The Competitive Effects of Linking Electricity Markets Across Space*

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Abstract

Locational marginal pricing (LMP) is the standard US wholesale electricity market design. Major barriers to implementation in other countries are the perceived effects on the liquidity of forward markets and the distribution effects of charging customers at different locations different prices for electricity. Yet, forward contracts in LMP markets often clear against a quantity-weighted average of locational prices, and customers are also charged average LMPs. We demonstrate market performance implications of these practices: A forward contract that clears against the average of a set of LMPs increases competition in short-term markets relative to forward contracts that clear against individual LMPs.

Key Words: Electricity market design, equity, regional prices, market power.

JEL: C72, D43, G10, G13, L13

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1 Introduction

Locational marginal pricing or nodal pricing is used to operate all offer-based wholesale electricity markets in the United States.¹ Locational marginal prices (LMPs) are computed by minimizing the as-offered cost of serving demand at all locations in the transmission network subject to all relevant generation unit and transmission network constraints. The LMP at a location or node in the transmission network is equal to the increase in the minimized as-offered cost of withdrawing an additional megawatt-hour (MWh) at that node. This process can give rise to thousands of potentially different LMPs within the geographic footprint of the wholesale market each pricing period.² If all suppliers submit each generation unit’s marginal cost as its offer price, then each LMP is the economically efficient price signal for that location in the transmission network during that pricing period. Specifically, these LMPs make it unilaterally profit-maximizing for each generation unit to produce at a level of output that minimizes the total variable cost of serving demand at all locations in the transmission network.

Restructured wholesale electricity markets in many other parts of the world rely on much less sophisticated methods than locational marginal pricing for allocating resources across the system. In such markets, the algorithms used for calculating market prices routinely fail to account for important bottlenecks in the transmission network. However, many of these jurisdictions are becoming increasingly aware of the inefficiencies associated with market designs that neglect these fundamental constraints on system operation. The costs of operating such electricity markets have increased as a consequence of energy policies through which countries become more and more reliant on variable renewable energy. The variability of wind and solar generation units accentuates network capacity constraints, which substantially increases the costs of achieving a reliable supply of electricity unless handled efficiently. Yet, it has been extremely difficult to implement market designs with granular prices outside the US despite the efficiency properties of wholesale market designs based on locational marginal prices.³

²Bohn et al. (1984) characterize the mathematical programming problem solved to compute market-clearing quantities and LMPs.
³Wolak (2011) finds that the transition from a zonal to a nodal market design in California was associated with annual savings in the variable cost of serving demand of over $100 million annually. Triolo and Wolak (2021) estimate that the transition to an LMP market design in Texas produced more than $300 million in annual variable cost savings during the first twelve months of operation.
A fundamental barrier to adoption has been the potential of the LMP market design to set different prices at different locations in the transmission network. A major argument against this design rests on the notion that locational wholesale market prices will reduce the liquidity of financial markets and make it more expensive for consumers to hedge electricity prices in the forward market. A second argument against charging final consumers a wholesale price that reflects the LMP at their location in the transmission network is based on the view that it is unfair to charge customers in major load centers higher wholesale prices than customers at less supply-constrained locations in the transmission network.\footnote{More sophisticated versions of this argument claim that a very different transmission network would have been built had a locational marginal pricing market design been in place when the network was originally constructed.}

Actual LMP markets address liquidity and equity issues to varying degrees. Forward contracts often clear against trading-hub prices instead of individual LMPs in order to increase liquidity in this market. A trading-hub price is calculated as the volume-weighted average of the LMPs at all locations that jointly form the trading hub. This enables all retailers within the geographical region covered by the trading hub to purchase forward contracts at the same \textit{regional forward price}. Regulators and market operators have addressed equity concerns in LMP markets by requiring that all customers within a given geographical area purchase wholesale electricity at a single \textit{regional consumer price} based on the volume-weighted average of all locational prices in that geographic area.\footnote{For example, all retail customers in the service territory of each of the three large investor-owned utilities in California pay a wholesale price equal to the quantity-weighted average of LMPs at all load withdrawal locations in that utility’s service territory. All customers of each utility purchase their wholesale electricity at the same Load Aggregation Point (LAP) price for their utility regardless of where they are located in the utility’s service territory. Singapore operates a nodal pricing market, and all loads purchase their wholesale electricity at the Uniform Singapore Electricity Price (USEP) which is equal to the quantity-weighted average of the LMPs in Singapore.} We show that these regional features of LMP market designs have important consequences for the performance of imperfectly competitive short-term wholesale electricity markets that employ location-based pricing.

Our basic insight is that linking $M$ local markets through a single fixed-price forward contract that clears against the quantity-weighted average of the locational short-term prices over all $M$ markets, increases the equilibrium quantity of fixed-price forward contracts held by suppliers, retailers and large consumers beyond what would occur if there were $M$ independent forward markets with a forward contract price in each local market clearing against the locational short-term price in that market. As is well-known, fixed-price forward contracts can improve short-term market performance because any change in the short-term price only applies to the quantity sold in the short-term market beyond the supplier’s fixed-price
forward contract obligation. The associated increase in the amount of forward contracts reduces short-term prices below the level that would exist under forward contracts cleared against individual locational short-term prices in local markets.

We analyze a two-stage game where producers sell fixed-price forward contracts in a first stage and produce for the short-term market in a second stage, as in Allaz and Vila (1993). The demand for forward contracts comes from retailers and large consumers who are forward-looking and anticipate that buying such contracts will reduce the short-term price and therefore the cost of electricity consumption (Ruddell et al., 2018). The short-term market consists of $M$ markets that have local market-clearing prices because of transmission network constraints. There is a fixed number of generators with market power in each local short-term market $m$. Each producer with market power owns generation capacity only in one local market.

The equilibrium amount of forward contracting that emerges from this model balances a number of opposing forces. Because of the associated short-term market efficiency benefits, consumers are willing to pay a premium on forward contracts. This forward premium makes it profitable for producers to participate in the forward market. However, more forward contracting reduces the forward price, which constrains the volume of forward contracts sold in equilibrium. An additional strategic effect makes it profitable to increase forward contracting beyond the level that maximizes forward profit if the local short-term market is an oligopoly. A larger forward quantity then acts as a commitment device for a producer with market power to supply more electricity to the short-term market. This commitment increases profit in the short-term market through a strategic response that causes competitors to reduce their own production.

The trade-offs facing producers regarding how much electricity to sell in the forward market depends on whether the forward contract sold by a producer located in market $m$ clears against the local short-term price $p_m$ or against the volume-weighted average $\bar{p}$ of short-term prices. In the first case of local forward markets, there is one separate forward price $f_m$.

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6Wolak (2000) demonstrates the empirical relevance of this mechanism for a large supplier in an Australian wholesale electricity market. United States regulators also recognize that suppliers with the ability to exercise unilateral market power submit offer prices into the short-term market closer to their marginal cost if they have substantial fixed-price forward contract commitments. As Wolak (2003) notes, this mechanism is a major lesson from the California Electricity Crisis.

7Concentrated ownership is increasingly relevant to the extent that local markets are defined at the nodal level, where each generation unit interconnection point is a separate market.

8This assumption is a reflection of the geographically concentrated ownership of generation assets found in most restructured electricity markets.

9Even a local monopoly producer will sell forward contracts in this model, unlike in Allaz and Vila (1993) where a monopoly would not sell any forward contracts.
for every local market. In the second case of a regional forward market, all forward contracts are traded at the same forward price $\bar{f}$. Under regional forward contracting, an increase in the volume of forward contracts sold by a producer located in market $m$ has a negative spill-over effect on the producers located in other local markets through a reduction in the forward price $\bar{f}$. This spill-over effect does not occur when forward markets are local. Consequently, producers will sell relatively more forward contracts in equilibrium when forward markets are regional because producers internalize a smaller fraction of the negative forward price effect. The pro-competitive effect in the short-term market of forward contracting is weaker under a regional forward contract because the short-term price $p_m$ only constitutes a fraction of the clearing price $\bar{p}$ of the forward contract. We show that the volume increase attributed to the spill-over effect dominates the smaller pass-through of forward contract volumes to output. Our model therefore predicts LMP markets in which forward contracts clear against trading-hub prices to be more liquid and efficient than LMP markets where forward markets clear against individual LMPs.

The trade-offs facing independent retailers and large industrial consumers regarding how much electricity to purchase in the forward market depends on whether they pay the local short-term price $p_m$ for their electricity consumption or the volume-weighted average $\bar{p}$ of short-term prices. In the second case of a regional consumer price, forward purchases by a retailer located in local market $m$ has a positive spill-over effect on consumers in other local markets through a reduction in the common wholesale price $\bar{p}$ of electricity. This spill-over effect does not occur under a local consumer price $p_m$. Consequently, the willingness to pay for forward contracts is smaller under a regional consumer price compared to the case of local wholesale prices of electricity. The smaller inverse demand for forward contracts reduces the equilibrium volume of forward contracts. Our model therefore predicts a regulatory mandate that addresses equity concerns of the nodal market design by requiring all loads to purchase their wholesale electricity at a quantity-weighted average of LMPs within a service territory, to be less efficient than LMP markets where consumers pay the local short-term price for their electricity.

LMP markets that feature both trading-hub forward prices and geographically averaged consumer prices can be more or less efficient than LMP markets with local forward contracts and local consumer prices. The pro-competitive effects of regional forward contracting dominates, for instance, if markets are highly concentrated and sufficiently asymmetric in terms of size. We find that producers internalize more of the forward price effect under regional forward contracting if they own generation assets in more than one local market. Consequently, linking markets through a regional forward contract reduces prices in the short-term market.
compared to the case of local forward markets if and only if producers with the ability to exercise unilateral market power own production assets in a sufficiently small number of local markets.

The competitive benefits of linking forward markets across space can be substantial. In a symmetric quantity-setting model, linking together five local monopoly markets through a single forward contract can cause the short-term price-cost margins to drop by 5-15% in each local market. Notably, these price effects are purely the result of changes in the forward market, and do not rely on any changes in market structure.

Finally, we consider the case when producers and consumers both can write local and regional forward contracts, to see if the market can sustain both types of contracts in equilibrium. Typically, local forward premiums are above [below] the regional forward premium in markets with above [below] average demand. Therefore, producers with market power in large [small] markets will only offer local [regional] forward contracts. Adding a regional forward market to an existing set of local forward markets therefore has no effect on short-term prices in relatively large markets, but will reduce short-term prices in relatively small markets.

Allaz and Vila (1993) are the first to demonstrate the pro-competitive effects of forward contracting. However, the burgeoning literature on forward contracting under imperfect competition is based on the analysis of a single spot market. Motivated by standard design features of restructured electricity markets in the US, our contribution is to investigate how regional aspects of forward and wholesale markets affect market performance.\textsuperscript{10} Mahenc and Salanié (2004) find forward contracting to reduce market performance if firms compete in prices in the spot market. Holmberg (2011) establishes conditions under which forward contracting improves market performance when firms compete in supply functions. These papers suggest that results can be sensitive to the mode of competition in the short-term market. We establish fundamental results under local monopoly conditions that are robust to strategic interaction in the short-term market. Our extension to local oligopoly markets is based on the assumption of quantity-setting competition. This model has been used in empirical research to model strategic interaction among suppliers in many wholesale markets for electricity, including California, New England and PJM (Bushnell et al., 2008), the Midwest market (Mercadal, 2016), the German market (Willems et al., 2009) and the Nordic market (Lundin and Tangerås, 2020).

\textsuperscript{10}Green and Le Coq (2010) show that increasing the contract length (linking electricity markets across time) has ambiguous effects on the ability to sustain collusion. We consider unilateral market power and thus leave aside the question of how different market designs affect collusion.
The rest of the paper is organized as follows. Section 2 introduces our setup and illustrates the mechanisms in a symmetric example with two local markets and one producer with market power in each local market. Section 3 generalizes the model to an asymmetric setting with an arbitrary number of local markets and an arbitrary number of producers with market power in every local market. Section 4 considers the possibility that producers own generation capacity in multiple local markets. Section 5 considers the combined case of a regional and local forward markets. Section 6 concludes with a discussion of the implications of our results for the design of electricity markets. The proofs of a number of results presented in the text are given in an Appendix.

2 The mechanisms demonstrated in a simple model

We here introduce the modeling framework used throughout the paper, but in the simplest possible setting that generates the key mechanisms of our paper. This simple model features two symmetric local markets that are functionally separate from one another. There is one producer with market power in each local market. Electricity demand comes from a number of retailers and large consumers that also participate in the forward market modeled along the lines of Ruddell et al. (2018).

In this two-market model, linking markets through a regional forward contract that clears against the quantity-weighted average of the short-term prices in the two local markets, increases competition in the short-term market beyond what is possible under local forward contracts that clear against the local short-term prices. Under the regional forward contract, producers with market power sell more forward contracts than in the Allaz and Vila (1993) and the Ruddell et al. (2018) model, because producers ignore the negative short-term price effect in the other local market when they increase forward sales. The effect on equilibrium forward volumes is sufficient to render the total short-term price effect negative despite the fact that changes in forward contracting volumes have a smaller pass-through effect to output in the short-term market under regional compared to local forward contracting.

The model An electricity network is a set of nodes connected by a high-voltage transmission grid. There are two types of nodes. A generation injection node is a location at which a power plant feeds electricity into the grid. A load withdrawal node is a location at which either a large industrial plant pulls electricity from the grid or a substation converts electricity to lower voltages for distribution to smaller industries and households. A local market is a collection of generation injection and load withdrawal nodes with the property...
that the transmission grid has sufficient capacity to handle all power flows between all those
nodes. The free flow of power means that the short-term price of electricity in a locational
pricing market, the locational marginal price (LMP), will be the same at all nodes within the
local market.\footnote{This result holds only if the market does not include marginal losses in the LMPs.} The electricity market is the set of local markets that covers all nodes in the
network. The composition of local markets may change from period to period depending on
locational demand and supply conditions and capacity constraints on all of the links of the
transmission grid.

In the simple model, we consider an electricity supply industry that consists of two local
markets. These markets are symmetric and functionally separate from one another in the
sense that there is no flow of electricity between them. This assumption hugely simplifies
the analysis, but also reveals a key insight: There can be market performance gains from
financially linking markets even if there is no actual trade of goods between them. We
genralize the model to an arbitrary number of asymmetric local markets in Section 3.

We analyze a two-stage game in which firms compete in quantities in the forward market
in the first-stage and in quantities in the short-term market in the second stage.

To capture the fact that short-term demand for electricity is highly price inelastic, we
let total demand $D$ be constant and equal to $\frac{1}{2}D$ in each local market. This demand comes
from $H$ retailers and large consumers, half of which are located in each local market. We let
$D_h$ be the local demand by consumer $h \in \{1, \ldots, \frac{1}{2}H\}$. Local demand $\frac{1}{2}D = \sum_{h=1}^{\frac{1}{2}H} D_h$ must
be met entirely by local supply by our assumption of functionally separate local markets,
but local production is a homogeneous good by our assumption of a free flow of electricity
within each local market. We assume that consumption and production of electricity both
are deterministic. A deterministic setup means that there is no hedging motive on the part
of retailers and large consumers or suppliers for trading forward contracts in the first stage.

