who wears chunky shoes hardly stands out; these are almost *de rigeur* on many college campuses. It is reasonable to expect that sooner or later, a new style that currently "stands out" will take over from chunky shoes, and the cycle will repeat. Such fashion cycles are observed not only for a great variety consumer goods. Similar patterns can obtain for almost any product or behavior thought to connote or express personal style – for example, trends in the popularity of bars, niteclubs, restaurants, neighborhoods, styles of music, particular rock bands, or intellectual fashions in academia.

Matters of fashion and style are of tremendous importance in modern economies and societies. Firms spend billions of dollars each year advertising not primarily by announcing the prices and intrinsic qualities of their goods, but rather by sending the implicit message that "Buying this product makes you a member of such-and-such a group, and you may (or should) desire to be a part of this group." In other words, firms understand that consumers see owning many products as expressing or signaling something about "who they are" – that is, about their social or personal identity – and further that consumers have preferences over what they so express. The processes by which consumer goods and, more generally, styles of behavior or speech, acquire these social meanings is highly complex. Firms may attempt to define the social meaning of a product through advertising, but success requires the collective and coordinated assent of both buyers and nonbuyers. (Witness, for example, the Chrysler ad campaign pleading that "It's Not Your Father's Oldsmobile.") And many products appear to acquire the social meanings that determine their appeal not through any coordinated effort by a particular firm or industry, but rather through more decentralized processes of social interaction and imitation.[1]

[1]I believe the increasing popularity of chunky shoes is a case in point; see also chokers, tatoos, body piercing, hennaed hair, and trends in the shape of "cool," like grunge music. A *New York Times* article recently documented the increasing use by market research firms of "cool hunters" who are employed to spot trends emerging beyond the grip of advertising. "The hunters prowl inner-city basketball courts, fashionable nightclubs and houses in the Hamptons observing the arbiters of coolness whose tastes may eventually be adopted by the general populace. ... The firms say their hunters have a taste level that is avant-garde but not

No one individual (or at least, very few individuals) can unilaterally dictate the social meaning of a product or behavior, like the "hipness" of chunky shoes at a particular moment. Rather the social meaning or significance of particular goods and behaviors emerges out of and is fixed by multilateral social coordination and agreement.

Fashion trends and cycles are important examples of processes that determine the social or expressive meanings of goods or behaviors. This paper formally develops the implications of a simple intuition about a mechanism that may lie behind at least some fashion trends. Suppose that individuals vary in their preference over how much they stand out from or blend in with the crowd, or in the degree to which they want to be considered unusual or hip as opposed to normal. If such variation is continuous in a population, then many goods and behaviors may be subject to tipping processes like those described by Schelling (1980). When a new style is introduced, it will be taken up initially by those with the strongest preference for standing out. But as more people wear the new style, more "thresholds" are crossed and more buy the good, until it ceases to be hip or unusual to wear the style. This ultimately drives the nonconformists to seek a new style with which to distinguish themselves.

Formally, I will suppose that buying or expressing a particular style effectively makes one a member of a group – chunky shoe wearers, sneaker wearers, Coke drinkers, sport utility vehicle owners, Tiki Room patrons, Sonic Youth fans, Marxists, etc. – and that every individual has a most preferred style group size that he or she tries to attain when choosing among styles. Relatively "nonconformist" types prefer smaller style groups, while relatively conformist types prefer to blend in with the more popular styles.

I consider equilibria of the following dynamic process: In each successive moment, an individual is drawn at random to choose a style. The individual (myopically) chooses the style that puts him in the group whose current size is closest to his ideal group size. I show that different initial distributions of preferred group sizes yield essentially two types of dynamically stable equilibrium style distributions. If society has enough "conformist" types (who ideally want to be in a majority), then there will be a large conformist or "normal" style, along with a succession of style groups that are progressively smaller and more "different" or "radical." By contrast, if society has enough nonconformist types,

so far out that it won't become mainstream." Roy Furchgott, "For Cool Hunters, Tomorrow's Trend Is the Trophy," 28 June 1998.

then all style groups will be the same size in equilibrium.

With the introduction or innovation of a new style, a fashion trend occurs, in the sense that relative nonconformists increasingly adopt the new style as a means of "standing out." The trend entails tipping, however, only if there are individuals with a particular sort of asymmetry in their preferences for conformity and distinction – they must feel more discomfort from standing out than from blending in more than they would like. Such individuals will initially spurn a new style when it is too "radical," but are willing to buy when it becomes sufficiently popular.

Still, in the model as described a fashion trends cannot lead to a new style taking over in popularity from an old one, or to fashion cycles where two or more styles go "in" and "out" over time. I show that stochastic take-overs and fashion cycles can occur if we assume that individuals also have random aesthetic or other preferences for particular styles. The intuition is that as more marginal types idiosyncratically adopt the currently less popular style, more serious conformists become willing to consider wearing the less popular style on aesthetic grounds, which can then lead to a "take over" by this style. An implication of the model is that most of cycling between styles will take place in the minority of the population that wants to stand out (relative to the majority). Only rarely will a new style tip so as to change the majority choice.

Finally, I provide some limited results on social welfare and the effects of price competition. Analysis of examples with two and three styles suggests that the free choice of styles suboptimally frustrates a somewhat ironic common interest of the most extreme conformists and nonconformists in a society. The more extreme nonconformists don't want their hip styles or practices to spread, and neither do the most conformist types! With the free choice of styles, however, "wanna be's" tend to invade the hipsters styles, to the consternation of both hipsters, who want a smaller, more exclusive style group, and conformists, who want a larger one.

Price competition appears to worsen matters, as the producer of the more popular (conformist) brand chooses a higher equilibrium price in order to take to advantage of a larger consumer base. In the two-style case considered below, the effect is to create more wanna-bes who adopt the less popular style.

Section 2 compares the approach here to existing models of fashion and fashion cycles. Section 3 introduces the model, the equilibrium concept, and an analysis. Section 4 develops some specific examples of different sorts of equilibrium style configurations. Section 5 considers social welfare implications, and section 6 looks the effects of price competition in the two-style case. Section 7 shows how fashion cycles can arise, and section 8 develops an explanation for "image" advertising based on the model.

2 Related work on fashion and fashion cycles

A number of papers have considered models in which people have preferences over the number of others purchasing or using a good. Leibenstein (1950) introduced the idea of "snob" and "bandwagon" effects that occur when individuals prefer (respectively) smaller and larger numbers of consumers of the same item. Becker (1991) and Karni and Levin (1994) look at pricing for restaurants and other social events under the assumption that a person's utility can increase with the number of other diners, concert-goers, etc. Some of the agents in Karni and Schmeidler's (1990) model of fashion cycles just want to wear what the largest number of other agents are wearing. Relatedly, the literature on network externalities considers goods whose value increases with number buying the same type, such as television, computers or VCRs (e.g., Katz and Shapiro 1986).

These papers do not allow for a continuum of types with preferences over different-sized style groups, and with the exception of Karni and Schmeidler, none model fashion trends or cycles. A distinctive feature of the model here (I think) is the continuum of types, which is precisely what gives rise to the possibility of a fashion cycle as a Schelling-esque tipping process.

The logic that generates fashion cycles in Karni and Schmeidler (1990) is rather different. Similar to a "matching pennies" dynamic, their "upper class" types want to wear what other upper class types are wearing, while avoiding what the lower classes wear. The lower class wants to wear what the largest number of others wear, so they may end up chasing the upper class from one style to the next for certain parameter values. This strikes me as a plausible explanation for some fashion trends and cycles, such as the development of progressively refined etiquettes among the European nobility (Elias 1982), or the tendency of white teenagers to "chase" black styles (from jeans and blues to baggy jeans and rap, for example). But the argument seems to depend on the assumption that "types" can be distinguished independently of what they are wearing – class must be ascertainable independently of style in order for the upper class to know what to imitate and what to avoid. I would claim that it is often what one wears that makes one "cool," not who one is that makes the clothes (etc.) cool.

Some more recent work on fashion follows Veblen's argument that fashion is a means by which certain "good" types signal this credibly, in the presence of incentives for bad types to mimic the good. Thus, Pesendorfer (1995) imagines a matching game with "good" mates and "bad" mates, where the good ones use high-priced fashions to signal their quality. (Bad types are assumed to care relatively less about mate quality on whatever the relevant dimension is, so they are willing to buy cheaper styles.) Fashion cycles are then explained as a consequence of price discrimination. Firms introduce new designs at high prices to sell to the high valuation "good" types first, and then later flood the market to sell to the bad types. When the market is flooded, the good types need a new design to distinguish themselves, and the next cycle begins.

This is a clever argument and I think it has merit in explaining some fashion trends and cycles, especially those involving the "high fashion" of runways and designer houses. But fashion trends and cycles are ubiquitous in what might be called "low fashion" as well – trends of chokers, chunky shoes, tatoos, grunge music, and so on, would be hard to explain as effects of price discrimination. Relatedly, one might question the assumption that whatever it is that high fashion signals, it is something that everyone wants to be thought to possess.[2] The individuals in the model developed here can be interpreted as choosing styles in order to send signals about their types, as in the Veblen and Pesendorfer arguments. For instance, they may want to "stand out" or "blend in" because this is a signal of a personality trait or preference, and they are using publicly observable styles to facilitate meeting mates or friends of a similar type. But in contrast to the Veblen approach, there is no assumption here that everyone wants to be thought as one particular type (e.g., rich). Indeed, the continuum of preferences over conformity and distinction is precisely what allows for the separation of types through decentralized style choices rather than through price discrimination by firms.

[2]It can't be that buying high fashion goods is simply a means of signaling wealth a la Veblen (see also Bernheim and Douglass 199x), since for this purpose the generic Brooks Brothers design will serve just as well as the latest Armani.

3 The model

Imagine a large population from which one individual will be drawn at random every day (or hour, or minute) to choose from an exogenously given set of $k > 1$ styles. Choosing a style might involve purchasing a product, like a pair of shoes, a haircut, or a restaurant, or expressing a style through behavior, like wearing clothes or using a particular vocabulary or accent. Individuals wear or express the style at least until the next time they are drawn to make a new choice (e.g., when the shoes wear out). When making a choice individuals are assumed to know the current shares of the population wearing each style.

Considering the purest case first, I will assume that individuals care only about the relative size of the style group they buy into. They care only about how much they blend in or stand out; they have no idiosyncratic aesthetic or other preferences over the different styles available. In addition, assume for now that all styles have the same price and that individuals demand one unit whenever drawn to make a choice. Formally, each individual has an ideal point $x \in [0, 1]$ that represents his or her most preferred share of the population wearing the same style. For example, a person with ideal point $x = .1$ would ideally like to wear a style worn by exactly 10% of the population, while a person with ideal point $x > .5$ will always prefer the most popular style currently worn. It is natural to refer to individuals with $x > .5$ as *conformists.* At the other end of the spectrum, a person with $x = 0$ is a radical *nonconformist* who wants to wear or express a style that no one else has chosen.

