5 or even larger. We consider a case with the comparatively large core separation \( D = 5.5 \), to have negligible \( P_1/P_2 \). For \( p = 3.8 \mu \text{m} \) and \( \Delta = 0.005 \) (this gives \( V = 2.4 \) in the wavelength range of 1.5-1.6 \( \mu \text{m} \)), the coupling length will be about 16 cm. For nonlinear integrated optical waveguide couplers, there is always a constraint on the device length due to the nature of the integration and the fabrication technique. It seems impractical to fabricate integrated waveguide couplers with lengths over 10 cm. Hence, nonlinear couplers will probably have to be made with small core separations, and thus nonlinear modal effects will have to be taken into consideration. This will be particularly relevant in cases where optical circuits using integrated optical waveguide couplers are to be designed with small dimensions for optical signal processing or computing.

![Fig. 2 Nonlinear power transmission \( P_1/(P_1 + P_2) \) for \( n/2 \) (i.e., \( C_2 = C_1 + n/2 \)) coupler for various values of \( D \).](image)

![Fig. 3 Ratio of critical power to single fibre selfoscillating power \( P_1/P_2 \), in optical switching using directional couplers](image)

In summary, we have examined the nonlinear mode field effects in nonlinear waveguide couplers. We find that, even in cases where \( P_1/P_2 \leq 1 \), the nonlinear change in the fundamental mode field in the cladding region can be quite significant. This implies that the nonlinear mode field effect can be an important factor in designing and analysing nonlinear optical devices, particularly those that depend on the evanescent part of the guided waves.

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SURFACE EMITTING SECOND HARMONIC GENERATION IN VERTICAL RESONATOR

Indexing terms: Resonators, Harmonic generation, Waveguides

The efficiency of surface emitting second harmonic generation (SHG) in waveguides can be significantly increased by resonating the second harmonic field in a vertical cavity structure. The resonant device can be several orders of magnitude more efficient than a non-resonant device. Microwatt output can be expected for tens of milliwatts of infra-red input in a doubly resonant structure for reasonable device parameters.

Introduction: Several workers\(^3\) have demonstrated surface emitting second harmonic generation (SHG) in waveguide devices. The limited vertical interaction length reduces the efficiency of this interaction compared to collinear SHG. Even if quasi-phase-matching techniques are used to solve the interference problem, the efficiency will increase with the core thickness only up to the point where the mode size begins to scale with the waveguide thickness. Although a waveguide resonator\(^1\) which increases the circulating pump power, is clearly an improvement over a nonresonant structure, the theoretical second harmonic power output for a realistic circulating pump power is still too low for many applications.

Resonating the second harmonic wave also increases conversion efficiency, as shown by Ashkin et al.\(^2\) for the collinear case. The purpose of this Letter is to develop and apply a simple theoretical model for doubly resonant SHG with orthogonally propagating fundamental and second harmonic waves, as shown in Fig. 1.

Theory: Although it is possible to analyse this interaction exactly in the limit of no pump depletion by solving the appropriate boundary value problem,\(^3\) a coupled cavity mode formalism\(^3\) provides additional physical insight and allows treatment of pump depletion. For simplicity, we assume a slab waveguide geometry where both the fundamental and second

![Fig. 1 Side view of surface-emitting waveguide resonator geometry](image)
harmonic fields are TE. In this case, we may write the electric field as

\[
E_{z} = \left\{ a_{1}(t)N_{1}(x) \sin(\beta z - \omega t) + a_{2}(t)N_{2}(x) \sin(\gamma z + \omega t) \right\} + c.c.
\]

where \( f(x) \) is the transverse field distribution, \( \beta \) is the propagation constant of the fundamental mode, and the dimensions of the waveguide resonator are \( L \times w \times l \) (Fig. 1). The normalization constants \( N_{1} \) and \( N_{2} \) are chosen such that the circulating powers are given by \( P_{1e} = \frac{n_{1}n_{2}a_{1}^{2}N_{1}^{2}}{n_{2}^{2}N_{2}^{2}} \) and \( P_{2e} = \frac{n_{2}a_{2}^{2}N_{2}^{2}}{n_{2}^{2}N_{1}^{2}} \), where the effective index of the fundamental mode is \( n_{1} = \mu_{0} / \mu_{r} \), and the indices of the core at \( \omega \) and \( 2\omega \) are \( n_{1} \) and \( n_{2} \), respectively. The wavevector \( k_{2} = 2n_{2} \omega / c. \)

With this normalization, the coupled equations for the on-resonance cavity mode amplitudes \( a_{1} \) and \( a_{2} \) are

\[
\begin{align*}
a_{1} &= 3n_{2}^{2}k_{2}a_{2} - 3\gamma_{1}a_{1} + i_{1} \\
a_{2} &= -3n_{1}^{2}k_{1}a_{1} - 3\gamma_{2}a_{2} + i_{2}
\end{align*}
\]