There are two producers with market power: Producer 1 has market power in local
market 1, and producer 2 has market power in local market 2. Each producer is active in
one local market in the sense that it owns generation capacity only in one local market.
As noted earlier, geographical concentration of generation assets is realistic in electricity
markets where many companies are former monopolists with local production capacity and
distribution networks connected to local consumers. We consider the effects of producers
operating in multiple local markets in Section 4. We also assume that each local market has
a number of independent producers that behave as a competitive fringe by selling electricity
at marginal cost. Below we describe the interaction in local market 1. The description of
local market 2 is identical by symmetry, so we leave it aside.

The producer with market power in market 1 sells forward contracts for \( k \) MWh electricity in the first stage, and produces \( q \in [0, \frac{1}{2}D] \) MWh electricity in the second stage at constant marginal production cost \( c \). The competitive fringe supplies the residual demand in market 1 net of producer 1's supply, \( \frac{1}{2}D - q \), at upward sloping linear marginal cost \( b(\frac{1}{2}D - q) \), so the market-clearing short-term price equals \( p = b(\frac{1}{2}D - q) \). If we define \( a = \frac{bD}{2} > \frac{5}{3}c \), then the inverse demand curve facing the producer with market power equals \( P(q) = a - bq \). The slope of the inverse demand curve \( P(q) \) comes from the slope of the marginal cost curve of the competitive fringe. We solve for the unique subgame-perfect equilibrium of this two-stage game by backward induction. The equilibrium properties depends on the underlying assumptions about market design.

### 2.1 Spatially independent markets

We consider first the benchmark case of a spatially independent market. In this market design, consumers pay the local short-term price and producers receive the local short-term price. Forward markets are local in the sense that contracts clear against the local short-term price. This is the default market design for all LMP markets in the United States.

**Equilibrium in the short-term market** The second-stage profit of the producer with market power equals

\[
(f - P(q))k + (P(q) - c)q.
\]

(1)

The first term measures the forward profit if the forward price is \( f \) per MWh. The second term is the profit in the short-term market. The firm’s first-order condition for profit maximization equals

\[
- P'(q)k + P(q) - c + P'(q)q = 0
\]

(2)

in interior optimum. The producer has an incentive to withhold output \( q \) to sustain a higher short-term price and thereby increase profit in the short-term market. This incentive is muted if the producer has sold fixed-price forward contracts. An increase in output then increases the forward profit by reducing the short-term price \( P(q) \) that forward contracts clear against. The magnitude of this effect on the forward profit is larger when the producer has sold a larger quantity \( k \) of forward contracts. The production decision is independent of the forward price \( f \) because the forward revenue \( fk \) is sunk at the production stage.
By way of \( P(q) = a - bq \) and \( P'(q) = -b \), we can solve for the production

\[
q(k) = \frac{1}{2} a - \frac{c}{b} + \frac{1}{2} k
\]

of the producer with market power as a function of its forward position \( k \). The corresponding short-term price equals

\[
p(k) = P(q(k)) = \frac{a + c}{2} - \frac{b}{2} k.
\]

Production is larger and the short-term price is smaller when the producer with market power has sold a larger volume of forward contracts.

**Equilibrium in the forward market** The \( \frac{1}{2} H \) retailers and large consumers participate in the forward market. These consumers are strategic in the sense that they are forward-looking and anticipate the effect of forward contracting on short-term prices. A retailer or large consumer \( h \) that purchases \( k_h \) of the total volume \( k \) of forward contracts has profit:

\[
U(D_h) + (P(q(k)) - f)k_h - P(q(k))D_h.
\]

The first term, \( U(D_h) \), is the value of electricity consumption \( D_h \) to the retailer or large consumer, the second term is the forward profit (or deficit) and the third term is the cost of electricity consumption. By aggregating the first-order condition

\[
P(q(k)) - f + (k_h - D_h)P'(q(k))q'(k) = 0
\]

across all retailers and large consumers, and using \( P'(q) = -b \) and \( q'(k) = \frac{1}{2} \) from (3), we can solve for the inverse demand function \( f = F(k) \) for forward contracts:

\[
F(k) = P(q(k)) + \frac{\frac{1}{2} D - k}{H}.
\]

Retailers and large consumers purchase forward contracts in this model to drive down the short-term price of electricity, not because they want to hedge price uncertainty. Indeed, they pay a premium on forward contracts if contract coverage is incomplete, i.e. \( k < \frac{1}{2} D \) (Ruddell et al., 2018). The forward premium converges to zero as the number \( H \) of retailers and large consumers grows to infinity because then individual consumers have very little influence over the short-term price. This limiting case corresponds to the zero forward premium assumption in Allaz and Vila (1993).
The first-stage profit of the generator with market power equals

\[(F(k) - P(q(k)))k + (P(q(k)) - c)q(k)\]  \hspace{1cm} (8)

as a function of its forward position \(k\), where \(q(k)\) is given by (3), \(P(q(k))\) by (4) and \(F(k)\) by (7). The effect of a marginal increase in \(k\) can be written as

\[
\underbrace{F(k) - p(k) + F'(k)k}_{\text{Marginal forward profit}} + \underbrace{[-P'(q)k + P(q) - c + P'(q)q]q'(k)}_{\text{Marginal profit in the short-term market}}. \hspace{1cm} (9)
\]

The effect of forward contracts on the profit in the short-term market is of second-order importance by the first-order condition (2). Hence, the optimal forward position \(k^I\) from the viewpoint of the producer with market power, is found at the point

\[F(k^I) - p(k^I) + F'(k^I)k^I = 0\]

that maximizes the forward profit, where superscript \(I\) identifies the case of spatially independent markets. By using the functional forms (4) and (7), we obtain the volume

\[k^I = \frac{D}{H+4}\]  \hspace{1cm} (10)

of forward contracts sold in equilibrium by a producer with local market power. The producer’s equilibrium output is:

\[q^I = q(k^I) = \frac{1}{2b}\left(\frac{H+6}{H+4}a - c\right),\]  \hspace{1cm} (11)

and the corresponding price-cost margin in the short-term market is:

\[p^I - c = p(k^I) - c = \frac{a - c}{2} - \frac{a}{H+4} > 0.\]  \hspace{1cm} (12)

The forward premium arising from retailers’ and large consumers’ demand for forward contracts causes a producer with market power to supply forward contracts regardless of the fact that selling such contracts will reduce the short-term price-cost margin below the monopoly level \(\frac{a-c}{2}\). The pro-competitive effect of forward contracting is stronger if there are fewer retailers or large consumers in the market for forward contracts, i.e. \(H\) is smaller, because then the willingness to pay for forward contracts is larger. Specifically, the fixed-price forward contracts purchased by each retailer or large consumer conveys a positive benefit to
all retailers and large consumers in the form of lower short-term prices. To the extent that there are fewer retailers or large consumers in a local market, any retailer or large consumer that purchases a forward contract captures a greater share of the short-term price benefits from its forward contract purchases. Hence, the only case when forward contracting does not improve competition is when the number of retailers and large consumers in the local market is infinitely large ($H \to \infty$).

Notwithstanding the pro-competitive effects of forward contracting, the equilibrium short-term price remains inefficiently high. For the producer with market power to behave in a fully competitive manner, this would require full contract coverage, i.e. a forward contract position equal to the producer’s entire output. Instead, the equilibrium contract coverage is only partial:

$$\frac{k}{q} = \frac{4a}{(a-c)(H+6)+2c} < 1.$$  

This means there would be efficiency gains of reinforcing producers’ incentives to sell forward contracts. The key insight of this paper is that linking markets creates such an incentive.

2.2 Linking forward contracts across space

Consider the consequences of linking spatial markets through a regional forward contract. In our simple example, this is a forward contract that clears against the quantity-weighted average $\frac{1}{2}(p_1 + p_2)$ of the short-term market prices in the two local markets. As mentioned in the introduction, such contracts are common in US LMP markets where forward prices clear against trading-hub prices. Examples include the PJM Interconnection Western Hub and California ISO NP15 and SP15 EZ Gen Hub forward contracts. We refer to a forward market in which all contracts clear against the same quantity-weighted average of short-term prices as a regional forward market. We maintain the assumption that consumers pay the local short-term price for the electricity they use, and producers receive the local short-term price for the electricity they generate.

Equilibrium in the short-term market Let $k_1$ be the quantity of forward contracts sold by producer 1, and let $q_1 \in [0, \frac{1}{2}D]$ be the quantity it produces in local market 1. Similar notation applies for producer 2 located in local market 2. If $\bar{f}$ is the price of the regional forward contract, then the profit of producer 1 at the second stage of the game equals

$$\left(\bar{f} - \frac{1}{2}[P(q_1) + P(q_2)]\right)k_1 + (P(q_1) - c)q_1,$$
and its first-order condition for profit maximization in interior equilibrium is:

\[- \frac{1}{2} P'(q_1)k_1 + P(q_1) - c + P'(q_1)q_1 = 0. \quad (13)\]

The competitive effect of forward contracting in the short-term market is weaker when the forward contract clears against the average short-term price in the two markets compared to the case of local forward markets elucidated in (2). The reason is that the marginal effect on the clearing price of an increase in \( q_1 \) is smaller when the forward contract clears against the weighted average of multiple short-term prices. By comparing (13) with (2), we see that producer 1 must sell twice the amount of the regional forward contract relative to the local forward contract for the competitive effect to be the same. Hence, the output of producer 1 equals

\[ q_1 = q\left(\frac{k_1}{2}\right) = \frac{1}{2} \frac{a - c}{b} + \frac{1}{2} \frac{k_1}{2} \]

under the regional forward contract, and the short-term price in local market 1 is

\[ p_1 = P\left(q\left(\frac{k_1}{2}\right)\right) = \frac{a + c}{2} - \frac{b}{2} \frac{k_1}{2} \]

Analogous expressions hold for the producer with market power in local market 2 as a function of \( \frac{1}{2} k_2 \).

**Equilibrium in the forward market**  Consider now the first stage of the game where the two producers decide the forward contract volumes \( k_1 \) and \( k_2 \) to offer to the market, and retailers and large consumers choose the volume of regional forward contracts to purchase.

The profit of retailer or large consumer \( h \) located in market 1 equals

\[ U(D_h) + \left(\frac{1}{2}[P(q\left(\frac{k_1}{2}\right)) + P(q\left(\frac{k_2}{2}\right))] - \bar{f}\right)k_{1h} - P(q\left(\frac{k_1}{2}\right))D_h \quad (14)\]

if it buys a regional forward contract for \( k_{1h} \) MWh electricity from producer 1, and producer 1 [2] offers a total volume of \( k_1 \) [\( k_2 \)] regional forward contracts. This consumer buys no forward contracts from producer 2 because it then does not receive the benefit of a reduction in its electricity cost. Summing up the first-order condition

\[ \frac{1}{2}[P(q_1) + P(q_2)] - \bar{f} + \frac{1}{2} P'(q_1)q'\left(\frac{k_1}{2}\right)\left(\frac{k_{1h}}{2} - D_h\right) \]

for all retailers and large consumers in each local market and across both local markets,
returns the inverse demand

\[ \bar{F}(k_1, k_2) = \frac{1}{2} [P(q(\frac{k_1}{2})) + P(q(\frac{k_2}{2}))] + \frac{b}{4H} (D - \frac{k_1 + k_2}{2}). \tag{15} \]

for the regional forward contract as a function of the two producers’ forward positions \( k_1 \) and \( k_2 \). Retailers’ or large consumers’ willingness to pay for a regional forward contract is smaller than the willingness to pay for a local forward contract even if the volume of regional forward contracts is so large relative to local forward contracts, \( k_1 = k_2 = 2k \), that the short-term price would be the same under both types of forward contracts:

\[ \bar{F}(2k, 2k) - P(q(k)) = \frac{b}{2} \frac{D - k}{H} < b \frac{1}{2} \frac{D - k}{H} = F(k) - P(q(k)). \tag{16} \]

This is because the marginal pro-competitive effect in the short-term market of an increase in the volume of forward contracts is weaker under regional than local forward contracting.

Turning to the profit-maximizing forward quantities, producer 1’s first-stage profit equals

\[ (\bar{F}(k_1, k_2) - \frac{1}{2} [P(q(\frac{k_1}{2})) + P(q(\frac{k_2}{2}))])k_1 + (P(q(\frac{k_1}{2})) - c)q(\frac{k_1}{2}) \tag{17} \]

under the regional forward contract. As in the case of local forward contracts, we can partition the producer’s marginal profit into an expression for marginal forward profit and an expression for marginal profit in the short-term market:

\[ \begin{aligned} &\bar{F}(k_1, k_2) - \frac{1}{2} [P(q_1) + P(q_2)] + \frac{\partial \bar{F}(k_1, k_2)}{\partial k_1} k_1 + \left[ -\frac{1}{2} P'(q_1)k_1 + P(q_1) - c + P'(q_1)q_1 \frac{1}{2} q'(\frac{k_1}{2}) \right] \end{aligned} \]

Marginal forward profit

\[ \begin{aligned} &\bar{F}(k_1, k_2) - \frac{1}{2} [P(q_1) + P(q_2)] + \frac{\partial \bar{F}(k_1, k_2)}{\partial k_1} k_1 + \left[ -\frac{1}{2} P'(q_1)k_1 + P(q_1) - c + P'(q_1)q_1 \frac{1}{2} q'(\frac{k_1}{2}) \right] \end{aligned} \]

Marginal profit in the short-term market

Again, the second term is of second-order effect on producer 1’s profit. Producer 1’s marginal profit therefore equals

\[ \bar{F}(2k^I, 2k^I) - p(k^I) + \frac{\partial \bar{F}(2k^I, 2k^I)}{\partial k_1} 2k^I \]

evaluated at \( k_1 = k_2 = 2k^I \). On the one hand, the smaller forward premium under the regional forward contract, see equation (16), tends to reduce forward contracting below the level \( 2k^I \) that would yield the same short-term price under a regional forward contract as under spatially independent markets. On the other hand, the marginal reduction in the forward profit associated with the decrease in the forward price is smaller in magnitude
under a regional forward contract than one that clears against the local price:

\[
\frac{\partial \tilde{F}(2k^I, 2k^I)}{\partial k_1}2k^I = -\frac{b}{2}\frac{H + 1}{2H}k^I > -\frac{b}{2}\frac{H + 2}{H}k^I = F'(k^I)k^I.
\] (18)

By construction of the regional forward contract, a share of the negative price effect of an increase in \( k_1 \) spills over to local market 2, where producer 1 has no market presence. This spill-over effect creates an incentive for firm 1 to increase \( k_1 \) above \( 2k^I \). The spill-over effect dominates the effect of a smaller forward premium, and therefore (superscript RF identifies the case of a regional forward market):

**Proposition 1** Consider an electricity market with two symmetric local markets and one producer with market power in each local market. Linking the two local markets through a regional forward contract that clears against the quantity-weighted average of the short-term prices in those two markets, more than doubles the volume \( k^{RF} \) of forward contracts sold by each producer with market power,

\[
k^{RF} = \frac{2D}{H + 3} > \frac{2D}{H + 4} = 2k^I,
\] (19)

compared to the benchmark of spatially independent markets. This increase in the volume of forward contracts has a pro-competitive effect in the short-term market:

\[
0 < p^{RF} - c = \frac{a - c}{2} - \frac{a}{H + 3} < \frac{a - c}{2} - \frac{a}{H + 4} = p^I - c.
\] (20)

Proposition 1 predicts forward contracts that clear against the volume-weighted average of the short-term prices across multiple local markets to be substantially more liquid than forward contracts that clear against local short-term prices. Moreover, the forward premiums on regional forward contracts will be relatively smaller. This increase in liquidity will improve the performance of the short-term market, even in the case where the local market is characterized by one single generator with the ability to exercise market power.