Let $S = \{1, 2, \ldots, k\}$ represent the set of available styles, and s_i the set of types that choose style $i \in S$ in some equilibrium. Let α_i represent the share (measure) of the population choosing style i, where we number the styles in order of increasing popularity in equilibrium (thus, $\alpha_1 \leq \alpha_2 \leq \ldots \leq \alpha_k$). For now, I will assume that an individual's utility for wearing style i decreases symmetrically with the distance of α_i from her ideal point $x \in [0, 1]$.

The convention of labelling styles in order of increasing popularity in equilibrium implies that any equilibrium is unique only up to the naming of the styles. For instance, if there is an equilibrium in which chunky shoes are worn by a few nonconformists and hi-tops by the majority, then there will also be an equilibrium in which the nonconformists wear hi-tops and the majority wears chunky shoes. Historical accidents and path dependence will decide which obtains. Because my focus here is on the relative popularity of different styles in equilibrium and the conditions under which fashion cycles occur, I assume that the styles have no idiosyncratic consequences for individual or social welfare. This is in contrast to the related work on goods with network externalities, where normative concern focuses on the possibility of an inferior product becoming "locked in" (e.g., QWERTY keyboards, or DOS versus Apple operating systems). If one thinks that there are objective aesthetic criteria for evaluating fashion styles (for instance, clogs are just ugly in all contexts) or if one brought in welfare considerations specific to particular styles (for instance, the probability of a twisted ankle), then the "lock in" problem would be relevant here as well.

3.1 Equilibrium concept

For any given distribution of ideal points, we would like to be able to characterize long-run equilibria of the dynamic process wherein individuals are drawn at random to (myopically) choose the style group whose current size is closest to their ideal. We would also like to know what would happen if a new style is introduced or innovated. Will there be a fashion trend? Answering either question requires both a characterization of the distribution of ideal points and an appropriate equilibrium concept.

Let $F(x)$ be the cumulative distribution function that gives the share of the population with ideal points less than or equal to $x \in [0, 1]$. Intuitively, we might consider a vector of group sizes $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_k)$ to be in equilibrium if it satisfied the following property: Given α, no individual in the population would, when drawn to make a choice, change styles. I will refer to an assignment of types to styles as a Nash equilibrium if this condition holds.[3]

For some distributions, however, no such equilibria exist. Suppose that there are two styles available for a population of 100 people, 70 of whom have $x = 0$ and 30 have $x \geq .5$. Unequally sized style groups cannot form an equilibrium by this definition, since this would require that some type $x = 0$ persons wear the more popular style, and they would then want to switch when drawn. But neither would two groups of 50 each be an equilibrium, since a person with $x > .51$ will want to switch to the other group to make of group of 51 versus 49.

[3]This is an abuse of terminology, since the players here are not strategic; but the condition is clearly in the spirit of Nash equilibrium.

One way to address this problem is to consider an infinitely large population represented as a continuum of ideal points on $[0, 1]$. In this case, (.5, .5) will be a Nash equilibrium in the case of two styles for *any* distribution of ideal points, since individuals obtain the same payoff from either style and cannot change a style's share by joining it. But considering Nash equilibria of the continuum case also implies the possibility of "equilibria" that are not equilibria in a finite approximation of the same ideal point distribution. For example, suppose all individuals in the population have $x > .5$. Then for any large but finite and even population size n, two groups of $n/2$ each is not Nash, even though $\alpha = (.5, .5)$ would be Nash in the case of $n = \infty$.

Intuitively, the case of two equal-sized groups with a continuum of conformists is not *dynamically stable* in the sense that a small perturbation of style shares from (.5, .5) would send the system to the more robust equilibrium of (0, 1). These examples suggest that we should introduce a dynamic stability condition like the following: *A size distribution α is dynamically stable if a small perturbation from α tends to return towards α.*

I will provide formal conditions for dynamic stability below. Note for now that the condition also addresses the non-existence of a pure-strategy equilibrium in the example of 100 people with 70 nonconformist types. While the size of the groups would never "fix" at (50, 50), it would always tend to return towards (50, 50) following departures, since the probability of drawing a nonconformist type exceeds the probability of drawing a conformist, and nonconformists will always head for the smaller group.

3.2 General results

We will look, then, for dynamically stable Nash equilibria in the case of a continuum of consumers with ideal points distributed by $F(x)$ on [0, 1]. This section offers results for the general case of $F(x)$ in the form of a series of propositions. Proofs are in the appendix.

Proposition 1. (No unused styles.) If $F(x)$ is strictly increasing for all $x \in [0, 1]$, then all styles are worn by a positive share of the population in any equilibrium (that is, $\alpha_i > 0 \; \forall i \in S$).

Intuitively, if society contains "all types of people" in the sense that there are always some radical nonconformists, these will let no styles go unused. (If we allowed individuals to innovate styles, the implication would be that the most radical nonconformists would be continually innovating new styles.)

Proposition 2. (More popular styles are chosen by more conformist types.) Suppose that in an equilibrium $\alpha_j < \alpha_{j+1}$. Then $x < y$ for all $x \in s_j$, $y \in s_{j+1}$, except for type $z = (\alpha_j + \alpha_{j+1})/2$ who is indifferent between the two styles.

Thus, if there are different sized style groups in some equilibrium, then individuals have self-selected or sorted themselves between these according to their preference for standing out or blending in. The larger of any two groups contains the more conformist types.

Call an equilibrium *strict* if no two style groups are the same size. Proposition 2 implies that we can characterize such an equilibrium (if one exists) by a sequence of cut points $x_1, x_2, x_3, \ldots, x_{k-1}$, where x_j is the type that is indifferent between styles j and $j+1$. Defining $x_0 = 0$ and $x_1 = 1$, the share of style j, α_j, is just $F(x_j) - F(x_{j-1})$. Proposition 3 gives conditions that define the cutpoints of any strict equilibrium and a set of inequalities that must be satisfied for such an equilibrium to exist. The cutpoints represent the types that are indifferent between the immediately smaller and larger style groups; the inequalities ensure that larger groups are populated by more conformist types, as required by Proposition 2. Proposition 4 shows that, somewhat surprisingly, there can be at most one strict equilibrium for any given distribution of preferences over conformity and distinction.

Proposition 3. (Conditions for an equilibrium with different sized style groups.) Let $x_0 = 0$ and $x_k = 1$. If, for a given $F(x)$, there exists a strict equilibrium, then the cutpoints $x = (x_1, x_2, x_3, \ldots, x_{k-1})$ that define it satisfy the following equality constraints for $0 < j < k$,

$$x_j = \frac{F(x_{j+1}) - F(x_j) + F(x_j) - F(x_{j-1})}{2} = \frac{F(x_{j+1}) - F(x_{j-1})}{2}. \tag{1}$$

The following inequality constraints must also be satisfied for $0 < j < k$: $F(x_{j+1}) - F(x_j) > F(x_j) - F(x_{j-1})$, or

$$F(x_j) < \frac{1}{2}(F(x_{j+1}) + F(x_{j-1})). \tag{2}$$

Proposition 4. (At most one equilibrium with different-sized groups). For any given $F(x)$, there can be at most one strict equilibrium.

In the next section I provide examples of ideal point distributions that give rise to strict equilibria. The remainder of this section considers *nonstrict* equilibria in which at least two style groups are the same size. This implies indifference for those that choose one or the other of the equally popular groups, which raises the question of dynamic stability.

Proposition 5 confirms that without a dynamic stability condition, one can support k equal-sized style groups in a Nash equilibrium for any distribution of ideal points.

Proposition 5. If $F(x)$ is strictly increasing on [0, 1] then any allocation of types to k style groups is a Nash equilibrium if all k groups have the same population share (measure) of $1/k$.

The proof is immediate, since if all styles have measure $1/k$ then all types get the same payoff regardless of what style they choose. (Note that it is never possible to support an equilibrium with $j < k$ equal-sized style groups, since by Proposition 1 no styles can go unused in equilibrium).

But as the example given earlier suggested, such nonstrict equilibria may not be robust in the dynamic setting either because they depend on the assumption of a continuum of consumers or because they would disappear with a small perturbation from equality.

Consider the case of $k = 2$ with a Nash equilibrium at $\alpha_1 = \alpha_2 = .5$. Suppose that at some moment in time a perturbation yields the shares $(.5 - \epsilon, .5 + \epsilon)$, where $\epsilon > 0$ is very small. *Dynamic stability requires that the rate of flow into the smaller of the two groups be greater than the rate of flow into the larger.* In a finite approximation to the continuum case, this would imply that the probability of drawing an individual with $x < .5$ must be greater than the probability of drawing a person with $x > .5$. Thus, in the $k = 2$ case dynamic stability requires $F(1/2) > 1/2$.[4]

The condition for dynamic stability for the case of three or more equal-sized groups is trickier. Suppose there are $k \geq 3$ equal-sized groups, and some small perturbation makes group j the largest of the set. Then flow into j will be proportional to $1 - F(1/k)$, since all types $x > 1/k$ will prefer the largest group (along with a negligeable fraction below $1/k$). Flow into the smallest group will *initially* be proportional to $F(1/k)$, but this will quickly make the smallest group equal in size to the second smallest, and the third smallest, and so on. Thus, flow into the smallest groups will be on average proportional to $F(1/k)/(k-1)$. Equality of style sizes will tend to be restored – the equilibrium will be dynamically stable – when $F(1/k)/(k-1) > 1-F(1/k)$, or $F(1/k) > (k-1)/k$.

In words, this says that if there are sufficiently many nonconformist types, we can support a dynamically stable equilibrium in which all styles are worn equally. Equal shares are robust in this case for the same reason that supermarket check-out lines tend to be of roughly the same length – people desert or avoid longer lines in favor of shorter ones. Likewise, if one style happens to become momentarily less popular than others, then nonconformists flock into it till it regains equality. Further, there aren't enough conformist types to make a marginally more popular style grow permanently larger than any other.

Proposition 6 goes further, establishing that if $F(1/k) > (k-1)/k$, then the *unique* dynamically stable equilibrium is one where all groups are the same size.

Proposition 6. When $F(1/k) > (k-1)/k$, a style distribution is a dynamically stable equilibrium if and only if all groups are of size $1/k$.

Proposition 6 implies that, for any given distribution of ideal points, either all styles are equally patronized or some styles are more popular than others – multiple dynamically stable equilibria involving both possibilities cannot occur.

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[4]In the continuum case, I guess this is done in terms of rates of flow? Flow into the smaller of the perturbed groups is proportional to $F(1/2)$, which must be greater than the flow into the larger, which is proportional to $1 - F(1/2)$?