The interaction frequencies are assumed to be far enough from all material resonances so that we can write the total nonlinear polarization as \( P_{NL} = \beta_{12}a_{1}a_{2}^{*}f(x) \) where \( \beta_{12} \) accounts for the projection of the modal fields on the SHG tensor \( \beta \). We then find that the nonlinear coupling coefficient \( k \) is real, due to the phase conversion in eqn. 1, and is given by

\[
k = \frac{\beta_{12}k_{1}k_{2}f(x)}{n_{2}^{2}L}
\]

where \( Z_{0} \) is the impedance of free space, and the dimensionless overlap integral \( I \)

\[
I = \frac{1}{L} \int f(x) N_{1}(x) \sin(\beta z + \omega t) \, dx
\]

where \( N_{1} = \frac{\sin(\beta x)}{\beta} \) by definition. The linear loss parameters are

\[
\begin{align*}
\gamma_{1} &= \frac{e}{2n_{1}L} (1 - \sqrt{R^{2}T^{2}e^{-2\alpha_{1}}}) \\
\gamma_{2} &= \frac{e}{2n_{2}L} (1 - \sqrt{R^{2}T^{2}e^{-2\alpha_{2}}})
\end{align*}
\]

and the driving term \( i \) is

\[
i = \frac{e}{2n_{2}L} \frac{\beta_{12}k_{1}k_{2}f(x)}{n_{2}^{2}L}
\]

where \( P_{m} \) is the incident power. All mirrors satisfy \( R + T + A = 1 \), where \( R \) is the retransmit- tance, and \( A \) is the loss of the mirror. The superscript and subscript notation for mirror quantities is shown on Fig. 1. The power attenuation coefficients for the fundamental and second harmonic modes are \( \alpha_{1} \) and \( \alpha_{2} \), respectively, Eqs. 2 and 3 correctly model the steady state behaviour for arbitrary input and output power, provided all other linear losses are small. If, in addition, the linear coupling is small, the simpler approximate forms for \( \gamma_{1} \) and \( \gamma_{2} \) given in eqns. 6 and 7 apply, where \( \Delta \alpha = T^{2} + A^{2} - 2T^{2}A^{2} \) and \( \Delta \beta^{2} = 1 - T^{2} - A^{2} + 2T^{2}A^{2} + 2A^{2}L^{2}. \)

The steady state form of eqns. 2 and 3 is

\[
a_{1} = \left( \frac{a_{1} - i_{1}}{\gamma_{1} - i_{1}} \right)
\]

and

\[
a_{2} = \left[ 1 + \frac{a_{1}^{2}}{\gamma_{1} - i_{1}} \right] a_{2}^{*} - \frac{i_{2}}{\gamma_{2}}
\]

Because \( a_{1} \) is in phase with \( a_{2} \), we lose no generality in taking \( a_{1} \) to be real in eqn. 8. Eqn. 10 for real \( a_{1} \) is a cubic with a unique solution.

In the low conversion limit, i.e. where \( a_{1} \approx a_{2} \gamma_{1} / i_{1} \), we find, using eqns. 8, 9, and 10 and the relation \( P_{m} = \frac{1}{2} T P_{1}^{2} P_{2}^{2} \), that the surface- emitted second harmonic power \( P_{2e} \) is

\[
P_{2e} = \frac{128\pi^{2} \beta_{12}^{2} k_{1}^{2} f^{2}}{\gamma_{2}^{2}} \left[ \frac{1}{T} \right]^{2} \frac{4T^{2}}{T^{2} + \alpha^{2}}
\]

\[
\times \left[ \frac{1}{T} \frac{4T^{2}}{T^{2} + \alpha^{2}} \right]^{2} \text{ single-pass pump}
\]

\[
\times \left[ \frac{1}{T} \frac{4T^{2}}{T^{2} + \alpha^{2}} \right] \text{ resonant pump}
\]

where \( \alpha \) is the free space wavelength of the fundamental, and the low loss forms given in eqns. 6 and 7 are assumed to be valid. Since we see that the conversion efficiency in this limit is maximized by the usual low cavity impedance matching conditions for the resonant modes, namely \( T = \frac{\alpha}{\alpha} \), and, if the pump is resonant, \( \alpha = \alpha \), where \( T \) is the optimal coupling. If material losses are the dominant losses in the system, then \( \alpha = \alpha \), \( \alpha = \alpha \), and the dependence of the conversion efficiency of an optimally coupled device on the material parameters is of the form \( \beta_{12} L_{1} / n_{2}^{2} L_{2}^{2} \).