### 2.3 Linking forward contracts and consumer prices across space

In the above market design, generators receive the local short-term price for their production and consumers pay the local short-term price for their consumption. Many LMP markets are partitioned into service territories within which all consumption is charged the same price based on the quantity-weighted average of the LMP prices across the service territory.
Singapore operates a nodal pricing market and all loads purchase their wholesale electricity at the Uniform Singapore Electricity Price (USEP) which is equal to the quantity-weighted average of the LMPs in Singapore. We here consider the implications of such regional consumer prices for competition in the short-term market. We maintain the assumption of the previous section of a regional forward market.

**Equilibrium in the short-term market** Generators are paid the local short-term price for their output, whereas the forward contract clears against the average short-term price. Hence, the producer with market power in local market 1 supplies \( q(k_1/2) \) MWh electricity to the short-term market as a function of its forward position \( k_1 \), and the short-term price in that market equals \( p_1 = P(q(k_1/2)) \). Similar expressions hold for local market 2.

**Equilibrium in the forward market** The profit of retailer or large consumer \( h \) located in local market 1 equals

\[
U(D_{1h}) + \left( \frac{1}{2}[P(q(k_1/2)) + P(q(k_2/2))] - \hat{f} \right) k_{1h} - \frac{1}{2}[P(q(k_1/2)) + P(q(k_2/2))] D_{1h}.
\]  

(21)

In this equation, \( \hat{f} \) is the price of the forward contract under regional forward contracting and regional consumer prices. The difference between this expression and (14), is that the consumer here pays the quantity-weighted average of the two local prices for its consumption instead of the short-term price in market 1.

By taking the first-order condition of the profit expression (21), and aggregating over all consumers in both markets, we can solve for the inverse demand

\[
\hat{F}(k_1, k_2) = \frac{1}{2}[P(q(k_1/2)) + P(q(k_2/2))] + \frac{b D - k_1 - k_2}{H}.
\]

for the regional forward contract under regional consumer prices. The forward price effect of an increase in forward contracting is the same as before.\(^\text{12}\) However, the forward price is different. If consumption clears against the local short-term price, then consumer \( h \) located in market 1 benefits from the full reduction in the short-term price \( p_1 \) if it purchases a forward contract from producer 1. If consumers instead pay the quantity-weighted average of all short-term prices, then consumers in both local markets benefit from the reduction in the short-term price \( p_1 \) if consumer \( h \) located in market 1 purchases a forward contract from producer 1. Since the consumer only reaps a fraction of the benefits of forward contracting, the willingness

\[\text{Specifically, } \frac{\partial \hat{F}(k_1, k_2)}{\partial k_1} = -\frac{H+1}{H} \frac{b}{8} = \frac{\partial \hat{F}(k_1, k_2)}{\partial k_2}.\]

\(^{12}\)
to pay for forward contracts is smaller under regional consumer prices compared to the case when the consumer pays the local short-term price for its consumption:

$$\hat{F}(k_1, k_2) - \bar{F}(k_1, k_2) = -\frac{b}{8H}D.$$  

This reduction in the forward price will have important consequences for producers’ incentives to sell forward contracts.

Producer 1’s profit equals

$$(\hat{F}(k_1, k_2) - \frac{1}{2}[P(q(\frac{k_1}{2}) + P(q(\frac{k_2}{2})))k_1 + (P(q(\frac{k_1}{2}) - c)q(\frac{k_1}{2})$$

under regional forward contracting with regional consumer prices. By invoking the first-order condition (13), the equilibrium forward positions $k_1 = k_2 = k^R$ of the two producers with market power under regional forward contracting with regional consumer prices, are again found at the point at which the marginal forward profit is zero:

$$\hat{F}(k^R, k^R) - \frac{1}{2}[P(q(\frac{k^R}{2}) + P(q(\frac{k^R}{2})))k^R + (P(q(\frac{k^R}{2}) - c)q(\frac{k^R}{2})$$

Superscript $R$ signifies the ”regional” market design under which consumers in both local markets pay the same price for their consumption and all forward contracts have the same regional price. We can then use the functional forms to derive the following straightforward result:

**Proposition 2** Consider an electricity market with two symmetric local markets and one producer with market power in each local market. Let the two markets be linked through a regional forward contract that clears against the quantity-weighted average of the short-term prices in those two markets, and let consumers in both markets pay that quantity-weighted average price for their consumption. Each producer with market power sells

$$k^R = \frac{D}{H + 3} \in (k^l, 2k^l)$$  

forward contracts in symmetric equilibrium. The equilibrium short-term price $p^R$ is higher than in the case of independent markets:

$$p^R - c = \frac{a - c}{2} - \frac{1}{2H + 3} \frac{a}{2} > \frac{a - c}{H + 4} - \frac{a}{H + 4} = p^l - c.$$  

$$23$$
Regional forward contracting increases liquidity in the forward market compared to the case of independent markets also when consumer prices are required to be identical in both local markets. However, this increased liquidity is insufficient to compensate for the effect that regional forward contract volumes have a weaker effect on prices in the short-term market than forward contracts that clear against the local short-term prices. Therefore, short-term prices are higher than under spatially independent markets.

### 2.4 Linking consumer prices across space

The final market design we consider, is the one in which producers receive the local short-term price for the electricity they generate, consumers pay the quantity-weighted average of short-term prices for the electricity they consume, and forward contracts are local in the sense that they clear against the local short-term prices. This market design is consistent with the current California market design where all customers of each of the three large investor-owned utilities purchase wholesale energy at a regional price, yet all generators are paid their local price. A similar design exists in the Italian wholesale market, where all consumers pay a national price and all generators are paid the price at their location.\(^{13}\)

**Equilibrium in the short-term market** Since forward contracts clear against the local short-term price, producer 1 supplies \(q(k_1)\) and producer 2 supplies \(q(k_2)\) to the short-term market. The corresponding short-term prices are \(P(q(k_1))\) and \(P(q(k_2))\).

**Equilibrium in the forward market** The profit of retailer or large consumer \(h\) located in local market 1 equals

\[
U(D_{1h}) + (P(q(k_1)) - \tilde{f}_1)k_{1h} - \frac{1}{2}[P(q(k_1)) + P(q(k_2))]D_{1h}. \tag{24}
\]

In this equation, \(\tilde{f}_1\) is the price of the forward contract when the forward market is local, but consumer prices are regional. We can use the retailers’ and large industrial consumers’ first-order conditions to solve for the inverse demand, \(\tilde{f}_1 = \tilde{F}(k_1)\), for the local forward contract in market 1:

\[
\tilde{F}(k_1) = P(q(k_1)) + b_1^4D - k_1 \frac{1}{H}.
\]

\(^{13}\)See Graf and Wolak (2020) for a description of the operation of the Italian market.
Substituting this expression into (1), delivers the first stage profit of producer 1:

\[(\tilde{F}(k_1) - P(q(k_1)))k_1 + (P(q(k_1)) - c)q(k_1).\]

By way of the first-order condition (2), we obtain the equilibrium forward volume \(k^{RC}\) (superscript \(RC\) identifies the case of a regional consumer price) as the solution to:

\[\tilde{F}(k^{RC}) - p(k^{RC}) + \tilde{F}'(k^{RC})k^{RC} = 0.\]

It is then straightforward to verify the following result:

**Proposition 3** Consider an electricity market with two symmetric local markets and one producer with market power in each local market. Linking the two local markets through a regional consumer price that clears against the quantity-weighted average of the short-term prices in those two markets, reduces the volume \(k^{RC}\) of forward contracts sold by each producer with market power,

\[k^{RC} = \frac{1}{2} \frac{D}{H + 4} < \frac{D}{H + 4} = k^I,\]

compared to the benchmark of spatially independent markets. This reduction in the volume of forward contracts has an anti-competitive effect in the short-term market:

\[p^{RC} - c = \frac{a - c}{2} - \frac{1}{2} \frac{a}{H + 4} > \frac{a - c}{2} - \frac{1}{2} \frac{a}{H + 3} = p^R - c.\]

Proposition 3 establishes that regulatory mandated ”equity-based” consumer prices come at the cost of increased production inefficiency in an imperfectly competitive market.

### 2.5 Discussion

In the previous sections, we examined a taxonomy of market designs that differ in the extent to which forward markets or consumer prices are linked across local markets. All market designs have properties that correspond with those found in actual LMP markets. We display those designs in the below matrix, along with the relevant price comparisons.

<table>
<thead>
<tr>
<th></th>
<th>Local forward market</th>
<th>Regional forward market</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local consumer price</td>
<td>(p^I)</td>
<td>(p^{RF} &lt; p^I)</td>
</tr>
<tr>
<td>Regional consumer price</td>
<td>(p^{RC})</td>
<td>(p^I &lt; p^R &lt; p^{RC})</td>
</tr>
</tbody>
</table>
The two columns display market designs under which forward contracts either clear against the local short-term price or the quantity-weighted average of short-term prices. The two rows show market designs under which consumers either pay the local short-term price or the quantity-weighted average of short-term prices for their electricity usage.

Introducing a regional forward market is pro-competitive \( (p_{RF} < p^I \text{ and } p^R < p^{RC}) \) because the spill-over effect of lower forward prices into the other markets causes producers with market power to sell more forward contracts than would otherwise be the case. Introducing an "equity-based" regional consumer price is anti-competitive \( (p^{RC} > p^I \text{ and } p^R > p^{RF}) \) because the spill-over effect of lower consumer prices into the other markets reduces the demand for forward contracts. We can also rank the four different market designs in terms of their competitiveness. The most competitive market is the one in which consumers pay the local short-term price for their electricity and forward markets are regional. Number two is the case of spatially independent markets, followed by the design where both forward markets and consumer prices are regional. The least competitive market design is the one with regional consumer prices and local forward prices \( (p_{RF} < p^I < p^R < p^{RC}) \).

Propositions 1-3 are not merely artifacts of assuming local monopoly production or symmetry. We establish in the next section the robustness of these results in an asymmetric model featuring an arbitrary number of local markets and producers with market power. However, asymmetries have implications for competition in the short-term market that makes an analysis of asymmetric markets interesting in its own right.

3 Multiple local markets

We now generalize the highly stylized model in Section 2 to an arbitrary number \( M \) of local markets that can be asymmetric. We show that creating a regional forward contract that clears against the quantity-weighted average of the short-term price in all \( M \) local markets has competitive effects in the short-term market by its effect on firms’ unilateral incentives to sell forward contracts. In particular, we demonstrate that the regional forward contract reduces the volume-weighted average of the short-term prices compared to the case of \( M \) local forward markets, if market concentration is sufficiently large in each local market. We also show that establishing an additional consumer regional price equal to the quantity-weighted average of the local short-term prices yields an average short-term price above the level that would occur in spatially independent markets if local markets are similar in size. This result is reversed if markets are sufficiently asymmetric in size and sufficiently concentrated.
The model We index local markets by \( m \in \{1, \ldots, M\} = \mathcal{M} \) and an individual producer with market power in local market \( m \) by \( l \in \{1, \ldots, L_m\} \).

In the first stage, each producer \( l \) with market power supplies \( k_{lm} \) MWh of forward contracts, taking the aggregate forward positions \( K_{-lm} = \sum_{i \neq l} k_{im} \) of the other \( L_m - 1 \) producers with market power as given. Let \( K_m = k_{lm} + K_{-lm} \) be the total supply of forward contracts by the \( L_m \) producers with market power in local market \( m \). These forward contracts are purchased by \( H_m \) retailers or large consumers located in local market \( m \).

In the second stage, each producer \( l \) in market \( m \) observes \( K_{-lm} \) and decides how much electricity, \( q_{lm} \), to produce for the short-term market at constant marginal cost \( c_{lm} \), taking the production \( Q_{-lm} = \sum_{i \neq l} q_{im} \) of the other producers with the ability to exercise market power as given. The total production of electricity in local market \( m \) of firms with market power equals \( Q_m = q_{lm} + Q_{-lm} \). Retailer or large consumer \( h \) located in market \( m \) has demand \( D_{mh} \) for electricity, all of which is purchased in the short-term market. Total demand in short-term market \( m \) equals \( D_m = \sum_{h=1}^{H_m} D_{mh} \). The residual demand \( D_m - Q_m \) is covered by a competitive fringe that supplies electricity at linear marginal cost \( b_m (D_m - Q_m) \). The inverse demand curve facing the \( L_m \) producers with the ability to exercise unilateral market power in short-term market \( m \) can then be written as \( p_m = P_m(Q_m) = a_m - b_m Q_m \), where \( a_m = b_m D_m \).

### 3.1 Spatially independent markets

We first consider the benchmark case where consumers pay the local short-term price for the amount of electricity they use, and producers receive the local short-term price for the amount of electricity they generate. Forward markets are local in the sense that all forward contracts clear against the local short-term price.

**Equilibrium in the short-term market** The second-stage profit of producer \( l \) equals

\[
(f_m - P_m(Q_m))k_{lm} + (P_m(Q_m) - c_{lm})q_{lm}. \tag{25}
\]

The first term measures the forward profit when forward contracts are sold at the local forward price \( f_m \). The second term is the profit in the short-term market. The \( L_m \) first-order conditions

\[
-P'_m(Q_m)k_{lm} + P_m(Q_m) - c_{lm} + P'_m(Q_m)q_{lm} = 0 \tag{26}
\]
for profit maximization solve for the equilibrium production \((q_1, \ldots, q_{lm}, \ldots, q_{Lm})\) of the producers with market power. Sum up those first-order conditions to obtain the total production

\[ Q_m(K_m) = \frac{L_m}{L_m + 1} \frac{a_m - c_m}{b_m} + \frac{K_m}{L_m + 1} \]  

(27)
of the \(L_m\) producers with market power in local market \(m\) as a function of the total volume \(K_m\) of forward contracts sold by these producers, and the average marginal production cost \(c_m = \frac{1}{L_m} \sum_{l=1}^{L_m} c_{lm}\). The markup of the short-term price over \(c_m\) equals

\[ p_m(K_m) - c_m = P_m(Q_m(K_m)) - c_m = \frac{a_m - c_m}{L_m + 1} - \frac{b_m K_m}{L_m + 1}. \]  

(28)

We can then back out the production of producer \(l\),

\[ q_{lm}(k_{lm}, K_{-lm}) = \frac{1}{L_m + 1} \frac{a_m - c_m}{b_m} + \frac{c_m - c_{lm}}{b_m} + \frac{L_m k_{lm}}{L_m + 1} - \frac{K_{-lm}}{L_m + 1}, \]  

(29)

and the residual output of all producers other than \(l\) in market \(m\),

\[ Q_{-lm}(k_{lm}, K_{-lm}) = \frac{L_m - 1}{L_m + 1} \frac{a_m - c_m}{b_m} - \frac{c_m - c_{lm}}{b_m} - \frac{L_m - 1}{L_m + 1} k_{lm} + \frac{2K_{-lm}}{L_m + 1}, \]  

(30)

if \(L_m \geq 2\). Producer \(l\) supplies more electricity to the short-term market when it has sold more forward contracts. The increase in forward contracting creates a strategic effect by which the other producers reduce their own output. The net effect is a reduction in the short-term price.