Another implication is that the greater the number of available styles to choose from, the greater the density of radical nonconformists is necessary for an equilibrium with equal-sized groups to be supportable. This implication may be stated more formally as follows.

Corollary to Proposition 6. For any given distribution of ideal points $F(x)$ with full support on $[0,1]$, there exists a number $\bar{k}$ such for all $k \geq \bar{k}$, a dynamically stable equilibrium entails that some styles are strictly more popular than others. (That is, there are relatively "conformist" and "nonconformist" styles.)

Propositions 5 and 6 raise the question of whether there is a unique dynamically stable equilibrium for any given distribution of ideal points in the population. Note that this has *not* already been proved. There are many more possible configurations besides a strict equilibrium on the one hand, and an equilibrium with k equal-sized groups on the other. With three styles, for example, there might be an equilibrium with two equal-sized small styles and one larger style. Or there might be one with two equal-sized large styles, and one small style.[5] With k styles there are 2^{k-1} possible configurations in this sense, because each group s_i must either be equal in size or strictly larger than the one before it (by Proposition 2).

To answer the question about multiple equilibria and also for sake of completeness, I should state the dynamic stability condition more generally. Suppose that there are $l > 1$ styles of the same size in some equilibrium. By Proposition 2, it must be that $\alpha_{j-1} < \alpha_j = \alpha_{j+1} = \alpha_{j+2} = \ldots = \alpha_{j+l-1} < \alpha_{j+l} \ldots$, where $\alpha_j, \alpha_{j+1}, \ldots, \alpha_{j+l-1}$ denote the l equal sized groups. Proposition 2 further implies that the size of each of these groups is $\bar{\alpha} \equiv (F(x_{j+l-1}) - F(x_{j-1}))/l$. Dynamic stability requires that for each such set of equal sized groups,

$$\frac{F(\bar{\alpha}) - F(x_{j-1})}{l-1} > F(x_{j+l-l}) - F(\bar{\alpha}),$$

or

$$F(\bar{\alpha}) > \frac{(l-1)F(x_{j+l-1}) + F(x_{j-1})}{l}. \tag{3}$$

[5]A simple ideal point distribution that yields the former is 40% of the population with $x < .4$ and 60% with $x > .5$; for the latter, a population with 10% at $x < .2$ and 90% at $x = .45$.

(Note that this yields the condition given earlier for the case of $k = l$, $j = 1$.)

Returning to the question of uniqueness, multiple dynamically stable equilibria are in fact possible, although the conditions under which these can occur appear to be rather limited. It is trivial to show with two styles there is a unique set of equilibrium style shares (see the next section). But multiple equilibria for a given $F(x)$ may occur when there are more than two styles on the market. To get an intuition for how this is possible, consider $k = 3$ and a society in which every person has ideal point $x = .3$. Then both $(0, .5, .5)$ and $(1/3, 1/3, 1/3)$ are dynamically stable equilibria. In the first case, which is Pareto inefficient, no one wants to "defect" to the unused style because he or she would be the only one wearing it, and this would entail standing out too much ($|.3-0| > |.3-.5|$).

Having an equilibrium with an unused style – $(0, .5, .5)$ – depends on assuming an ideal point distribution that does not have full support (i.e., "all types of people"). But the multiple equilibria result does not. Consider the general conditions for a $k = 3$ equilibrium in which there is a nonconformist style and two larger, equal-sized conformist styles. Using Proposition 2, such an equilibrium will be defined by a cutpoint x_1 such that type x_1 is indifferent between the nonconformist style that has share $F(x_1)$, and the conformist styles that each have share $(1 - F(x_1))/2$. This implies the equality condition

$$x_1 = \frac{F(x_1) + \frac{1-F(x_1)}{2}}{2} = \frac{1}{4}(1 + F(x_1)).$$

Since the right-hand side is strictly increasing and varies between 1/4 and 1/2 for $x_1 \in [0, 1]$, it is entirely possible for this condition to be satisfied by more than one value of x_1 for a given $F(x)$. Moreover, nothing prevents multiple such values from satisfying the dynamic stability condition (from above, with $j = 2$ and $l = 2$)

$$F(\frac{1 - F(x_1)}{2}) > \frac{1}{2}(1 + F(x_1)).$$

For an example, let $F(x)$ be a normal distribution with mean .32 and variance .001. Then there are three solutions to the equality above, which are approximately .254, .287, and .499. The first two satisfy the dynamic stability condition, and yield equilibrium style shares of roughly $(.02, .49, .49)$ and $(.147, .427, .427)$ respectively.

In my analysis of the three-style case, distributions of this sort – societies dominated by "wanna be's" with ideal points in $[.25, .5]$ – are the only ones that can support multiple equilibria. Proposition 7 asserts that if there is a strict equilibrium with $k = 3$, then it is unique.

Proposition 7. Suppose $k = 3$, and suppose that $F(x)$ supports a dynamically stable strict equilibrium $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. Then no other style distribution $\alpha' \neq \alpha$ can be a dynamically stable equilibrium.

I suspect this claim extends to the general case of any k. It is worth noting that many different ideal point distributions can yield identical sets of equilibrium style shares. For example, suppose that for some $F(x)$ there is a strict equilibrium with cutpoints $x_1, x_2, \ldots, x_{k-1}$. Then any distribution $G(x)$ that redistributes mass only between the cutpoints and not across them will give rise to the same equilibrium style shares, since the conditions in Proposition 2 will still be satisfied.

I conclude this section with one stray result of some interest: It is possible to show that the equilibrium share of the most popular style (or styles) has to be at least as large as the population share of conformists. Thus, it is not possible to have a dynamically stable equilibrium where the conformists (types with $x > .5$) fail to coordinate for mutual advantage on one style.

Proposition 8. (The largest style group is always at least as large as the population share of conformists.) In any dynamically stable equilibrium, the largest style group has measure of at least $1 - F(1/2)$.

4 Examples

This section provides examples of dynamically stable style shares for particular distributions of individual preferences about conformity and distinction.

4.1 Two styles, any distribution of ideal points

Suppose $k = 2$ and consider any strictly increasing distribution of ideal points $F(x)$ on $[0, 1]$. If there is a strict equilibrium – which here implies an equilibrium with a more popular and a less popular style, $\alpha_1 < \alpha_2$, then by Proposition 3 the cutpoint is

$$x_1 = \frac{1 - F(x_1) + F(x_1) - 0}{2} = \frac{1}{2}.$$

Thus with two styles the conformists ($x > 1/2$) choose the more popular style and those who ideally prefer a group with less than half the population choose the less popular style. The size of the style groups will be $\alpha_1 = F(1/2)$ and $\alpha_2 = 1 - F(1/2)$. For the conformists' choice to in fact be more popular it must be that $1 - F(1/2) > F(1/2)$ or $F(1/2) < 1/2$, which is just the inequality condition from Proposition 3. If this condition does not hold, then the unique dynamically stable equilbrium involves two equal-sized groups of 50% each (by Proposition 6 for the case of $k = 2$). Deviations from (.5, .5) tend to return towards (.5, .5) because a majority of the population desires to wear the less common style.

4.2 $k > 2$ styles and a uniform distribution of ideal points

Now suppose that there are $k > 2$ styles available and that ideal points are distributed uniformly, so that $F(x) = x$ for $x \in [0, 1]$. If there exists a strict equilibrium, then by Proposition 3 the cutpoints that define it satisfy the following $k - 1$ linear equations with $k - 1$ unknowns:

$$\begin{aligned} x_1 &= x_2/2, \\ x_2 &= (x_3 - x_1)/2, \\ x_3 &= (x_4 - x_2)/2, \\ &\vdots \\ x_{k-2} &= (x_{k-1} - x_{k-3})/2, \\ x_{k-1} &= (1 - x_{k-2})/2. \end{aligned}$$

The system can easily be solved without recourse to linear algebra by recognizing its kinship with a Fibonacci-like series of numbers. Consider the series 0, 1, 2, 5, 12, 29, 70, ... Each term in the series equals twice the prior term plus the term before that. Formally, $a_0 = 0$, $a_1 = 1$, and $a_i = 2a_{i-1} + a_{i-2}$

for $i > 1$.[6] Now define $x_i = a_i/a_k$, where k is the number of styles available. By the definition of the series,

$$x_i = \frac{a_i}{a_k} = \frac{2a_{i-1} + a_{i-2}}{a_k} = 2x_{i-1} + x_{i-2},$$

which can be rearranged to yield

$$x_{i-1} = \frac{1}{2}(x_i - x_{i-2}).$$

Since $x_k = 1$ and $x_0 = 0$, it is evident that the series $x_1, x_2, x_3, \ldots, x_{k-1}$ will solve the system of linear equations above. Table 1 gives the cutpoints and group sizes for several values of k.

Table 1: Cutpoints and style group shares for the uniform distribution

Equilibrium cutpoints

k	x_0	x_1	x_2	x_3	x_4	x_5	x_6
2	0	.5	1				
3	0	.2	.4	1			
4	0	.08	.16	.42	1		
5	0	.03	.07	.17	.41	1	
6	0	.01	.03	.07	.17	.41	1

Equilibrium style-group sizes

k	α_1	α_2	α_3	α_4	α_5	α_6
2	.5	.5				
3	.2	.2	.6			
4	.08	.08	.26	.58		
5	.04	.04	.10	.24	.59	
6	.01	.01	.04	.10	.24	.59

For a strict equilibrium to obtain, we need the inequalities in Proposition 3 to hold; that is, more conformist types have to be wearing more popular styles.

[6] Fibonacci numbers have the same start and $f_i = f_{i-1} + f_{i-2}$.

Inspection of the series (and Table 1) reveals that this will be true except for the equality of the smallest two style groups. So the equilibrium in the case of a uniform distribution is not strictly strict, as it were, but this equality of α_1 and α_2 poses no difficulty, as dynamic stability is weakly satisfied.[7]

Inspection of Table 1 suggests that the cutpoints and style group sizes tend to converge towards limit values as the number of styles k grows. To find these, consider again the series $a_i = 2a_{i-1} + a_{i-2}$. Letting $r_i = a_i/a_{i-1}$, we can rewrite this as $r_i = 2 + \frac{1}{r_{i-1}}$. If the ratio of two successive terms converges to a value r as i approaches infinity,[8] then this value must be $r = 2 + (1/r)$, or (taking the positive root of the quadratic equation that results), $r = 1 + \sqrt{2}$. Hence, the cutpoint for the largest group approaches $1/(1+\sqrt{2}) = \sqrt{2}-1$ as the number of styles gets large, which implies that its proportion of the population approaches $1 - (\sqrt{2} - 1)) = 2 - \sqrt{2}$, or about .586. The ratio of each group to the next smallest will be r all the way down, so we have that the j-th smallest group has a share of the population equal to

$$(2 - \sqrt{2})(\sqrt{2} - 1)^{j-1}.$$

Thus, with a large number of styles and uniformly distributed preferences over group sizes, in a dynamically stable equilibrium the largest group is approximately 60% of the population, with smaller style groups being each about 40% as large as the next largest. The actual shares are (roughly) .586, .242, .101, .042, .017, ...