Discussion: Owing to their large absorption losses, III-V semiconductor with 2\( \alpha \)-greater than the direct bandgap, which have been used in conventional surface emitting second harmonic generators, are clearly inappropriate for resonant second harmonic interactions. Nonlinear media with much lower absorption coefficients are better candidates for SHG of blue or green light, even if their nonlinear susceptibilities are somewhat smaller, e.g. wide-bandgap II-VI semiconductors. Certain organic crystals with very large nonlinear susceptibilities, such as 2-methyl-4-nitroaniline (MNA) has an extremely large nonlinear susceptibility, \( \beta_{12} = 250 \mu m / V \), is relatively transparent at 0.5-\( \mu m \), and has already been grown in appropriate waveguide geometries.

To calculate the efficiency of a resonant surface-emitting SHG device, we must define the overlap integral \( I \) used in eqn. 5. For simplicity, we analyse only the symmetric waveguide case, and assume the dielectric mirrors can be modelled as a uniaxial form-birefringent medium at \( \omega \) so that the cladding can be modelled with an index \( n_{3} \) for TE modes. As the details of the cladding fields are unimportant here, this is an adequate approximation for \( \omega \) when the mirrors are quarter wave stacks at 2\( \omega \). Finally, we assume the fundamental is propagating in the lowest-order transverse mode, so that

\[
f(x) = \begin{cases} 
\cos(2U \omega z) & |x| \leq L/2 \\
\delta \cos(2U \omega z) & |x| > L/2
\end{cases}
\]

Here the mode parameters \( U \) and \( W \) are given by \( U = \frac{\sqrt{W_{1} L_{1} n_{2}^{2} - n_{1}^{2} L_{2}^{2}}}{2} \), and \( 2W = \frac{\sqrt{W_{2} L_{2} n_{2}^{2} - n_{1}^{2} L_{1}^{2}}}{2} \), where \( \delta = \omega / c \). The eigenvalue equation for \( a_{2} \) is from Reference 9, \( W = U \)

Owing to the simple form of \( f(x) \), the overlap integral defined in eqn. 5 can easily be evaluated. Noting the cavity resonance condition for the second harmonic, \( k_{2} L = \pi m \), where \( m \) is an integer, where

\[
T = \frac{L}{\pi m} \left[ \frac{m \cos(2U \omega z)}{4U \omega z} \right]^{2}
\]

for odd \( m \) and \( I = 0 \) for even \( m \), where \( L_{m} = \frac{1}{2}[L + (1 + iW)] \) for this case. If quasi-phase-matching were obtained by reversing the sign of the nonlinear coefficient every half wavelength, the overlap integral for odd \( m \) would be approximately \( m \) times larger.

The fundamental mode of an MNA waveguide with \( k_{2} = \frac{1}{106 \mu m}, n_{1} = 1.8, n_{2} = 1.7, n_{3} = 2.2, \) and \( L = 30 \mu m \).
0.36 μm has an effective index of nₑ = 1.73. If we further assume a nonresonant pump, ωₒ = 2πc/nₑ, l = 100 μm, d₄₄ = d₂₂ = 250 pm/V, and l₁ = l₂ = 3 m, and use eqns. 10 and 11 we find Pₑ/αₑ = 0.05 W⁻¹. Because m = 3, we expect the efficiency to be approximately nine times larger if quasi-phasematching is employed (numerically we obtain a factor of 13.4), which gives Pₑ/αₑ = 0.66 W⁻¹.

If both the pump and the second harmonic are resonant, the assumption of low conversion efficiency is no longer valid for an incident power of the order of 100 mW, as is easily seen from the above numerical result. In such cases, eqn. 11 is no longer valid, and the exact solution to eqns. 9 and 10 must be used. We give a numerical example here, reserving a more detailed analysis for future work. Assuming A₁ = A₂ = 0.005, β₁ = 0.05, no quasi-phasematching, and Pₑ = 100 mW in addition to the above parameters, we find a maximum second harmonic power output of 36 mW for an optimal coupling given by t₁² = t₂² = 0.097, and t₁ = t₂ = 0.027. We see that high conversion efficiencies are possible for modest input power, and that the optimal coupling differs from that calculated in the low efficiency regime.

Conclusion: We have shown that vertical cavity resonant SHG structures can have significantly higher conversion efficiency than conventional surface emitting SHG devices, and that high-efficiency interaction. A phase-matchable nonlinearity is not necessary, as seen in the numerical example where tens of milliwatts of second harmonic were obtained for a 100 mW pump using the nonphasematchable d₄₄ coefficient of MNA. The efficiency may be significantly increased by calculating here if quasi-phasematching techniques can be applied to this resonant geometry. Extention of this theory to TM modes, multilayer waveguides and interactions such as asymmetric amplification and oscillation will be given elsewhere.

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