**Equilibrium in the forward market**  
Retailer or large consumer \(h\) in local market \(m\) obtains profit

\[ U(D_{mh}) + (p_m(K_m) - f_m) k_{mh} - p_m(K_m) D_{mh}, \]

where the first term is the value of consuming the electricity, the second term is the forward profit, or deficit, and the last term is the cost of electricity consumption. Summing up the first-order condition

\[ p_m(K_m) - f_m + p_m'(K_m)(k_{mh} - D_{mh}) = 0 \]

for the optimal purchase of forward contracts across all \(H_m\) retailers or large consumers in market \(m\) yields the inverse demand \(f_m = F_m(K_m)\) for forward contracts in local market \(m\).
as

\[ F_m(K_m) = p_m(K_m) + \frac{b_m}{L_m + 1} \frac{D_m - K_m}{H_m} \]  

(31)

when forward markets are local.

The first term in producer \( l \)'s first-stage profit expression

\[
(F_m(K_m) - p_m(K_m))k_{lm} + (p_m(K_m) - c_{lm})q_{lm}(k_{lm}, K_{-lm}).
\]

is the forward profit, and the second term is the profit in the short-term market. The marginal effect on profit of increasing \( k_{lm} \) can be written as

\[
\frac{F_m(K_m) - p_m(K_m) + F'_m(K_m)k_{lm} - (p_m(K_m) - c_{lm})}{\partial k_{lm}} \partial Q_{-lm}(k_{lm}, K_{-lm})
\]

(32)

after invoking the short-term market first-order condition (26). Compared to the marginal profit condition (9) in the single-producer case, an increase in \( k_{lm} \) has a first-order effect on producer \( l \)'s profit in the short-term market in oligopoly. As demonstrated above, an increase in producer \( l \)'s forward quantity is a credible commitment to increase output in the short-term market. Under quantity competition, this commitment triggers a strategic response by which the competing producers reduce their own output. The second term in (32) represents the marginal benefit to firm \( l \) of the competitors' aggregate output contraction.

By the properties of the price-cost margin \( p_m(K_m) - c_m \) established in (28), the quantity \( Q_{-lm}(k_{lm}, K_{-lm}) \) established in (30) and the inverse demand for forward contracts characterized in (31), we can solve for the per-firm quantity of forward contracts sold by the \( L_m \) producers with market power in local market \( m \in \mathcal{M} \),

\[
\frac{K'^l_m}{L_m} = \frac{(L_m + 1)D_m}{(H_m + 1)(L_m^2 + 1) + 2L_m} + \frac{H_m(L_m - 1)}{(H_m + 1)(L_m^2 + 1) + 2L_m} a_m - c_m \frac{b_m}{b_m},
\]

(33)

where \( K'^l_m \) is equilibrium quantity of total fixed-price forward contracts sold in market \( m \). Using this market-clearing fixed-price forward quantity, we can also solve for the equilibrium markup over the average marginal cost in short-term market \( m \):

\[
p'^l_m - c_m = \frac{(H_m + 1)(a_m - c_m) - L_m c_m}{(H_m + 1)(L_m^2 + 1) + 2L_m}.
\]

(34)

A strategic incentive makes it individually rational for producers to sell forward contracts
above the level they would choose if they were the only generator with market power in the local market because forward contracting commits the producer to aggressive behavior in the short-term market.

3.2 Linking forward contracts across space

Assume that consumers pay the local short-term price for the amount of electricity they use, and producers receive the local short-term price for the amount of electricity they generate. The forward market is regional in the sense that all forward contracts sold in all local markets clear against the same quantity-weighted average

$$\bar{P}(Q) = \sum_{m=1}^{M} \frac{D_m}{D} P_m(Q_m)$$

of the $M$ short-term prices. Each short-term price $P_m(Q_m)$ is weighted by the size of local market $m$ relative to the size of the whole market, measured in terms of the $D_m$ MWh electricity consumed in local market $m$ relative to system total demand $D = \sum_{m=1}^{M} D_m$. The vector $Q = (Q_1, ..., Q_m, ..., Q_M)$ is the total output by generators with market power in each of the $M$ local markets.

Equilibrium in the short-term market  Producers with the ability to exercise unilateral market power take into account how their forward contract position affects their output choice in the short-term market. The second-stage profit of producer $l$ in market $m$ thus becomes

$$(\bar{f} - \bar{P}(Q))k_{lm} + (P_m(Q_m) - c_{lm})q_{lm},$$

where $\bar{f}$ is the price of the regional forward contract, and we maintain the assumption that firms with market power are active in one local market only. The first-order condition

$$-\frac{D_m}{D} P'_m(Q_m)k_{lm} + P_m(Q_m) - c_{lm} + P'_m(Q_m)q_{lm} = 0$$

for producer $l$’s quantity choice differs from the case of spatially independent markets, see equation (26), by an increase in production now having a relatively smaller positive effect on forward profit because of the $\frac{D_m}{D}$ term. Holding the forward contract quantity constant, the short-term market behavior by generators with market power is less competitive than under local forward markets.
Solving for the $L_m$ linear first-order conditions yields the total output

$$Q_m(D_m K_m) = \frac{L_m}{L_m + 1} \frac{a_m - c_m}{b_m} + \frac{1}{L_m + 1} \frac{D_m}{D} K_m$$

(35)

of the $L_m$ producers with market power in short-term market $m$ as a function of the total volume $K_m$ of forward contracts sold by those producers, the average markup

$$p_m\left(\frac{D_m}{D} K_m\right) - c_m = P_m(Q_m\left(\frac{D_m}{D} K_m\right)) - c_m = \frac{a_m - c_m}{L_m + 1} - \frac{b_m}{L_m + 1} \frac{D_m}{D} K_m,$$

(36)

the production of generator $l$,

$$q_{lm}\left(\frac{D_m}{D} k_{lm}, \frac{D_m}{D} K_{-lm}\right) = \frac{1}{L_m + 1} \frac{a_m - c_m}{b_m} + \frac{c_m - c_{lm}}{b_m} + \frac{L_m}{L_m + 1} \frac{D_m}{D} k_{lm} - \frac{1}{L_m + 1} \frac{D_m}{D} K_{-lm},$$

(37)

and of all other producers in local market $m$:

$$Q_{-lm}\left(\frac{D_m}{D} k_{lm}, \frac{D_m}{D} K_{-lm}\right) = \frac{L_m - 1}{L_m + 1} \frac{a_m - c_m}{b_m} - \frac{c_m - c_{lm}}{b_m} - \frac{L_m - 1}{L_m + 1} \frac{D_m}{D} k_{lm} + \frac{2}{L_m + 1} \frac{D_m}{D} K_{-lm}.$$

(38)

Note that the strategic effect is weaker than in equations (29) and (30) because an increase in $k_{lm}$ now has a smaller effect on output $q_{lm}$.

**Equilibrium in the forward market** Retailers and large consumers anticipate the short-term price $p_m\left(\frac{D_m}{D} K_m\right)$ in each short-term market $m$ and the clearing price

$$\bar{p}(K) = \sum_{m=1}^{M} \frac{D_m}{D} p_m\left(\frac{D_m}{D} K_m\right)$$

(39)

of the regional forward contract, where $K = (K_1, ..., K_m, ..., K_M)$ is the vector of forward contract quantities in the $M$ local markets. Retailer or large consumer $h$ in market $m$ has profit

$$U(D_{mh}) + (\bar{p}(K) - \bar{f})k_{mh} - p_m(D_m K_m)D_{mh}.$$ 

Aggregating the first-order condition

$$\bar{p}(K) - \bar{f} - \frac{D_m}{D} p_m'\left(\frac{D_m}{D} K_m\right) (D_{mh} - \frac{D_m}{D} k_{mh}) = 0$$

24
for profit maximization across all \( H_m \) retailers or large consumers in market \( m \) and across all short term markets, yields the inverse demand function

\[
F(K) = \bar{p}(K) + \frac{1}{H} \sum_{m=1}^{M} \frac{D_m}{D} \frac{b_m}{L_m} + 1 (D_m - \frac{D_m}{D} K_m) \tag{40}
\]

for the regional forward contract. The regional forward premium is a function of the total number \( H = \sum_{m=1}^{M} H_m \) of retailers or large consumers in the overall market. To focus on supply-side heterogeneity within each local market, we assume that the number of retailers or large consumers is uniformly distributed across all local markets: \( H_m = \frac{H}{M} \) for all \( m \in M \). We allow all other parameters to vary across markets.

In the first stage, each producer \( l \) in local market \( m \) chooses its forward position \( k_{lm} \) to maximize

\[
(F(K) - \bar{p}(K))k_{lm} + (p_m(D_m K_m) - c_{lm})q_{lm}(D_m K_{lm}, D_m K_{-lm}).
\]

The marginal effect

\[
\bar{F}(K) - \bar{p}(K) + \frac{\partial \bar{F}(K)}{\partial K_m} k_{lm} - (p_m(D_m K_m) - c_{lm}) \frac{\partial Q_{-lm}(D_m K_{lm}, D_m K_{-lm})}{\partial k_{lm}} \tag{41}
\]

on profit of increasing forward sales \( k_{lm} \) trades off the marginal forward profit against the strategic effect in the short-term market. We then obtain the aggregate results (the proof is in the Appendix):

**Proposition 4** Consider an electricity market with \( M \geq 2 \) local markets and \( L_m \geq 1 \) producers with market power in each local market \( m \in M \). Assume that each producer is active only in one local market. Linking the \( M \) local markets through a regional forward contract that clears against the quantity-weighted average of the short-term prices in those local markets, yields a quantity-weighted average of the price-cost margins in the \( M \) short-term markets equal to

\[
\sum_{m=1}^{M} \frac{D_m}{D} (p_{RF} - c_m) = \sum_{m=1}^{M} \frac{\Psi(L_m) H + 1}{L_m} \frac{D_m a_m - c_m}{D L_m + 1} - \sum_{i=1}^{M} \frac{\Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} \sum_{m=1}^{M} \frac{D_m c_m}{D L_m + 1}, \tag{42}
\]

and where

\[
\Psi(L) = \frac{L(L + 1)}{H(L^2 + 1) + L + 1}. \tag{43}
\]
Linking local markets through a regional forward contract increases competition in the short-term markets by reducing the quantity-weighted average of the short-term prices,

\[
\bar{p}^{RF} = \frac{1}{M} \sum_{m=1}^{M} D_m p_m^{RF} < \frac{1}{M} \sum_{m=1}^{M} D_m p_m^I = \bar{p}^I,
\]

compared to the benchmark of spatially independent markets, if the local markets are sufficiently concentrated in the sense that

\[
L_m \leq \bar{L} = 1 + \frac{1}{2} \left[ \sqrt{(M - H - 3)^2 + 8(M - 1)} + M - H - 3 \right] \forall m \in \mathcal{M}. \quad (44)
\]

Creating a regional forward contract may increase or decrease competition in a local short-term market compared to the case of spatially independent markets, depending on the local market conditions compared to those in the other markets. This is why we consider a measure of average market performance, in which the price-cost margin in each local short-term market is weighted by the relative size of that local market.

To assess the incentives to sell forward contracts in a regional forward market, consider the case in which each producer takes a regional forward position that yields the same markup over the average marginal cost as under local forward markets, i.e., \( k_m = \frac{D_m K_m}{L_m} f^I_m \) for all \( L_m \) producers with market power in all \( M \) local markets. This yields, \( p_m(D_m K_m) = p_m^I \) for all \( m \in \mathcal{M} \), and the regional forward price satisfies

\[
F(K) - \sum_{m=1}^{M} D_m f^I_m = -\frac{M - 1}{H} \sum_{m=1}^{M} D_m \frac{b_m}{L_m + 1} (D_m - K_m^I) < 0,
\]

where \( f^I_m = F_m(K_m^I) \) is the equilibrium forward price in local market \( m \) under spatially independent markets. Regional forward positions have a weaker effect on competition in the short-term market than forward positions that clear against the local short-term price. This implies a willingness to pay for the regional contract that is smaller than the average willingness to pay for a local forward contract, as reflected by the above price difference. The lower profitability of the regional forward contracts tends to reduce forward contracting compared to the case of spatially independent markets.

If local market \( m \) features multiple producers with market power, \( L_m \geq 2 \), then the impact of additional sales of fixed-price forward contracts by firm \( l \) in market \( m \) implies a smaller short-term market response from its competitors under the regional versus local...
forward contract,

\[-\frac{\partial Q_{-t_m}(D_m k_{lm}, D_m K_{lm})}{\partial k_{lm}} = \frac{D_m L_m - 1}{D L_m + 1} \frac{L_m - 1}{L_m + 1} = \frac{\partial Q_{-t_m}(k_{lm}, K_{lm})}{\partial k_{lm}},\]

which also tends to reduce the level of fixed-price forward contracting.

Those two effects are offset by the reduced price sensitivity of the regional forward contract with respect to increases in the forward position \(k_{lm}\),

\[\frac{\partial F(K)}{\partial K_m} k_{lm} = -\frac{H + 1}{H} D_m b_m \frac{b_m}{L_m + 1} k_{lm} > -\frac{H + M}{H} D_m b_m \frac{b_m}{L_m + 1} k_{lm} = \frac{D_m}{D} \frac{\partial F_m(K_m)}{\partial K_m} k_{lm},\]

which tends to increase forward contracting.

The third effect dominates, and the regional forward contract reduces short-term prices, precisely in the circumstances under which competition is weak, i.e. when the short-term market consists of a few producers with market power. Condition (44) is satisfied if each local market features one single generator with market power. If we consider instead the demand side and assume that there is one large retailer or large consumer in each local market, \(H_m = 1\) for all \(m \in M\), then \(\bar{L} = 2\) for \(M = 3\), \(\bar{L} = 3\) for \(M = 6\), and \(\bar{L} = 4\) for \(M = 10\). Few local short-term markets have more than 4 producers with market power. Condition (44) is more restrictive if \(H\) is larger, but this is only a sufficient condition for regional forward contracting to reduce prices. For instance, Proposition 4 holds for arbitrary \(L_m\) if the average marginal cost \(c_m\) of producers with market power is sufficiently small relative to the demand intercept \(a_m\).\(^{14}\) It also holds if local markets are sufficiently similar, as we shall see in the next Section.

### 3.3 Linking forward contracts and consumer prices across space

Assume that retailers and large consumers pay the quantity-weighted average of the \(M\) short-term prices for their consumption. Generators are still paid the local short-term price for their output, and forward markets clear against the quantity-weighted average of the short-term prices.

**Equilibrium in the short-term market** Equations (35)-(38) characterize quantities and prices in the short-term market as functions of the forward positions taken by producers.

\(^{14}\)The demand intercept \(a_m = b_m D_m\) in market \(m\) is the competitive fringe’s marginal cost of supplying demand if firms with market power produce zero output, i.e. \(Q_m = 0\).
with market power in the different local markets because forward contracts clear against the quantity-weighted average of all short-term prices.

**Equilibrium in the forward market**  The profit of retailer or large consumer $h$ located in local market $m$ equals

$$U(D_{mh}) + (\bar{p}(K) - \hat{f})k_{mh} - \bar{p}(K)D_{mh},$$

where $\bar{p}(K)$ defined in (39) characterizes the clearing price of the regional forward contract as well as the regional consumer price.