This same set of equilibrium style group shares will obtain for any ideal point distribution $F(x)$ that is uniform up to $x = 1/2$ (that is, any $F(x)$ such that $F(x) = x$ for $x \in [0, 1/2)$). Changing the distribution of preferences among conformist types won't matter, since they will all sort themselves into the strictly largest group regardless.

Still, we would like to know how the equilibrium style shares change when the underlying distribution of ideal points is not uniform. The analysis above suggested the following loose generalization: The more nonconformists, the greater the tendency towards equal-sized groups; the more conformists, the

[7] More needed on this knife edge case.

[8] Show or find book that shows this for Fib sequences

greater the tendency towards a large conformist group and smaller nonconformist groups.

This tendency can be illustrated by looking at the family of distributions characterized by $F(x) = x^\beta$, $\beta > 0$, for $k = 3$ styles. $\beta = 1$ is the uniform distribution we have just analyzed. As β falls below 1, nonconformists increasingly dominate in the population; the larger β is above 1, the more the distribution is skewed in favor of conformists. It is not difficult to show that

a. When $\beta \leq \ln(2/3)/\ln(1/3) \approx .37$, there are three equal-sized groups in equilibrium.[9]

b. When $\beta \in (\frac{\ln(2/3)}{\ln(1/3)}, 1]$, there are two equal-sized smaller groups and one strictly larger, conformist style. As β increases in this range (more conformists), the nonconformist styles shrink relative to the more popular style.

c. For $\beta > 1$ the equilibrium is strict with three different sized groups, the share of the largest tending towards 1 as β grows.

For instance, when $F(x) = \sqrt{x}$ there are two styles with shares .297 each, and one larger style with share .407. When $F(x) = x$, equilibrium shares are .2, .2, .6, as shown earlier. When $F(x) = x^2$, they are .015, .233, .752.

5 Social welfare

Consider a large but finite population with ideal points described by $F(x)$. Could a social planner increase social welfare by assigning types to styles in a manner different from a dynamically stable equilibrium? Answering this question is also a good way to learn who benefits and who loses from having styles allocated by free choice.

A necessary condition for an assignment of individuals to styles to be efficient is that no two types can, by swapping style choices, make at least one of them strictly better off without making either worse off. A version of Proposition 2 shows that if some allocation has $\alpha_i > \alpha_j$ for styles i and j,

[9] The condition on β assures that $F(1/3) > 2/3$.

then Pareto efficiency requires that everyone wearing style i be weakly more conformist than everyone wearing style j. This fact implies in turn that in an efficient allocation a style set s_i must be convex whenever there is no other style of the same size. That is, in an efficient allocation, if no other style has share α_i, then $x, y \in s_i$, $x < y$, implies that $z \in s_i$ for all $z \in (x, y)$.[10] And clearly, when two styles are of equal size in an allocation, no two persons wearing them could do strictly better by swapping.

It follows that an efficient allocation must "look like" a dynamically stable equilibrium in the sense that (a) sets of individuals wearing different sized styles are convex, and (b) more popular styles are worn by more popular types. In other words, an allocation will satisfy the "no profitable swapping" condition if and only if there is a series of cutpoints that mark off the style groups of different sizes, with the more conformist types in the more popular styles.

But the argument so far shows only that such a configuration is necessary for Pareto efficiency – we now need to ask if some choices for cutpoints Pareto dominate others. The answer here is No: If the distribution of types has full support, then no matter where one puts the cutpoints, there will be types who get their first-best outcome (e.g., types with $x = \alpha_i$). Thus, once the no-profitable-swapping condition is met, any movement of the cutpoints makes some types worse off. We can conclude that *any dynamically stable equilibrium is Pareto efficient.*[11]

We might still ask, however, if a social planner can increase a social welfare function that weights all types' utility equally by assigning types to styles differently than in a dynamically stable equilibrium. Suppose that a type x individual's loss is $(\alpha_i - x)^2$ when the individual chooses style i, and that the social planner tries to minimize aggregate social loss weighting everyone equally. Pareto efficiency is clearly necessary for an allocation of types to styles to be socially efficient in this sense, so we can restrict attention to partitions of $[0, 1]$ such that more conformist types wear styles whose shares are at least as large or

[10] To see this, suppose to the contrary that there is a set of types in (x, y) that belong to some different-sized s_j in an efficient allocation. If $\alpha_j < \alpha_i$, then there are less conformist types in a neighborhood close enough to x who are wearing a more conformist style than more conformist types. Similarly for $\alpha_j > \alpha_i$.

[11] It might be helpful to provide an example of an allocation that is *not* Pareto efficient. Consider a uniform distribution with types $[0, .6]$ wearing style 1 and types $(.6, 1]$ wearing style 2. Shifting types $(.6, 1]$ into style 1 and types $[0, .4]$ into style 2 makes everyone weakly better off and a set of measure .8 strictly better off.

larger than those chosen by less conformist types. The question now is whether a different set of cutpoints can yield a lower social loss than the cutpoints implied by dynamic stability.

Consider the case of two styles. The social planner's problem is to choose the cutpoint $x_1 \in [0,1]$ to minimize

$$\int_0^{x_1} (F(x_1) - s)^2 f(s) ds + \int_{x_1}^1 (1 - F(x_1) - s)^2 f(s) ds,$$

where $f(x)$ is the density function associated with $F(x)$. After much algebra, the associated first-order condition solves to

$$\int_{x_1}^1 s f(s) ds - \int_0^{x_1} s f(s) ds = \frac{1}{2}(3 - 2x_1)(1 - 2F(x_1)).$$

In the case of the uniform distribution $F(x) = x$, this reduces to the quadratic $0 = 3x_1^2 - 4x_1 + 1 = (3x - 1)(x - 1)$, where the solution at $x_1 = 1$ is clearly a maximum. $x_1^* = 1/3$ minimizes the social loss function.

Thus, in the case of uniformly distributed style preferences and two available styles, a social planner maximizing a social welfare function creates style groups of size 1/3 and 2/3, while the "free market" produces two groups of equal size. In this case, then, *the market creates both less conformity and less nonconformity than is socially optimal.* By moving the cutpoint from 1/2 to 1/3, the social planner makes the relative nonconformists in $[0, 5/12]$ better off by providing them with a more exclusive group. At the same time, the relatively conformist types in $[7/12, 1]$ are made better off by the formation of a larger group and a more homogenous society overall. Only the types in $[5/12, 7/12]$ are made worse off.

Put differently, this analysis suggests that the more extreme conformist and nonconformist types in a society have an ironic common interest: both want nonconformist styles to remain relatively uncommon. For example, "real" punks deplored the spread of punk styles to the unwashed (washed?) suburban masses, while parents of the suburban masses often did the same.[12] In the free

[12]For examples of grumbling by original or "true" punks, note the TV Personalities song "Part-time Punks," or Frank (1997) (who, it should be noted, attributes the spread of punk styles more to advertising than anything else).

market of style choices, the problem is that “wanna be’s” – loosely, individuals between .3 (say) and .5 – cannot be stopped from imitating the nonconformist’s efforts to distinguish themselves. The Coasian solution, I suppose, would be to have the parents and the original punks pay the suburbanites not to wear spiked hair, and the urban hipsters to pay the yuppies not to move into Wicker Park (a formerly artsy neighborhood in Chicago whose yuppification has been much deplored by the “original” (white) artsy pioneers). But this would seem rather hard to organize, and in any event it might often be cheaper for nonconformists to invent new styles or move on to new marginal neighborhoods. And note that this last alternative will give rise to fashion cycles and style trends, discussed in the next section.

I am not sure how far this analysis generalizes. The results for three styles and a uniform distribution are qualitatively similar. However, distributions other than the uniform (or distributions close to it, like a normal centered at .5 with large enough variance) may yield different results, even with two styles. For example, for $F(x) = x^2$ the dynamically stable equilibrium cutpoint of $x_1 = .5$ also happens to be the outcome that maximizes aggregate social welfare. Evidently, as a distribution puts more weight on “wanna be’s,” the social planner will cater more to them.

6 Price competition

The assumption that all styles have the same price may be defensible if there are multiple producers of each style or in the case of styles as behaviors rather than products for sale. But if each style is a brand produced by one firm, this is now a problem of oligopolistic competition in a market with social influences on price. I briefly consider the polar cases of markets with all conformists and all nonconformists before turning to the mixed case.

6.1 All conformists (the network externalities case)

Consider a society of Chairman Maos: Everyone wants to wear what everyone else is wearing and all like full coordination on one style best of all ($x = 1$ for everyone). This is identical to preferences over goods with strong network externalities, where everyone wants a larger number of others to adopt the same network standard.

Let $v(\alpha_i, p_i)$ be an individual's utility for purchasing style (or product) i, where α_i is the market share of style i and p_i is its price. We assume in this case that $v(\alpha_i, p_i)$ is increasing in its first argument and decreasing in its second.[13] The firms have constant marginal cost $c \geq 0$. Considering the simplest case of two brands, I will say that a pair of prices (p_1, p_2) is in equilibrium if three conditions obtain. First, no firm can increase its expected revenue on the next consumer randomly drawn to buy, given the other firm's price. Second, consumers are expected to buy to maximize one-period utility given the prevailing distribution of styles and prices. And third, the distribution of styles is dynamically stable given the price vector.

The proof of Proposition 10 demonstrates that the only such equilibria involve one of the two firms getting the whole market and charging a price $p^* > c$ such that consumers are indifferent between paying the 'monopoly' price for the "in" style and being the only person buying the "out" style at its cost of production c. The "in" style firm's profits $p^* - c$ increase as consumers put more value on conformity (or when the importance of the network externalities is greater).

Proposition 10. The only price pairs that are consistent with the three equilibrium conditions satisfy p_i^* such that $v(1, p_i^*) = v(0, c)$ and $p_j = c$. In such an equilibrium $p_i^* > c$.

Note that if consumers could reliably coordinate their purchases in response to the prices posted by the two firms, then they could threaten to all buy from whichever firm charged less, and so support the competitive outcome of $p_1 = p_2 = c$. The practical obstacles to such coordination seem insuperable, especially in the case of products with some durability, like computers or clothing.

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[13] I am still assuming a continuum of consumers, so $\alpha_i \in [0, 1]$.

