Noncritical Quasi-Phase-Matched Second Harmonic Generation in an Annealed Proton-Exchanged LiNbO₃ Waveguide


Abstract—We report the demonstration of dimensional noncritical phase matching, a phase-matched interaction length exceeding 10 mm, and an internal conversion efficiency of 204%/W for second harmonic generation of 976 nm radiation in a periodically poled, annealed proton-exchanged LiNbO₃ waveguide. Using models for the linear and nonlinear optical properties of annealed proton-exchanged LiNbO₃ waveguides and the observed ferroelectric domain grating, the phase-matching wavelength was predicted to within several nm and the conversion efficiency to within ±20% of the measured values. Optimization of waveguide second harmonic generation devices is discussed.

I. INTRODUCTION

QUASI-PHASE-MATCHED (QPM) second harmonic generation (SHG) in LiNbO₃ waveguides is an attractive method for the generation of blue light from infrared diode lasers. Many different techniques have been used to form the ferroelectric domain grating necessary for QPM. Waveguide confinement has been achieved exclusively using the annealed proton exchange (APE) process. While normalized conversion coefficients exceeding 600%/W cm² have been reported [1], [2], modeling of the experimentally observed phase-matching wavelengths and normalized conversion efficiencies has not appeared in the literature. The observed phase-matching wavelengths deviate significantly from those predicted from the QPM period and the bulk LiNbO₃ refractive indexes. The reported normalized conversion efficiencies, varying from 40 to 700%/W cm² [1]–[5], are substantially lower than predicted theoretically. The variations in device performance are due to a combination of factors, including the use of first, second, and third QPM grating orders, different ferroelectric domain grating fabrication processes, and different APE waveguide processing conditions. The discrepancy between the observed and theoretically expected performance is due to inadequate device characterization and an incomplete understanding of the linear and nonlinear optical properties of APE–LiNbO₃. The linear optical properties of the waveguide are necessary for calculation of the effective indexes of the guided modes for prediction of the phase-matching wavelength. Both the linear and nonlinear optical properties of the waveguide are necessary for the prediction of the guided wave conversion efficiency. Recently published models for the evolution of the refractive index profile [6] and dₑₑ nonlinear coefficient [7] with annealing in APE–LiNbO₃ waveguides have yet to be applied to frequency conversion devices.

An additional complication in understanding waveguide frequency conversion devices is the presence of axial phase velocity inhomogeneities. These inhomogeneities can arise from axial variations in the waveguide lithography or processing conditions. Such inhomogeneities reduce the maximum conversion efficiency and distort the ideal sinc⁴ tuning curve [8]. Previous work showed that waveguide designs exist that are relatively insensitive to waveguide dimensional fluctuations [9]. This insensitivity arises from eliminating the first-order dependence of the phase velocity mismatch on waveguide dimension, and is termed dimensional noncritical phase-matching in analogy to certain bulk, birefringently phase-matched, nonlinear optical interactions which are insensitive to variations in angle or temperature. Experimentally, noncritically phase-matched interactions should display tuning curves close to ideal, and enhanced interaction lengths over critically phase-matched interactions.

In this paper, we discuss the fabrication, characterization, and analysis of guided wave QPM–SHG devices for λₐ≈976 nm fundamental radiation. Phase-matching wavelengths and wavelength tuning curves versus waveguide width are presented that show the evolution towards dimensional noncritical phase matching. Experimental verification of the noncritical phase-matching condition and a phase-matched interaction length exceeding 10 mm are presented. In such a waveguide, an internal conversion efficiency of 204%/W was observed, with a corresponding internal normalized conversion efficiency of 185%/W cm². The observed phase-matching wavelengths and conversion efficiencies are in good agreement with the expected performance based on recently published models for the linear [6] and nonlinear [7] optical properties of APE–LiNbO₃ waveguides. Comparison between observed and expected performance clearly demonstrates the effects that reduce the conversion efficiency of QPM–SHG devices in APE–LiNbO₃, and provides guidance towards improvement of the overall device performance.