Taking the first-order condition of the above profit expression, summing up across all retailers and large consumers in all local markets yields the inverse demand

$$\hat{F}(K) = \bar{p}(K) + \frac{1}{H} \sum_{m=1}^{M} \left( \frac{D_{m}}{D} \right)^{2} \frac{b_{m}}{L_{m} + 1} (D_{m} - K_{m}).$$

for the regional forward contract under regional consumer prices. The marginal effect on the forward price $\hat{F}(K)$ of an increase in $K_{m}$ is the same as for $\bar{F}(K).$\(^{15}\) The forward price $\hat{F}(K)$ is smaller than $\bar{F}(K)$ because the spillover effects of forward contracting into the consumer price in the other local markets reduces the willingness to pay for forward contracts:

$$\hat{F}(K) - \bar{F}(K) = -\frac{1}{H} \sum_{m=1}^{M} \frac{D_{m}}{D} \left( 1 - \frac{D_{m}}{D} \right) \frac{a_{m}}{L_{m} + 1}.$$

The profit of producer $l$ with market power in local market $m$ equals

$$(\hat{F}(K) - \bar{p}(K))k_{lm} + (p_{m}(\frac{D_{m}}{D}K_{m}) - c_{lm})q_{lm}(\frac{D_{m}}{D}k_{lm}, \frac{D_{m}}{D}K_{lm}).$$

The smaller forward price paid by consumers reduces the volume of forward contracts sold in equilibrium and increases short-term prices under regional forward contracting and a consumer regional price mandate, compared to the case when consumers pay the local short-term price for their electricity consumption under regional forward contracting (the proof is in the Appendix):

**Proposition 5** Consider an electricity market with $M \geq 2$ local markets and $L_{m} \geq 1$ producers with market power in each local market $m \in \mathcal{M}$. Assume that each producer is active

\(^{15}\)Specifically, $\frac{\partial \hat{F}(K)}{\partial K_{m}} = -\frac{H+1}{H} \left( \frac{D_{m}}{D} \right)^{2} \frac{b_{m}}{L_{m} + 1} = \frac{\partial \bar{F}(K)}{\partial K_{m}}$.
only in one local market. Let the $M$ markets be linked through a regional forward contract that clears against the quantity-weighted average of the short-term prices in those $M$ markets, and let consumers in the $M$ markets pay that quantity-weighted average price for their consumption. In equilibrium, the quantity-weighted average of the prices in the $M$ short-term markets equals

$$
\bar{p}^R = \sum_{m=1}^{M} \frac{D_m}{D} p_m = \bar{p}^{RF} + \frac{\sum_{i=1}^{M} \Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} \sum_{m=1}^{M} \frac{D_m}{D} (1 - \frac{D_m}{D}) \frac{a_m}{L_m + 1}.
$$

(45)

Regional forward contracting and regional consumer prices reduce competition in the short-term markets by increasing the quantity-weighted average of the short-term prices, $\bar{p}^R > \bar{p}^I$, compared to the benchmark of spatially independent markets, if the local markets are similar in size ($\frac{D_m}{D}$ is sufficiently close to $\frac{1}{M}$ for all $m \in M$). This result is reversed, $\bar{p}^R < \bar{p}^I$, if markets are concentrated and market sizes are sufficiently asymmetric.

Under regional consumer prices, the spill-over effect of forward contracting into other local markets are large when all markets are similar in size since each local short-term price then enters with a relatively small weight in the regional consumer price. This weakens the incentive of producers with market power to sell forward contracts so much that the quantity-weighted average of the short-term prices in equilibrium is higher than in the benchmark of spatially independent markets. This result generalizes Proposition 2 to the case of asymmetric market conditions. However, the result is not universal. If one local market is large relative to the others, then consumers in that market internalize most of the effects of buying forward contracts because that local market carries so much weight in the calculation of the regional consumer price. In fact, when one market is very large and the others very small, then the quantity-weighted average of the short-term prices is nearly the same as when consumption is cleared against the local short-term price; see (45). If market concentration then is sufficiently large, competition under a regional forward contract and a regional consumer price mandate is stronger than in the benchmark of spatially independent markets.

### 3.4 Linking consumer prices across space

In this final case, we assume that producers receive the local short-term price for their generation, that consumers pay the quantity-weighted average of all short-term prices for their electricity consumption and that forward contracts clear against local short-term prices.
Equilibrium in the short-term market Since forward contracts clear against local short-term prices, quantities produced and short-term prices are given by (27)-(30) as functions of forward positions.

Equilibrium in the forward market The profit of retailer or large consumer \( h \) located in local market \( m \) equals

\[
U(D_{mh}) + (p_m(K_m) - \tilde{f}_m)k_{mh} - \sum_{i=1}^{M} \frac{D_i}{D} p(K_i) D_{mh},
\]

where \( \tilde{f}_m \) is the forward price of the local forward contract sold by producers in local market \( m \). Optimizing behavior of retailers and large industrial consumers yield the inverse demand

\[
\tilde{F}_m(K_m) = p_m(K_m) + \frac{b_m}{L_m + 1} \frac{D_m}{D} D_m - K_m
\]

for forward contracts in local market \( m \). The first-stage profit of producer \( l \) with market power in local market \( m \) equals

\[
(p_m(K_m) - c_{lm})q_{lm}(k_{lm}, K_{-lm})
\]

as a function of forward positions. An increase in forward sales \( k_{lm} \) by producer \( l \) in market \( m \) affects forward profit and has a positive effect on profit in the short-term market through the strategic effect:

\[
\tilde{E}_m(K_m) - p_m(K_m) + \tilde{F}'_m(K_m)k_{lm} - (p_m(K_m) - c_{lm}) \frac{\partial Q_{-lm}(k_{lm}, K_{lm})}{\partial k_{lm}}.
\]

Proposition 6 Consider an electricity market with \( M \geq 2 \) local markets and \( L_m \geq 1 \) producers with market power in each local market \( m \in M \). Assume that each producer is active only in one local market. Let consumers in the \( M \) markets pay the quantity-weighted average of the short-term prices in those \( M \) markets for their consumption. The average volume of forward contracts sold by producers with market power in local market \( m \) equals:

\[
\frac{K_m^{RC}}{L_m} = \frac{D_m}{D} \frac{(L_m + 1) D_m}{(H_m + 1)(L_m^2 + 1) + 2L_m} + \frac{H_m(L_m - 1)}{(H_m + 1)(L_m^2 + 1) + 2L_m} \frac{a_m - c_m}{b_m}.
\]
The equilibrium short-term price is higher than in the case of independent markets:

\[ p_{RC}^m - c_m = \frac{(H_m + L_m + 1)(a_m - c_m) - L_m P_m a_m}{(H_m + 1)(L_m^2 + 1) + 2L_m} > p_{I}^m - c_m \forall m \in M. \]

Equilibrium prices are higher than in the benchmark case of spatially independent markets because the regional consumer price generates positive spill-over effects on retailers in other markets that reduces the demand for forward contracts.

### 3.5 Discussion

Important results of the model with two symmetric markets in Section 2 carry over to the model with multiple asymmetric markets. In concentrated markets, the most competitive market design is the one in which consumers pay the local short-term price for electricity, and all forward contracts clear against the same quantity-weighted average of short-term electricity prices. Equity-based consumer prices reduces competition, all else equal (\( \bar{p}_{CR}^R > \bar{p}_{I}^I \)) and (\( \bar{p}_{R}^R > \bar{p}_{RF}^R \)). However, the analysis of asymmetric markets also provides new insights. In particular, a market with regional forward prices and regional consumer prices is more competitive than the benchmark of spatially independent markets if market concentration is high and local markets are sufficiently asymmetric in size.

### 4 Producers active in multiple local markets

We now explore the market design of Section 3.2 in larger detail by allowing producers with market power to own generation assets in more than one local market. This change in ownership structure implies that producers internalize more of the negative price effects of selling forward contracts. We show that a regional forward contract reduces short-term prices relative to local market forward contracts if and only if asset ownership is sufficiently concentrated.\(^{16}\)

To maintain tractability, we reimpose perfect symmetry on the model, similar to Section 2. Let there be \( S \) large producers in the overall market, each of which owns generation capacity and exercises market power in \( \hat{M} \) of the \( M \) local markets. These producers are symmetrically located, so that the number \( L = S \frac{M}{M} \) of producers with market power is the

\(^{16}\)Under regional consumer prices, the equilibrium short-term price can never be smaller under regional forward contracting than in the benchmark of spatially independent markets regardless of the ownership structure, in the symmetric setting we consider here.
same in each local market. Multi-market presence does not matter when the local markets are spatially independent because each local market then is functionally independent from all the other local markets. Imposing symmetry on (34) yields:

\[ p^I - c = \frac{(H + M)(a - c) - MLC}{(H + M)(L^2 + 1) + 2ML}. \]

The ownership structure of generation assets does not affect competition in the short-term market under regional forward contracting, because these markets clear independently of one another. Hence, the total production in short-term market \( m \) is given by

\[ Q(\frac{K_m}{M}) = \frac{L}{L+1} \frac{a - c}{b} + \frac{1}{L+1} \frac{K_m}{M} \]

as a function of the total volume \( K_m \) of forward contracts sold by producers with market power in that market. The price-cost margin equals

\[ p(\frac{K_m}{M}) - c = P(Q(\frac{K_m}{M})) - c = \frac{a - c}{L+1} - \frac{b}{L+1} \frac{K_m}{M}, \]

in short-term market \( m \), the production of a generator \( s \in \{1, ..., S\} \) with production assets in local market \( m \) is

\[ q_s(\frac{k_{sm}}{M}, \frac{K_{-sm}}{M}) = \frac{1}{L+1} \frac{a - c}{b} + \frac{L}{L+1} \frac{k_{sm}}{M} - \frac{1}{L+1} \frac{K_{-sm}}{M}, \]

and of all other producers in local market \( m \):

\[ Q_{-s}(\frac{k_{sm}}{M}, \frac{K_{-sm}}{M}) = \frac{L}{L+1} \frac{a - c}{b} - \frac{L-1}{L+1} \frac{k_{sm}}{M} + \frac{2}{L+1} \frac{K_{-sm}}{M}. \]

The demand for the regional forward contract is also unaffected by the generation ownership structure, and equals

\[ \bar{F}(\mathbf{K}) = \frac{1}{M} \sum_{m=1}^{M} p(\frac{K_m}{M}) + \frac{b}{M(L+1)H}(D - \frac{K}{M}) \]

after simplification of (40), where \( K = \sum_{m=1}^{M} K_m \) is the volume of regional forward contracts.
Producer $s$ chooses its retail portfolio $(k_{s1}, \ldots, k_{sm}, \ldots, k_{sM})$ to maximize profit

$$(\bar{F}(K) - \frac{1}{M} \sum_{m=1}^{M} p(K_m))k_s + \sum_{m=1}^{M} \beta_{sm}(p(K_m) - c)q_s(k_{sm} M, K_{sm} M),$$

where $\beta_{sm}$ is an indicator function taking the value 1 if producer $s$ owns generation capacity in local market $m$ and 0 if not. The variable $k_s = \sum_{m=1}^{M} k_{sm}$ denotes the position of firm $s$ in the regional forward market. The producer only takes a forward position in those markets where it owns generation capacity.

The marginal effect on profit of increasing $k_{sm}$ is

$$\bar{F}(K) - \frac{1}{M} \sum_{m=1}^{M} p(K_m M) + \frac{\partial \bar{F}(K)}{\partial K_m} k_s - (p(K_m M) - c) \frac{\partial Q_{s}(k_{sm} M, K_{sm} M)}{\partial k_{sm}}.$$ 

Compared to the case in which producers are active in only one market, each producer $s$ with market power now takes into account the spill-over effects of the forward price reduction in the other markets in which it is present, as measured by the total forward position $k_s$. Set the marginal profit to zero, use the functional form expressions and apply symmetry to solve for the equilibrium price-cost margin

$$p^{RF} - c = \frac{\bar{M}(H + 1)(a - c) - MLc}{H(L^2 + 1) + (ML + 1 + (M - 1)(H + 1))(L + 1)}$$

under the regional forward contract. Producers internalize relatively more of the negative forward price effect when they own generation capacity in more local markets, which weakens the incentive to sell forward contracts. Hence, the effect on short-term prices of linking electricity markets through a regional forward contract is ambiguous:

**Proposition 7** Consider an electricity market with $M \geq 2$ symmetric local markets. Assume that each of the $S$ producers with market power is active in $\bar{M}$ local markets. Linking the $M$ local electricity markets through a regional forward contract that clears against the quantity-weighted average of the short-term prices in those markets, reduces short-term prices compared to the benchmark of spatially independent markets if and only if the geographical concentration of generation ownership is sufficiently high $[p^{RF} < p^I$ if and only if $\bar{M} < \frac{H+M}{H+1}]$.

**Proof.** Algebraic simplification of $p^{RF}$ and $p^I$ yield

$$p^I - p^{RF} = \frac{[(H(L - 1) + M(L + 1))(a - c) + M(L + 1)c][H + M - (H + 1)\bar{M}]L}{[(H + M)(L^2 + 1) + 2ML][H(L^2 + 1) + (ML + 1 + (M - 1)(H + 1))(L + 1)]}. $$
The denominator and the first term in square brackets in the numerator are both positive. Hence, the sign of \( p^I - p^{RF} \) is identical to the sign of \( H + M - (H + 1)\bar{M} \). ■

Producers that own generation capacity and exercise market power in multiple local markets account for a larger share of the spill-over effects of changes in the regional forward price on the other local markets. This increased internalization softens the incentive to participate in the forward market, potentially to such an extent that the regional forward contract is anti-competitive. However, this effect is unlikely to be substantial. If, for instance, all producers are active in all local markets, \( \bar{M} = M \), then there are as many producers in each local market as there are producers in the overall market, \( L = S \). Competition would then be quite intense in the short-term market even if producers did not sell any forward contracts at all.

**Quantitative effects of forward contracting** The price effects of forward contracting depend on local demand and cost conditions, market structure in the forward and short-term market and the number of local markets. Calculating the competitive effects is complicated even in the symmetric case, \( a_m = a, b_m = b, c_m = c \) and \( L_m = L \) for all \( m \), by the fact that we still need demand and cost estimates to calculate the equilibrium short-term prices \( p^I \) and \( p^{RF} \). However, because of the linear structure of the model, we can derive lower and upper boundaries to the competitive effects that do not depend on these characteristics. Under symmetry,

\[
I = 100 \times \left[ 1 - \frac{p^{RF} - c}{p^I - c} \right]
\]

measures the percentage reduction in the price cost margin in the short term market associated with linking local markets through a regional forward contract compared to the benchmark of spatially independent markets. On the basis of the above expressions for \( p^I \) and \( p^{RF} \), it is straightforward to verify the lower boundary

\[
I \geq I(h, L, M) = 100 \times \frac{M - 1}{h + 1} \frac{(L + 1 + h(L - 1))L}{Mh(L^2 + 1) + (ML + 1)(L + 1)}
\]

on the competitive effect if all producers with market power are active in one local market \( (\bar{M} = 1) \), and the price-cost margin is non-negative \( (p^{RF} \geq c) \). This boundary is a function of the number of retailers or large consumers per local market, \( h = \frac{H}{M} \), the number \( L \) of producers with market power in each local market and the number \( M \) of local markets linked through the regional forward contract, but is independent of the demand and cost conditions \( (a, b, c) \). If we hold \( (L, M) \) constant and require non-negative price-cost margins for all \( h \geq 1 \),

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then we obtain the upper boundary

\[ I \leq \bar{I}(h, L, M) = 100 \times \frac{M - 1}{Mh - 1} \frac{(M(L + 1) + 1)(L + 1) + h(L - 1)ML}{Mh(L^2 + 1) + (ML + 1)(L + 1)}. \]

on the competitive effect.