6.2 A group of would-be nonconformists

Now suppose that there are two styles or brands and that a consumer's utility for buying style i is $v(\alpha_i, p_i)$, but that utility is *decreasing* in both arguments. That is, consumers want to wear or use the *less* popular style or good (other things equal, like prices). This implies that with equal prices, the unique dynamically stable equilibrium has $\alpha_i = \alpha_j = 1/2$. By lowering its price a bit, a firm attracts more consumers, but only up to a point where the next consumer is not willing to give up the greater exclusiveness of the less popular brand for the lower price of the more popular brand.

Define $\alpha(p_1, p_2)$ to be the popularity of brand 1 such that when prices are p_1 and p_2, consumers are indifferent between the two brands. Formally, $v(\alpha(p_1, p_2), p_1) \equiv v(1 - \alpha(p_1, p_2), p_2)$. By differentiating both sides it is easily shown that $\alpha_i \equiv \frac{\partial \alpha(p_1,p_2)}{\partial p_i}$ is negative and finite as long as $v_1 < 0$, as assumed. Raising one's price loses customers, but as long as they care about exclusiveness ($v_1 < 0$), a higher price does not mean that *everyone* buys the cheaper brand, as in the perfectly competitive case. So this is equivalent to a duopoly with differentiated products (e.g., Mas-Colell et al., 395ff.). In equilibrium, both styles will be equally popular, the firms will charge the same price $p^* > c$, and they will make duopoly profits that increase the more consumers care about exclusiveness.[14]

6.3 The mixed case: conformists and nonconformists

Now consider a continuum of consumers with ideal points distributed by $F(x)$ on $[0, 1]$, and two available brands. If the probability that the next consumer buys brand 1 is $\alpha(p_1, p_2)$, then the firms have the problem

$$\max_{p_1} \alpha(p_1, p_2)(p_1 - c),$$

and

$$\max_{p_2} (1 - \alpha(p_1, p_2))(p_2 - c).$$

[14] Of course, the firms could make even more if they could collude on a price above this Cournot-like outcome, but they would then have incentives to lower prices to attract more customers.

Taking derivatives and dividing the first first-order condition by the second yields

$$-\frac{\alpha_1}{\alpha_2}\frac{p_1 - c}{p_2 - c} = \frac{\alpha(p_1, p_2)}{1 - \alpha(p_1, p_2)},$$

where the prices are now at equilibrium values. This expression implies that if $\alpha_1 = -\alpha_2$, then *the more popular brand has the higher equilibrium price.* Intuitively, the firm selling the more popular style has a steeper marginal revenue curve.

It remains to show that $\alpha_1 = -\alpha_2$ and what gives rise to $\alpha(p_1, p_2)$ in the first place. This will hold if (but certainly not only if) consumers preferences depend negatively on price and negatively on distance between a style's share and the individual's ideal point. For example, let individual utility for buying style i when it has share α_i and price p_i be $v(|\alpha_i - x|, p_i)$, decreasing in both arguments. Given prices and letting α be the share of brand 1, let $x(p_1, p_2)$ solve $v(|x(p_1, p_2) - \alpha|, p_1) = v(|1 - \alpha - x(p_1, p_2)|, p_2)$.[15] In a dynamically stable equilibrium where brand 1 is the less popular style, it must be that $\alpha = F(x(p_1, p_2))$ and $x(p_1, p_2) > \alpha$. Defining $\alpha(p_1, p_2) = F(x(p_1, p_2))$, we can write the identity $v(F^{-1}(\alpha) - \alpha, p_1) = v(1 - \alpha - F^{-1}(\alpha), p_2)$ (where α stands for $\alpha(p_1, p_2)$ here. By taking first derivatives with respect to each price it is straightforward to show that as claimed,

$$\alpha_1 = -\frac{v_2}{2v_1 f^{-1}(\alpha(p_1, p_2))} = -\alpha_2 < 0.$$

In words, if consumers have a common utility function that depends negatively on price and negatively on the distance between ideal point and the share wearing a style, then with two firms producing two brands *the more popular brand will be more expensive.* In this respect, the mixed case is a less extreme version of the "all conformist" world considered above, where the maker of the more popular style was able to charge a higher price and still get all the demand. But with a continuum of types, this higher price means that in comparison to the competitive case considered in section 3, the equilibrium share of the less popular style will be greater. The cheaper price will tip more wanna be's into the relatively nonconformist style.

[15] It is straightforward to show that for $\alpha < 1/2$ $x(p_1, p_2)$ is either uniquely defined in $[0, 1]$ or there is a corner solution. Corner solutions can't be part of an equilibrium here ...

It would be interesting to see how this analysis extends to cases with more than two brands. It seems clear that more popular styles will still have higher equilibrium prices, although the effect of price competition on relative popularity is probably indeterminate for all but the most and least popular styles. It would also be interesting to consider diversity in the intensity of individuals preferences for blending in or standing out. Would having some individuals who fervently desire to stand out and a lot of others who don't much care about style one way or the other imply that nonconformist goods would be more expensive?

7 Fashion trends, tipping, and fashion cycles

Consider a dynamically stable equilibrium with k styles, and suppose a new style is innovated or offered for sale. What will happen? The new style begins with a share of zero, so it will initially attract the most extreme nonconformists, those in the interval $[0, \alpha_j/2]$ where α_j is the currently most "radical" style. One might think that as more nonconformists choose the new style, it will become desirable for more wanna be's, which may lead to further expansion. This suggests the possibility of a fashion trend as a tipping process like those studied by Schelling (1980).

In fact, with preferences that are symmetric around ideal points as above, it is not correct to describe what happens with the introduction of a new style as a tipping process. Consider the simplest case of a population that begins the experiment all wearing one style, call it A. Now introduce a new style, B. All types $x \in [0, 1/2)$ strictly prefer the new style, and all types $x \in (1/2, 1]$ strictly prefer the old style. Thus the new style B will grow steadily towards share $F(1/2)$ as more and more types in $[0, 1/2)$ make choices. We have, then, a model of a *fashion trend* – a new style becomes increasingly popular, and it is bought by those in society who wish to use it to stand out from the crowd. However, no types ever "tip" into the new style, in the sense that they want to buy it only when enough others are wearing it.

Nor does adding a new style C when there are already two change this observation, though matters are now slightly more complicated. In the case of a uniform distribution of ideal points, the third style will initially be strictly preferred by all types in $[0, .25)$. Here, however, the initial growth of style C is at the expense of the second style B, whose share will begin to shrink below

.5. This induces types with ideal points just less than .5 to return to the old style A, so the old conformist style gains back popularity. Equilibrium, as we have seen above, obtains when B and C have equal shares at .2 each, and style A has .6 (all types above $x = .4$). At no point does the increasing popularity of the new style C lead to its purchase by consumers who initially found it too radical. Still, the process looks like a fashion trend, albeit one that affects style choices only among the more fashionably minded nonconformists of the society. The conformists of society continue wearing what they have always worn.

The effect of adding further styles is predictable: The most radical nonconformists choose the new style, and this has (increasingly minor) effects on the shares of styles already present in the population. So while the model as given does yield fashion trends with the introduction of new styles, it does not capture the empirical intuition that they are the result of a tipping process.[16] The basic model is also at odds with the fact that fashions can "go out of style" and that a new style can replace or "take over" from an older dominant style. Empirically, we often observe both *take overs*, where an old style loses its dominance to a newer one, and *fashion cycles*, where a given set of styles fluctuate in popularity over time.

The rest of this section shows how minor modifications or additions to the basic model yield results that plausibly explain these empirical observations. I begin with tipping and next consider take overs and fashion cycles.

7.1 Asymmetric conformity preferences and tipping

For simplicity, the preceding analysis assumed individuals to be indifferent between style groups whose sizes were equidistant from their most preferred size. I show here that the presence of people who feel more discomfort from wearing a too-unusual style than a too-popular one will give rise to fashion tipping.

To convey the intuition, imagine a population of individuals with symmetric preferences as above, except for one person who is indifferent between any style worn by *at least* 30% of the population. The person dislikes any style

[16] To defend this stylized fact I can only appeal to the reader's own experience or intuition. I maintain that reasoning like this is common: At one time, "I can't or won't wear that because it would be too radical for me"; at a later time, "Lots of people [in my comparison set] are wearing this. I'll wear it too."

with less than a 30% share, the more so the less popular the style. Now consider again the experiment above: Everyone begins wearing style A, and a new style B is introduced. Initially, the asymmetric-preference type prefers to keep wearing the old style A, until 30% have bought the new style, at which point this person is willing to change styles. If there are idiosyncratic style preferences, the person may positively desire to "tip" into the new style. (Recall that with symmetric preferences this person would have wanted to switch to the new style as soon as it was introduced.)

More generally, consider a population with preferences over style sizes defined as follows. An individual with ideal point x suffers utility loss $a(x-\alpha_i)$ when wearing styles with share $\alpha_i < x$, and loss $b(\alpha_i - x)$ when wearing styles with share $\alpha_i > x$, where $a > b > 0$. Thus, individuals have most preferred style group sizes, as before, but now find the same deviation from their ideal more uncomfortable if it is in the direction of "standing out" more than they would ideally like.[17]

Type $\hat{x} \in (\alpha_i, \alpha_j)$ is now indifferent between styles i and j when $a(\hat{x}-\alpha_i) = b(\alpha_j - \hat{x})$, or

$$\hat{x} = \frac{a\alpha_i + b\alpha_j}{a+b}.$$

So in the case of $k = 2$, type

$$\hat{x} = \frac{a\alpha + b(1-\alpha)}{a+b} = \frac{b + \alpha(a-b)}{a+b}$$

is indifferent between the two styles when the less popular style has share α. This implies that a fashion trend as a tipping process occurs with the introduction of a new style. When the new style B is introduced, types $x < b/(a+b) < 1/2$ want to buy it. As more buy, $\hat{x}$ increases, implying that increasingly conformist wanna-be's want to buy as style B becomes less radical.[18] For the uniform case, in the limit as a grows large (for any fixed b), only the most extreme

[17] The assumption of linear preferences is not highly restrictive; any monotonic transformation of these will yield the same results reported here.

[18] In the $k = 2$ case, equilibrium is reached when all types less than the cutpoint defined implicitly by $x = (b + F(x)(a-b))/(a+b)$ are wearing the new style. When $F(x) = x$, this reduces to $x = 1/2$ for any $b > 0$, as in the symmetric-preferences case considered earlier. However, for other ideal point distributions the equilibrium cutpoint with asymmetric preferences will be less than 1/2; that is, making people more sensitive to "standing out"

nonconformists are initially willing to buy the new style, which nonetheless gradually spreads through tipping to half the population.

It seems plausible that people may have any variety of symmetric or asymmetric preferences over style group sizes, in addition to varying in their ideal points. For example, some nonconformists may care little about just how radical (unpopular) a style is, just so long as it is *not* being worn by more than some threshold share of the population (i.e., a close to zero, $b > a$). These people will simply be quicker to jump into new styles than they would if they had more symmetric preferences. Provided there are enough people with the asymmetry described above – more sensitive to excess standing out than excess blending in – then the introduction of new styles can activate a tipping process that spreads the new fashion.