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II. EXPERIMENTAL

A titanium indiffusion process was used to fabricate ferroelectric domain inversion gratings for quasi-phase-matching [2]. A 50-Å thick Ti film was patterned by wet etching into a grating with 5.00 μm period and 1.0 μm wide lines on the +z face of LiNbO₃. Domain inversion was performed by placing the sample on top of congruent LiNbO₃ powder in a closed Al₂O₃ crucible and annealing with an approximately linear 2 h ramp to 1100°C for a 4 min soak, after which the furnace was turned off and cooled at approximately 8°C/min. Annealed proton-exchanged waveguides were fabricated after domain inversion by patterning a 2000-Å thick SiO₂ mask with a variety of channel widths, exchanging in pure benzoic acid at 158°C for 90 min to an initial proton exchange depth of 0.22 μm, and annealing for 3 h at 333°C. All process temperatures were measured at the sample position. Waveguide widths quoted in the text are the corresponding mask widths. The sample had multiple waveguides with the same width, and following end face polishing, the device was 10.5 mm long.

The pump laser for SHG measurements was a standing wave titanium–sapphire laser, turned by a Lyot filter and a 3-Å free spectral range etalon, which oscillated in a single longitudinal mode. An isolator was used to prevent feedback into the laser. Laser wavelength was measured to an accuracy of 0.01 Å using a wavemeter. To circumvent complications due to variable input coupling and weak Fabry–Perot resonance effects within the waveguide at the fundamental wavelength (λₛ), all measurements were performed by monitoring the transmitted fundamental (Pₛ) and second harmonic (P₂ₛ) powers separately and recording the ratio P₂ₛ/Pₛ, corresponding to the external conversion efficiency. Absolute power measurements are corrected for Fresnel reflections at the exit endface of the waveguide to yield internal conversion efficiencies. Unless noted, all measurements were made with fundamental powers <1 mW to minimize photorefractive or thermal effects in the waveguide. We define the phase-matching wavelength (λₚₘ) as the wavelength of the observed maximum conversion efficiency, which can differ from the wavelength at which the maximum second harmonic (SH) power is observed due to resonances in the waveguide at λₛ. The transverse mode structure of the waveguide at the fundamental and the second harmonic (λ₂ₛ) wavelengths was monitored using a CCD array, and all the waveguides studied were single transverse mode at the fundamental wavelength.

III. MEASUREMENTS

Quasi-phase-matched SHG was observed at λₛ = 1026 nm with the SH in the transverse TM₀ mode and at λₛ ≈ 976 nm with the SH in the transverse TM₁ mode (one node in depth), with the exact phase-matching wavelength dependent on waveguide channel width. For comparison, the phase-matching wavelength in bulk LiNbO₃ with a 5.00 μm QPM grating is λₚₘ = 968 nm [10]. Fig. 1 shows the measured λₚₘ, with the SH in the transverse TM₀ mode, as a function of waveguide width (the dashed curve is meant to guide the eye). The scatter in the data is due to two sources: different waveguides of the same nominal width had slightly different phase-matching wavelengths, and the ambient temperature varied by 5°C on different days, represented by the open and closed circles. A maximum in the measured λₚₘ occurs in waveguides fabricated with mask openings between 5.00 and 5.25 μm. At this maximum, the phase mismatch has no first-order dependence on waveguide width fluctuations, so in the presence of width fluctuations, the maximum phase-matched interaction length should occur in the waveguides with the largest λₚₘ.

Increased interaction length due to noncritical phase matching was confirmed by measuring the wavelength tuning curves for the TM₀ mode interaction in waveguides with mask widths of 4.25, 4.50, 5.00, and 5.25 μm, as shown in Fig. 2. The wavelength tuning curves from the waveguides with 4.25 and 4.50 μm widths are far from ideal, with significant conversion efficiency in the wings of the tuning curve and reduced efficiency in the central lobe. As waveguide width increases, the peak conversion increases and the tuning curve narrows. The tuning curve from the 5.00 μm wide waveguide is symmetric and nearly an ideal sinc², with a measured wavelength FWHM of 1.1 Å. For wider waveguides, the tuning curves are asymmetric and the peak wavelength remains nearly constant, as shown in Fig. 2(d) for a 5.25 μm wide waveguide. This behavior is consistent with an interpretation of the deviations from the ideal sinc² behavior as being due to axial variations in waveguide lithography, which scale with waveguide width.

A useful check of the consistency of these measurements is comparison of the integrated areas under the tuning curves of waveguides of different widths. In a nonlinear optical interaction, the area under the tuning curve is independent of axial phase velocity inhomogeneities [11], which distribute efficiency away from the central lobe into the wings of the tuning curve. The tuning curves shown in Fig. 2 have
integrated areas that are constant to within 20%, with the area increasing with waveguide width.