The solid line in Figure 1 plots the lower boundary \( \underline{I}(h, 1, 5) \) of the competitive effect in the symmetric model with five local markets \( (M = 5) \), one producer with market power in each local market \( (L = 1) \), and under the assumption that all producers are active in one local market \( (\bar{M} = 1) \). The dashed line plots the corresponding upper boundary \( \bar{I}(h, 1, 5) \) of the competitive effect. The \( x \)-axis in Figure 1 measures the competitiveness of the forward market in terms of the number \( h \) of strategic consumers in each local market.

In the polar case of \( h = 1 \), linking the five local electricity markets through a regional forward contract causes price-cost margins to drop by at least 18% in the short-term market compared to the benchmark of spatially independent markets. The forward premiums are smaller if \( h \) is larger. For instance for \( h = 3 \), price-cost margins drop between 5% and 15% in the symmetric model depending on the cost and demand conditions. The competitive effects of regional forward contracting are negligible if \( h \) is large and forward premiums therefore very small.

5 Combined local and regional forward markets

In this section, we examine the effects of combining a regional forward market with local forward markets. The question is whether pro-competitive regional forward markets will emerge in equilibrium, or if they necessitate a regulatory mandate.

Producers with market power can sell forward contracts that clear against the short-term price in the local market where they own production capacity as well as forward contracts that clear against the quantity-weighted average of all short-term market prices. Consumers pay a wholesale price equal to the volume-weighted average of the short-term prices. We find that both types of forward markets can be sustained in equilibrium, and that producers generally will participate either in the local or the regional forward market, but not both. This configuration has no effect on short-term competition in locations sustained by local forward contracts and has positive effects on short-term competition in locations sustained by regional forward contracting compared to the benchmark of local forward markets.
Equilibrium in the short-term market  The second-stage profit of producer $l$ active in market $m$ is

$$\left(\hat{f}_m - P_m(Q_m)\right)z_{lm}^l + \left(\hat{f} - \bar{P}(Q)\right)z_{lm}^R + \left(P_m(Q_m) - c_{lm}\right)q_{lm},$$

where $z_{lm}^l$ is its volume of local forward contracts (i.e. that clear against $p_m$), and $z_{lm}^R$ is its volume of regional forward contracts. The first-order condition

$$-P_m'(Q_m)\left(z_{lm}^l + \frac{D_m}{D}z_{lm}^R\right) + P_m(Q_m) - c_{lm} + P_m'(Q_m)q_{lm} = 0$$

identifies producer $l$’s optimal production as a function of total output $Q_m$ and its composite forward position $k_{lm} = z_{lm}^l + \frac{D_m}{D}z_{lm}^R$.

Let $K_m = \sum_{l=1}^{L_m} k_{lm}$ be the amount of composite forward contracts sold in local market $m$ and $K_{-lm} = K_m - k_{lm}$ the forward contracts sold by producers with market power other than $l$. The total output $Q_m(K_m)$ in short-term market $m$ is given by (27), the average price-cost margin $p_m(K_m) - c_m$ by (28), producer $l$’s output by (29) and the residual output of all producers other than $l$ by (30).
Forward market equilibrium  Retailer or large consumer $h$ located in local market $m$ has profit

$$U(D_{mh}) + (p_m(K_m) - \tilde{f}_m)z^I_{mh} + \left( \sum_{i=1}^M D_i \frac{D_i}{D} p_i(K_i) - \hat{f} \right)z^R_{mh} - \sum_{i=1}^M D_i \frac{D_i}{D} p_i(K_i) D_{mh}$$

if it purchases a volume of $z^I_{mh}$ MWh forward contracts that clear against the short-term price $p_m$ and a volume of $z^R_{mh}$ MWh regional forward contracts. By way of the two first-order conditions

$$p_m(K_m) - \tilde{f}_m + p'_m(K_m) (z^I_{mh} + D_m \frac{D_m}{D} z^R_{mh} - D_m \frac{D_m}{D} D_{mh}) = 0$$

and

$$\sum_{i=1}^M D_i \frac{D_i}{D} p_i(K_i) - \hat{f} + \frac{D_m}{D} p_m(K_m) (z^I_{mh} + D_m \frac{D_m}{D} z^R_{mh} - D_m \frac{D_m}{D} D_{mh}) = 0$$

for the retailer or large consumers’ profit maximization problems, we obtain the inverse demand

$$\tilde{F}_m(K_m) = p_m(K_m) + \frac{M \sum_{m=1}^M D_m}{H L_m + 1} (\frac{D_m}{D} D_m - K_m)$$

(47)

for the local forward contract and the inverse demand

$$\hat{F}(K) = \sum_{m=1}^M \frac{D_m}{D} p_m(K_m) + \frac{1}{H} \sum_{m=1}^M \frac{D_m}{D} b_m L_m + 1 (\frac{D_m}{D} D_m - K_m)$$

(48)

for the regional forward contract. These demand functions depend only on producers’ composite forward positions. The smaller effect of a regional forward position on competition in the short-term market drives the regional forward premium down below those in the local forward markets. Specifically, the regional forward premium is proportional to the quantity-weighted average of the local forward premiums:

$$\hat{F}(K) - \sum_{m=1}^M \frac{D_m}{D} p_m(K_m) = \frac{1}{M} \sum_{m=1}^M \frac{D_m}{D} (F_m(K_m) - p_m(K_m))$$

(49)

The first stage profit of producer $l$ equals

$$(\tilde{F}_m(K_m) - p_m(K_m)) z^I_{lm} + (\hat{F}(K) - \sum_{i=1}^M \frac{D_i}{D} p_i(K_i)) z^R_{lm} + (p_m(K_m) - c_{lm}) q_{lm}(k_{lm}, K_{-lm})$$

where the first term is the profit of selling forward contracts that clear against $p_m$ for $z^I_{lm}$
MWh electricity, the second term is the profit from selling contracts for $z_{lm}^R$ MWh electricity in the regional forward market, and the final term is the profit in the short-term market.

Consider producer $l$’s profit-maximizing choice $z_{lm}^R$ versus $z_{lm}^I$ subject to holding the composite forward position constant at $k_{lm} = z_{lm}^I + \frac{D_m}{D} z_{lm}^R$. Rewrite the profit expression as:

$$
\left[ (\tilde{F}_m(K_m) - p_m(K_m)) k_{lm} + (p_m(K_m) - c_{lm}) q_{lm}(k_{lm}, K_{-lm}) 
+ \left[ \hat{F}(K) - \sum_{i=1}^M D_i \frac{D}{D} p_i(K_i) - \frac{D_m}{D} (\tilde{F}_m(K_m) - p_m(K_m)) \right] z_{lm}^R \right].
$$

(50)

All terms on the first row and all terms inside the square brackets on the second row of (50) depend on $l$’s forward contracting only through the composite forward position $k_{lm}$. For constant $k_{lm}$, firm $l$’s profit function therefore is linear in $z_{lm}^R$. By way of (49), the expression inside the square brackets of (50) is strictly positive for some local markets and strictly negative for other local markets unless all $\frac{D_i}{D}(F_i(K_i) - p_i(K_i))$ are identical. Such symmetry will not generally hold in equilibrium. Producers in some local markets therefore would seem to be able to make huge arbitrage profits from taking positive and very large regional forward positions $z_{lm}^R$ and negative and very small local forward positions $z_{lm}^I$, whereas arbitrage profits would arise from taking the opposite positions in other local markets. However, such arbitrage profits would translate into equally huge and unsustainable arbitrage losses for retailers or large consumers. One way of closing the model would be to impose break-even constraints on retailers or large consumers. We take a simpler approach by assuming that retailers or large consumers do not sell local or regional forward contracts, and that any given supply of forward contracts first is allocated to retailers or large consumers. By implication, producers cannot take negative forward positions, i.e. each producer $l$ maximizes its profit subject to $z_{lm}^I \geq 0$ and $z_{lm}^R \geq 0$. Under these assumptions,

$$
\sum_{l=1}^{L_m} z_{lm}^I = 0 \text{ if } \frac{D_m}{D}(\tilde{F}_m(K_m) - p_m(K_m)) < \hat{F}(K) - \sum_{i=1}^M D_i \frac{D}{D} p_i(K_i)
$$

(51)

and

$$
\sum_{l=1}^{L_m} z_{lm}^R = 0 \text{ if } \frac{D_m}{D}(\tilde{F}_m(K_m) - p_m(K_m)) > \hat{F}(K) - \sum_{i=1}^M D_i \frac{D}{D} p_i(K_i)
$$

(52)

are optimal. This means that producers with market power in each local market either sell local or regional forward contracts, but not both. To characterize the equilibria and say something about which markets will feature local versus regional forward contracting, we
restrict attention to the case with two local markets \((M = 2)\) and one producer with market power in each local market \((L_1 = L_2 = 1)\). We prove the following result in the Appendix:

**Proposition 8** Consider an electricity market with two local markets and one producer with market power in each local market. Each producer can supply local forward contracts that clear against the short-term price in its own local market and regional forward contracts that clear against the quantity-weighted average of the short-term prices in the two local markets. Consumers pay the quantity-weighted average of short-term prices for their electricity. Assume that producers cannot take negative forward positions. Then, there exists an equilibrium in which the producer in local market \(i\) exclusively supplies local forward contracts and the producer in local market \(m \neq i\) exclusively supplies regional forward contracts if and only if

\[
\frac{a_i D_i^2}{a_m D_m^2} \geq 2 \sqrt{\frac{H + 4}{H + 2}} - \frac{H + 4}{H + 2}. \tag{53}
\]

In the combined forward market, the equilibrium price-cost margin equals

\[
p_i^C - c_i = \frac{a_i - c_i}{2} - \frac{D_i}{D} \frac{a_i}{H + 4} = p_i^{RC} - c_i \tag{54}
\]

in short-term market \(i\) and

\[
p_m^C - c_m = \frac{a_m - c_m}{2} - \frac{1}{2} \frac{D_m}{D} \frac{a_m}{H + 2} - \frac{1}{2} \frac{D_i}{D} \frac{1}{D_m} a_i D_i \leq p_m^{CR} - c_m \tag{55}
\]

in short-term market \(m\).

The volume-weighted local forward premium in the larger market \(i\) tends to be larger than the regional forward premium, where relative market size is measured by \(\frac{a_i D_i^2}{a_m D_m^2}\). Therefore, the producer in this local market tends to be better off by selling local forward contracts than participating in the regional forward market. The opposite is true in the smaller market \(m\).\(^{17}\) Introducing a regional forward contract to an existing market for local forward contracts has no effect on the larger market. However, the producer with market power in the smaller market will start trading in the regional forward market instead. This leads to a substantial increase in the volume of forward contracts sold in that market and a corresponding reduction in the short-term price. Hence, short-term prices are unaffected in some local markets and fall in other local markets as a consequence of allowing producers to trade also in regional markets.

\(^{17}\)By condition (53), this type of forward market specialization is sustainable also for \(a_i D_i^2 < a_m D_m^2\) if the two markets are similar in size. The game thus has multiple equilibria in this case.
6 Concluding policy discussion

A key problem with market performance in restructured electricity markets is the high degree of market concentration that sometimes arises when transmission constraints divide a region into smaller local markets with one or a few large producers in each. Increased market concentration strengthens suppliers’ incentives to exercise market power in the wholesale market. Improving competition through entry or market integration is problematic in many electricity markets because of economic or political barriers to large supplier entry or network investment to expand the size of the geographic market.

We show that market design can substantially reduce market power without involving supplier entry or network investment. Specifically, a single forward market in which contracts clear against the quantity-weighted average of a set of locational marginal prices (LMP) is pro-competitive compared to multiple local forward markets in which forward contracts clear against the individual location-specific short-term prices.

"Equity-based" pricing under which consumers pay the quantity-weighted average of a set of locational marginal prices (LMP) increases short-term prices compared to the case when consumers pay individual LMP prices for their electricity. However, this market rule is likely to facilitate retailer entry into more local markets by vertically integrated retailers because it mitigates a major source of risk they face in entering a local market where they do not own generation units: spatial price risk between where they own generation units and this local market. This spatial price risk has led many vertically integrated firms to focus their retailing efforts on the local markets where they own generation units to avoid such risk. Moreover, an effective entry deterrence strategy by vertically integrated retailers with generation units in the same local market as their retail customers, is to use their ability to exercise unilateral market power to spike the local wholesale price and effectively eliminate any retail profit margin a new entrant without local generation capacity could earn from selling retail electricity. Requiring all retailers to purchase the wholesale electricity

\[18\] If we instead assume that consumers pay the local short-term price for their consumption, then allowing regional forward contracting on top of local forward contracting has the same effect on equilibrium prices compared to the benchmark of spatially independent markets.

\[19\] Consistent with such entry deterrence incentives, Wolak (2009) presents empirical evidence demonstrating that over the sample period he studies, the four large vertically integrated retailers in the New Zealand wholesale electricity market concentrated their retailing activities in the regions where they owned generation units.
sold to final consumers at a quantity-weighted average of all LMPs significantly limits the spatial price risk any supplier faces from entering any local market, which should increase the competitiveness of retail markets in particular.

The recent experience of many European countries illustrates the policy relevance of our findings. Aggressive renewables policies in a number of European countries have significantly increased the cost of making generation schedules that emerge from the day-ahead market in those countries operational in real-time. Transitioning to an LMP market design would eliminate the vast majority of these physical feasibility costs. For example, suggestions have been made to break the single German price area into smaller ones to reduce these physical feasibility costs for Germany.

Major barriers to transitioning to more locational pricing in these markets are fears that LMP prices increase consumers’ costs of hedging electricity prices and the perceived unfairness of charging different wholesale prices to consumers at different locations in the transmission network. Such arguments received a lot of public attention following the division of the Swedish day-ahead market into four price areas in 2011. Previously, Sweden had constituted a single price area.

Our results suggest that dividing Germany and other countries into multiple price areas while allowing producers and consumers to write forward contracts based on the quantity-weighted average of those area prices could improve short-term market efficiency and reduce prices in all local markets by improving local competition. More generally, our results argue that introducing an LMP market where all relevant operating constraints are explicitly priced, all generation units are paid their locational marginal price, forward contracts clear against the quantity quantity-weighted average of LMPs, and all loads pay that quantity-weighted average for their consumption, can increase market efficiency relative to an LMP market where all suppliers and loads face their local price. Since the default LMP design with local prices is more efficient than any non-LMP market design, our results show that it is possible to increase market efficiency through locational pricing, while still ensuring liquid forward markets and equity-based consumer prices.

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20ENTSO-E (2018) notes the annual costs of making day-ahead generation schedules feasible for real-time system operation in 2017 was more than 1 billion Euros in Germany, more than 400 million in the United Kingdom, and 80 million in Spain.