7.2 Take-overs and Fashion Cycles

Even with asymmetric preferences, however, the model as given does not yield take-overs or fashion cycles. If a large number of styles are already present in the population, only the currently most avant-garde style will be substantially affected by the introduction of a new style. While I think it is empirically accurate to say that most fashion trends occur among and influence the consumption behavior of only the most nonconformist types in society, nonetheless these trends *do* sometimes spill out of the avant-garde and take-over the dominant conformist styles, as with chunky shoes. It is reasonable to ask whether and how this could arise in the present framework.

It proves sufficient to drop the unrealistic assumption that people care *only* about the size of style group they are buying into. Assume instead that people have at least some idiosyncratic style preferences: certain individuals happen to find certain fashions or behaviors particularly appealing, independent of how many others are wearing it. Formally, we might suppose that a type x individual who buys at moment t gains utility $-(x-\alpha_j^t)^2+\epsilon_j^t$ for buying style j, where ϵ_j^t is a symmetrically distributed random variable with zero mean.[19] Such idiosyncratic style preferences mean that conformists will sometimes "go out on

implies more conformity in equilibrium. In the case of $k > 2$ styles and a uniform distribution of ideal points, the analogous Fibonacci-like generating sequence is $a_i = 2a_{i-1} + \frac{a}{b}a_{i-2}$.

[19]Alternatively, we could assume that an individual's idiosyncratic preferences are constant across purchases. An example would be a personal rule such as "I look bad in red" (as opposed

limb," and nonconformists will sometimes buy relatively conformist styles just because they like them.

Idiosyncratic style preferences create the possibility of a less popular style periodically changing places with a more popular style – that is, of take overs and fashion cycles. To see how, consider a finite population choosing between two styles, A and B, and a distribution of ideal points that is approximated by $F(x) = x^{1.3}$. If these people cared only about conformity and distinction (no idiosyncratic style preferences), this would yield in equilibrium a less popular style with about 40% of the population, and a more popular style with about 60%. Introducing a very small degree of idiosyncratic style preferences will make the style shares fluctuate some around these levels, more widely the less people care about blending in or standing out.

However, as idiosyncratic preferences become more important to people (the variance of ϵ_j^t increases), the probability increases that there will be a run of buyers wearing the currently more popular style who idiosyncratically prefer the less popular style, thus raising its market share. Suppose that A is currently the less popular, nonconformist style, and B is currently more popular. The consumers wearing B who are most likely to happen to "defect" to A are those with ideal points closest to the equilibrium cut point (here, 1/2), since they are almost indifferent between the two styles on grounds of conformity/distinction preferences. If a number of these marginal conformists happen to run into the less popular style A, the shares of the two styles become more equal, so that an increasing number of more seriously conformist types become more open to trying A. At the same time, as A becomes more popular, more of the nonconformist types wearing A also become willing to consider B. But since there were more people wearing style B to begin with, the likely number of defectors to A is greater. This means that there is some tendency for runs into the less popular style to be self-reinforcing; each new A-wearer increases the probability that another conformist will allow herself to be swayed by an idiosyncratic preference for the less popular style. As the shares of the two styles approach equality, choice between them comes to be determined almost entirely by idiosyncratic "personal" preference for *all* types of consumer, so there approaches a 50% likelihood that the "tip" will continue, leading to a take-over by the formerly less popular style.

This argument shows how the combination of concerns about conformity

to "I don't feel like red today").

and distinction and idiosyncratic style preferences creates the possibility of stochastic transitions that look like fashion cycles.[20] Eventually there will be a run of buyers wearing the currently more popular style who idiosyncratically prefer the less popular style, which generate "tipping" that turns the less popular style into the more popular.

If a new style is introduced or innovated, idiosyncratic style preferences imply that its spread need not be limited to wearers of the previously most avant-garde style, as in the deterministic case above. Instead, idiosyncratic preferences may "tip" the new style so that it replaces the previously most avant-garde style, possibly continuing to invade more conformist styles further up the continuum of types. If new styles are continually being innovated by the most radical nonconformist types, then periodically an innovation will be (idiosyncratically) preferred by enough marginal types on the relevant cutpoint boundaries, leading a formerly obscure style or fashion to "enter the mainstream."

The model with idiosyncratic style preferences yields some specific empirical predictions. First, if equilibrium without idiosyncratic preferences implies a large gap in the shares of two styles, then stochastic transitions between them will be very rare. For example, if there are two styles and the deterministic equilibrium would have 10% in one and 90% in the other, then it would require a very long (and unlikely) run of relative conformists idiosyncratically preferring the less popular style to yield a take-over. Thus, more conformist societies – that is, societies where greater stress is placed on blending in – should see fewer take-overs and fashion cycles that tip into the most popular styles.

Second, as the size of the relevant population increases, the law of large numbers takes over, so that runs into a less popular style become much less likely. This implies that take-overs and fashion cycles will be more common among the relative nonconformists of society, since these types wear the least popular – and thus least populated – styles.

Given the scale of modern economies and the reach of the media that disseminate information on who is wearing (and buying) what, it may judged

[20] I have confirmed this with simulations; it may be possible to show it analytically with a Markov analysis. A simple example of a simulation that shows fashion cycles has 100 people, 40 with ideal point .4, 60 with ideal point .6, and quadratic loss functions summed with a random variable that is uniformly distributed on $[-.8, .8]$. This system transitions periodically between approximately 40% on style A, 60% on B, and 60% on A, 40% on B.

implausible that "runs" of the sort described here are what lie behind many takeovers and fashion cycles. In concluding this section I note two modifications that might yield fashion cycles even with large numbers and relatively unimportant idiosyncratic preferences. First, if people are organized in small but overlapping reference groups, then stochastic tipping dynamics might propagate across such local style communities through the population as a whole.[21] I have not yet investigated this.

Second, if we add the assumption that individuals will not return for some length of time to a style they have just abandoned, then it is easy to generate fashion cycles under a much broader range of parameter conditions. The reason is straightforward – now individuals will stick with a style for some length of time if they switch to it, so that idiosyncratic preferences tend to work much more uniformly in the direction of favoring the currently less popular style. For example, if a new style B is introduced to a population wearing style A, then style B can only grow in popularity. It may "stall out" if the society is sufficiently conformist and if people put sufficiently little weight on idiosyncratic preferences, but now there will be a much greater tendency for the new style ultimately to take over style B's popularity. Style A may be driven from the scene entirely, or it may begin a "come back" when nonconformists start to be willing to consider wearing it again.

The assumption that people will not return immediately to a style they have just abandoned is clearly ad hoc, but it is also empirically plausible. Style and fashion choices are understood as expressions or signals of "identity," which in turn suppose some continuity. A very few individuals may constantly and rapidly cycle through a fixed set of styles or modes of self-presentation, but this would be viewed either as psychotic or itself a sort of fashion statement that has its own continuity. If we think of choices between intellectual styles, approaches, and theories along the lines developed here, then it is well known that academics who constantly shift between espousing opposed theories or positions risk undermining their credibility.[22] Such considerations go beyond

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[21] Cf. Ellison, etc.

[22] At least in political science, my impression is that intellectual fashion cycles like those described here emerge from a competitive job market that drives graduate students to differentiate themselves by doing something "new," but not so new and different as to make them seem quirky. The differential willingness of graduate students to run risks of being too "far out" implies tipping dynamics, which in turn imply that what seemed quirky yesterday (rational choice, constructivism) becomes tomorrow's "cutting edge."

the very simple psychology supposed in the basic model here, where people care only about finding the right-sized group. It would be interesting to try to incorporate concerns about continuity of identity in a more systematic manner.

8 Advertising and the social construction of identities

Producers of many consumer goods spend billions of dollars each year advertising their products in a manner inexplicable from the perspective of the neoclassical theory of the consumer. In textbook neoclassical theory, consumers care only about the price and intrinsic properties of goods. By "intrinsic properties" I mean properties of goods that do not depend on what anyone thinks about the good. For example, a car's gas mileage or turning radius, the hazard rate of a refridgerator or light bulb, or the efficacy of a brand of insect repellent are intrinsic properties in this sense. "Social properties" of goods are properties that depend on how people think about the good – for example, whether Fruitopia is cool, whether Oldsmobiles and Cadillacs are associated with older people, or the association of tye-dye with the 1960s and things the 1960s is thought to stand for.

Under neoclassical consumer theory, the implied role of advertising would be to inform consumers about prices and to make (verifiable) claims about products' intrinsic qualities and capacities.[23] Without doubt a great deal of advertising does announce prices, and probably a large majority of ads offer some claims about intrinsic properties. But there can also be no doubt that many ads make no mention of price and only incidental or even no reference to intrinsic qualities. For a partial exception that really proves the rule, consider the "Obey Your Thirst!" campaign for the soft drink Sprite. These ads counsel viewers *not* to pay attention to the advertising strategy of all the other soft drinks, which (the ad implies) is to suggest that drinking soft drink X is stylish, cool, and confers a desirable social image on the drinker. Instead, the Obey Your Thirst commandment is intended to say: No one really thinks a soft drink will determine whether one is judged cool; drink Sprite because it quenches thirst well (a claim about a more intrinsic property). A possibly intended irony of the commercial is that this anti-"image" message is often delivered in the high MTV style of a rapid-fire montage of cool images. The subtext is clearly,

[23] Of course, in the most bare bones neoclassical theory consumers have perfect information about the prices and qualities of goods, so there is no need for advertising to inform them.

“when it comes to soft drinks, it is cool to appear not to care about image and to drink Sprite.” This is a far cry from The Pepsi Challenge, where blindfolded consumers discovered they preferred Pepsi to Coke on grounds of taste.

The model and analysis above suggest a natural explanation for “image” advertising. I will first give a brief informal statement of the argument and then connect it more closely to the model above.

If consumer goods can acquire social meanings and if consumers often care passionately about what these social meanings are, then an important rationale for some advertising may simply be *to coordinate beliefs about the social meaning of different goods.* This allows buyers both to ascertain and have some confidence in what social message a product sends. If I want to buy a car and I care not only about its price, reliability, size, etc., but also about what other people will infer about a person who drives this style of car, then advertising may help direct me to the car that sends the social message I desire. Since I know that the advertising is public, I can have some confidence that other people will understand what message is intended.[24] And to the extent that I can expect that other types “like me” will also see the advertisements and self-select into this style of car, the firm’s bid to associate this product with a particular social property may be self-confirming and self-sustaining.