The tuning curve for the 5.00 μm wide waveguide indicates that axial inhomogeneities in the grating or the phase velocities are negligible, so this waveguide is ideal for accurate measurements of the normalized conversion efficiency. In this waveguide, the measured internal conversion efficiency was 9.6%/W for SHG into the TM_{00} mode and 204%/W for SHG into the TM_{01} mode. We estimate the accuracy of these measurements to be ±10%. Assuming a phase-matched interaction length equal to the 10.5 mm device length, the measured internal normalized conversion efficiencies are \( \eta_{TM_{00}} = 8.7% / W \cdot cm^2 \) and \( \eta_{TM_{01}} = 185% / W \cdot cm^2 \). The conversion efficiency of 204%/W is the highest conversion efficiency reported to date in an APE–LiNbO\(_3\) waveguide.

IV. COMPARISON BETWEEN MEASUREMENTS AND CALCULATIONS

Difficulties in understanding QPM frequency conversion in LiNbO\(_3\) waveguides have been exacerbated by inadequate knowledge of the modal properties and the \( d_{33} \) nonlinear coefficient of APE–LiNbO\(_3\) waveguides. Models for the linear [6] and nonlinear [7] optical properties of APE–LiNbO\(_3\) waveguides versus processing conditions were used to predict the phase-matching wavelengths and the normalized conversion efficiencies of the QPM-SHG devices discussed in this work from the QPM grating and waveguide fabrication parameters. Reference [6] contains models for the refractive index profile of APE–LiNbO\(_3\) waveguides and material dispersion of APE–LiNbO\(_3\) that were used to calculate the two-dimensional refractive index profile of the waveguide [12]. The effective indexes and waveguide mode profiles at \( \lambda_\omega \) and \( \lambda_{2\omega} \) were then calculated using the effective index method. For a given QPM grating period, the effective indexes determine the wavelength for phase-matching between any set of transverse modes. The normalized conversion efficiencies were calculated using the modes at \( \lambda_\omega \) and \( \lambda_{2\omega} \), the depth-dependent Fourier component of the QPM grating derived from microscopy of the ferroelectric domain grating, and the variation in the \( d_{33} \) coefficient with depth described in [7]. In all cases, the calculations were performed using only the fabrication conditions, with no free parameters. More complete discussion of the linear and nonlinear optical properties of APE–LiNbO\(_3\) waveguides will appear elsewhere [13].

The solid line in Fig. 1 shows the phase-matching wavelengths for waveguides formed with a 5.00 μm period QPM grating, calculated using the annealed proton exchange processing parameters described above and mask openings varying from 3.5 to 7 μm. The calculation reproduces the phase-matching wavelength to within several nm, and the calculated maximum at the width of 5.50 μm is in good agreement with the observed maximum between 5.00 and 5.25 μm. For narrower widths, the variation in \( \lambda_{pm} \) with width increases, indicating a large first-order dependence of phase velocity mismatch on waveguide width. The slight offset between the data and the calculation corresponds to an error of about \( 10^{-1} \) in the prediction of the effective index dispersion of the APE–LiNbO\(_3\) waveguide, which could be due to variations in processing conditions or errors in the model described in [6]. For a 5.00 μm wide waveguide, we calculated \( \lambda_{pm} = 1006 \) nm and \( \lambda_{pm} = 975.4 \) nm with the SH in the TM_{00} and TM_{01} modes, respectively, in good agreement with the observed values of \( \lambda_{pm} = 1026 \) nm and \( \lambda_{pm} = 976.1 \) nm. The increasing discrepancy at longer wavelengths may be due to increasing error in the effective index method used to calculate the effective indexes as the waveguide approaches cutoff.

Using the model for the APE–LiNbO\(_3\) waveguide, we computed the tuning bandwidth for 976 nm frequency doubling with the SH in the TM_{01} mode. Evaluating the waveguide effective indexes \( N_{\omega} \) at \( \lambda_{pm} = 976 \) nm and \( N_{2\omega} \) at \( \lambda_{2\omega} = 485 \) nm yields \( dN_{\omega}/d\lambda_\omega = 7.2 \cdot 10^{-5} \) nm\(^{-1}\) and \( dN_{2\omega}/d\lambda_{2\omega} = 6.3 \cdot 10^{-4} \) nm\(^{-1}\), about 10% greater than the bulk LiNbO\(_3\) wavelength derivatives. Using these values, we compute a wavelength FWHM tuning bandwidth of 12.6 Å ± mm for \( \lambda_{pm} = 976 \) nm. From the tuning curve shown in Fig. 2(c), the observed FWHM bandwidth indicates an interaction length of 11.4 mm, in reasonable agreement with the actual 10.5 mm device length.