21See, for instance, Egerer et al. (2016) and references therein.
Appendix

A.1 Proof of Proposition 4

Characterization  Substitute

\[-\frac{\partial Q_{-l_m}(D_m k_{lm}, D_m K_{lm})}{\partial k_{lm}} = L_m - 1 \frac{D_m}{L_m + 1} D\]

into marginal profit (41), set the expression equal to zero and sum up over all \(L_m\) producers to obtain the aggregate first-order condition

\[L_m(\bar{F}(K^{RF}) - \bar{p}(K^{RF})) + \frac{\partial \bar{F}(K^{RF})}{\partial K_m} K^{RF}_m + \frac{L^2_m - L_m D_m}{L_m + 1} \frac{D_m (p^{RF}_m - c_m)}{L_m + 1} = 0\]

in local market \(m\). By way of (28) and (40),

\[\bar{F}(K^{RF}) - \bar{p}(K^{RF}) = \frac{1}{H} \sum_{m=1}^{M} D_m \left( p^{RF}_m - c_m + \frac{c_m}{L_m + 1} \right)\]

and

\[\frac{\partial \bar{F}(K)}{\partial K_m} = \frac{\partial \bar{p}(K)}{\partial K_m} - \frac{1}{H} \left( \frac{D_m}{D} \right)^2 \frac{b_m}{L_m + 1} = \frac{H - 1}{H} \left( \frac{D_m}{D} \right)^2 \frac{b_m}{L_m + 1}\]

Substitute these expressions into the aggregate first-order condition above, and simplify expressions to get the modified optimality condition

\[L_m \sum_{i=1}^{M} \frac{D_i}{D} (p^{RF}_i - c_i) + \frac{L_m}{\Psi(L_m)} \frac{D_m}{D} (p^{RF}_m - c_m) = (H + 1) \frac{D_m a_m - c_m}{L_m + 1} - L_m \sum_{i=1}^{M} \frac{D_i c_i}{D L_i + 1},\]

where \(\Psi(L)\) was defined in (43). Multiply the modified optimality condition through by \(\frac{\Psi(L_m)}{L_m}\), sum up across all \(M\) local markets and rewrite to get (42).

Comparative statics  Use (34) to get the weighted average

\[\sum_{m=1}^{M} \frac{D_m}{D} (p^{RF}_m - c_m) = \sum_{m=1}^{M} \frac{D_m (H_m + 1)(a_m - c_m) - L_m c_m}{(H_m + 1)(L_m^2 + 1) + 2L_m}\]
of the price-cost margin when all forward markets are spatially independent. Subtract (42) from this expression to obtain

\[
\sum_{m=1}^{M} \frac{D_m}{D} (p_m^I - p_m^{RF}) = \sum_{m=1}^{M} \left[ \frac{(H_m + 1)(H(L_m^2 + 1) + L_m + 1)}{(H_m + 1)(L_m^2 + 1) + 2L_m} - \frac{H + 1}{1 + \sum_{i=1}^{M} \Psi(L_i)} \right] \frac{\Psi(L_m)}{L_m} \frac{D_m a_m - c_m}{D_m} \frac{1}{L_m + 1} + \sum_{m=1}^{M} \left[ \frac{\sum_{i=1}^{M} \Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} - \frac{L_m(L_m + 1)}{(H_m + 1)(L_m^2 + 1) + 2L_m} \right] \frac{D_m}{D} \frac{c_m}{L_m + 1}.
\]

We can simplify the first expression in square brackets to:

\[
\frac{(H_m + 1)(H(L_m^2 + 1) + L_m + 1) \sum_{i \neq m} \Psi(L_i) - 2(H - H_m)L_m}{[(H_m + 1)(L_m^2 + 1) + 2L_m][1 + \sum_{i=1}^{M} \Psi(L_i)]}
\]

The denominator is positive. The numerator is positive by

\[
\Psi(L) - \frac{1}{H + 1} = \frac{(H + L + 1)(L - 1)}{(H + 1)(H(L^2 + 1) + L + 1)} \geq 0
\]

and

\[
(H_m + 1)(H(L_m^2 + 1) + L_m + 1) \frac{M - 1}{H + 1} - 2(H - H_m)L_m = \frac{M - 1}{M} \left[ \frac{M - 1}{H + 1} (H(L_m^2 + 1) + L_m + 1) + H(L_m - 1)^2 + L_m + 1 \right] > 0,
\]

where we have used the assumption that \( H_m = \frac{H}{M} \). To evaluate the second expression in square brackets, observe first that

\[
\frac{\sum_{i=1}^{M} \Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} - \frac{M}{M + H + 1} = \frac{(H + 1) \sum_{i=1}^{M} \Psi(L_i) - \frac{1}{H + 1}}{(M + H + 1)(1 + \sum_{i=1}^{M} \Psi(L_i))} \geq 0.
\]

Next

\[
\frac{M}{M + H + 1} - \frac{L_m(L_m + 1)}{(H_m + 1)(L_m^2 + 1) + 2L_m} = M \frac{(2M - L_m - 1)L_m - (H + M)(L_m - 1)}{[M + H + 1][(H + M)(L_m^2 + 1) + 2ML_m]},
\]

where the right-hand side expression follows from substituting in \( H_m = \frac{H}{M} \). The denominator is positive. The numerator is non-negative if and only if \( L_m \leq \bar{L} \), where we defined \( \bar{L} \) in equation (44).
A.2 Proof of Proposition 5

Following the same steps as in the proof of Proposition 4, we get the aggregate first-order condition

\[ L_m (\hat{F}(K^R) - \bar{p}(K^R)) + \frac{\partial \hat{F}(K^R)}{\partial K_m} K_m + \frac{L_m^2 - L_m}{L_m + 1} \frac{D_m}{D} (p_m - c_m) = 0 \]

in local market \( m \), and where the forward market premium can be written as:

\[ \hat{F}(K^R) - \bar{p}(K^R) = \frac{1}{H} \sum_{m=1}^{M} \frac{D_m}{\Psi(L_m)} (p_m^R - c_m) + \frac{D_m}{L_m + 1} a_m - \frac{a_m - c_m}{L_m + 1}. \]

Substitute this expression and \( \frac{\partial \hat{F}(K^R)}{\partial K_m} = -\frac{H+1}{H} \left( \frac{D_m}{D} \right)^2 \frac{b_m}{L_m + 1} \) into the first-order condition above to get the modified first-order condition:

\[ L_m \sum_{i=1}^{M} \frac{D_i}{\Psi(L_i)} (p_i^R - c_i) + \frac{L_m}{\Psi(L_m)} \frac{D_m}{D} (p_m^R - c_m) = (H+1) \frac{D_m}{L_m + 1} a_m - \frac{a_m - c_m}{L_m + 1} + \sum_{i=1}^{M} \frac{D_i}{\Psi(L_i)} (a_i - c_i) - \frac{D_i}{D} \frac{a_i}{L_i + 1}. \]

Multiply through by \( \frac{\Psi(L_m)}{L_m} \), sum up across all \( M \) local markets and rewrite to get

\[ \sum_{m=1}^{M} \frac{D_m}{D} (p_m^R - c_m) = \sum_{m=1}^{M} \frac{\Psi(L_m)^{H+1}}{1 + \sum_{i=1}^{M} \Psi(L_i)} \frac{D_m}{L_m + 1} a_m - \frac{a_m - c_m}{L_m + 1} + \sum_{i=1}^{M} \frac{\Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} \sum_{m=1}^{M} \frac{D_m}{D} \left( \frac{a_m - c_m}{L_m + 1} - \frac{D_m}{D} \frac{a_m}{L_m + 1} \right). \]

This expression characterizes the volume-weighted price cost margin under regional forward contracting and regional consumer prices. By way of the characterization in (42), this expression can be written as (45).

**Similar market sizes** Subtract the volume-weighted average of the short-term prices under independent markets characterized in equation (56) from the volume-weighted average of short-term prices under regional forward contracting and regional consumer prices char-
characterized above to get

\[
\sum_{m=1}^{M} \frac{D_m}{D} (p^R_m - p^L_m) = \sum_{m=1}^{M} \left[ \frac{M L_m (L_m + 1)}{(H + M)(L^2_m + 1) + 2ML_m} - \frac{1}{M} \sum_{i=1}^{M} \Psi(L_i) \right] \frac{D_m}{D} \frac{c_m}{L_m + 1}
\]

\[
+ \sum_{m=1}^{M} \left[ \frac{\Psi(L_m) H + 1}{L_m} + \frac{\sum_{i=1}^{M} \Psi(L_i) M - 1}{1 + \sum_{i=1}^{M} \Psi(L_i)} - \frac{(H + M)(L_m + 1)}{(H + M)(L^2_m + 1) + 2ML_m} \right] \frac{D_m}{D} \frac{a_m - c_m}{L_m + 1}
\]

\[
+ \sum_{i=1}^{M} \frac{\Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} \sum_{m=1}^{M} \frac{a_m}{L_m + 1} \left( 1 - \frac{D_m}{D} \right)
\]

The term inside the square brackets on the first row is strictly positive. To see this, notice that

\[
\frac{2}{H} - \Psi(L) = \frac{H((L - 1)L + 2) + 2(L + 1)}{H(H(L^2 + 1) + L + 1)} > 0
\]

implies

\[
\frac{2}{H + 2M} - \frac{1}{M} \sum_{i=1}^{M} \frac{\Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} = \frac{H \sum_{i=1}^{M} \left( \frac{2}{H} - \Psi(L_i) \right)}{M(H + 2M)(1 + \sum_{i=1}^{M} \Psi(L_i))} > 0.
\]

The result then follows from

\[
\frac{ML(L + 1)}{(H + M)(L^2 + 1) + 2ML} - \frac{2}{H + 2M} = \frac{(M - 2)((H + M)(L + 1) + 2ML) \gamma + (L - 1)(2(H + M) + M^2L)}{(H + 2M)((H + M)(L^2 + 1) + 2ML)} \geq 0.
\]

The term inside the square brackets on the second row is also strictly positive, but is more complicated to evaluate. If \(L_m = 1\), then this expression reduces to

\[
\frac{1}{1 + \sum_{i=1}^{M} \Psi(L_i)} + \frac{\sum_{i=1}^{M} \Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} - \frac{H + M}{M} = \frac{M - 2}{H + 2M} + \frac{2}{H + 2M} - \frac{1}{1 + \sum_{i=1}^{M} \Psi(L_i)} > 0.
\]

If \(L_m = 2\), then the same expression becomes

\[
\frac{3 \frac{H + 1}{M + 3}}{1 + \sum_{i=1}^{M} \Psi(L_i)} + \frac{\sum_{i=1}^{M} \Psi(L_i)}{1 + \sum_{i=1}^{M} \Psi(L_i)} - \frac{3(H + M)}{5H + 9M} = \frac{1}{1 + \sum_{i=1}^{M} \Psi(L_i)} \frac{1}{5H + 9M} \left[ \frac{6(M - 2)(H + 1) + M + 1}{5H + 3} \right.
\]

\[
+ \frac{H}{M} \sum_{i=1}^{M} \left( \frac{6}{H} - \Psi(L_i) \right) + [2(M - 2)(H + 3) + 3 \sum_{i=1}^{M} \Psi(L_i)],
\]

45
which is also strictly positive.

Assume finally that \( L_m \geq 3 \). We can rewrite the expression inside the square brackets on the first row of the above equation as

\[
1 + \frac{1}{\sum_{i=1}^{M} \Psi(L_i)} \frac{1}{(H + M)(L_m^2 + 1) + 2ML_m} \\
\times \left[ \frac{[(H + M)(L_m - 1) + 2(H + 1)M]L_m(L_m + 1)}{(H(L_m^2 + 1) + L_m + 1)} - \frac{4H + M}{H} + (H + M) \frac{2}{M} \left( \frac{2}{H} - \sum_{i=1}^{M} \Psi(L_i) \right) \\
+ [(H + M)((M - 2)L_m + L_m - 1)(L_m - 1) + 2(M - 1)ML_m] \frac{1}{M} \sum_{i=1}^{M} \Psi(L_i) \right].
\]

The sum of the two first terms inside the square brackets is strictly positive if \( L_m \geq 3 \):

\[
H[(H + M)(L - 1) + 2(H + 1)M]L(L + 1) - 4(H + M)(H(L^2 + 1) + L + 1)
= H[(H + M)(L - 1) + 2(H + 1)M]L(L - 3)
+ 4(H + 1)[((M - 1)(2H - 1) + H - 1)(L - 1) + 2H(M - 2) + 2(H - M)].
\]

The other terms are strictly positive. We have now demonstrated that the terms on the first two rows in the expression characterizing the average price difference between regional forward contracting with regional consumer prices and spatially independent markets, are strictly positive. The whole expression is strictly positive if \( \frac{D_m}{D} = \frac{1}{M} \) for all \( m \in \mathcal{M} \) because then the expression on the third row vanishes. By continuity, the result extends to \( \frac{D_m}{D} \neq \frac{1}{M} \) as long as \( \frac{D_m}{D} \) is sufficiently close to \( \frac{1}{M} \) for all \( m \in \mathcal{M} \).

**Dissimilar market sizes**  Assume that market concentration is high in the sense that \( L_m = 1 \) for all \( m \in \mathcal{M} \). In this case,

\[
\sum_{m=1}^{M} \frac{D_m}{D} (p_m^I - p_m^R) = \frac{M}{2} \sum_{m=1}^{M} \left[ \frac{1}{H + 1 + M} \frac{D_m}{D} - \frac{1}{H + 2M} \right] \frac{D_m}{D} a_m.
\]
Assume also that \( \frac{D}{D_j} = 1 - \varepsilon, \varepsilon \in (0, 1) \), and \( \sum_{j \neq i} \frac{D}{D_j} = \varepsilon \). Let \( a^\text{max}_{-i} = \max_{j \neq i} a_j \), and let \( M^\text{max}_{-i} \) be the number of local markets for which \( a_j = a^\text{max}_{-i} \). For \( \varepsilon \) sufficiently small:

\[
\sum_{m=1}^{M} \frac{D_m}{D} (p_m^I - p_m^R) \geq \frac{M}{2} \left[ \frac{1}{H + 1 + M} M^\text{max}_{-i} \varepsilon - \frac{1}{H + 2M} \varepsilon a^\text{max}_{-i} \right] + \frac{M}{2} \left[ \frac{M - 1}{(H + 1 + M)(H + 2M)} - \frac{\varepsilon}{H + 1 + M} \right] (1 - \varepsilon) a_i.
\]

This expression is strictly positive for \( \varepsilon \) sufficiently close to zero, but positive. ■

### A.3 Proof of Proposition 8

Let \( (k_1^{CI}, k_1^{CR}) \) be an equilibrium portfolio of forward positions taken by the producer with market power in local market 1, and define \( (k_2^{CI}, k_2^{CR}) \) correspondingly in local market 2. The composite forward positions in the two markets are \( k_1^C = k_1^{CI} + \frac{D_1}{D} k_1^{CR} \) and \( k_2^C = k_2^{CI} + \frac{D_2}{D} k_2^{CR} \) in equilibrium. The equilibrium prices of the local forward contracts are \( f_1^C = \hat{F}_1(k_1^C) \) and \( f_2^C = \hat{F}_2(k_2^C) \), and the equilibrium short-term prices are \( p_1^C = p_1(k_1^C) \) and \( p_2^C = p_2(k_2^C) \). The equilibrium price of the regional forward contract is \( \hat{F}_2(k_1^C, k_2^C) \). Let

\[
\Pi_1(z_1^I, z_1^R) = (\hat{F}_1(k_1) - p_1(k_1))z_1^I + (\hat{F}(k_1, k_2^C) - \frac{D_1}{D} p_1(k_1) - \frac{D_2}{D} p_2^C)z_1^R + (p_1(k_1) - c_1)q_1(k_1)
\]

be the first stage profit of producer 1 as a function of its forward portfolio \((z_1^I, z_1^R)\) evaluated at producer 2’s equilibrium portfolio \((k_2^{CI}, k_2^{CR})\), and where \( k_1 = z_1^I + \frac{D_1}{D} z_1^R \) is 1’s composite forward position. We define \( \Pi_2(z_2^I, z_2^R) \) in an analogous manner. Recall also that

\[
\hat{F}(k_1, k_2) - \frac{D_1}{D} p_1(k_1) - \frac{D_2}{D} p_2(k_2) = \frac{1}{2} [\frac{D_1}{D} (\hat{F}_1(k_1) - p_1(k_1)) + \frac{D_2}{D} (\hat{F}_2(k_2) - p_2(k_2))] \quad (57)
\]

from (49). We first derive three properties of equilibrium forward positions in three claims. We then establish necessary and sufficient equilibrium conditions.