Language provides a useful analogy. People communicate not only with words but also through behaviors and choices of consumer goods that come to connote style or type. There is a constantly evolving “language” of styles and social properties of consumer goods that constitutes a significant part of a society’s culture. For such styles and goods, advertising can act as *a publicly available dictionary of social meanings.* In contrast to the case of language, this “dictionary” does not just attempt to codify the results of a largely decentralized, societal process. Instead, it continually creates neologisms and bids for their popular acceptance. Social properties are conferred on goods both by decentralized processes of interaction and exchange within society, and the more centralized, “top down” efforts of advertisers.

[24]Chwe (1998) has stressed that for a “social good” characterized a consumption value that increases with the number of others consuming it, advertising may be desirable for its ability to create common knowledge about a product’s existence among a field of potential buyers. The argument here in effect extends this insight to cases where different types of buyers want to separate or distinguish themselves through their purchases. I would argue that matters of fashion and style are always about conforming with some and distinguishing oneself from others.

The model analyzed above depicts a decentralized process by which goods or behaviors take on social meanings as “conventional,” “radical,” and the like. For example, in the case of two goods or styles, a path-dependent process of decentralized individual choices decides which one comes to be regarded as the radical style, and which the conventional or “normal” style. But the model also suggests how there could be incentives for firms to advertise to try to coordinate consumer expectations about the social meaning of a product.

Consider a population 80% of which are conformist types ($x \geq .5$), who are currently choosing between two styles of some good. Suppose that goods of each style are produced by several firms, and consider the problem faced by an entrant who would like to begin competing in the market for the more popular style. If it cannot be made unambiguous from design or packaging that its product is intended as conformist, then, without advertising, conformist types may avoid the new brand because they fear that it will send the wrong message. At the same time, without advertising, nonconformists will take up the new brand as a means of standing out, in which case the decentralized social process of assigning meanings would have stuck the firm with the “wrong” (less profitable) market.[25] The entering firm therefore has an incentive to introduce the product with an ad campaign designed to associate it with conformity and normalness rather than radicalness and standing out. Moreover, the firm could reasonably expect such an ad campaign to be effective. Nonconformists will not want to buy the brand if they expect that enough conformists are going to buy it, and conformists will want to buy the brand in this same circumstance. Advertising thus serves as a public signal that creates demand by assuring the “right type” of consumer that other right types will be buying it, or by creating public knowledge about the message the product is intended to convey.

The argument can be extended beyond the case of consumers with preferences over the dimension of standing out versus blending in. Let the unit interval $[0, 1]$ represent any dimension over which consumers come to have preferences for distinguishing themselves. For example, the continuum could refer to ideological orientation (left-right) if people come to care about signaling their political preferences; an age continuum if people come to care about particular goods saying “youngster,” “old-timer” or “30-something,” etc.; an urban/countryside continuum if people come to care about signaling leisure-time

[25]Of course, either the nonconformist or the conformist market could be more profitable for an entrant; this would depend on industry specifics, such as level of competition in each sector.

preferences through consumer goods. Suppose that individuals choose among a fixed set of styles or brands and that they wish to use these publicly observable style choices to communicate their privately known types. For example, they might wish to communicate their type through style choices because they are playing a matching game in pursuit of mates or friends, and they want to associate with similar types. Thus, an individual will want to buy the style that is worn by other consumers with types most similar to his or her own.[26]

Consider now what happens if a new brand is introduced without advertising into a market currently in equilibrium.[27] If the variance of the estimate of a type wearing a new style is large enough, then no one will want to buy the new brand. People want to communicate their type or "identity" through their style choices, so if a new style sends no clear signal, they will avoid it. By contrast, publicly advertising the good as (in effect) "For types $x \in [a, b] \subset [0, 1]$" can effectively create demand for the product by providing a signal around which these types can coordinate, assured that others will understand the message they are trying to send.[28]

This analysis might shed light on an old debate in economics over the purpose and effects of advertising. Becker and Stigler (1977) criticized Galbraith (1958) and many others who argued that the point of advertising is to

[26]There are several ways "most similar" might be formalized. Perhaps the most natural is to have each individual choose the style that minimizes the expected distance between her type and the type of a randomly selected individual wearing the same style. If s_i is the (measurable) set of types choosing style i and $f_i(x)$ is the density function for these types, then type x wants to choose the style that minimizes $\int_{s_i} (x - r)^2 f_i(r) dr$. This implicitly assumes that individuals *know* the distribution of types wearing each style, which might arise if people get observations of type from meeting people wearing different styles or through various media reporting on styles; it would be interesting to model explicitly the process by which people drew inferences about what types are wearing what styles.

[27]In an equilibrium, people correctly estimate the distribution of types buying each style, and no type wishes to choose a style different than expected

[28]The use of celebrities as product spokespersons can be explained in these terms as well: The choice of spokesperson signals what types the product is intended for, and what social message it is intended to convey. Celebrities are useful for this purpose because by definition they are widely known and associated with particular activities, qualities, preferences, dispositions, and so on. This is in contrast to what I would guess are the received theories of product spokespersons: (a) magical thinking ("If I drink this soda, I will acquire Michael Jordan's shooting ability or personal charisma"), or (b) information transmission ("Michael Jordan has valuable private information about this soda").

change consumer preferences, “manufacturing demand” in Galbraith’s words. Instead, Becker and Stigler proposed that advertising does not change preferences but merely supplies consumers with information about products, which in turn affects their demand. “If [a consumer] does not know whether berries are poisonous, they are not food” (p. 205), they observe. However, the Becker and Stigler view is hard to reconcile with the sort of advertising described above, which makes only incidental or inessential reference to the intrinsic qualities of the goods. Do soft drink or athletic footwear commercials always work by conveying information about the intrinsic quality of the product?

The argument here might be seen as reconciling the Galbraith and the Becker/Stigler views. On the one hand, the analysis can assume fixed underlying preferences. On the other, it is not incorrect to say that advertising is, in a sense, “manufacturing demand” without providing any real information about the product itself. In this argument, advertising manufactures demand not by providing information or by engaging in some sort of nonrational brainwashing, but rather by acting as a coordinating device. One can go further: Advertising in this argument can even help manufacture “identities” in the sense of social categories, since there will often be many different ways that groups of similar types might be marked off, and how they are will depend on industry specifics and history.

9 Extensions

In place of a conclusion, I briefly note three questions suggested by the analysis that extensions might help resolve. First, I have taken the set of styles to be finite. But consider hair or hem length, both of which have long been observed to be subject to fashion trends (for an early study of hem lengths, see Kroeber 1919). How might the approach be extended to cases where the set of styles are naturally ranged on continuous dimension like this? One possibility would be consider individuals who have preferences over the number of standard deviations they are from the current average hair or hem length, and ask about the limiting population distribution that would result from the dynamic process analyzed above. It seems likely that with enough individuals wanting to be close to the average, the result will be total conformity on one hair length. If enough want to be far enough away from the mean, then, combined with exogenously determined limits on how short or long hair/hems can be, we might get fashion cycles. These are just conjectures, of course.

Second, the analysis here assumed that everyone knows how to distinguish between the set of styles from which choices are made. But the process of saying where one style ends and another begins is itself a matter of collective determination and convention. Consider the colors of cars, which show tremendous variety if one considers all the various subtle shadings and hues. Somehow, we group together all sorts greens, reds, blues, and silvers as "normal" colors that basically "blend in," while certain colors like lime green, yellow or pink, especially if they are bright, really "stand out." How is it that multiple shades of green and red become grouped together under the meaning "normal" despite the fact that they are distinct colors? In principle, it seems to me that the approach taken here might be extended to address this question, but I don't see how yet.

Third (and relatedly), there are chunky shoes, and then there are *really* chunky shoes. And one can combine different sorts of chunky shoes with an almost infinite variety of other style signals of almost infinite gradation to send very personal, idiosyncratic fashion messages. While people certainly choose to put themselves in particular broad social categories by how they dress and act, within these broad categories they may align themselves with, or stake out, more subtle subcategories (a stockbroker with a Brooks Brothers suit and an earring, for example). I think it would be possible to allow for subcategories in the model above by having individuals choose not only a broad style but also from a set of options "within" each style. For instance, the types wearing the recognizably most conformist broad style might engage in more subtle differentiation at the "next level" of style choices, in effect replicating a sequence of more and less popular substyles within the broader conformist style.

10 Appendix

Proof of Proposition 1. If not, then there can be in an equilibrium with at least one style that no one wears. Let $\alpha_j > 0$ be the size of the smallest strictly positive group in this equilibrium. Then types $x < \alpha_j/2$ strictly prefer to deviate to an unused style, and this set has positive measure by the assumption on $F(x)$. Q.E.D.

Proof of Proposition 2. Since preferences depend only on distance between an individual's ideal point and the size of a style group, an individual is indifferent between α_j and α_{j+1} when $|x - \alpha_j| = |\alpha_{j+1} - x|$, which is solved uniquely by $x = (\alpha_j + \alpha_{j+1})/2$ when $\alpha_j < \alpha_{j+1}$. Types $x' > x$ strictly prefer α_{j+1}, since it is closer to their ideal points, and likewise types $x' < x$ strictly prefer α_j. Any distribution in which a type $x' > x$ ($x' < x$) chooses style α_j (α_{j+1}) is thus not Nash, and the proposition follows immediately. Q.E.D.

Proof of Proposition 3. Condition (1) defining the cutpoints follows immediately from Proposition 2. If any inequality constraint is not satisfied, then either there are two groups of the same size (and thus the equilibrium is not "strict" as defined) or there are two groups such that $\alpha_j > \alpha_{j+1}$. This is not possible in equilibrium since all types in each group (except for x_j) would strictly prefer to deviate to the other style. By contrast, if the inequalities are satisfied, then by Proposition 2 no type strictly prefers to deviate to another group. QED.

Proof of Proposition 4. Condition 1 implies that in a strict equilibrium $x_1 = F(x_2)/2$, and thus we can write $x_2(x_1) = F^{-1}(2x_1)$. Since $F(x)$ is strictly increasing, so is F^{-1}, implying that $x_2(x_1)$ is a strictly increasing in x_1 for $x_1 \in [0, 1/2]$. In addition, $x_2(0) = 0$. More generally , writing $x_j(x_{j-1}) = F^{-1}(2x_{j-1} + F(x_{j-2}))$ implies that x_j is a strictly increasing function of x_{j-1}. Since x_{j-1} is likewise a strictly increasing function of x_{j-2}, we can also define implicitly $x_j(x_{j-2})$ is also strictly increasing in x_{j-2}, and on back to $x_j(x_1)$.