We also compared the experimental normalized conversion efficiencies to the theoretical values. With the direction of propagation along the x axis over a phase-matched interaction length \( l \), the ideal, lossless, internal normalized conversion


\[ \eta_0 = \frac{8\pi^2 d_{\text{eff}}^2}{N_2^2 N_{\text{d}} c t_0 \lambda_0^2} \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dG_m(z) d\bar{z} \right)^2 \cdot E_{2\omega}(y, z) E_{2\omega}(y, z) \right)^2. \]  

(1)

In this expression, \( d_{\text{eff}} = 2 \cdot d_{\text{d}} / \pi m \) is the order of the quasi-phase matching, \( d(\bar{z}) = d_{\text{d}} \text{APE}(z) / d_{\text{d}} \text{N} \) is the \( d_{\text{d}} \) nonlinear coefficient in the APE-LiNbO\(_3\) waveguide normalized to the bulk LiNbO\(_3\) value, and \( E_w \) and \( E_{2\omega} \) are the optical modes normalized to carry unity power. \( P_\omega \) is the fundamental power coupled into the waveguide. We have explicitly the depth dependence of the \( d_{\text{d}} \) nonlinear coefficient within the overlap integral. The term \( G_m(z) \) is the depth-dependent Fourier coefficient of the QPM grating defined as

\[ G_m(z) = \frac{m \pi}{2} \left( \frac{1}{\Lambda_\phi} \int_{0}^{\Lambda_\phi} g(x, z) e^{-i(2m\pi / \Lambda_\phi) x} \, dx \right) \]  

(2)

where \( \Lambda_\phi \) is the QPM grating period and \( g(x, z) \) is the function that describes the spatial variation of the ferroelectric domain grating, taking a value \(+1(-1)\) in +\( z \) (−\( z \))-oriented domains. For the first-order QPM \( (m = 1) \) devices described in this work, \( G(z) = \sin \{ \pi \xi(z) \} \), where \( \xi(z) \) is the depth-dependent duty cycle of the grating which takes a value of \( 1/2 \) for ideal first-order QPM. In subsequent calculations, we take 34.5 pm/N as the value of the \( d_{\text{d}} \) nonlinear coefficient in bulk LiNbO\(_3\), and use the measurements of \( d_{\text{d}} \) for the depth dependence of \( d(\bar{z}) \), the nonlinear coefficient in the APE-LiNbO\(_3\) waveguide.

Assuming the modes are separable (i.e., \( E(y, z) = E_y(y) E_z(z) \)), the above expression for \( \eta_0 \) simplifies to

\[ \eta_0 = \frac{8\pi^2 d_{\text{eff}}^2}{N_2^2 N_{\text{d}} c t_0 \lambda_0^2} \left( I_z I_y \right)^2 \]  

with

\[ I_z = \int_{-\infty}^{\infty} G_m(z) \bar{d}(z) E_{2\omega}(z) E_{2\omega}(z) \, dz \]

\[ I_y = \int_{-\infty}^{\infty} E_{2\omega}(y) E_{2\omega}(y) \, dy. \]  

(3)

In the limit of negligible loss and ignoring the Fabry–Perot resonances within the waveguide (these effects and their influence on measurement of the normalized conversion efficiency are discussed in the Appendix), the experimentally observed normalized conversion efficiency and the ideal, theoretical normalized conversion efficiency defined in (1)–(3) are equivalent quantities suitable for comparison.