**Claim 1** The positions \((k_1^{CI}, k_1^{CR})\) and \((k_2^{CI}, k_2^{CR})\) constitute equilibrium forward portfolios only if \( \frac{D_1}{D} (\hat{f}_1^C - p_1^C) \neq \frac{D_2}{D} (\hat{f}_2^C - p_2^C) \).

**Proof.** If \( \frac{D_1}{D} (\hat{f}_1^C - p_1^C) = \frac{D_2}{D} (\hat{f}_2^C - p_2^C) \), then

\[
\Pi_1(k_1^C, k_1^{CR}) = \left( \frac{D_1}{D} z_1^R, z_1^I \right) = (\hat{f}_1^C - p_1^C) k_1^C + (p_1^C - c_1) q_1^C,
\]

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which is independent of \( z_i^R \). Hence, \((z_i^I, z_i^R) = (k_i^C, 0)\) is optimal. Moreover, \(\Pi_i(k_i^C, 0) \leq \Pi_i(\hat{k}_i^I, 0) = \hat{\pi}_i^I\) implies \(k_i^C = \hat{k}_i^I = \frac{D_i a_i}{H + 1}\). The premium on the local forward contract in market \(i\) equals
\[
\hat{f}_i^I - \hat{p}_i^I = \frac{b_i}{H} \left( \frac{D_i}{D} D_i - \hat{k}_i^I \right) = \frac{1}{H} \frac{H + 2}{H + 4} a_i.
\]
(58)
If \(k_1^C = \hat{k}_1^I\) and \(k_2^C = \hat{k}_2^I\), then
\[
\frac{D_1}{D} (\tilde{f}_1^C - p_1^C) - \frac{D_2}{D} (\tilde{f}_2^C - p_2^C) = \frac{1}{D^2 H} \frac{H + 2}{H + 4} (a_1 D_1^2 - a_2 D_2^2),
\]
which is different from zero unless \(a_1 D_1^2 = a_2 D_2^2\). To close the proof, consider the knife-edge case \(a_1 D_1^2 = a_2 D_2^2\), and assume that 1 deviates from \((\hat{k}_1^I, 0)\) to \(z_1^I = 0\) and \(\frac{D_1}{D} z_1^R = k_1 \geq \hat{k}_1^I\). The marginal profit of this deviation evaluated at \(k_1 = \hat{k}_1^I\) is
\[
\frac{\partial \Pi_1(0, \frac{D_1}{D} \hat{k}_1^I)}{\partial z_1^R} = a_1 D_1 \frac{1}{D H} \frac{H}{H + 4} > 0.
\]
Hence, \(\frac{D_1}{D} (\tilde{f}_1^C - p_1^C) = \frac{D_2}{D} (\tilde{f}_2^C - p_2^C)\) cannot be sustained as an equilibrium even under symmetry \(a_1 D_1^2 = a_2 D_2^2\).

**Claim 2** The positions \((k_i^{CI}, 0)\) and \((0, k_m^{CR})\), \(i \neq m\), constitute equilibrium forward portfolios only if \(\frac{D_i}{D} (\tilde{f}_i^C - p_i^C) > \frac{D_m}{D} (\tilde{f}_m^C - p_m^C)\).

**Proof.** If \(\frac{D_i}{D} (\tilde{f}_i^C - p_i^C) < \frac{D_m}{D} (\tilde{f}_m^C - p_m^C)\), then it is optimal for producer \(i\) to deviate to \(z_i^I = 0\) and \(z_i^R = \frac{D_i}{D} k_i^{CI}\) by (50) and
\[
\tilde{f}_i^C - \frac{D_i}{D} p_i^C - \frac{D_i}{D} p_i^C = \frac{1}{2} \left[ \frac{D_m}{D} (\tilde{f}_m^C - p_m^C) - \frac{D_i}{D} (\tilde{f}_i^C - p_i^C) \right] > 0,
\]
where we have invoked (57). Hence, \(\frac{D_i}{D} (\tilde{f}_i^C - p_i^C) > \frac{D_m}{D} (\tilde{f}_m^C - p_m^C)\) by Claim 1.

**Claim 3** The positions \((k_i^{CI}, 0)\) and \((0, k_m^{CR})\), \(i \neq m\), constitute equilibrium forward portfolios only if
\[
k_i^{CI} = \hat{k}_i^I \quad \text{and} \quad \frac{D_m}{D} k_m^{CR} = \frac{D_m}{D} \frac{D_m}{H + 2} + \frac{D_i}{D} \frac{1}{H + 4} \frac{a_i D_i}{a_m}.
\]
(59)

**Proof.** The producer with market power in market \(i\) only participates in the local forward market if \(k_i^{CR} = 0\). If \(k_i^{CI} \neq \hat{k}_i^I\), then producer \(i\) can increase profit by a marginal increase or reduction in \(z_i^I\) from \(k_i^{CI}\) without violating the necessary condition from Claim 2. This leaves \(k_i^{CI} = \hat{k}_i^I\) as the only equilibrium candidate for producer \(i\). Consider next producer \(m\). Assume that producer \(i\) plays \((\hat{k}_i^I, 0)\) and that producer \(m\) does not participate in local
The profit-maximizing regional forward position \( k^*_m \) solves the first-order condition

\[
\frac{D_m}{D} \frac{\partial \Pi_m(0, \frac{D_m}{D} k^*)}{\partial z^*_m} = \hat{F}(k^*_m, k^*_i) - \frac{D_m}{D} p_m(k^*_m) - \frac{D_i}{D} p^C_i - \frac{H + 1}{H} b_m D_m k^*_m = 0, \tag{60}
\]

where we have substituted in \( \frac{\partial \hat{F}(k_1, k_2)}{\partial k_m} = -\frac{H + 1}{H} b_m D_m \). Apply (57) to obtain the modified first-order condition

\[
\frac{1}{2}\left[ \frac{D_m}{D} (\hat{F}_m(k^*_m) - p_m(k^*_m)) + \frac{D_i}{D} (\hat{f}^C_i - p^C_i) \right] - \frac{H + 1}{H} D_m b_m k^*_m = 0.
\]

We can then use \( \hat{F}_m(k^*_m) - p_m(k^*_m) = \frac{b_m}{H}(\frac{D_m}{D} D_m - k^*_m) \) and \( \hat{f}^C_i - p^C_i = \hat{f}^C_i - p^C_i \) characterized in (58) to solve for

\[
k^*_m = \frac{D_m}{D} H + 2 + \frac{D_i}{D} H + 4 \frac{a_i D_i}{a_m}.
\]

If \( \frac{D_m}{D} k^C_m \neq k^*_m \), then producer \( m \) can increase profit by a marginal increase or reduction in \( z^*_m \) from \( k^C_m \) without violating the necessary condition from Claim 2. This leaves \( \frac{D_m}{D} k^C_m = k^*_m \) as the only equilibrium candidate for producer \( m \).

The forward positions characterized in (59) yield the price-cost margins characterized in (54) and (55). We now show that condition (53) is necessary and sufficient for the equilibrium characterized in (59) to exist.

**Necessity** We first characterize the equilibrium profits. By way of the price-cost margin identified in (54) and \( q^C_i = \frac{1}{2}(a_i - c_i + k^C_i) \), see (27), it follows that producer \( i \)'s profit in the short-term market equals

\[
(p^C_i - c_i)q^C_i = \frac{b_i}{2} \left( a_i - c_i - k^C_i \right) \frac{1}{2} \left( \frac{a_i - c_i}{b_i} + k^C_i \right) = \frac{(a_i - c_i)^2}{4b_i} - \frac{b_i}{4}(k^C_i)^2.
\]

From producer \( i \)'s first-order condition

\[
\hat{f}^C_i - p^C_i - \frac{H + 2}{H} \frac{b_i}{2} k^C_i = 0
\]

in the forward market, we retrieve \( i \)'s forward profit

\[
(\hat{f}^C_i - p^C_i)k^C_i = \frac{H + 2}{H} \frac{b_i}{2}(k^C_i)^2.
\]
Adding the two profit expressions returns i’s total profit

\[
\pi_i^C = (f_i^C - p_i^C)k_i^C + (p_i^C - c_i)q_i^C = \left(\frac{a_i - c_i}{4b_i} + \frac{b_i H + 4}{4H}(k_i^C)^2\right) = \frac{(a_i - c_i)^2}{4b_i} + \frac{1}{H}(\frac{D_i}{D})\frac{a_i D_i}{H + 4} = \hat{\pi}_i^C.
\]

Producer m’s profit in the short-term market is qualitatively similar to that of producer i. From the first-order condition (60) we get m’s forward profit

\[
(f^C - \frac{D_1}{D}p_1^C - \frac{D_2}{D}p_2^C)k_m^C = \frac{b_m}{2}(H + \frac{1}{H})(k_m^C)^2.
\]

Hence,

\[
\pi_m^C = (f^C - \frac{D_1}{D}p_1^C - \frac{D_2}{D}p_2^C)k_m^C + (p_m^C - c_m)q_m^C = \frac{(a_m - c_m)^2}{4b_m} + \frac{b_m}{4H}(k_m^C)^2.
\]

Producer m can always deviate to \((z_m^I, z_m^R) = (\hat{k}_m^I, 0)\) and obtain profit \(\hat{\pi}_m^I\). The net benefit of this deviation is

\[
\frac{(H + 4)^2}{H + 2} \frac{4HD^2}{b_mD_m^4} (\hat{\pi}_m^I - \pi_m^C) = \left[2\sqrt{\frac{H + 4}{H + 2} + \frac{4}{H + 2}}\frac{D}{D_m^2}k_m^C\right]2\sqrt{\frac{H + 4}{H + 2} - \frac{H + 4}{H + 2}} = \frac{D}{D_m^2}k_m^C.
\]

In particular,

\[
\frac{2}{\sqrt{\frac{H + 4}{H + 2} - \frac{H + 4}{H + 2}}} \frac{D}{D_m}k_m^C = 2\sqrt{\frac{H + 4}{H + 2} - \frac{H + 4}{H + 2}} - \frac{a_i D_i^2}{a_m D_m^2}
\]

is strictly positive if condition (53) is violated. In that case, \(\hat{\pi}_m^I > \pi_m^C\), so the forward positions characterized in Claim 3 cannot be sustained in equilibrium. As this is the only equilibrium candidate with \(k_m^{CR} = k_m^{CI} = 0\), there can be no equilibrium with \(k_m^{CR} = k_m^{CI} = 0\) if (53) is violated.

**Sufficiency** Consider first producer m’s incentive to deviate from \((k_m^{CI}, k_m^{CR}) = (0, \frac{D}{D_m}k_m^*)\) to some arbitrary \((z_m^I, z_m^R)\), where \(k_m = z_m^I + \frac{D}{D_m}z_m^R\). It is always optimal to set either \(z_m^R = 0\) or \(z_m^I = 0\) conditional on \(k_m\), and therefore \(\Pi_m(z_m^I, z_m^R) \leq \max\{\Pi_m(k_m, 0); \Pi_m(0, \frac{D}{D_m}k_m^*)\}\). By the definitions of \(\hat{k}_m^I\) and \(k_m^*\), \(\Pi_m(k_m, 0) \leq \Pi_m(\hat{k}_m^I, 0) = \hat{\pi}_m^I\) and \(\Pi_m(0, \frac{D}{D_m}k_m^*) \leq \Pi_m(0, \frac{D}{D_m}k_m^*) = \pi_m^C\). Hence, \(\Pi_m(z_m^I, z_m^R) \leq \max\{\hat{\pi}_m^I, \pi_m^C\}\). If condition (53) is satisfied, then \(\pi_m^C \geq \hat{\pi}_m^I\), in which case \(\Pi_m(z_m^I, z_m^R) \leq \pi_m^C\). This establishes \((k_m^{CI}, k_m^{CR}) = (0, \frac{D}{D_m}k_m^*)\) as a best reply to \((k_i^{CI}, k_i^{CR}) = (\hat{k}_i^I, 0)\) if condition (53) is met.

By analogous arguments, a deviation by producer i from \((k_i^{CI}, k_i^{CR}) = (\hat{k}_i^I, 0)\) to some
arbitrary \((z^l_i, z^R_i)\) yields profit \(\Pi_i(z^l_i, z^R_i) \leq \max\{\pi^C_i, \pi^*_i\}\), where \(\pi^*_i = \max_k \Pi_i(0, \frac{D}{D^i} k_i)\). We conclude the proof by demonstrating \(\pi^C_i > \pi^*_i\) if condition (53) is met.

Let \(k^*_i = \arg \max_k \Pi_i(0, \frac{D}{D^i} k_i)\). This optimal forward position is found as the solution to \(i\)'s first-order condition.

\[
\dot{F}(k^*_i, k^C_m) - \frac{D_i}{D} p_i(k^*_i) - \frac{D_m}{D} p^C_m - \frac{H + 1}{2} \frac{D_i}{D} k^*_i = 0.
\]

We can then solve for

\[
k^*_i = \frac{D_i}{D} \frac{H + 3}{H + 4} \frac{H + 2}{2} + \frac{D_m}{D} \frac{H + 1}{(H + 2)^2} \frac{a_m D_m}{a_i}.
\]

by following the same procedure as for \(k^*_m\). Producer \(i\)'s profit of pursuing this strategy is

\[
\pi^*_i = (\dot{F}(k^*_i; k^C_m) - \frac{D_i}{D} p_i(k^*_i) - \frac{D_m}{D} p^C_m) \left( \frac{D_i}{D^i} k^*_i + (p_i(k^*_i) - c_i)q_i(k^*_i) = \frac{(a_i - c_i)^2}{4b_i} + \frac{b_i H + 2}{4H} (k^*_i)^2.\right.
\]

The net benefit of playing the equilibrium strategy relative to deviating to \((0, \frac{D}{D^i} k^*_i)\) is:

\[
\frac{4H(\pi^C_i - \pi^*_i)}{b_i(H + 2)} = \left( \frac{D_i}{D} \frac{2D_i}{\sqrt{(H + 4)(H + 2)}} \right) \left[ \frac{D_i}{D} \frac{2D_i}{\sqrt{(H + 4)(H + 2)}} - k^*_i \right].
\]

After manipulating terms, we finally get

\[
\frac{a_i D}{a_m D_m^2} (H + 2) \left[ \frac{D_i}{D} \frac{2D_i}{\sqrt{(H + 4)(H + 2)}} - k^*_i \right] = \left[ 2\sqrt{H + 2} - H + 3 \right] \frac{a_i D^2}{a_m D_m^2} - H + \frac{H + 1}{H + 2} = \left[ 2\sqrt{H + 2} - H + 3 \right] \frac{a_i D^2}{a_m D_m^2} - H + \frac{H + 1}{H + 2} = \left[ 2\sqrt{H + 2} - H + 3 \right] \left[ \frac{a_i D^2}{a_m D_m^2} + \frac{H + 4}{H + 2} - 2\sqrt{H + 4} \right] + \frac{2H + 5}{H + 4} \sqrt{H + 4} \left[ \sqrt{H + 4} - \frac{2H + 7}{2H + 5} \right],
\]

which is strictly positive if condition (53) is met. ■

References


Bohn, Roger E., Michael C. Caramanis, and Fred C. Schweppe (1984): Optimal pricing in