Condition (1) implies that in a strict equilibrium $x_{k-1} = (1 - F(x_{k-2}))/2$. But by the above logic, x_{k-1} and x_{k-2} can each be written as strictly increasing functions of x_1, yielding

$$x_{k-1}(x_1) = \frac{1 - F(x_{k-2}(x_1))}{2}.$$

At $x_1 = 0$ the left-hand side is 0 and strictly increases with x_1; the right-hand side is $1/2$ at $x_1 = 0$ and strictly decreases with x_1. At x_1 such that $x_{k-1}(x_1) = 1$, it must be that $x_{k-2}(x_1) < 1$ which implies that the curves cross just once; there is a unique solution in x_1 to the above equality, which then uniquely determines the other cutpoints $x_2, x_3, \ldots$.

To complete the proof, ask whether it is necessarily the case that the k in the above equality be the same as the exogenously given number of styles – there will be an x_1 that satisfies this equality for any $k' > 0$, and not only for the actually available number k. But if $k' < k$, then there must be an unused style, which contradicts Proposition 1. $k' > k$ is obviously not feasible since only k are available to be chosen. QED (needs tightening up).

Proof of Proposition 6. Suppose not. Then under the condition in the proposition we can support a dynamically stable equilibrium in which there is a strictly largest group, or there is a set of $l > k$ equal-sized groups that are strictly larger than the next largest group ($1 < l < k$). Consider the case of a single strictly largest group first.

If this were the case, then by Proposition 2 all types $x > x_{k-1}$ choose this style, where x_{k-1} is a cut point strictly less than 1. Further, the size of the largest group, $1 - F(x_{k-1})$, must be strictly larger than $1/k$, or else the group sizes would not sum to 1. $1 - F(x_{k-1}) > 1/k$ implies that $F(x_{k-1}) < (k-1)/k$. Combined with the dynamic stability condition, it follows that

$$F(x_{k-1}) < \frac{k-1}{k} < F(1/k),$$

and thus $x_{k-1} < 1/k$.

By Proposition 2,

$$x_{k-1} = \frac{1}{2}(1 - F(x_{k-1}) + \alpha_{k-1}),$$

where α_{k-1} is the size of the second largest group. $\alpha_{k-1} \geq F(x_{k-1})/(k-1)$, since otherwise the group sizes could not sum to 1. Thus we have

$$x_{k-1} \geq \frac{1}{2}\left(1 - F(x_{k-1}) + \frac{F(x_{k-1})}{k-1}\right),$$

which simplifies to

$$x_{k-1} \geq \frac{1}{2}\left(1 - \frac{F(x_{k-1})(k-2)}{k-1}\right).$$

Using the earlier implication that $F(x_{k-1}) < (k-1)/k$, the last inequality implies $x_{k-1} > 1/k$, which contradicts $x_{k-1} < 1/k$ above. Thus $F(1/k) > (k-1)/k$ implies that there cannot be a strictly largest group.

Now suppose that there are $l > 1$ equal sized groups that are strictly larger than the next largest group α_{k-l}. Let x_{k-l} be the cut point that divides the types joining the largest groups from the next largest. Since the group sizes must sum to 1 (by Proposition 1), we have $(1 - F(x_{k-l}))/l > 1/l$, or $F(x_{k-l}) < (k-l)/k$. Combining this with the hypothesis that $F(1/k) > (k-1)/k$ implies that $F(1/k) > F(x_{k-l})$ and thus $x_{k-l} < 1/k$.

By Proposition 2, the condition

$$x_{k-l} = \frac{\frac{1-F(x_{k-l})}{l} + \alpha_{k-l}}{2}$$

defines x_{k-l}, and it must be that $\alpha_{k-l} \geq F(x_{k-l})/(k-l)$ for the shares to sum to 1. These two facts imply that

$$x_{k-l} \geq \frac{1}{2}\left(\frac{1-F(x_{k-l})}{l} + \frac{F(x_{k-l})}{k-l}\right) = \frac{1}{2}\left(\frac{k-l-(k-2l)F(x_{k-l})}{k-l}\right)$$

Using the fact that $F(x_{k-l}) < (k-l)/k$ implies that

$$x_{k-l} > \frac{1}{2}\left(1 - \frac{k-2l}{k}\right) = \frac{l}{k},$$

which contradicts the previous implication that x_{k-l} must be strictly less than $1/k$. Q.E.D.

Proof of Proposition 7. By Proposition 2, in any Nash equilibrium $\alpha_j \leq \alpha_{j+1}$ for all $j \in S$. For $k = 3$, this implies that there are four (2^{k-1}) possible "configurations," that can be represented by the set $\{==, =+, +=, ++\}$. $==$

refers to the case of all three groups of the same size, and $++$ to a strict equilibrium with each s_j strictly larger than the group immediately before it. $=+$ is the case of two equal sized small groups and one larger group; $+=$ is one smaller group and two equal-sized larger groups.

By Proposition 6, if there is a dynamically stable equilibrium of the form $==$, no other equilibria are possible. By Proposition 4, there is at most one equilibrium of the form $++$. So it remains to show that the follow sorts of multiple equilibria are impossible: Pairs of the form $(++,=+)$ and $(++,+=)$.

Beginning with $(++,=+)$, if such a pair of equilibria are possible for a given distribution of preferences $F(x)$, then there are pairs of cutpoints (x_1,x_2) and (x_1',x_2') that simultaneously satisfy the following conditions. For the $=+$ equilibrium,

$$x_2 = \frac{1-F(x_2)+F(x_2)/2}{2} = \frac{1}{2}(1-\frac{F(x_2)}{2}). \tag{4}$$

$$F(\frac{F(x_2)}{2}) > F(x_2) - F(\frac{F(x_2)}{2}).$$

The latter is the dynamic stability condition, which can be rewritten

$$F(\frac{F(x_2)}{2}) > \frac{1}{2}F(x_2). \tag{5}$$

For the $++$ equilibrium we have that

$$x_1' = \frac{1}{2}F(x_2') \tag{6}$$

$$x_2' = \frac{1-F(x_1')}{2} \tag{7}$$

$$F(x_1') < \frac{1}{2}F(x_2') \tag{8}$$

6 and 7 imply that

$$x_2' = \frac{1}{2}(1-F(\frac{F(x_2')}{2})), \tag{9}$$

and 6 and 8 imply that

$$F(\frac{F(x_2')}{2}) < \frac{1}{2}F(x_2'). \tag{10}$$

In turn, 9 and 10 together imply that $x_2' > \frac{1}{2}(1 - \frac{1}{2}F(x_2'))$, which together with 4 implies that $x_2' > x_2$ since $F(x)$ is strictly increasing. But 4 and 5 imply that $x_2 > \frac{1}{2}(1 - F(F(x_2)/2))$, which together with 9 implies that $x_2 > x_2'$, a contradiction. Thus it is impossible to have equilibria of the forms $++$ and $=+$ for a given distribution of ideal points.

The proof that equilibria of the forms $++$ and $+=$ cannot coexist has the same form. If they could, then there would be cutpoints x_1 for the $+=$ equilibrium and (x_1, x_2) for the strict equilibrium such that following expressions are satisfied. For the $+=$ equilibrium,

$$x_1 = \frac{1}{2}(\frac{1 - F(x_1)}{2} + F(x_1)) = \frac{1}{4}(1 + F(x_1)). \tag{11}$$

$$F(\frac{1 - F(x_1)}{2}) - F(x_1) > 1 - F(\frac{1 - F(x_1)}{2}),$$

or

$$F(\frac{1 - F(x_1)}{2}) > \frac{1}{2}(1 + F(x_1)). \tag{12}$$

For the $++$ equilibrium,

$$x_1' = \frac{1}{2}F(x_2') \tag{13}$$

$$x_2' = \frac{1 - F(x_1')}{2} \tag{14}$$

$$1 - F(x_2') > F(x_2') - F(x_1'), \text{ or}$$

$$F(x_2') < \frac{1}{2}(1 + F(x_1')). \tag{15}$$

11 and 12 together imply that

$$x_1 = \frac{1}{4}(1 + F(x_1)) < \frac{1}{4}(1 + 2F(\frac{1 - F(x_1)}{2}) - 1) = \frac{1}{2}F(\frac{1 - F(x_1)}{2}),$$

which together with 13 and 14 implies that $x_1 < x_1'$. But 13, 14, and 15 together imply that

$$x_1' = \frac{1}{2}F(\frac{1-F(x_1')}{2}) < \frac{1}{4}(1+F(x_1')),$$

which together with 11 implies that $x_1' > x_1$, a contradiction. This implies that we cannot have equilibria of the forms ++ and += at the same time. Q.E.D.

Proof of Proposition 8. If there is a strictly largest style group in equilibrium, then the claim follows immediately from Proposition 3, since $x_{k-1} = (1-F(x_{k-2}))/2 < 1/2$ implies that $1-F(x_{k-1}) > 1/2$. If there are $l > 1$ equal-sized groups that are larger than any other, then the dynamic stability condition for these l groups is

$$\frac{F(\frac{1-F(x_{k-l}}{l}) - F(x_{k-l})}{l-1} > 1 - F(\frac{1-F(x_{k-l}}{l}),$$

which reduces to

$$F(x_{k-l}) < lF(\frac{1-F(x_{k-l}}{l}) - (l-1).$$

If the claim is false, then it is possible that

$$\frac{1-F(x_{k-l})}{l} < 1 - F(1/2),$$

which is rearranged as $F(x_{k-l}) > lF(1/2) - (l-1)$. Combining this with the inequality from the dynamic stability condition implies that

$$lF(1/2) - (l-1) < lF(\frac{1-F(x_{k-l}}{l}) - (l-1),$$

which is impossible for $l > 1$ since $F(x_{k-l}) > 0$. Q.E.D.

Proof of Proposition 9. An equilibrium in which $v(\alpha_i, p_i) = v(\alpha_j, p_j)$, $p_i, p_j > c$, $\alpha_i, \alpha_j \in (0,1)$ is impossible, since a firm could then cut price a tiny bit and get a discontinuous jump in the probability of the next purchase (all would want to buy from the newly cheaper firm). But an equilibrium with $v(1/2, c) = v(1/2, c)$, $\alpha_i, \alpha_j \in (0,1)$ won't work, since this violates dynamic stability – a small perturbation would send all customers into the favored brand. If in an equilibrium, $v(\alpha_i, p_i) > v(\alpha_j, p_j)$, then it must be that $\alpha_i = 1$ and $\alpha_j =$

0. But if $v(1, p_i) > v(0, p_j)$, then firm i has an incentive to raise p_i until $v(1, p_i) = v(0, p_j)$. But this will support an equilibrium only if firm j cannot lower its price marginally and attract all customers, which entails that $p_j = c$. So an equilibrium in which firms simultaneously maximize their probability of selling to the next consumer, consumers maximize given the current style distribution, and this distribution is dynamically stable given prices, requires that $v(1, p^*) = v(0, c)$. Since v increases in the first argument and decreases in the second, this implies that $p^* > c$. Q.E.D. (needs elaboration)

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