Fig. 3(a) shows a schematic of the periodic QPM ferroelectric domain grating used in this work, revealed after endface polishing by etching in pure HF for 5 min. The grating angle \( \theta \) is 24° and the buried grating depth \( a \) is 1.1 \( \mu \)m. Fig. 3(b) and (c) show the factors entering into the integrand of \( I_z \) and \( I_y \) for phase-matching into the TM\(_{0}\) and TM\(_{01}\) SH modes for the 5.00 \( \mu \)m wide noncritically phase-matched waveguide previously discussed. The integrands of \( I_y \) are not shown for brevity since they depend only on the optical modes.
soak temperature of shortening the soak times of the Ti diffusion. Another domain inversion grating was fabricated with a soak temperature of 1090°C rather than 1100°C, with all other diffusion parameters the same as previously described. This domain grating had a shape nearly identical to that shown in Fig. 3(a) with a buried depth \( a = 0.8 \mu m \). APE waveguides were fabricated as previously discussed. In the 5.00 \( \mu m \) wide waveguide, a near ideal tuning curve was observed with a FWHM of 3.0 \( \AA \), corresponding to an interaction length of 4.2 mm, in good agreement with the 4.4 mm sample length. The measured normalized conversion efficiencies in this waveguide were \( \eta_{\text{TM00}} = 36 \) and \( \eta_{\text{TM01}} = 177\% / W \cdot cm^2 \) for SHG into the TM00 and TM01 SH modes, respectively. Fig. 4(a) shows the measured and calculated normalized conversion efficiencies in the 5.00 \( \mu m \) wide waveguide versus \( a \). Fig. 4(b) shows the measured and calculated ratio \( \eta_{\text{TM00}}/\eta_{\text{TM01}} \). Modifying the grating position increases \( \eta_{\text{TM00}} \) and decreases the ratio \( \eta_{\text{TM01}}/\eta_{\text{TM00}} \) by a factor of 4, in good agreement with the theoretical results. The agreement between experiment and theory when the actual domain grating is used in the calculation substantially confirms the previously presented models for the linear and nonlinear optical properties of APE–LiNbO3. In this sample, we measured a maximum blue output power of 5.1 mW with 163 mW of fundamental radiation exciting the waveguide. This corresponds to a conversion efficiency about 1/2 that measured at low input powers.

Fig. 4 indicates that the Ti-diffusion process for domain inversion and an adequate reduction of the grating depth should result in conversion efficiencies approaching 150%/W \cdot cm^2 for SHG into the TM00 mode. For phase matching 976 nm radiation, this requires a grating period shorter than
the 5.00 μm period discussed above. Preliminary experiments for SHG of 976 nm radiation with a 4.5 μm ferroelectric domain grating and a grating depth of α = 0.3 μm show normalized conversion efficiencies 140%/W·cm²; these devices will be discussed along with high-power effects in a future publication. Further efficiency gains may be achieved by a deeper, depth-independent domain grating and by tighter waveguide confinement, as is clear from the low modal overlap and extended fundamental mode shown in Fig. 3(b) and (c). Normalized conversion efficiencies of 150%/W·cm² in a TM00 mode and interaction lengths exceeding 10 mm should prove sufficient for many device applications that require 488 nm radiation.

V. CONCLUSION

We have demonstrated noncritical phase matching and interaction lengths exceeding 10 mm in a quasi-phase-matched annealed proton-exchanged LiNbO3 waveguide. An internal conversion efficiency of 204%/W over a 10 mm interaction length was achieved using the Ti-indiffusion process for ferroelectric domain inversion and tailoring the APE-LiNbO3 waveguide for dimensional noncritical phase matching. We have accurately modeled the phase-matching wavelengths and conversion efficiencies of these devices using recently available models for the linear and nonlinear optical properties of APE-LiNbO3. The degradation of the d33 coefficient does not affect these devices, but may play a role in limiting the conversion efficiencies of other guided wave SHG devices, especially those with tighter modal confinement than was used here. Dimensional noncritical phase-matching and the accompanying reduction in axial phase velocity inhomogeneities allow a detailed examination of device performance, and are essential for understanding and improving guided wave QPM frequency conversion devices.

APPENDIX

Measurement of normalized conversion efficiencies in waveguide frequency conversion devices is complicated by a variety of factors, including grating and/or waveguide inhomogeneities, propagation loss in the waveguide, resonance effects within the waveguide, the mode structure of the pump laser, and the magnitude of the relevant material parameters such as the bulk nonlinear coefficient. Accurate measurement and analysis of the normalized conversion efficiency is important for device engineering and optimization. This Appendix treats some of these effects.

Grating and waveguide inhomogeneities can have a large effect on conversion efficiency. Some types of inhomogeneities do not modify the tuning curves, but reduce the device conversion efficiency [8]. In general, these inhomogeneities locally reduce the magnitude of the pertinent grating Fourier component. As an example of this type of inhomogeneity, we consider duty cycle variations in the ferroelectric domain grating, observed as a slightly varying shape of the Ti-diffused domain grating shown in Fig. 3(a) and recently in electric field periodically poled LiNbO3 [1]. In the case of random duty cycle errors with a fixed period defined by lithography, the expectation value of the conversion efficiency (η) normalized to η0, the ideal, internal normalized conversion efficiency calculated from (1)–(3), is given by [14]

$$\frac{\eta}{\eta_0} \approx \exp\left[-\left(\sqrt{2\pi \sigma / \Lambda_g}\right)^2\right]$$

(A1)

where Λg is the QPM grating period and σ is the rms error in the duty cycle, assumed to be normally distributed. rms errors as large as 1/6 of Λg reduce the conversion efficiency by only 50%. The duty cycle effect is probably smaller than other measurement and analysis errors in this work, but may be relevant for the devices described in [1].

The annealed proton-exchanged waveguide fabrication process can yield low-loss waveguides. However, even loss values of ≃1–2 dB/cm can have a significant effect of the nonlinear frequency conversion process. In the presence of phase velocity mismatch and propagation loss, the conversion efficiency normalized to η0 is given by [15]

$$\frac{\eta(\Delta k, \alpha_w, \alpha_{2w})}{\eta_0} = 2e^{-\alpha_w (\alpha_w + \alpha_{2w} / 2) / \Lambda_g} \cdot \frac{\cosh(\Delta k l) - \cos(\Delta k l)}{(\Delta k l)^2}$$

(A2)

where αw and α2w are the exponential power loss coefficients at λw and λ2w, and Δk = α2w/2 - αw. The phase-matching bandwidth is a function of the combined loss parameter Δα. Fig. 5 shows the FWHM bandwidth normalized to the lossless case versus Δα. There is a negligible effect on the bandwidth and little distortion of the ideal sinc² tuning curve for Δαl < 2, a condition obeyed by typical APE waveguides. However, this is not the case for the phase-matched conversion efficiency, given by

$$\frac{\eta(\Delta k = 0, \alpha_w, \alpha_{2w})}{\eta_0} = e^{-2\alpha_w (1 - e^{-\Delta k l})}$$

(A3)

Fig. 6 shows the conversion efficiency normalized to η0 versus αwl for different values of α2w/αw. As an example, a 1 dB single-pass loss at both λw and λ2w leads to a ≃25% reduction in the peak conversion efficiency.

The above expressions for conversion efficiency in the presence of loss are referenced to the fundamental power coupled into the waveguide, while the experimental conversion efficiencies quoted in this work and in most of the literature are referenced to the fundamental power after propagation over the length of the waveguide (there is a further distinction between the internal and external conversion efficiencies, with the former correcting the measured powers for Fresnel reflections at the waveguide endface). In the presence of propagation loss, the fundamental power coupled into the waveguide and the fundamental power remaining in the waveguide after propagating over a length l are related by P2(l) = P2(0)e^{-αwl}. The ratio of the phase-matched conversion efficiency, referenced to the fundamental power after propagation over a length l, to η0 is given by

$$\frac{\eta(\Delta k = 0, \Delta \alpha)}{\eta_0} = \left(1 - e^{-\Delta \alpha l}\right)^2.$$  

(A4)
\[
\eta = \frac{\sin^2 \left( \beta \Delta \lambda_\omega \left( N_{2\omega}(\lambda_{2\omega}) - N_\omega(\lambda_\omega) - 1/\Lambda_\omega \right) \right)}{\left( (1 - R_\omega)^2 + 4R_\omega \sin^2 (\beta N_\omega(\lambda_\omega)) \right)^2 \left( (1 - R_{2\omega})^2 + 4R_{2\omega} \sin^2 (2\beta N_{2\omega}(\lambda_{2\omega})) \right)}
\] (A5)

Fig. 5. Increase in FWHM bandwidth versus combined loss parameter \( \Delta \alpha = \alpha_\omega - \alpha_{2\omega} \). Annealed proton-exchanged LiNO3 waveguides can be fabricated with low enough losses that the bandwidth increase is negligible.

Fig. 6. Reduction in phase-matched conversion efficiency, normalized to the lossless case, versus \( \alpha_\omega \ell \) for different values of \( \alpha_{2\omega}/\alpha_\omega \). Even high-quality, low-loss APE-LiNO3 waveguides will exhibit a reduced conversion efficiency due to loss.

This expression gives the surprising result that the error between \( \eta_0 \) and the measured conversion efficiency when normalized to the transmitted fundamental power depends on the combined loss parameter \( \Delta \alpha \) rather than \( \alpha_\omega \) and \( \alpha_{2\omega} \) considered separately. \( \eta/\eta_0 = 1 \) when \( \alpha_{2\omega} = 2 \cdot \alpha_\omega \) and decreases with increasing loss at the second harmonic. Since \( \Delta \alpha \) depends on the exponential loss coefficients, even losses of 3 dB at the SH and 1 dB at the fundamental only result in \( \approx 10\% \) error when comparing \( \eta \) to the ideal expression \( \eta_0 \).

The insensitivity of \( \eta/\eta_0 \) to loss arises from the competing effects of loss on the SHG interaction; loss reduces the driving nonlinear polarization along the waveguide, resulting in less SH output, but loss also reduces the throughput of the fundamental, increasing the apparent conversion efficiency when referenced to the transmitted power. When \( \Delta \alpha \) is small, the effects essentially offset each other, and reasonably accurate measurements of \( \eta_0 \) can be obtained by using the transmitted fundamental power to determine the conversion efficiency.

If a waveguide device has polished endfaces with the surface normal nearly aligned to the waveguide propagation direction, the waveguide acts as a weak resonator at \( \lambda_\omega \) and \( \lambda_{2\omega} \). The transmitted and circulating power at the fundamental wavelength oscillate with waveguide temperature or incident wavelength due to the Fabry–Perot resonance. There is a similar effect at \( \lambda_{2\omega} \). In the presence of resonance, the ratio of the conversion efficiency, referenced to the fundamental power coupled into the waveguide, to \( \eta_0 \) is given by (A5), shown at the top of the page, where \( R_\omega \) and \( R_{2\omega} \) are the waveguide endface power reflectivities and \( \beta = 2\pi f/\lambda_\omega \). A tuning curve of \( R_{2\omega} \) versus \( \Delta k \) will have fringes due to resonance at \( \lambda_\omega \) and \( \lambda_{2\omega} \). The \( \sin^2 \) phase-matching envelope is modulated by fringes that, depending on device length and \( \Lambda_\omega \), can have a periodicity ranging from much less to much greater than the FWHM phase-matching bandwidth. By recording the transmitted powers at \( \lambda_\omega \) and \( \lambda_{2\omega} \) and referencing the conversion efficiency to the transmitted fundamental power, the fringes due to the fundamental wavelength resonance may be removed. Fringes due to the second harmonic resonance remain, but are usually less of a problem due to increased loss at shorter wavelengths. Accurate measurements of the conversion efficiency and phase-matching wavelengths are facilitated by waveguide endface angle polishing and anti-reflection coating to reduce the resonance effects.

The expressions for the conversion efficiency used in this work assume a single axial mode pump source. Multiple axial pump modes can mix in the nonlinear frequency conversion interaction with a complicated effect on conversion efficiency. In the case of a pump source with multiple axial modes with uncorrelated phases and a total bandwidth significantly less than the phase-matching bandwidth, the ratio of the conversion efficiency to \( \eta_0 \) is given by [16]

\[
\frac{\eta_{\text{multimode}}}{\eta_0} = \frac{2}{N} - \frac{1}{N}
\] (A6)

where \( N \) is the total number of axial modes. As an example, a standing wave Ti : sapphire laser with a 200 MHz free spectral range may have 50 modes oscillating in a 10 GHz bandwidth, considerably less than a typical 1-Å SHG device bandwidth. In this case, one may expect an enhancement of \( \approx 2 \) over the single mode case. However, in a typical laser diode, the output power may be evenly distributed among four axial modes with
a mode spacing of ≈ 5 Å, far greater than the SHG phase-matching bandwidth. The maximum conversion efficiency in this case would be reduced by a factor of 16 compared to the ideal single-mode case. Thus, a single SHG device can exhibit conversion efficiencies that easily vary by over an order of magnitude due to the spectral content of typical laboratory pump sources.

Finally, analysis of quasi-phase-matched LiNbO₃ frequency conversion devices requires a value for the d₃₃ nonlinear coefficient of bulk LiNbO₃. However, values in the literature range from 27 [17] to 34.5 pm/V [18], resulting in analysis errors of ≈ 50%. Recent measurements of 23.7 pm/V [19] reemphasize the continuing need for accurate measurements of bulk nonlinear coefficients.

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